Computer Algebra
Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/30-1.1.4.2-c-x-^m-a-x^j+b-x^n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [454]. This is test number [30].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (454)	0.00 (0)
Mathematica	100.00 (454)	0.00 (0)
Maple	85.02 (386)	14.98 (68)
Fricas	71.59 (325)	28.41 (129)
Giac	57.49 (261)	42.51 (193)
Mupad	42.51 (193)	57.49 (261)
Maxima	33.70 (153)	66.30 (301)
Sympy	27.53 (125)	72.47 (329)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

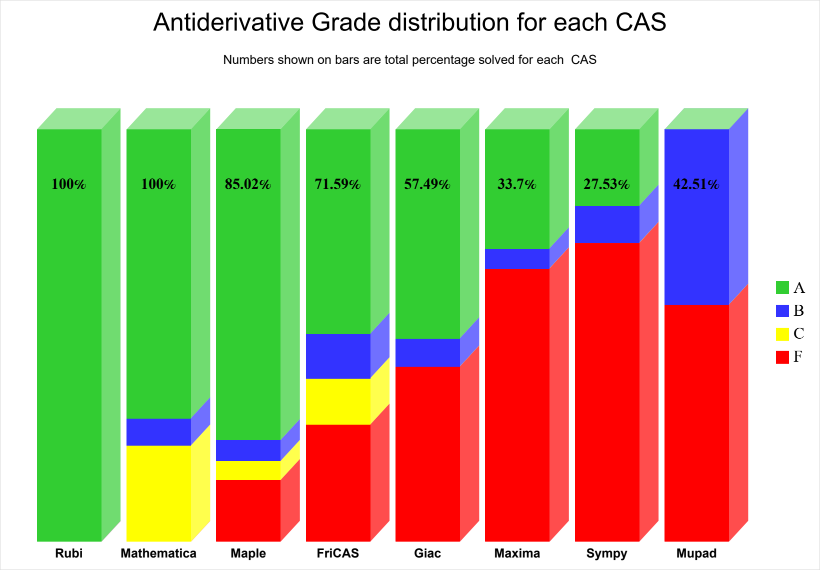
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

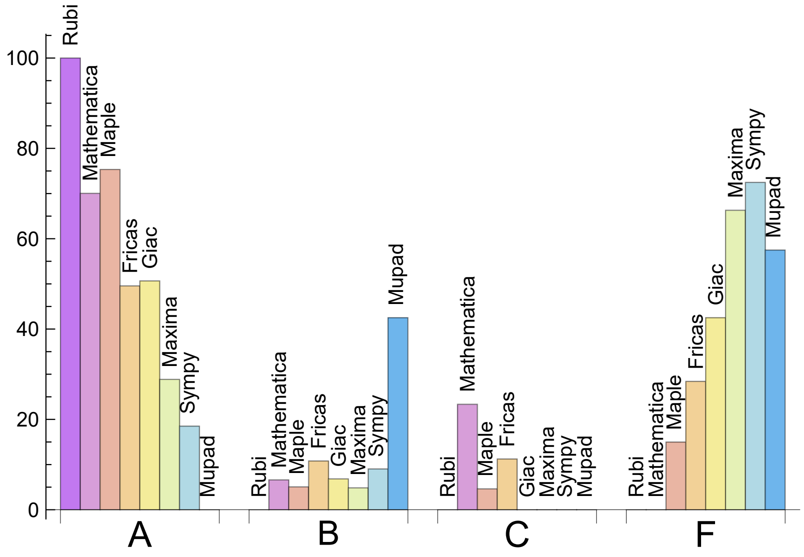
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	75.330	5.066	4.626	14.978
Mathematica	70.044	6.608	23.348	0.000
Giac	50.661	6.828	0.000	42.511
Fricas	49.559	10.793	11.233	28.414
Maxima	28.855	4.846	0.000	66.300
Sympy	18.502	9.031	0.000	72.467
Mupad	0.000	42.511	0.000	57.489

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	68	100.00	0.00	0.00
Fricas	129	39.53	25.58	34.88
Giac	193	97.41	0.52	2.07
Mupad	261	0.00	100.00	0.00
Maxima	301	100.00	0.00	0.00
Sympy	329	92.10	7.60	0.30

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.10
Maxima	0.22
Giac	0.30
Sympy	1.44
Maple	2.25
Mathematica	2.41
Mupad	5.95
Fricas	8.92

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	42.06	1.64	27.00	0.89
Mupad	46.89	1.45	37.00	0.87
Sympy	65.68	2.60	31.00	0.93
Mathematica	67.24	1.14	59.00	0.92
Rubi	119.28	1.00	74.00	1.00
Maple	122.80	1.21	58.50	0.87
Fricas	126.86	1.80	57.00	1.00
Giac	274.17	8.09	53.00	0.96

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

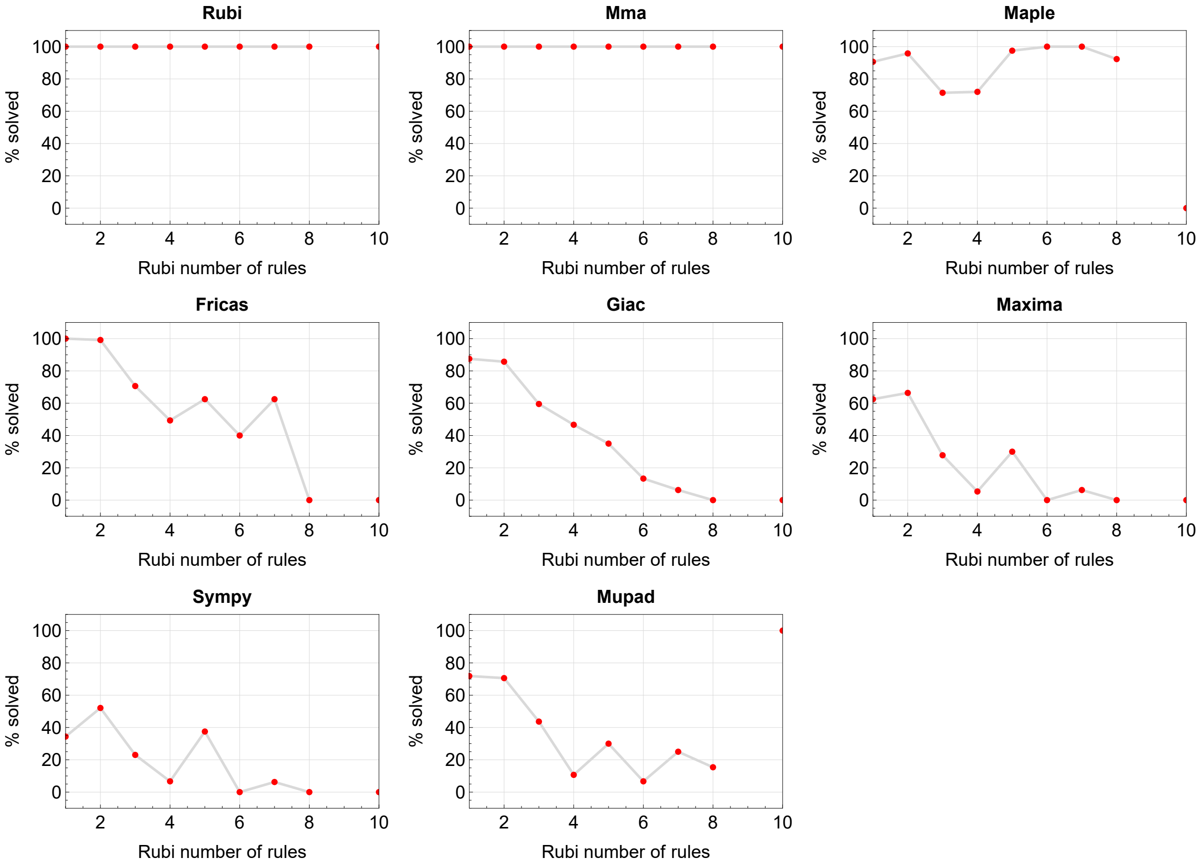


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

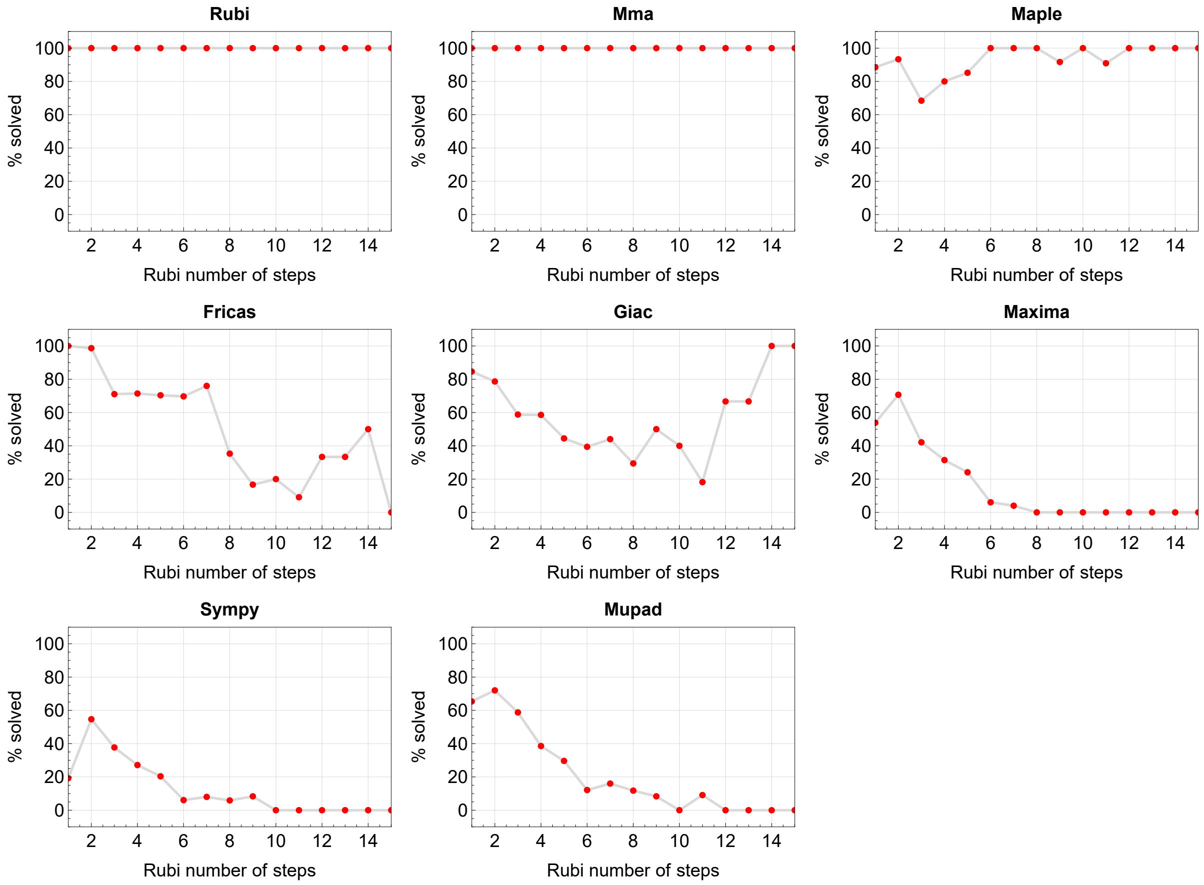


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

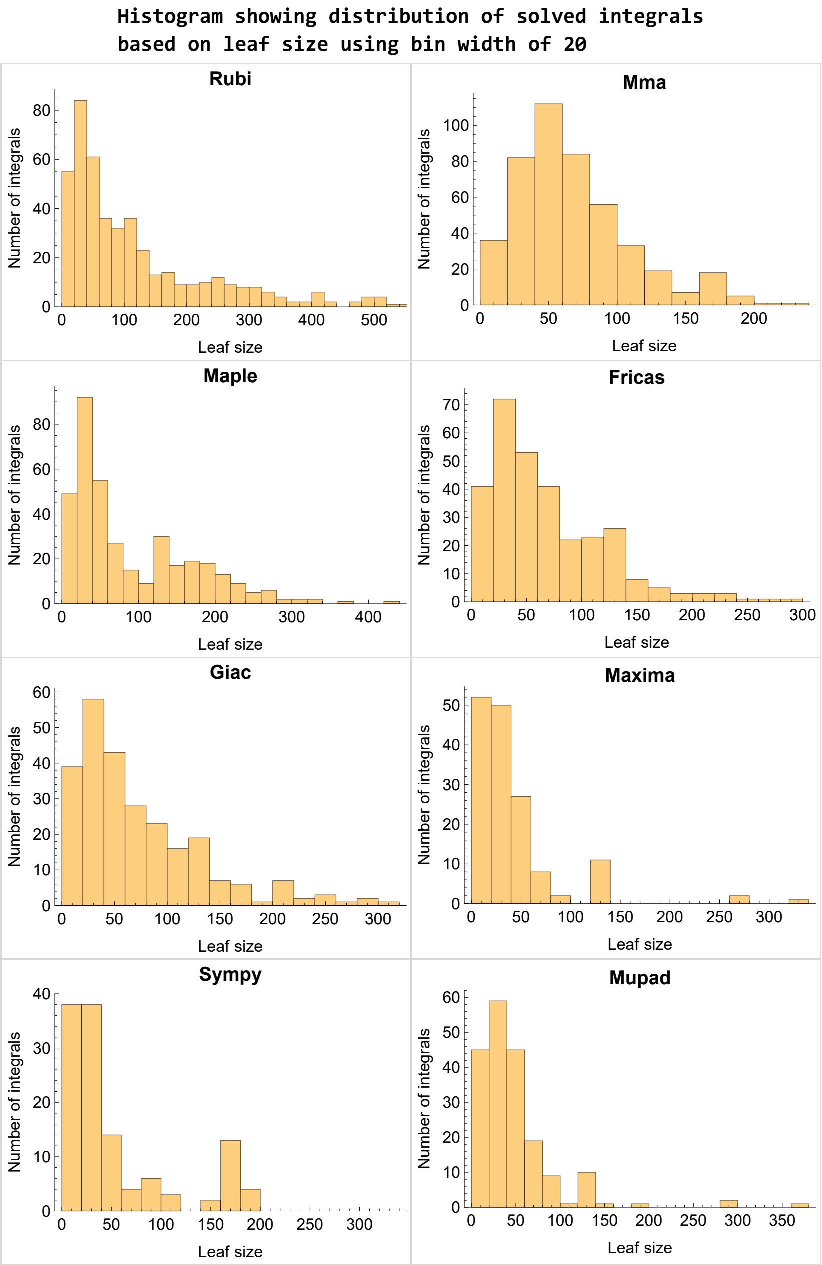
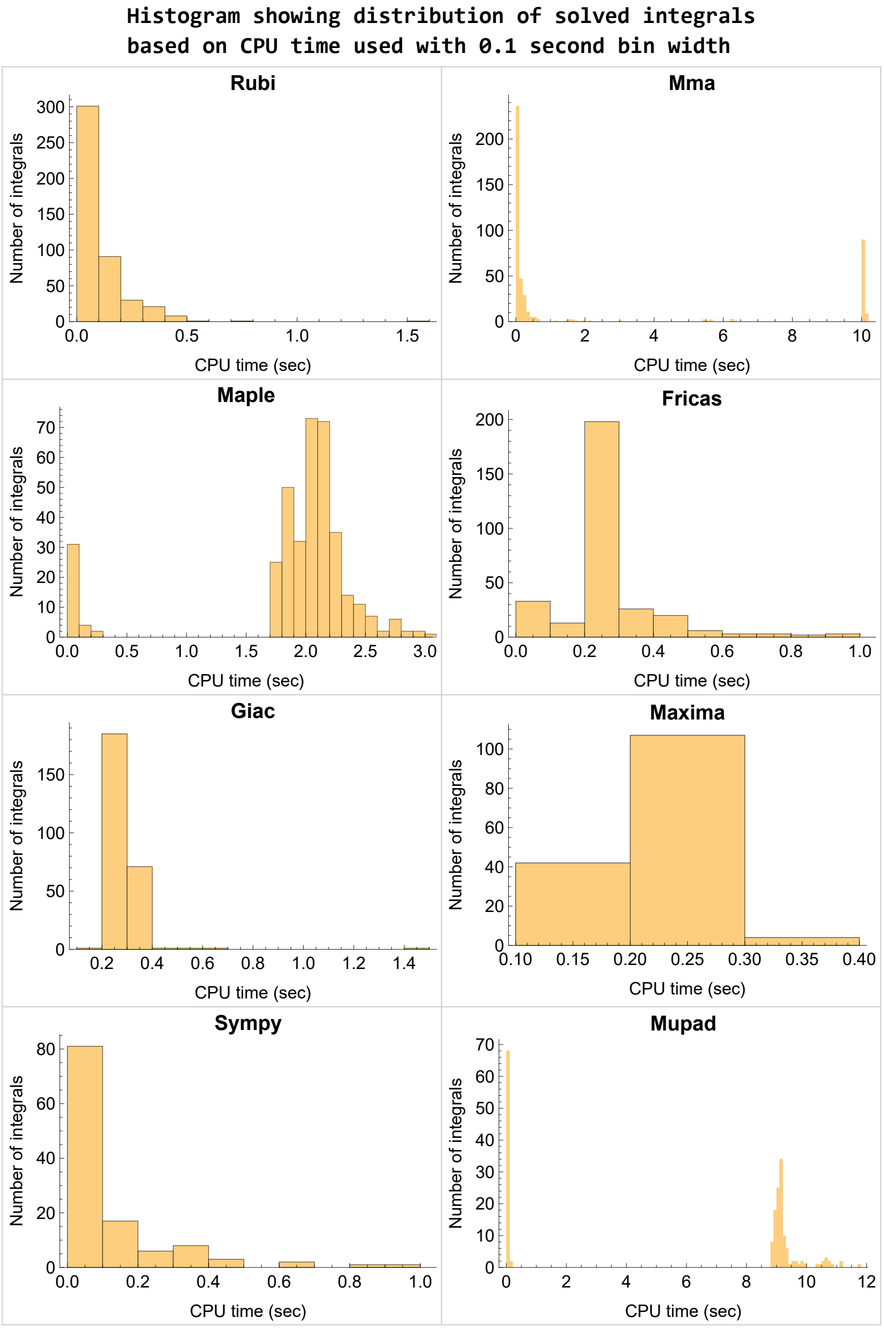


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.



Histogram showing distribution of solved integrals based on CPU time used with 0.1 second bin width

Rubi

Mma

Maple

Fricas

Giac

Maxima

Sympy

Mupad

Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

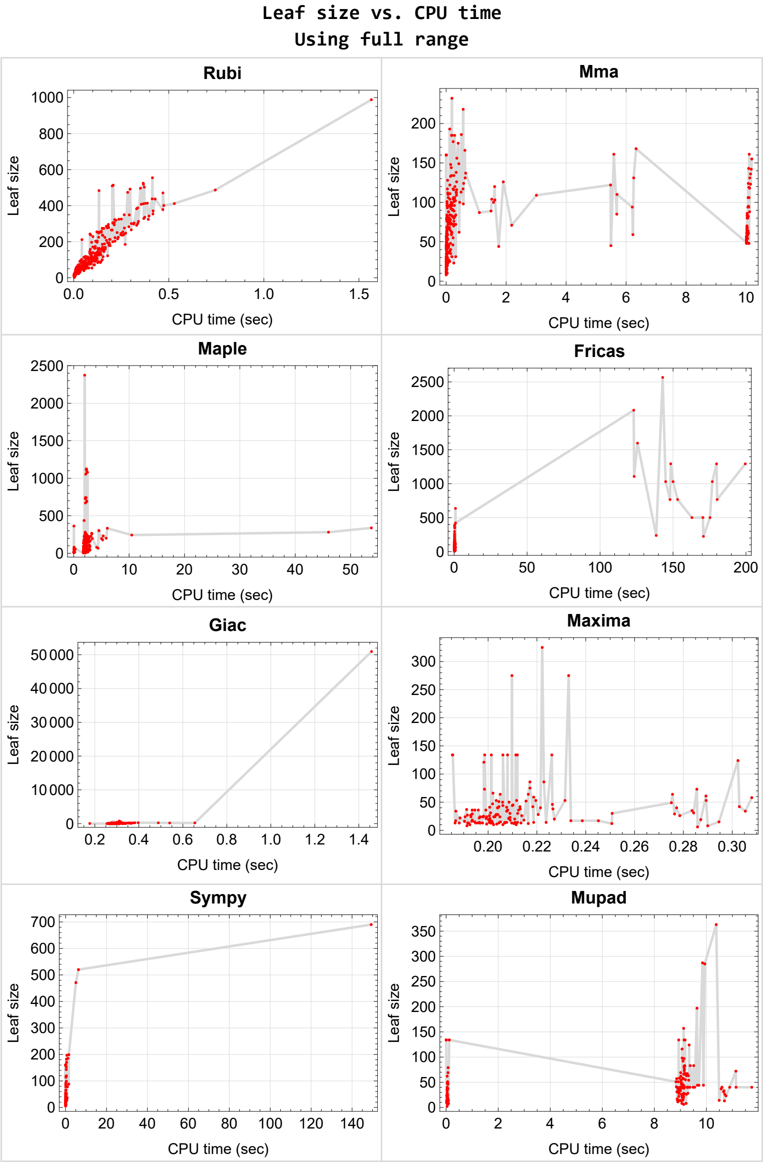


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

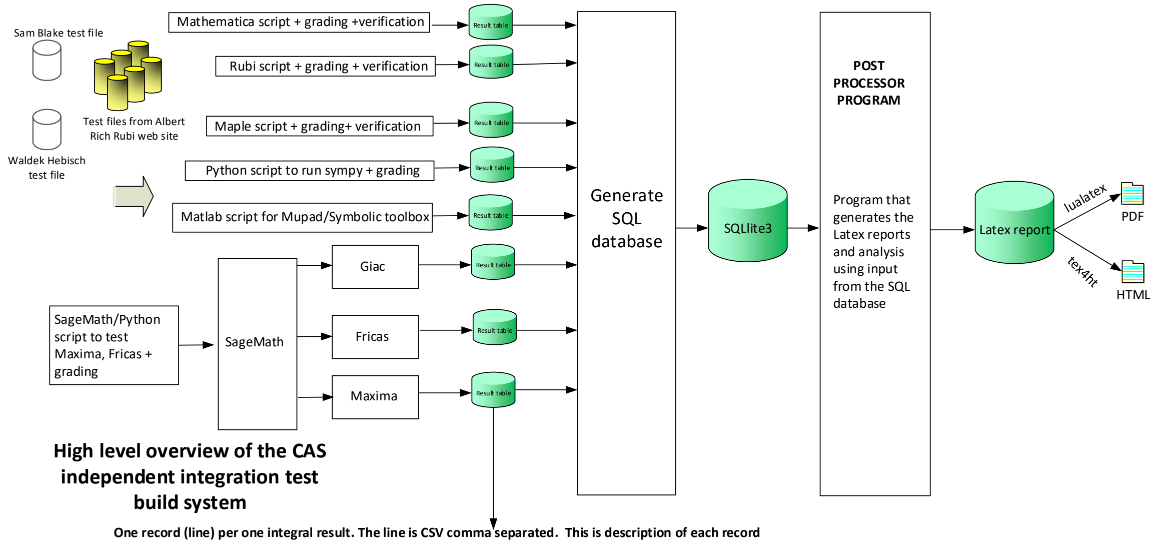
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 331, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade { 26, 28, 30, 32, 34, 328, 329, 330, 332, 333, 334, 335, 336, 348, 349, 350, 351, 352, 353, 354, 392, 410, 414, 415, 416, 417, 418, 419, 420, 421 }

C grade { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 98, 99, 100, 101, 102, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 181, 182, 183, 184, 200, 201, 202, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 411, 412, 413, 437, 438 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 30, 31, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 297, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 349, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 382, 383, 384, 385, 386, 388, 389, 394, 402, 408, 412, 431, 432, 433, 434, 435, 436, 448, 449, 450 }

B grade { 86, 112, 113, 126, 272, 300, 328, 329, 330, 331, 332, 333, 334, 335, 336, 348, 350, 351, 353, 387, 407, 411, 454 }

C grade { 26, 28, 32, 34, 97, 98, 99, 100, 101, 102, 243, 295, 296, 298, 299, 301, 302, 304, 305, 307, 347 }

F normal fail { 277, 278, 279, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 409, 410, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 108, 109, 113, 114, 115, 116, 121, 122, 123, 128, 129, 130, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 280, 281, 282, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 338, 339, 340, 344, 345, 346, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 382, 383, 384, 385, 386, 387, 388, 389, 394, 402, 407, 408, 409, 410, 411, 412, 413, 414, 416, 419, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade { 28, 30, 32, 34, 79, 84, 127, 167, 168, 169, 170, 171, 176, 177, 178, 179, 185, 186, 187, 188, 189, 194, 195, 196, 197, 283, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 341, 342, 343, 347, 348, 349, 350, 351, 352, 353, 354 }

C grade { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 98, 99, 102, 289, 290, 291, 292, 293, 294, 301, 302, 304, 305, 307 }

F normal fail { 97, 100, 101, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 277, 278, 279, 295, 296, 298, 299, 439, 440, 441, 442, 443 }

F(-1) timedout fail { 103, 104, 105, 110, 111, 112, 117, 118, 119, 120, 124, 125, 126, 172, 173, 174, 175, 180, 181, 182, 183, 184, 190, 191, 192, 193, 198, 199, 200, 201, 202, 437, 438 }

F(-2) exception fail { 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 415, 417, 418, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 34, 35, 36, 37, 94, 95, 96, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 252, 253, 254, 255, 260, 261, 262, 263, 284, 285, 286, 303, 306, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 338, 339, 340, 344, 345, 346, 347, 349, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 383, 387, 394, 402, 433, 434, 435, 436, 448, 449, 450 }

B grade { 28, 30, 32, 283, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 341, 342, 343, 348, 350, 351, 353 }

C grade { }

F normal fail { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 236, 237, 238, 239, 246, 247, 248, 249, 250, 251, 256, 257, 258, 259, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 384, 385, 386, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453, 454 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 283, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 352, 354, 360, 361, 382, 384, 386, 387, 388, 408, 412, 432, 433, 434, 435, 436 }

B grade { 26, 28, 30, 32, 34, 176, 177, 178, 179, 235, 240, 241, 242, 243, 244, 245, 328, 329, 330, 331, 332, 333, 334, 336, 348, 350, 351, 353, 362, 383, 431 }

C grade { }

F normal fail { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 99, 100, 101, 102, 114, 115, 116, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 277, 278, 279, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 325, 326, 327, 337, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 410, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

F(-1) timedout fail { 335 }

F(-2) exception fail { 407, 409, 411, 413 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 41, 48, 61, 72, 94, 95, 96, 98, 105, 113, 134, 142, 153, 162, 170, 178, 189, 198, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 240, 241, 242, 243, 244, 245, 252, 253, 254, 255, 258, 260, 261, 262, 263, 266, 268, 280, 281, 282, 283, 284, 285, 286, 291, 308, 309, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 371, 382, 383, 384, 386, 387, 388, 392, 415, 416, 417, 418, 419, 420, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 444 }

C grade { }

F normal fail { }

F(-1) timedout fail { 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 235, 237, 238, 239, 246, 247, 248, 249, 250, 251, 256, 257, 259, 264, 265, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312, 313, 364, 365, 366, 367, 368, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 421, 422, 423, 424, 425, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 16, 18, 20, 22, 23, 24, 25, 27, 29, 31, 33, 34, 35, 36, 37, 103, 104, 105, 117, 118, 119, 120, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 308, 309, 319, 320, 321, 322, 324, 338, 339, 340, 344, 345, 346, 347, 360, 362, 363, 369, 377, 394, 433 }

B grade { 9, 13, 15, 17, 19, 21, 26, 28, 30, 32, 283, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 341, 342, 343, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 402 }

C grade { }

F normal fail { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 364, 365, 366, 367, 368, 370, 371, 372, 373, 375, 376, 378, 379, 381, 382, 383, 385, 386, 387, 389, 390, 391, 392, 393, 395, 396, 397, 398, 400, 401, 403, 404, 406, 407, 408, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 447, 451, 452, 453 }

F(-1) timedout fail { 75, 76, 77, 78, 79, 80, 81, 82, 83, 148, 184, 374, 380, 384, 388, 399, 405, 409, 413, 434, 446, 448, 449, 450, 454 }

F(-2) exception fail { 337 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.002	0.204	0.195	0.253	0.019	0.271	0.024

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.002	0.100	0.213	0.261	0.016	0.285	0.023

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.002	0.000	0.104	0.196	0.250	0.016	0.278	0.020

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.002	0.001	0.044	0.212	0.279	0.017	0.261	0.018

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	14	11
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	1.08	0.85
time (sec)	N/A	0.003	0.002	0.044	0.204	0.262	0.031	0.255	0.024

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.011	0.002	1.963	0.205	0.398	0.017	0.263	0.042

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.013	0.001	1.987	0.198	0.381	0.017	0.269	0.034

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.008	0.006	1.998	0.203	0.343	0.017	0.261	0.032

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	24	24	24	24	24
N.S.	1	1.00	1.00	0.94	1.50	1.50	1.50	1.50	1.50
time (sec)	N/A	0.005	0.005	1.973	0.191	0.370	0.019	0.277	0.035

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.008	0.002	1.975	0.208	0.383	0.020	0.295	0.032

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	36	36	42	36	36
N.S.	1	1.00	1.00	0.80	0.78	0.78	0.91	0.78	0.78
time (sec)	N/A	0.010	0.005	2.121	0.212	0.238	0.023	0.269	0.043

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	23	22	20	24	22
N.S.	1	1.00	1.00	0.85	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.016	0.007	2.036	0.209	0.258	0.067	0.277	0.042

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	23
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74
time (sec)	N/A	0.010	0.011	2.192	0.279	0.263	0.064	0.278	10.752

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.005	0.002	2.117	0.197	0.256	0.046	0.270	10.693

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.006	0.005	2.130	0.295	0.256	0.055	0.276	0.048

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	18	15	24	18
N.S.	1	1.00	1.00	0.95	0.91	0.82	0.68	1.09	0.82
time (sec)	N/A	0.010	0.006	2.088	0.211	0.263	0.093	0.278	0.065

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	26
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76
time (sec)	N/A	0.010	0.015	2.102	0.276	0.264	0.083	0.264	10.709

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	33	31	43	31
N.S.	1	1.00	1.00	0.91	0.89	0.94	0.89	1.23	0.89
time (sec)	N/A	0.018	0.008	2.059	0.201	0.245	0.130	0.263	10.658

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	40	106	87	40	37
N.S.	1	1.00	1.00	0.91	0.93	2.47	2.02	0.93	0.86
time (sec)	N/A	0.014	0.024	2.076	0.277	0.253	0.102	0.269	10.567

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	44	45	42	57	46
N.S.	1	1.00	1.00	0.90	0.90	0.92	0.86	1.16	0.94
time (sec)	N/A	0.021	0.009	2.138	0.210	0.246	0.149	0.265	0.067

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.009	0.028	2.074	0.284	0.415	0.101	0.272	10.680

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	34	47	34	47	34
N.S.	1	1.00	0.87	0.92	0.89	1.24	0.89	1.24	0.89
time (sec)	N/A	0.020	0.019	2.081	0.208	0.368	0.136	0.268	0.048

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	54	45	49	136	92	47	44
N.S.	1	1.00	0.95	0.79	0.86	2.39	1.61	0.82	0.77
time (sec)	N/A	0.013	0.043	2.144	0.275	0.437	0.138	0.274	9.655

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	52	50	73	51	51	51
N.S.	1	1.00	0.84	1.06	1.02	1.49	1.04	1.04	1.04
time (sec)	N/A	0.027	0.040	2.487	0.209	0.804	0.188	0.273	0.058

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	67	55	64	172	114	59	58
N.S.	1	1.00	0.99	0.81	0.94	2.53	1.68	0.87	0.85
time (sec)	N/A	0.021	0.045	2.834	0.276	0.256	0.182	0.276	8.961

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	29	21	21	21	19	23	11
N.S.	1	1.00	2.23	1.62	1.62	1.62	1.46	1.77	0.85
time (sec)	N/A	0.008	0.007	2.153	0.199	0.249	0.039	0.270	8.903

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	18	14	14	15	14
N.S.	1	1.00	0.90	0.75	0.90	0.70	0.70	0.75	0.70
time (sec)	N/A	0.010	0.008	2.194	0.209	0.244	0.032	0.278	0.038

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	22	14	16	16	14	18	6
N.S.	1	1.00	3.67	2.33	2.67	2.67	2.33	3.00	1.00
time (sec)	N/A	0.006	0.004	2.121	0.190	0.259	0.043	0.275	0.060

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	13	8	8	15	8
N.S.	1	1.00	1.00	0.75	1.08	0.67	0.67	1.25	0.67
time (sec)	N/A	0.004	0.002	2.098	0.204	0.264	0.039	0.278	0.030

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	2
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00
time (sec)	N/A	0.003	0.003	2.086	0.188	0.425	0.051	0.276	0.035

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	11	10	16	11
N.S.	1	1.00	1.00	0.80	1.00	0.73	0.67	1.07	0.73
time (sec)	N/A	0.006	0.005	2.160	0.205	0.472	0.048	0.268	0.051

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	24	16	18	20	15	20	8
N.S.	1	1.00	3.00	2.00	2.25	2.50	1.88	2.50	1.00
time (sec)	N/A	0.005	0.005	2.063	0.194	0.350	0.048	0.259	0.034

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	20	24	17	26	16
N.S.	1	1.00	1.00	0.77	0.91	1.09	0.77	1.18	0.73
time (sec)	N/A	0.010	0.007	2.333	0.227	0.752	0.043	0.265	0.034

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	31	22	25	30	24	27	13
N.S.	1	1.00	2.07	1.47	1.67	2.00	1.60	1.80	0.87
time (sec)	N/A	0.007	0.006	2.210	0.198	0.259	0.059	0.270	9.045

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	27	30	22	33	23
N.S.	1	1.00	1.00	0.79	0.93	1.03	0.76	1.14	0.79
time (sec)	N/A	0.011	0.006	2.267	0.192	0.252	0.055	0.276	0.034

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	18	14
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	1.20	0.93
time (sec)	N/A	0.007	0.005	2.244	0.207	0.265	0.060	0.279	10.491

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	15	15	12	18	16
N.S.	1	1.00	1.00	0.89	0.83	0.83	0.67	1.00	0.89
time (sec)	N/A	0.008	0.009	2.250	0.194	0.253	0.069	0.278	0.056

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	95	158	0	59	0	0	0
N.S.	1	1.00	0.58	0.97	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.126	10.081	2.388	0.000	0.087	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	80	197	0	57	0	0	0
N.S.	1	1.00	0.28	0.70	0.00	0.20	0.00	0.00	0.00
time (sec)	N/A	0.176	10.039	2.420	0.000	0.089	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	79	146	0	49	0	0	0
N.S.	1	1.00	0.58	1.07	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.084	0.031	2.357	0.000	0.157	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	51	175	0	43	0	0	40
N.S.	1	1.00	0.20	0.69	0.00	0.17	0.00	0.00	0.16
time (sec)	N/A	0.122	0.015	2.223	0.000	0.138	0.000	0.000	11.139

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	48	124	0	34	0	0	0
N.S.	1	1.00	0.42	1.10	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.055	0.014	2.182	0.000	0.094	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	51	177	0	40	0	0	0
N.S.	1	1.00	0.21	0.71	0.00	0.16	0.00	0.00	0.00
time (sec)	N/A	0.131	10.017	2.127	0.000	0.083	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	53	123	0	36	0	0	0
N.S.	1	1.00	0.46	1.06	0.00	0.31	0.00	0.00	0.00
time (sec)	N/A	0.059	10.017	2.162	0.000	0.076	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	53	195	0	54	0	0	0
N.S.	1	1.00	0.19	0.69	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.182	10.021	2.190	0.000	0.164	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	94	169	0	70	0	0	0
N.S.	1	1.00	0.51	0.91	0.00	0.38	0.00	0.00	0.00
time (sec)	N/A	0.150	10.077	2.109	0.000	0.197	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	84	210	0	68	0	0	0
N.S.	1	1.00	0.28	0.69	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.205	10.047	2.131	0.000	0.256	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	83	158	0	60	0	0	40
N.S.	1	1.00	0.53	1.00	0.00	0.38	0.00	0.00	0.25
time (sec)	N/A	0.087	10.036	2.060	0.000	0.087	0.000	0.000	8.985

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	52	195	0	58	0	0	0
N.S.	1	1.00	0.19	0.71	0.00	0.21	0.00	0.00	0.00
time (sec)	N/A	0.150	10.019	2.114	0.000	0.089	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	49	143	0	47	0	0	0
N.S.	1	1.00	0.37	1.07	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.091	0.019	2.129	0.000	0.138	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	52	188	0	52	0	0	0
N.S.	1	1.00	0.19	0.69	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.158	10.018	2.126	0.000	0.176	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	54	139	0	45	0	0	0
N.S.	1	1.00	0.40	1.04	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.088	10.019	2.148	0.000	0.267	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	54	189	0	51	0	0	0
N.S.	1	1.00	0.19	0.68	0.00	0.18	0.00	0.00	0.00
time (sec)	N/A	0.177	10.019	2.192	0.000	0.086	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	54	139	0	44	0	0	0
N.S.	1	1.00	0.39	1.01	0.00	0.32	0.00	0.00	0.00
time (sec)	N/A	0.096	10.022	2.245	0.000	0.092	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	54	210	0	67	0	0	0
N.S.	1	1.00	0.18	0.69	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.214	10.020	2.307	0.000	0.090	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	54	160	0	59	0	0	0
N.S.	1	1.00	0.33	0.98	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.110	10.021	2.283	0.000	0.127	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	80	147	0	48	0	0	0
N.S.	1	1.00	0.57	1.05	0.00	0.34	0.00	0.00	0.00
time (sec)	N/A	0.101	10.051	2.162	0.000	0.146	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	66	178	0	43	0	0	0
N.S.	1	1.00	0.26	0.69	0.00	0.17	0.00	0.00	0.00
time (sec)	N/A	0.144	10.031	2.268	0.000	0.088	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	64	127	0	34	0	0	0
N.S.	1	1.00	0.55	1.09	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.061	10.033	2.122	0.000	0.091	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	53	158	0	22	0	0	0
N.S.	1	1.00	0.23	0.69	0.00	0.10	0.00	0.00	0.00
time (sec)	N/A	0.101	10.020	2.037	0.000	0.093	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	49	108	0	14	0	0	40
N.S.	1	1.00	0.53	1.17	0.00	0.15	0.00	0.00	0.43
time (sec)	N/A	0.032	10.027	2.037	0.000	0.163	0.000	0.000	10.598

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	48	182	0	42	0	0	0
N.S.	1	1.00	0.19	0.72	0.00	0.17	0.00	0.00	0.00
time (sec)	N/A	0.132	10.017	2.193	0.000	0.139	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	53	129	0	36	0	0	0
N.S.	1	1.00	0.45	1.08	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.062	10.019	2.188	0.000	0.085	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	53	195	0	56	0	0	0
N.S.	1	1.00	0.19	0.68	0.00	0.20	0.00	0.00	0.00
time (sec)	N/A	0.172	10.019	2.252	0.000	0.081	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	80	172	0	80	0	0	0
N.S.	1	1.00	0.50	1.07	0.00	0.50	0.00	0.00	0.00
time (sec)	N/A	0.135	10.051	2.709	0.000	0.087	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	68	200	0	76	0	0	0
N.S.	1	1.00	0.24	0.72	0.00	0.27	0.00	0.00	0.00
time (sec)	N/A	0.184	10.041	2.731	0.000	0.121	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	67	147	0	68	0	0	0
N.S.	1	1.00	0.49	1.07	0.00	0.50	0.00	0.00	0.00
time (sec)	N/A	0.093	10.032	2.590	0.000	0.206	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	57	182	0	61	0	0	0
N.S.	1	1.00	0.23	0.72	0.00	0.24	0.00	0.00	0.00
time (sec)	N/A	0.141	10.026	2.118	0.000	0.219	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	54	130	0	51	0	0	0
N.S.	1	1.00	0.47	1.13	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.068	10.029	2.105	0.000	0.083	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	56	184	0	60	0	0	0
N.S.	1	1.00	0.22	0.72	0.00	0.24	0.00	0.00	0.00
time (sec)	N/A	0.145	10.019	2.112	0.000	0.085	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	54	132	0	51	0	0	0
N.S.	1	1.00	0.47	1.16	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.054	0.027	2.056	0.000	0.138	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	51	206	0	72	0	0	40
N.S.	1	1.00	0.19	0.75	0.00	0.26	0.00	0.00	0.15
time (sec)	N/A	0.150	10.017	2.790	0.000	0.146	0.000	0.000	9.136

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	56	150	0	68	0	0	0
N.S.	1	1.00	0.40	1.08	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.088	10.020	2.797	0.000	0.343	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	56	228	0	90	0	0	0
N.S.	1	1.00	0.18	0.75	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.207	10.020	2.627	0.000	0.085	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	128	212	0	376	0	100	0
N.S.	1	1.00	0.81	1.33	0.00	2.36	0.00	0.63	0.00
time (sec)	N/A	0.158	0.048	2.110	0.000	0.294	0.000	0.312	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	68	70	0	108	0	80	0
N.S.	1	1.00	0.54	0.56	0.00	0.86	0.00	0.63	0.00
time (sec)	N/A	0.125	0.023	2.062	0.000	0.271	0.000	0.279	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	117	198	0	348	0	86	0
N.S.	1	1.00	0.90	1.52	0.00	2.68	0.00	0.66	0.00
time (sec)	N/A	0.125	0.041	2.079	0.000	0.421	0.000	0.303	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	64	59	0	97	0	64	0
N.S.	1	1.00	0.63	0.58	0.00	0.96	0.00	0.63	0.00
time (sec)	N/A	0.100	0.023	2.003	0.000	0.421	0.000	0.276	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	0	61	0	17	0
N.S.	1	1.00	1.00	1.08	0.00	2.44	0.00	0.68	0.00
time (sec)	N/A	0.022	0.018	1.983	0.000	0.483	0.000	0.302	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	46	48	0	86	0	50	0
N.S.	1	1.00	0.61	0.63	0.00	1.13	0.00	0.66	0.00
time (sec)	N/A	0.069	0.020	2.028	0.000	0.572	0.000	0.294	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	38	37	0	76	0	29	0
N.S.	1	1.00	0.75	0.73	0.00	1.49	0.00	0.57	0.00
time (sec)	N/A	0.047	0.017	2.048	0.000	0.281	0.000	0.329	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	37	0	75	0	33	0
N.S.	1	1.00	0.69	0.73	0.00	1.47	0.00	0.65	0.00
time (sec)	N/A	0.048	0.017	2.152	0.000	0.282	0.000	0.280	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	56	48	0	87	0	43	0
N.S.	1	1.00	0.74	0.63	0.00	1.14	0.00	0.57	0.00
time (sec)	N/A	0.084	0.020	2.190	0.000	0.279	0.000	0.300	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	0	63	0	23	0
N.S.	1	1.00	1.00	1.08	0.00	2.52	0.00	0.92	0.00
time (sec)	N/A	0.025	0.016	2.164	0.000	0.266	0.000	0.295	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	58	59	0	95	0	55	0
N.S.	1	1.00	0.57	0.58	0.00	0.94	0.00	0.54	0.00
time (sec)	N/A	0.101	0.022	2.176	0.000	0.272	0.000	0.283	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	106	217	0	360	0	114	0
N.S.	1	1.00	0.82	1.67	0.00	2.77	0.00	0.88	0.00
time (sec)	N/A	0.134	0.069	2.155	0.000	0.280	0.000	0.289	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	68	70	0	110	0	90	0
N.S.	1	1.00	0.54	0.56	0.00	0.87	0.00	0.71	0.00
time (sec)	N/A	0.132	0.020	2.044	0.000	0.415	0.000	0.312	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	122	234	0	396	0	104	0
N.S.	1	1.00	0.77	1.47	0.00	2.49	0.00	0.65	0.00
time (sec)	N/A	0.165	0.123	2.164	0.000	0.385	0.000	0.289	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	79	81	0	121	0	147	0
N.S.	1	1.00	0.52	0.53	0.00	0.80	0.00	0.97	0.00
time (sec)	N/A	0.151	0.021	2.090	0.000	0.667	0.000	0.310	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	134	247	0	422	0	138	0
N.S.	1	1.00	0.71	1.31	0.00	2.23	0.00	0.73	0.00
time (sec)	N/A	0.188	0.257	2.124	0.000	0.952	0.000	0.312	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	90	92	0	132	0	202	0
N.S.	1	1.00	0.50	0.51	0.00	0.73	0.00	1.12	0.00
time (sec)	N/A	0.185	0.210	2.063	0.000	0.374	0.000	0.324	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	82	45	0	133	0	45	0
N.S.	1	1.00	1.49	0.82	0.00	2.42	0.00	0.82	0.00
time (sec)	N/A	0.053	0.289	2.417	0.000	0.361	0.000	0.306	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	62	25	0	94	0	23	0
N.S.	1	1.00	1.94	0.78	0.00	2.94	0.00	0.72	0.00
time (sec)	N/A	0.028	0.433	2.199	0.000	0.341	0.000	0.294	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	19	0	14	19
N.S.	1	1.00	1.00	0.87	1.13	0.83	0.00	0.61	0.83
time (sec)	N/A	0.021	0.254	2.409	0.203	0.265	0.000	0.275	9.113

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	31	28	38	29	0	30	27
N.S.	1	1.00	0.65	0.58	0.79	0.60	0.00	0.62	0.56
time (sec)	N/A	0.044	0.320	2.588	0.227	0.267	0.000	0.281	9.123

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	44	41	50	40	0	47	40
N.S.	1	1.00	0.59	0.55	0.68	0.54	0.00	0.64	0.54
time (sec)	N/A	0.073	1.754	2.934	0.206	0.265	0.000	0.274	9.550

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	64	688	0	0	0	0	0
N.S.	1	1.00	0.29	3.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	10.034	2.342	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	49	671	0	16	0	39	40
N.S.	1	1.00	0.25	3.41	0.00	0.08	0.00	0.20	0.20
time (sec)	N/A	0.094	10.016	2.119	0.000	0.073	0.000	0.177	9.257

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	53	696	0	43	0	0	0
N.S.	1	1.00	0.24	3.09	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.131	10.019	2.360	0.000	0.077	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	503	66	1079	0	0	0	0	0
N.S.	1	1.00	0.13	2.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	10.031	2.491	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	53	1054	0	0	0	0	0
N.S.	1	1.00	0.11	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	10.022	2.170	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	497	497	48	1083	0	24	0	0	0
N.S.	1	1.00	0.10	2.18	0.00	0.05	0.00	0.00	0.00
time (sec)	N/A	0.351	10.017	2.368	0.000	0.075	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	113	151	0	0	182	109	0
N.S.	1	1.00	0.65	0.87	0.00	0.00	1.05	0.63	0.00
time (sec)	N/A	0.113	0.333	2.215	0.000	0.000	0.422	0.325	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	89	99	0	0	153	81	0
N.S.	1	1.00	0.77	0.85	0.00	0.00	1.32	0.70	0.00
time (sec)	N/A	0.073	0.262	2.207	0.000	0.000	0.329	0.324	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	62	50	0	0	119	52	72
N.S.	1	1.00	1.11	0.89	0.00	0.00	2.12	0.93	1.29
time (sec)	N/A	0.038	0.190	2.156	0.000	0.000	0.278	0.324	11.133

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	0	19	0	25	0
N.S.	1	1.00	1.00	0.80	0.00	0.76	0.00	1.00	0.00
time (sec)	N/A	0.025	0.122	2.220	0.000	0.320	0.000	0.301	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	48	67	0	42	0	84	0
N.S.	1	1.00	0.57	0.80	0.00	0.50	0.00	1.00	0.00
time (sec)	N/A	0.071	0.144	2.225	0.000	0.314	0.000	0.299	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	72	119	0	64	0	146	0
N.S.	1	1.00	0.51	0.84	0.00	0.45	0.00	1.03	0.00
time (sec)	N/A	0.125	0.152	2.219	0.000	0.333	0.000	0.307	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	96	171	0	86	0	208	0
N.S.	1	1.00	0.48	0.86	0.00	0.43	0.00	1.04	0.00
time (sec)	N/A	0.206	0.201	2.195	0.000	0.342	0.000	0.294	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	137	227	0	0	0	148	0
N.S.	1	1.00	0.70	1.15	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.123	0.650	2.191	0.000	0.000	0.000	0.345	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	113	175	0	0	0	120	0
N.S.	1	1.00	0.81	1.26	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.086	0.499	2.155	0.000	0.000	0.000	0.339	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	84	124	0	0	0	92	0
N.S.	1	1.00	1.09	1.61	0.00	0.00	0.00	1.19	0.00
time (sec)	N/A	0.050	0.319	2.292	0.000	0.000	0.000	0.327	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	31	45	0	36	0	34	40
N.S.	1	1.00	1.24	1.80	0.00	1.44	0.00	1.36	1.60
time (sec)	N/A	0.004	0.191	2.291	0.000	0.506	0.000	0.285	9.488

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	55	46	0	63	0	0	0
N.S.	1	1.00	0.70	0.58	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.074	0.232	2.232	0.000	0.945	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	81	98	0	87	0	0	0
N.S.	1	1.00	0.59	0.72	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.127	0.253	2.183	0.000	0.315	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	105	150	0	109	0	0	0
N.S.	1	1.00	0.54	0.77	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.186	0.255	2.185	0.000	0.314	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	126	177	0	0	199	123	0
N.S.	1	1.00	0.62	0.87	0.00	0.00	0.98	0.60	0.00
time (sec)	N/A	0.124	0.349	2.180	0.000	0.000	1.605	0.332	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	102	125	0	0	170	95	0
N.S.	1	1.00	0.70	0.86	0.00	0.00	1.16	0.65	0.00
time (sec)	N/A	0.084	0.308	2.284	0.000	0.000	0.487	0.335	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	80	74	0	0	143	67	0
N.S.	1	1.00	0.92	0.85	0.00	0.00	1.64	0.77	0.00
time (sec)	N/A	0.055	0.258	2.410	0.000	0.000	0.326	0.332	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	40	32	0	0	99	35	0
N.S.	1	1.00	1.18	0.94	0.00	0.00	2.91	1.03	0.00
time (sec)	N/A	0.034	0.150	2.158	0.000	0.000	0.398	0.339	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	35	41	0	29	0	53	0
N.S.	1	1.00	0.65	0.76	0.00	0.54	0.00	0.98	0.00
time (sec)	N/A	0.055	0.151	2.133	0.000	0.442	0.000	0.288	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	59	93	0	50	0	115	0
N.S.	1	1.00	0.53	0.83	0.00	0.45	0.00	1.03	0.00
time (sec)	N/A	0.100	0.177	2.133	0.000	0.345	0.000	0.312	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	83	145	0	72	0	177	0
N.S.	1	1.00	0.49	0.85	0.00	0.42	0.00	1.04	0.00
time (sec)	N/A	0.152	0.192	2.159	0.000	0.368	0.000	0.300	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	124	201	0	0	0	134	0
N.S.	1	1.00	0.73	1.18	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.101	0.584	2.138	0.000	0.000	0.000	0.344	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	100	148	0	0	0	106	0
N.S.	1	1.00	0.88	1.31	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.074	0.451	2.141	0.000	0.000	0.000	0.356	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	72	101	0	0	0	71	0
N.S.	1	1.00	1.20	1.68	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.045	0.277	2.136	0.000	0.000	0.000	0.365	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	25	0	54	0	26	0
N.S.	1	1.00	1.53	0.83	0.00	1.80	0.00	0.87	0.00
time (sec)	N/A	0.032	0.224	2.184	0.000	0.459	0.000	0.289	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	70	72	0	79	0	0	0
N.S.	1	1.00	0.65	0.67	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.101	0.251	2.166	0.000	0.492	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	96	124	0	101	0	0	0
N.S.	1	1.00	0.58	0.75	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.159	0.275	2.174	0.000	0.459	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	120	176	0	123	0	0	0
N.S.	1	1.00	0.54	0.79	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.217	0.290	2.214	0.000	0.423	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	155	264	0	0	0	0	0
N.S.	1	1.00	0.51	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.336	10.191	2.062	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	136	273	0	0	0	0	0
N.S.	1	1.00	0.33	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.367	10.149	2.096	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	118	198	0	0	0	0	0
N.S.	1	1.00	0.55	0.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.191	10.119	2.078	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	94	207	0	0	0	0	40
N.S.	1	1.00	0.29	0.64	0.00	0.00	0.00	0.00	0.12
time (sec)	N/A	0.215	10.062	2.035	0.000	0.000	0.000	0.000	9.151

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	54	132	0	0	0	0	0
N.S.	1	1.00	0.44	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	10.048	2.046	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	59	213	0	0	0	0	0
N.S.	1	1.00	0.18	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	10.051	1.982	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	59	179	0	0	0	0	0
N.S.	1	1.00	0.31	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	10.053	2.093	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	59	281	0	0	0	0	0
N.S.	1	1.00	0.14	0.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	10.053	2.079	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	59	245	0	0	0	0	0
N.S.	1	1.00	0.21	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	10.055	2.370	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	142	196	0	0	0	0	0
N.S.	1	1.00	0.48	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	10.176	3.363	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	123	261	0	0	0	0	0
N.S.	1	1.00	0.30	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.355	10.151	2.222	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	106	163	0	0	0	0	40
N.S.	1	1.00	0.51	0.78	0.00	0.00	0.00	0.00	0.19
time (sec)	N/A	0.172	10.100	2.043	0.000	0.000	0.000	0.000	9.089

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	60	205	0	0	0	0	0
N.S.	1	1.00	0.19	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	10.053	2.051	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	60	130	0	0	0	0	0
N.S.	1	1.00	0.42	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	10.057	1.996	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	62	235	0	0	0	0	0
N.S.	1	1.00	0.18	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	10.054	2.018	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	62	168	0	0	0	0	0
N.S.	1	1.00	0.29	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.188	10.060	3.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	62	301	0	0	0	0	0
N.S.	1	1.00	0.14	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	10.056	4.563	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	62	201	0	0	0	0	0
N.S.	1	1.00	0.21	0.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.304	10.065	5.863	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	161	196	0	0	0	0	0
N.S.	1	1.00	0.53	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	10.107	5.022	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	414	143	261	0	0	0	0	0
N.S.	1	1.00	0.35	0.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.388	10.087	3.283	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	124	163	0	0	0	0	0
N.S.	1	1.00	0.57	0.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	10.076	2.109	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	106	210	0	0	0	0	0
N.S.	1	1.00	0.33	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	10.069	1.996	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	80	127	0	0	0	0	40
N.S.	1	1.00	0.63	1.01	0.00	0.00	0.00	0.00	0.32
time (sec)	N/A	0.090	10.043	2.011	0.000	0.000	0.000	0.000	9.264

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	54	195	0	0	0	0	0
N.S.	1	1.00	0.18	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	10.052	2.002	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	59	142	0	0	0	0	0
N.S.	1	1.00	0.36	0.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.135	10.048	2.022	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	59	262	0	0	0	0	0
N.S.	1	1.00	0.15	0.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.336	10.053	3.265	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	59	179	0	0	0	0	0
N.S.	1	1.00	0.24	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	10.049	5.210	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	131	303	0	0	0	0	0
N.S.	1	1.00	0.30	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.428	10.125	4.530	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	119	228	0	0	0	0	0
N.S.	1	1.00	0.50	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	10.095	3.109	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	94	237	0	0	0	0	0
N.S.	1	1.00	0.27	0.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	10.088	2.148	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	82	160	0	0	0	0	0
N.S.	1	1.00	0.55	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	10.074	2.075	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	62	197	0	0	0	0	40
N.S.	1	1.00	0.21	0.67	0.00	0.00	0.00	0.00	0.14
time (sec)	N/A	0.182	10.033	2.031	0.000	0.000	0.000	0.000	9.284

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	62	162	0	0	0	0	0
N.S.	1	1.00	0.39	1.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	10.065	2.076	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	64	267	0	0	0	0	0
N.S.	1	1.00	0.17	0.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.331	10.061	3.246	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	64	231	0	0	0	0	0
N.S.	1	1.00	0.26	0.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	10.059	5.330	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	471	471	64	333	0	0	0	0	0
N.S.	1	1.00	0.14	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	10.057	6.066	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	185	156	0	1293	0	396	0
N.S.	1	1.00	0.50	0.42	0.00	3.49	0.00	1.07	0.00
time (sec)	N/A	0.417	0.179	2.084	0.000	179.802	0.000	0.299	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	133	123	0	1031	0	312	0
N.S.	1	1.00	0.47	0.43	0.00	3.64	0.00	1.10	0.00
time (sec)	N/A	0.290	0.147	2.072	0.000	150.030	0.000	0.297	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	96	90	0	767	0	228	0
N.S.	1	1.00	0.49	0.46	0.00	3.93	0.00	1.17	0.00
time (sec)	N/A	0.178	0.109	2.037	0.000	147.950	0.000	0.297	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	74	57	0	501	0	143	40
N.S.	1	1.00	0.68	0.52	0.00	4.60	0.00	1.31	0.37
time (sec)	N/A	0.093	0.073	2.065	0.000	162.933	0.000	0.292	11.743

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	0	224	0	23	0
N.S.	1	1.00	1.00	1.17	0.00	9.74	0.00	1.00	0.00
time (sec)	N/A	0.026	0.063	2.051	0.000	170.839	0.000	0.275	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	76	79	0	0	0	72	0
N.S.	1	1.00	0.84	0.88	0.00	0.00	0.00	0.80	0.00
time (sec)	N/A	0.094	0.257	2.054	0.000	0.000	0.000	0.314	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	112	125	0	0	0	126	0
N.S.	1	1.00	0.63	0.70	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.191	0.340	2.029	0.000	0.000	0.000	0.324	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	149	167	0	0	0	177	0
N.S.	1	1.00	0.56	0.63	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.302	0.422	2.011	0.000	0.000	0.000	0.369	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	354	186	209	0	0	0	228	0
N.S.	1	1.00	0.53	0.59	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.416	0.506	2.098	0.000	0.000	0.000	0.377	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	168	145	0	1293	0	770	0
N.S.	1	1.00	0.49	0.42	0.00	3.77	0.00	2.24	0.00
time (sec)	N/A	0.396	6.333	2.034	0.000	199.502	0.000	0.310	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	131	112	0	1031	0	602	0
N.S.	1	1.00	0.51	0.44	0.00	4.04	0.00	2.36	0.00
time (sec)	N/A	0.258	6.254	2.015	0.000	144.930	0.000	0.317	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	94	79	0	768	0	434	40
N.S.	1	1.00	0.56	0.47	0.00	4.54	0.00	2.57	0.24
time (sec)	N/A	0.166	6.208	2.088	0.000	153.105	0.000	0.311	10.894

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	59	48	0	501	0	265	0
N.S.	1	1.00	0.70	0.57	0.00	5.96	0.00	3.15	0.00
time (sec)	N/A	0.092	6.227	2.048	0.000	175.324	0.000	0.305	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	88	67	0	0	0	83	0
N.S.	1	1.00	1.13	0.86	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	0.090	10.118	2.050	0.000	0.000	0.000	0.305	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	61	93	0	0	0	92	0
N.S.	1	1.00	0.54	0.82	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.119	10.070	1.749	0.000	0.000	0.000	0.311	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	61	139	0	0	0	143	0
N.S.	1	1.00	0.30	0.68	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.217	10.068	1.797	0.000	0.000	0.000	0.332	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	61	181	0	0	0	194	0
N.S.	1	1.00	0.21	0.62	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.331	10.074	1.846	0.000	0.000	0.000	0.540	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	379	61	223	0	0	0	245	0
N.S.	1	1.00	0.16	0.59	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.469	10.079	1.882	0.000	0.000	0.000	0.488	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	185	167	0	1294	0	206	0
N.S.	1	1.00	0.46	0.42	0.00	3.23	0.00	0.51	0.00
time (sec)	N/A	0.474	0.193	2.068	0.000	148.429	0.000	0.654	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	148	134	0	1031	0	164	0
N.S.	1	1.00	0.47	0.43	0.00	3.29	0.00	0.52	0.00
time (sec)	N/A	0.340	0.150	1.765	0.000	176.923	0.000	0.279	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	111	101	0	768	0	122	0
N.S.	1	1.00	0.49	0.45	0.00	3.41	0.00	0.54	0.00
time (sec)	N/A	0.234	0.129	2.077	0.000	180.269	0.000	0.292	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	74	68	0	502	0	80	0
N.S.	1	1.00	0.54	0.50	0.00	3.66	0.00	0.58	0.00
time (sec)	N/A	0.126	0.086	1.790	0.000	170.471	0.000	0.289	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	36	36	0	238	0	36	40
N.S.	1	1.00	0.77	0.77	0.00	5.06	0.00	0.77	0.85
time (sec)	N/A	0.034	0.057	1.793	0.000	138.458	0.000	0.283	9.364

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	61	0	0	0	51	0
N.S.	1	1.00	1.00	1.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.066	0.183	1.809	0.000	0.000	0.000	0.300	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	101	123	0	0	0	109	0
N.S.	1	1.00	0.66	0.80	0.00	0.00	0.00	0.71	0.00
time (sec)	N/A	0.155	0.270	2.078	0.000	0.000	0.000	0.329	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	138	183	0	0	0	160	0
N.S.	1	1.00	0.57	0.76	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.265	0.333	2.183	0.000	0.000	0.000	0.326	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	175	243	0	0	0	211	0
N.S.	1	1.00	0.53	0.74	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.369	0.400	10.475	0.000	0.000	0.000	0.354	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	161	143	0	2566	0	214	0
N.S.	1	1.00	0.48	0.43	0.00	7.64	0.00	0.64	0.00
time (sec)	N/A	0.374	5.592	1.924	0.000	142.885	0.000	0.313	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	122	110	0	2083	0	163	0
N.S.	1	1.00	0.49	0.44	0.00	8.40	0.00	0.66	0.00
time (sec)	N/A	0.273	5.482	2.757	0.000	123.024	0.000	0.296	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	85	77	0	1598	0	112	0
N.S.	1	1.00	0.53	0.48	0.00	9.99	0.00	0.70	0.00
time (sec)	N/A	0.165	5.689	1.997	0.000	125.662	0.000	0.291	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	45	45	0	1107	0	60	0
N.S.	1	1.00	0.66	0.66	0.00	16.28	0.00	0.88	0.00
time (sec)	N/A	0.057	5.494	1.837	0.000	123.329	0.000	0.289	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	71	56	0	0	0	71	40
N.S.	1	1.00	1.18	0.93	0.00	0.00	0.00	1.18	0.67
time (sec)	N/A	0.037	2.186	2.286	0.000	0.000	0.000	0.286	9.343

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	110	88	0	0	0	105	0
N.S.	1	1.00	0.75	0.60	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.159	5.698	1.787	0.000	0.000	0.000	0.314	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	48	126	0	0	0	156	0
N.S.	1	1.00	0.20	0.53	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.256	10.092	2.033	0.000	0.000	0.000	0.329	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	48	159	0	0	0	207	0
N.S.	1	1.00	0.15	0.49	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.392	10.087	1.812	0.000	0.000	0.000	0.362	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	412	48	192	0	0	0	258	0
N.S.	1	1.00	0.12	0.47	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.529	10.096	1.991	0.000	0.000	0.000	0.382	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.006	0.003	0.038	0.198	0.279	0.017	0.281	0.023

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.007	0.001	0.034	0.192	0.289	0.016	0.281	0.021

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.002	0.000	0.030	0.186	0.271	0.016	0.279	0.021

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.006	0.001	0.027	0.202	0.277	0.018	0.284	0.021

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.003	0.001	0.024	0.193	0.257	0.017	0.276	0.018

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.022	0.003	1.820	0.193	0.235	0.017	0.283	0.040

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.013	0.003	1.877	0.197	0.246	0.017	0.265	0.033

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.010	0.004	1.769	0.199	0.240	0.018	0.272	0.032

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.012	0.002	2.625	0.198	0.250	0.018	0.276	0.033

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.012	0.002	2.598	0.202	0.249	0.018	0.273	0.033

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	52	52	49	53	51
N.S.	1	1.00	1.00	0.91	0.91	0.91	0.86	0.93	0.89
time (sec)	N/A	0.025	0.006	2.537	0.216	0.251	0.060	0.276	8.831

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	42	41	37	43	40
N.S.	1	1.00	1.00	0.93	0.95	0.93	0.84	0.98	0.91
time (sec)	N/A	0.018	0.004	2.158	0.204	0.246	0.052	0.287	0.040

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94
time (sec)	N/A	0.015	0.004	1.780	0.193	0.242	0.050	0.290	0.041

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	18
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00
time (sec)	N/A	0.011	0.003	1.815	0.215	0.243	0.040	0.275	0.036

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.005	0.001	1.767	0.202	0.240	0.018	0.286	0.021

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	18	16	10	20	15
N.S.	1	1.00	1.00	0.89	1.00	0.89	0.56	1.11	0.83
time (sec)	N/A	0.005	0.004	2.029	0.201	0.261	0.059	0.271	8.898

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	28	26	19	30	25
N.S.	1	1.00	1.00	0.93	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.009	0.005	1.862	0.203	0.258	0.074	0.268	0.055

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	38
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90
time (sec)	N/A	0.015	0.006	1.751	0.221	0.280	0.097	0.288	0.058

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	51	54	44	56	48
N.S.	1	1.00	1.00	0.95	0.91	0.96	0.79	1.00	0.86
time (sec)	N/A	0.020	0.007	1.783	0.204	0.275	0.118	0.270	0.060

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	59	73	54	62	62
N.S.	1	1.00	0.93	0.98	1.02	1.26	0.93	1.07	1.07
time (sec)	N/A	0.026	0.030	1.907	0.219	0.245	0.093	0.266	0.039

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	47	62	44	48	50
N.S.	1	1.00	0.93	0.98	1.02	1.35	0.96	1.04	1.09
time (sec)	N/A	0.024	0.015	1.968	0.202	0.248	0.088	0.266	0.046

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	36	47	31	34	36
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.03	1.09
time (sec)	N/A	0.016	0.015	1.828	0.199	0.266	0.071	0.295	0.040

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	26	28	20	24	23
N.S.	1	1.00	0.87	1.04	1.13	1.22	0.87	1.04	1.00
time (sec)	N/A	0.013	0.009	2.178	0.201	0.247	0.056	0.277	0.037

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	13	10	12	12
N.S.	1	1.00	1.00	1.08	1.08	1.08	0.83	1.00	1.00
time (sec)	N/A	0.005	0.002	2.295	0.210	0.246	0.053	0.283	0.032

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	28	39	22	31	26
N.S.	1	1.00	0.83	1.03	0.97	1.34	0.76	1.07	0.90
time (sec)	N/A	0.014	0.012	2.312	0.204	0.277	0.095	0.284	0.046

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	45	41
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.07	0.98
time (sec)	N/A	0.017	0.043	2.358	0.210	0.260	0.126	0.280	8.845

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	53	57	64	86	54	64	57
N.S.	1	1.00	0.91	0.98	1.10	1.48	0.93	1.10	0.98
time (sec)	N/A	0.023	0.052	1.974	0.205	0.248	0.146	0.278	8.858

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	73	95	66	73	69
N.S.	1	1.00	0.96	0.99	1.06	1.38	0.96	1.06	1.00
time (sec)	N/A	0.027	0.062	4.385	0.199	0.261	0.156	0.286	0.069

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	86	108	80	86	79
N.S.	1	1.00	0.94	0.94	1.02	1.29	0.95	1.02	0.94
time (sec)	N/A	0.038	0.050	4.141	0.217	0.269	0.171	0.281	0.074

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	53	32	53	62	0	131	62
N.S.	1	1.00	0.50	0.30	0.50	0.59	0.00	1.25	0.59
time (sec)	N/A	0.083	0.019	2.575	0.212	0.252	0.000	0.263	9.017

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	42	21	42	51	0	108	51
N.S.	1	1.00	0.52	0.26	0.52	0.64	0.00	1.35	0.64
time (sec)	N/A	0.048	0.013	1.864	0.218	0.263	0.000	0.273	9.026

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	41	13	30	39	0	81	39
N.S.	1	1.00	0.79	0.25	0.58	0.75	0.00	1.56	0.75
time (sec)	N/A	0.029	0.008	1.914	0.212	0.263	0.000	0.285	8.936

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	25	12	26	0	50	0
N.S.	1	1.00	0.92	1.00	0.48	1.04	0.00	2.00	0.00
time (sec)	N/A	0.023	0.008	1.890	0.211	0.264	0.000	0.290	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	36	0	111	0	67	73
N.S.	1	1.00	1.04	0.71	0.00	2.18	0.00	1.31	1.43
time (sec)	N/A	0.033	0.030	1.909	0.000	0.250	0.000	0.288	9.147

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	64	50	0	127	0	45	0
N.S.	1	1.00	1.23	0.96	0.00	2.44	0.00	0.87	0.00
time (sec)	N/A	0.036	0.026	1.971	0.000	0.256	0.000	0.294	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	81	61	0	149	0	72	0
N.S.	1	1.00	0.96	0.73	0.00	1.77	0.00	0.86	0.00
time (sec)	N/A	0.063	0.039	2.392	0.000	0.270	0.000	0.291	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	93	72	0	175	0	92	0
N.S.	1	1.00	0.83	0.64	0.00	1.56	0.00	0.82	0.00
time (sec)	N/A	0.091	0.040	1.998	0.000	0.271	0.000	0.316	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	80	32	86	95	0	282	80
N.S.	1	1.00	0.50	0.20	0.53	0.59	0.00	1.75	0.50
time (sec)	N/A	0.150	0.056	1.891	0.223	0.274	0.000	0.276	9.139

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	69	21	75	84	0	246	69
N.S.	1	1.00	0.51	0.15	0.55	0.62	0.00	1.81	0.51
time (sec)	N/A	0.119	0.048	1.855	0.217	0.262	0.000	0.282	9.010

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	58	13	64	73	0	210	58
N.S.	1	1.00	0.54	0.12	0.59	0.68	0.00	1.94	0.54
time (sec)	N/A	0.089	0.019	1.818	0.216	0.256	0.000	0.290	9.086

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	47	35	53	62	0	173	47
N.S.	1	1.00	0.59	0.44	0.66	0.78	0.00	2.16	0.59
time (sec)	N/A	0.086	0.018	1.823	0.220	0.263	0.000	0.290	9.034

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	35	41	50	0	136	36
N.S.	1	1.00	0.60	0.67	0.79	0.96	0.00	2.62	0.69
time (sec)	N/A	0.058	0.015	1.838	0.208	0.252	0.000	0.292	8.923

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	28	37	0	89	28
N.S.	1	1.00	0.92	1.08	1.12	1.48	0.00	3.56	1.12
time (sec)	N/A	0.026	0.010	2.421	0.221	0.254	0.000	0.277	9.287

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	68	61	0	130	0	85	0
N.S.	1	1.00	0.92	0.82	0.00	1.76	0.00	1.15	0.00
time (sec)	N/A	0.061	0.048	2.020	0.000	0.282	0.000	0.280	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	66	70	0	136	0	62	0
N.S.	1	1.00	0.90	0.96	0.00	1.86	0.00	0.85	0.00
time (sec)	N/A	0.060	0.045	1.888	0.000	0.284	0.000	0.299	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	82	67	0	154	0	70	0
N.S.	1	1.00	1.01	0.83	0.00	1.90	0.00	0.86	0.00
time (sec)	N/A	0.061	0.039	1.943	0.000	0.273	0.000	0.336	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	81	0	175	0	92	0
N.S.	1	1.00	0.86	0.74	0.00	1.61	0.00	0.84	0.00
time (sec)	N/A	0.088	0.050	2.248	0.000	0.281	0.000	0.297	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	104	92	0	197	0	109	0
N.S.	1	1.00	0.76	0.67	0.00	1.44	0.00	0.80	0.00
time (sec)	N/A	0.119	0.236	2.148	0.000	0.276	0.000	0.304	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	116	103	0	219	0	126	0
N.S.	1	1.00	0.70	0.62	0.00	1.33	0.00	0.76	0.00
time (sec)	N/A	0.156	0.252	2.873	0.000	0.277	0.000	0.313	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	53	52	53	51	0	64	51
N.S.	1	1.00	0.51	0.50	0.51	0.50	0.00	0.62	0.50
time (sec)	N/A	0.094	0.018	1.877	0.232	0.263	0.000	0.272	8.919

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	42	41	42	40	0	52	40
N.S.	1	1.00	0.56	0.55	0.56	0.53	0.00	0.69	0.53
time (sec)	N/A	0.068	0.014	1.834	0.213	0.260	0.000	0.282	8.878

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	30	30	30	28	0	38	31
N.S.	1	1.00	0.61	0.61	0.61	0.57	0.00	0.78	0.63
time (sec)	N/A	0.039	0.011	1.846	0.251	0.262	0.000	0.272	8.866

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	21	12	21	0	27	17
N.S.	1	1.00	0.91	0.91	0.52	0.91	0.00	1.17	0.74
time (sec)	N/A	0.008	0.006	2.034	0.216	0.268	0.000	0.271	8.878

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	13	0	74	0	45	0
N.S.	1	1.00	1.53	0.43	0.00	2.47	0.00	1.50	0.00
time (sec)	N/A	0.008	0.010	1.830	0.000	0.265	0.000	0.276	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	60	18	0	127	0	51	0
N.S.	1	1.00	1.11	0.33	0.00	2.35	0.00	0.94	0.00
time (sec)	N/A	0.036	0.021	2.158	0.000	0.259	0.000	0.312	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	83	36	0	153	0	73	44
N.S.	1	1.00	0.95	0.41	0.00	1.76	0.00	0.84	0.51
time (sec)	N/A	0.060	0.031	2.494	0.000	0.268	0.000	0.293	9.120

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	96	56	0	175	0	88	0
N.S.	1	1.00	0.83	0.49	0.00	1.52	0.00	0.77	0.00
time (sec)	N/A	0.091	0.031	2.027	0.000	0.280	0.000	0.304	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	50	56	41	60	0	79	57
N.S.	1	1.00	0.51	0.57	0.42	0.61	0.00	0.81	0.58
time (sec)	N/A	0.094	0.013	1.916	0.201	0.263	0.000	0.284	9.118

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	39	46	30	49	0	64	47
N.S.	1	1.00	0.54	0.64	0.42	0.68	0.00	0.89	0.65
time (sec)	N/A	0.068	0.011	1.991	0.209	0.260	0.000	0.269	9.074

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	26	34	19	38	0	48	35
N.S.	1	1.00	0.55	0.72	0.40	0.81	0.00	1.02	0.74
time (sec)	N/A	0.039	0.010	1.849	0.287	0.283	0.000	0.280	8.991

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	27	12	29	0	27	28
N.S.	1	1.00	0.90	1.29	0.57	1.38	0.00	1.29	1.33
time (sec)	N/A	0.011	0.005	1.885	0.251	0.251	0.000	0.286	8.936

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	54	31	0	156	0	77	0
N.S.	1	1.00	1.04	0.60	0.00	3.00	0.00	1.48	0.00
time (sec)	N/A	0.038	0.015	1.774	0.000	0.271	0.000	0.271	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	62	20	0	189	0	72	0
N.S.	1	1.00	0.83	0.27	0.00	2.52	0.00	0.96	0.00
time (sec)	N/A	0.055	0.023	1.824	0.000	0.275	0.000	0.290	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	84	13	0	219	0	92	42
N.S.	1	1.00	0.76	0.12	0.00	1.99	0.00	0.84	0.38
time (sec)	N/A	0.064	0.027	1.829	0.000	0.279	0.000	0.311	9.139

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	96	31	0	241	0	107	0
N.S.	1	1.00	0.70	0.22	0.00	1.75	0.00	0.78	0.00
time (sec)	N/A	0.123	0.184	1.851	0.000	0.272	0.000	0.305	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	106	51	0	263	0	122	44
N.S.	1	1.00	0.64	0.31	0.00	1.58	0.00	0.73	0.27
time (sec)	N/A	0.165	0.243	1.866	0.000	0.296	0.000	0.313	9.888

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	110	100	0	180	0	83	0
N.S.	1	1.00	0.88	0.80	0.00	1.44	0.00	0.66	0.00
time (sec)	N/A	0.113	0.061	1.936	0.000	0.280	0.000	0.303	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	99	89	0	159	0	71	0
N.S.	1	1.00	1.04	0.94	0.00	1.67	0.00	0.75	0.00
time (sec)	N/A	0.078	0.032	2.066	0.000	0.272	0.000	0.280	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	80	78	0	131	0	55	0
N.S.	1	1.00	1.33	1.30	0.00	2.18	0.00	0.92	0.00
time (sec)	N/A	0.052	0.032	1.836	0.000	0.277	0.000	0.291	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	53	58	0	77	0	37	0
N.S.	1	1.00	1.56	1.71	0.00	2.26	0.00	1.09	0.00
time (sec)	N/A	0.029	0.014	1.769	0.000	0.267	0.000	0.283	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	25	0	21	0	34	0
N.S.	1	1.00	0.92	1.00	0.00	0.84	0.00	1.36	0.00
time (sec)	N/A	0.023	0.011	1.871	0.000	0.264	0.000	0.289	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	31	31	0	29	0	54	0
N.S.	1	1.00	0.55	0.55	0.00	0.52	0.00	0.96	0.00
time (sec)	N/A	0.047	0.014	1.804	0.000	0.269	0.000	0.310	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	44	44	0	40	0	81	0
N.S.	1	1.00	0.51	0.51	0.00	0.47	0.00	0.94	0.00
time (sec)	N/A	0.078	0.016	1.816	0.000	0.265	0.000	0.292	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	55	55	0	51	0	107	0
N.S.	1	1.00	0.47	0.47	0.00	0.44	0.00	0.92	0.00
time (sec)	N/A	0.100	0.019	2.790	0.000	0.268	0.000	0.314	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.048	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	61	59	0	0	0	0	0	0
N.S.	1	1.27	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.038	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	36	0	38	0	0	54
N.S.	1	1.00	0.94	1.12	0.00	1.19	0.00	0.00	1.69
time (sec)	N/A	0.019	0.044	1.934	0.000	0.274	0.000	0.000	9.031

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	44	50	0	70	0	0	98
N.S.	1	1.00	0.63	0.71	0.00	1.00	0.00	0.00	1.40
time (sec)	N/A	0.041	0.070	1.921	0.000	0.272	0.000	0.000	9.068

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	72	84	0	111	0	0	157
N.S.	1	1.00	0.62	0.72	0.00	0.96	0.00	0.00	1.35
time (sec)	N/A	0.068	0.061	1.937	0.000	0.274	0.000	0.000	9.126

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.007	0.011	1.803	0.196	0.258	0.168	0.289	8.945

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	46	43	46	42	0	65	42
N.S.	1	1.00	0.58	0.54	0.58	0.52	0.00	0.81	0.52
time (sec)	N/A	0.072	0.017	2.506	0.226	0.270	0.000	0.283	9.090

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	34	32	34	30	0	48	33
N.S.	1	1.00	0.65	0.62	0.65	0.58	0.00	0.92	0.63
time (sec)	N/A	0.041	0.012	2.275	0.208	0.256	0.000	0.275	8.999

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	14	21	0	29	21
N.S.	1	1.00	1.00	0.88	0.56	0.84	0.00	1.16	0.84
time (sec)	N/A	0.011	0.010	2.218	0.219	0.264	0.000	0.275	9.069

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	54	43	0	75	0	47	0
N.S.	1	1.00	1.69	1.34	0.00	2.34	0.00	1.47	0.00
time (sec)	N/A	0.007	0.012	2.144	0.000	0.274	0.000	0.289	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	76	66	0	127	0	55	0
N.S.	1	1.00	1.29	1.12	0.00	2.15	0.00	0.93	0.00
time (sec)	N/A	0.033	0.025	2.311	0.000	0.275	0.000	0.291	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	68	248	0	37	0	0	0
N.S.	1	1.00	0.29	1.04	0.00	0.16	0.00	0.00	0.00
time (sec)	N/A	0.089	10.036	2.256	0.000	0.074	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	52	231	0	14	0	0	0
N.S.	1	1.00	0.25	1.09	0.00	0.07	0.00	0.00	0.00
time (sec)	N/A	0.043	10.018	1.945	0.000	0.072	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	55	248	0	38	0	0	44
N.S.	1	1.00	0.23	1.02	0.00	0.16	0.00	0.00	0.18
time (sec)	N/A	0.086	10.016	2.025	0.000	0.077	0.000	0.000	9.711

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	514	514	68	25	0	45	0	0	0
N.S.	1	1.00	0.13	0.05	0.00	0.09	0.00	0.00	0.00
time (sec)	N/A	0.207	10.039	1.995	0.000	0.075	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	484	55	15	0	22	0	0	0
N.S.	1	1.00	0.11	0.03	0.00	0.05	0.00	0.00	0.00
time (sec)	N/A	0.133	10.021	1.859	0.000	0.076	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	510	510	50	20	0	46	0	0	0
N.S.	1	1.00	0.10	0.04	0.00	0.09	0.00	0.00	0.00
time (sec)	N/A	0.203	10.020	1.825	0.000	0.077	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	86	742	0	0	0	0	0
N.S.	1	1.00	0.32	2.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	10.038	2.232	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	525	525	70	1115	0	0	0	0	0
N.S.	1	1.00	0.13	2.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.366	10.031	2.248	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	82	79	0	148	0	52	0
N.S.	1	1.00	1.26	1.22	0.00	2.28	0.00	0.80	0.00
time (sec)	N/A	0.058	0.034	1.872	0.000	0.361	0.000	0.313	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	70	727	0	0	0	0	0
N.S.	1	1.00	0.30	3.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.151	10.034	2.080	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	492	492	57	2374	0	0	0	0	0
N.S.	1	1.00	0.12	4.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	10.024	1.977	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	60	59	0	101	0	40	0
N.S.	1	1.00	1.67	1.64	0.00	2.81	0.00	1.11	0.00
time (sec)	N/A	0.030	0.012	1.780	0.000	0.353	0.000	0.299	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	55	437	0	16	0	0	0
N.S.	1	1.00	0.27	2.15	0.00	0.08	0.00	0.00	0.00
time (sec)	N/A	0.115	10.016	1.873	0.000	0.079	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	519	55	1115	0	24	0	0	0
N.S.	1	1.00	0.11	2.15	0.00	0.05	0.00	0.00	0.00
time (sec)	N/A	0.367	10.020	2.352	0.000	0.078	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	26	21	0	28	0
N.S.	1	1.00	1.00	1.07	0.96	0.78	0.00	1.04	0.00
time (sec)	N/A	0.029	0.012	1.804	0.214	0.254	0.000	0.300	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	57	732	0	48	0	0	0
N.S.	1	1.00	0.24	3.11	0.00	0.20	0.00	0.00	0.00
time (sec)	N/A	0.154	10.019	2.164	0.000	0.074	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	555	555	57	1125	0	55	0	0	0
N.S.	1	1.00	0.10	2.03	0.00	0.10	0.00	0.00	0.00
time (sec)	N/A	0.414	10.018	2.285	0.000	0.076	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	35	37	38	31	0	43	0
N.S.	1	1.00	0.62	0.66	0.68	0.55	0.00	0.77	0.00
time (sec)	N/A	0.051	0.015	1.826	0.213	0.266	0.000	0.323	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	57	742	0	62	0	0	0
N.S.	1	1.00	0.22	2.80	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.198	10.019	2.108	0.000	0.085	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	28	26	19	30	25
N.S.	1	1.00	1.00	0.93	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.012	0.006	2.066	0.204	0.250	0.082	0.268	0.050

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	38
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90
time (sec)	N/A	0.013	0.005	2.001	0.204	0.259	0.092	0.285	9.032

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	91	0	171	0	104	0
N.S.	1	1.00	1.00	0.81	0.00	1.53	0.00	0.93	0.00
time (sec)	N/A	0.112	0.053	2.429	0.000	0.284	0.000	0.321	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	101	69	0	150	0	88	0
N.S.	1	1.00	1.17	0.80	0.00	1.74	0.00	1.02	0.00
time (sec)	N/A	0.083	0.029	2.145	0.000	0.281	0.000	0.287	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	82	47	0	122	0	69	0
N.S.	1	1.00	1.46	0.84	0.00	2.18	0.00	1.23	0.00
time (sec)	N/A	0.060	0.034	2.106	0.000	0.260	0.000	0.307	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	57	25	0	74	0	47	0
N.S.	1	1.00	1.78	0.78	0.00	2.31	0.00	1.47	0.00
time (sec)	N/A	0.022	0.012	2.006	0.000	0.279	0.000	0.307	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	21	20	0	21	0	27	21
N.S.	1	1.00	0.91	0.87	0.00	0.91	0.00	1.17	0.91
time (sec)	N/A	0.004	0.008	2.001	0.000	0.262	0.000	0.296	9.076

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	31	26	0	29	0	53	42
N.S.	1	1.00	0.60	0.50	0.00	0.56	0.00	1.02	0.81
time (sec)	N/A	0.028	0.012	2.012	0.000	0.285	0.000	0.302	9.051

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	42	39	0	40	0	82	40
N.S.	1	1.00	0.52	0.49	0.00	0.50	0.00	1.02	0.50
time (sec)	N/A	0.054	0.012	2.078	0.000	0.270	0.000	0.302	9.088

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	53	50	0	51	0	111	92
N.S.	1	1.00	0.49	0.46	0.00	0.47	0.00	1.03	0.85
time (sec)	N/A	0.085	0.016	2.122	0.000	0.279	0.000	0.314	9.101

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	64	61	0	62	0	140	116
N.S.	1	1.00	0.47	0.45	0.00	0.46	0.00	1.03	0.85
time (sec)	N/A	0.111	0.150	2.146	0.000	0.269	0.000	0.298	9.065

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	28	22	32	22
N.S.	1	1.00	1.00	0.88	0.85	1.08	0.85	1.23	0.85
time (sec)	N/A	0.013	0.007	1.787	0.209	0.279	0.099	0.275	0.051

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	22	28	22	32	22
N.S.	1	1.00	1.00	0.85	0.81	1.04	0.81	1.19	0.81
time (sec)	N/A	0.014	0.008	1.716	0.196	0.270	0.103	0.294	9.074

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	14	9	14	8	5	15	8
N.S.	1	1.00	1.75	1.12	1.75	1.00	0.62	1.88	1.00
time (sec)	N/A	0.003	0.004	0.019	0.212	0.268	0.022	0.275	9.058

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	16	18	15	16	10
N.S.	1	1.00	1.00	1.10	1.60	1.80	1.50	1.60	1.00
time (sec)	N/A	0.002	0.004	0.017	0.202	0.275	0.024	0.278	0.034

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	16	26	27	16	26
N.S.	1	1.00	1.00	0.92	1.33	2.17	2.25	1.33	2.17
time (sec)	N/A	0.003	0.005	0.020	0.193	0.256	0.028	0.276	9.086

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.00	1.00
time (sec)	N/A	0.001	0.001	1.754	0.206	0.248	0.022	0.282	0.027

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	21	22	32	0	19
N.S.	1	1.00	1.00	0.95	1.05	1.10	1.60	0.00	0.95
time (sec)	N/A	0.005	0.004	0.041	0.201	0.248	0.259	0.000	9.080

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	40	36	82	0	21
N.S.	1	1.00	1.00	1.05	2.00	1.80	4.10	0.00	1.05
time (sec)	N/A	0.006	0.004	0.031	0.204	0.258	0.375	0.000	9.080

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	53	52	119	0	21
N.S.	1	1.00	1.00	1.05	2.65	2.60	5.95	0.00	1.05
time (sec)	N/A	0.007	0.004	0.038	0.203	0.256	0.453	0.000	9.096

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.003	0.005	1.791	0.211	0.258	0.042	0.281	9.142

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.009	0.005	1.959	0.212	0.252	0.046	0.272	9.154

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.010	0.005	2.210	0.208	0.245	0.048	0.283	0.119

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	338	275	231	520	285	287
N.S.	1	1.00	0.89	12.52	10.19	8.56	19.26	10.56	10.63
time (sec)	N/A	0.013	0.014	53.747	0.233	0.257	6.300	0.397	9.851

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.003	0.002	1.820	0.201	0.248	0.042	0.294	0.002

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.003	0.005	1.988	0.206	0.242	0.048	0.276	9.181

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.004	0.007	2.230	0.226	0.242	0.047	0.283	0.117

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	193	281	275	205	471	0	285
N.S.	1	1.00	7.15	10.41	10.19	7.59	17.44	0.00	10.56
time (sec)	N/A	0.006	0.119	46.017	0.210	0.256	5.034	0.000	9.937

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	88	120	121	93	197	93	124
N.S.	1	1.00	3.26	4.44	4.48	3.44	7.30	3.44	4.59
time (sec)	N/A	0.008	0.075	2.236	0.198	0.260	0.634	0.288	9.336

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	39	66	82	0	0	38
N.S.	1	1.00	1.00	1.44	2.44	3.04	0.00	0.00	1.41
time (sec)	N/A	0.011	0.084	1.850	0.202	0.247	0.000	0.000	9.132

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.004	0.003	0.020	0.192	0.246	0.047	0.283	0.044

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.004	0.005	0.027	0.201	0.237	0.064	0.274	8.926

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.004	0.006	0.023	0.201	0.237	0.078	0.289	0.045

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.004	0.013	0.033	0.208	0.238	0.126	0.282	0.040

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.004	0.011	0.045	0.193	0.235	0.228	0.278	0.060

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.004	0.014	0.056	0.195	0.237	0.320	0.277	8.905

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92
time (sec)	N/A	0.008	0.001	0.033	0.188	0.245	0.037	0.273	0.042

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	34	38	37	46	34
N.S.	1	1.00	1.00	0.88	0.85	0.95	0.92	1.15	0.85
time (sec)	N/A	0.018	0.009	0.074	0.187	0.247	0.060	0.271	0.037

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	44	48	49	45	47
N.S.	1	1.00	1.00	0.90	0.88	0.96	0.98	0.90	0.94
time (sec)	N/A	0.017	0.008	0.061	0.206	0.247	0.071	0.275	0.047

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	144	34	124	637	36	112	197
N.S.	1	1.00	0.78	0.18	0.67	3.44	0.19	0.61	1.06
time (sec)	N/A	0.271	0.171	0.064	0.302	0.900	0.812	0.291	9.639

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	232	363	325	297	690	50971	363
N.S.	1	1.00	8.00	12.52	11.21	10.24	23.79	1757.62	12.52
time (sec)	N/A	0.012	0.191	0.031	0.222	0.274	149.341	1.458	10.377

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.003	0.004	1.899	0.208	0.237	0.041	0.284	8.889

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.007	0.002	2.075	0.186	0.240	0.050	0.274	0.002

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.009	0.005	2.250	0.185	0.238	0.054	0.291	8.935

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.002	0.004	2.174	0.200	0.240	0.043	0.283	0.092

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
N.S.	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.006	0.003	2.202	0.199	0.281	0.050	0.284	0.002

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	160	15	14	134	160	14	14
N.S.	1	1.00	10.00	0.94	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.002	0.002	2.535	0.197	0.256	0.047	0.299	8.990

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	37	27	53	0	26
N.S.	1	1.00	1.00	1.17	1.61	1.17	2.30	0.00	1.13
time (sec)	N/A	0.008	0.022	1.804	0.191	0.258	0.328	0.000	9.039

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	29	27	28	49	0	31
N.S.	1	1.00	1.09	1.26	1.17	1.22	2.13	0.00	1.35
time (sec)	N/A	0.009	0.009	1.875	0.193	0.265	0.321	0.000	8.924

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	31	19	28	51	0	34
N.S.	1	1.00	1.00	2.07	1.27	1.87	3.40	0.00	2.27
time (sec)	N/A	0.006	0.005	1.777	0.200	0.259	0.353	0.000	8.953

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	24	25	16	26	31	0	26
N.S.	1	1.00	1.09	1.14	0.73	1.18	1.41	0.00	1.18
time (sec)	N/A	0.007	0.036	1.810	0.190	0.258	0.265	0.000	9.081

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	27	11	26	31	0	28
N.S.	1	1.00	1.00	1.80	0.73	1.73	2.07	0.00	1.87
time (sec)	N/A	0.005	0.024	1.769	0.199	0.258	0.293	0.000	9.130

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	8	8	8	9	8
N.S.	1	1.00	0.83	0.75	0.67	0.67	0.67	0.75	0.67
time (sec)	N/A	0.003	0.036	1.725	0.191	0.243	0.058	0.277	0.087

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	25	11	17	8	22	18	8
N.S.	1	1.00	1.79	0.79	1.21	0.57	1.57	1.29	0.57
time (sec)	N/A	0.004	0.020	1.744	0.187	0.256	0.106	0.293	9.221

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	32	8
N.S.	1	1.00	1.00	0.75	0.67	0.67	0.83	2.67	0.67
time (sec)	N/A	0.003	0.002	1.713	0.290	0.256	0.074	0.280	0.091

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	28	31	24	31	0	26
N.S.	1	1.00	1.00	1.17	1.29	1.00	1.29	0.00	1.08
time (sec)	N/A	0.013	0.047	1.906	0.284	0.263	0.256	0.000	9.230

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	104	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.271	0.000	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	109	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	0.344	0.000	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	103	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.112	1.626	0.000	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	99	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	0.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	100	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	1.579	0.000	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	45	39	58	97	78	0	0
N.S.	1	1.00	0.88	0.76	1.14	1.90	1.53	0.00	0.00
time (sec)	N/A	0.020	0.011	2.068	0.308	0.270	0.913	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.124	0.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	77	0	0	0	0	0	97
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	1.59
time (sec)	N/A	0.058	0.122	0.000	0.000	0.000	0.000	0.000	9.142

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.140	0.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	131	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	0.635	0.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	126	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.143	1.905	0.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	117	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.230	0.000	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	120	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.120	1.617	0.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	58	51	73	120	88	0	0
N.S.	1	1.00	0.79	0.70	1.00	1.64	1.21	0.00	0.00
time (sec)	N/A	0.026	0.029	2.300	0.286	0.711	1.578	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	97	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	0.152	0.000	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	94	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.118	0.149	0.000	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	100	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	0.168	0.000	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	96	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.150	0.152	0.000	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	58	47	0	93	0	67	67
N.S.	1	1.00	1.14	0.92	0.00	1.82	0.00	1.31	1.31
time (sec)	N/A	0.055	0.029	0.102	0.000	0.263	0.000	0.284	9.154

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	62	61	53	108	0	69	55
N.S.	1	1.00	1.48	1.45	1.26	2.57	0.00	1.64	1.31
time (sec)	N/A	0.017	0.029	0.052	0.289	0.486	0.000	0.308	9.196

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	66	55	0	104	0	71	63
N.S.	1	1.00	1.29	1.08	0.00	2.04	0.00	1.39	1.24
time (sec)	N/A	0.044	0.038	0.055	0.000	0.532	0.000	0.284	9.310

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	69	74	0	112	0	0	0
N.S.	1	1.00	1.13	1.21	0.00	1.84	0.00	0.00	0.00
time (sec)	N/A	0.055	0.028	2.213	0.000	0.478	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	66	55	0	98	0	61	67
N.S.	1	1.00	1.25	1.04	0.00	1.85	0.00	1.15	1.26
time (sec)	N/A	0.053	0.033	0.126	0.000	0.305	0.000	0.285	9.211

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	68	81	34	118	0	63	54
N.S.	1	1.00	1.58	1.88	0.79	2.74	0.00	1.47	1.26
time (sec)	N/A	0.018	0.065	0.063	0.305	0.286	0.000	0.296	9.321

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	73	73	0	109	0	65	63
N.S.	1	1.00	1.38	1.38	0.00	2.06	0.00	1.23	1.19
time (sec)	N/A	0.049	0.073	0.085	0.000	0.276	0.000	0.286	9.233

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	77	105	0	118	0	0	0
N.S.	1	1.00	1.22	1.67	0.00	1.87	0.00	0.00	0.00
time (sec)	N/A	0.065	0.082	2.098	0.000	0.279	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	98	0	0	0	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.572	0.000	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	89	0	0	0	0	0	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	1.497	0.000	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.010	0.103	0.000	0.000	0.000	0.000	0.000	9.193

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	87	0	0	0	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	1.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	42	76	27	0	0
N.S.	1	1.00	1.00	0.84	1.35	2.45	0.87	0.00	0.00
time (sec)	N/A	0.014	0.006	2.068	0.303	0.518	0.631	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	66	0	0	0	0	0	0
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.106	0.000	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	0.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	117	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	0.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	109	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	3.010	0.000	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	91	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	0.219	0.000	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	104	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	1.520	0.000	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	52	42	61	148	185	0	0
N.S.	1	1.00	0.96	0.78	1.13	2.74	3.43	0.00	0.00
time (sec)	N/A	0.024	0.032	1.756	0.289	0.409	1.165	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	78	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.129	0.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	74	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	78	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	74	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	0.137	0.000	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	64	56	0	102	0	0	0
N.S.	1	1.00	2.00	1.75	0.00	3.19	0.00	0.00	0.00
time (sec)	N/A	0.015	0.025	0.294	0.000	0.343	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	60	49	0	80	0	40	0
N.S.	1	1.00	1.88	1.53	0.00	2.50	0.00	1.25	0.00
time (sec)	N/A	0.012	0.023	0.071	0.000	0.289	0.000	0.297	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	64	0	0	102	0	0	0
N.S.	1	1.00	2.00	0.00	0.00	3.19	0.00	0.00	0.00
time (sec)	N/A	0.017	0.026	0.000	0.000	0.600	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	76	0	0	102	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	2.76	0.00	0.00	0.00
time (sec)	N/A	0.016	0.045	0.000	0.000	0.318	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	72	60	0	111	0	0	0
N.S.	1	1.00	2.18	1.82	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.010	0.236	2.440	0.000	0.573	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	68	51	0	88	0	47	0
N.S.	1	1.00	2.06	1.55	0.00	2.67	0.00	1.42	0.00
time (sec)	N/A	0.011	0.026	0.063	0.000	0.527	0.000	0.290	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	72	0	0	111	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.012	0.319	0.000	0.000	0.917	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	78	0	0	106	0	0	0
N.S.	1	1.00	2.05	0.00	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	0.017	0.094	0.000	0.000	0.262	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.017	0.105	0.000	0.000	0.000	0.000	0.000	9.269

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	109	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	2.95	0.00	0.00	1.81
time (sec)	N/A	0.017	0.017	0.000	0.000	0.281	0.000	0.000	9.190

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.013	0.016	0.000	0.000	0.000	0.000	0.000	9.150

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0	66
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.74
time (sec)	N/A	0.014	0.107	0.000	0.000	0.000	0.000	0.000	9.221

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	109	0	0	66
N.S.	1	1.00	2.11	0.00	0.00	2.87	0.00	0.00	1.74
time (sec)	N/A	0.015	0.017	0.000	0.000	0.442	0.000	0.000	9.126

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0	66
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.74
time (sec)	N/A	0.013	0.017	0.000	0.000	0.000	0.000	0.000	9.148

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	218	0	0	0	0	0	0
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.568	0.000	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	156	0	0	0	0	0	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.355	0.000	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	106	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	116	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.349	0.000	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	166	0	0	0	0	0	0
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.625	0.000	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	177	0	0	0	0	0	82
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.85
time (sec)	N/A	0.040	0.234	0.000	0.000	0.000	0.000	0.000	9.170

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	134	0	0	0	0	0	82
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.94
time (sec)	N/A	0.040	0.178	0.000	0.000	0.000	0.000	0.000	9.142

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	88	0	0	0	0	0	83
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.037	0.084	0.000	0.000	0.000	0.000	0.000	9.164

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	104	0	0	0	0	0	83
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.041	0.124	0.000	0.000	0.000	0.000	0.000	9.377

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	185	0	0	0	0	0	83
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.042	0.283	0.000	0.000	0.000	0.000	0.000	9.510

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	19	16	0	16	0	50	15
N.S.	1	1.00	1.06	0.89	0.00	0.89	0.00	2.78	0.83
time (sec)	N/A	0.005	0.024	0.058	0.000	0.731	0.000	0.296	9.101

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	0	19	0	11	27
N.S.	1	1.00	1.00	0.90	0.00	0.95	0.00	0.55	1.35
time (sec)	N/A	0.002	0.043	2.030	0.000	0.281	0.000	0.284	9.155

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.003	0.002	1.794	0.286	0.246	0.085	0.311	9.125

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	22
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	0.88
time (sec)	N/A	0.007	0.014	1.893	0.211	0.248	0.000	0.294	9.120

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	29
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	1.16
time (sec)	N/A	0.007	0.002	1.890	0.207	0.248	0.000	0.290	8.945

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	14	22
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	0.56	0.88
time (sec)	N/A	0.006	0.013	2.412	0.224	0.244	0.000	0.284	8.941

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	988	988	61	0	0	0	0	0	42
N.S.	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.04
time (sec)	N/A	1.565	10.024	0.000	0.000	0.000	0.000	0.000	9.109

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	487	487	61	0	0	0	0	0	42
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	0.09
time (sec)	N/A	0.744	10.021	0.000	0.000	0.000	0.000	0.000	9.100

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	92	92	0	0	0	0	0	0
N.S.	1	1.03	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	73	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	74	74	0	0	0	0	0	0
N.S.	1	1.12	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.142	0.000	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	82	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	83	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	64	0	0	76
N.S.	1	1.00	0.98	0.00	0.00	1.45	0.00	0.00	1.73
time (sec)	N/A	0.014	0.076	0.000	0.000	0.468	0.000	0.000	9.155

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	61	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.012	0.002	0.000	0.000	0.266	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	76	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.051	0.088	0.000	0.000	0.255	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	79	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	1.72	0.00	0.00	0.00
time (sec)	N/A	0.033	0.004	0.000	0.000	0.268	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	0
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	0.00
time (sec)	N/A	0.014	0.070	1.808	0.245	0.359	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	0
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	0.00
time (sec)	N/A	0.015	0.069	1.802	0.239	0.423	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	0
N.S.	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	0.00
time (sec)	N/A	0.014	0.067	1.829	0.234	0.690	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	47	0	0	47	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.020	0.052	0.000	0.000	0.253	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	45	0	0	54	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.018	0.031	0.000	0.000	0.259	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	40	0	0	76	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	1.95	0.00	0.00	0.00
time (sec)	N/A	0.035	0.144	0.000	0.000	0.267	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	38	86	0	64	0	0	0
N.S.	1	1.00	0.95	2.15	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.053	0.121	2.915	0.000	0.453	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [347] had the largest ratio of [.7778000000000000047]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	13	0.077
2	A	2	1	1.00	11	0.091
3	A	1	0	1.00	9	0.000
4	A	2	1	1.00	13	0.077
5	A	2	1	1.00	13	0.077
6	A	3	2	1.00	15	0.133
7	A	4	3	1.00	13	0.231
8	A	3	2	1.00	11	0.182
9	A	2	2	1.00	15	0.133
10	A	3	2	1.00	15	0.133
11	A	3	2	1.00	11	0.182
12	A	4	3	1.00	15	0.200
13	A	3	3	1.00	15	0.200
14	A	2	2	1.00	15	0.133
15	A	2	2	1.00	13	0.154
16	A	5	5	1.00	11	0.454
17	A	3	3	1.00	15	0.200
18	A	4	3	1.00	15	0.200
19	A	4	3	1.00	15	0.200
20	A	4	3	1.00	15	0.200
21	A	3	3	1.00	15	0.200
22	A	4	3	1.00	13	0.231
23	A	4	4	1.00	11	0.364
24	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	4	1.00	15	0.267
26	A	4	3	1.00	13	0.231
27	A	4	3	1.00	13	0.231
28	A	3	3	1.00	13	0.231
29	A	2	2	1.00	13	0.154
30	A	2	2	1.00	11	0.182
31	A	5	5	1.00	9	0.556
32	A	3	3	1.00	13	0.231
33	A	4	3	1.00	13	0.231
34	A	4	3	1.00	13	0.231
35	A	4	3	1.00	13	0.231
36	A	5	5	1.00	9	0.556
37	A	5	5	1.00	11	0.454
38	A	6	5	1.00	17	0.294
39	A	7	7	1.00	17	0.412
40	A	5	5	1.00	15	0.333
41	A	6	6	1.00	13	0.462
42	A	4	4	1.00	17	0.235
43	A	6	6	1.00	17	0.353
44	A	4	4	1.00	17	0.235
45	A	7	7	1.00	17	0.412
46	A	7	5	1.00	17	0.294
47	A	8	7	1.00	15	0.467
48	A	6	6	1.00	13	0.462
49	A	7	7	1.00	17	0.412
50	A	5	4	1.00	17	0.235
51	A	7	7	1.00	17	0.412
52	A	5	5	1.00	17	0.294
53	A	7	6	1.00	17	0.353
54	A	5	4	1.00	17	0.235
55	A	8	7	1.00	17	0.412
56	A	6	5	1.00	17	0.294
57	A	5	4	1.00	17	0.235
58	A	6	6	1.00	17	0.353
59	A	4	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	5	5	1.00	15	0.333
61	A	3	3	1.00	13	0.231
62	A	6	6	1.00	17	0.353
63	A	4	4	1.00	17	0.235
64	A	7	6	1.00	17	0.353
65	A	6	5	1.00	17	0.294
66	A	7	7	1.00	17	0.412
67	A	5	5	1.00	17	0.294
68	A	6	6	1.00	17	0.353
69	A	4	4	1.00	17	0.235
70	A	6	6	1.00	17	0.353
71	A	4	4	1.00	15	0.267
72	A	7	7	1.00	13	0.538
73	A	5	5	1.00	17	0.294
74	A	8	7	1.00	17	0.412
75	A	7	4	1.00	19	0.210
76	A	5	2	1.00	19	0.105
77	A	6	3	1.00	19	0.158
78	A	4	2	1.00	19	0.105
79	A	1	1	1.00	19	0.053
80	A	3	2	1.00	19	0.105
81	A	2	2	1.00	19	0.105
82	A	2	2	1.00	19	0.105
83	A	3	2	1.00	19	0.105
84	A	1	1	1.00	19	0.053
85	A	4	2	1.00	19	0.105
86	A	6	3	1.00	19	0.158
87	A	5	2	1.00	19	0.105
88	A	7	4	1.00	19	0.210
89	A	6	3	1.00	19	0.158
90	A	8	4	1.00	19	0.210
91	A	7	3	1.00	19	0.158
92	A	3	3	1.00	17	0.176
93	A	2	2	1.00	15	0.133
94	A	1	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	17	0.118
96	A	3	2	1.00	17	0.118
97	A	4	4	1.00	17	0.235
98	A	3	3	1.00	13	0.231
99	A	4	4	1.00	17	0.235
100	A	6	6	1.00	17	0.353
101	A	5	5	1.00	17	0.294
102	A	6	6	1.00	17	0.353
103	A	8	5	1.00	19	0.263
104	A	6	5	1.00	17	0.294
105	A	4	4	1.00	15	0.267
106	A	1	1	1.00	19	0.053
107	A	3	2	1.00	19	0.105
108	A	5	2	1.00	19	0.105
109	A	7	2	1.00	19	0.105
110	A	9	6	1.00	19	0.316
111	A	7	6	1.00	19	0.316
112	A	5	5	1.00	17	0.294
113	A	1	1	1.00	15	0.067
114	A	3	3	1.00	19	0.158
115	A	5	3	1.00	19	0.158
116	A	7	3	1.00	19	0.158
117	A	9	5	1.00	21	0.238
118	A	7	5	1.00	21	0.238
119	A	5	5	1.00	21	0.238
120	A	3	3	1.00	21	0.143
121	A	2	2	1.00	21	0.095
122	A	4	2	1.00	21	0.095
123	A	6	2	1.00	21	0.095
124	A	8	6	1.00	21	0.286
125	A	6	6	1.00	21	0.286
126	A	4	4	1.00	21	0.190
127	A	2	2	1.00	21	0.095
128	A	4	3	1.00	21	0.143
129	A	6	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	8	3	1.00	21	0.143
131	A	11	6	1.00	19	0.316
132	A	11	8	1.00	19	0.421
133	A	8	6	1.00	17	0.353
134	A	8	8	1.00	15	0.533
135	A	5	5	1.00	19	0.263
136	A	8	8	1.00	19	0.421
137	A	7	6	1.00	19	0.316
138	A	11	8	1.00	19	0.421
139	A	10	6	1.00	19	0.316
140	A	11	6	1.00	19	0.316
141	A	11	8	1.00	17	0.471
142	A	8	7	1.00	15	0.467
143	A	8	8	1.00	19	0.421
144	A	6	6	1.00	19	0.316
145	A	9	8	1.00	19	0.421
146	A	8	6	1.00	19	0.316
147	A	12	8	1.00	19	0.421
148	A	11	6	1.00	19	0.316
149	A	11	5	1.00	19	0.263
150	A	11	7	1.00	19	0.368
151	A	8	5	1.00	19	0.263
152	A	8	7	1.00	17	0.412
153	A	5	5	1.00	15	0.333
154	A	7	7	1.00	19	0.368
155	A	6	5	1.00	19	0.263
156	A	10	7	1.00	19	0.368
157	A	9	5	1.00	19	0.263
158	A	12	8	1.00	19	0.421
159	A	9	6	1.00	19	0.316
160	A	9	8	1.00	19	0.421
161	A	6	6	1.00	17	0.353
162	A	7	7	1.00	15	0.467
163	A	6	6	1.00	19	0.316
164	A	10	8	1.00	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	9	6	1.00	19	0.316
166	A	13	8	1.00	19	0.421
167	A	13	3	1.00	19	0.158
168	A	10	3	1.00	19	0.158
169	A	7	3	1.00	17	0.176
170	A	4	3	1.00	15	0.200
171	A	1	1	1.00	19	0.053
172	A	4	4	1.00	19	0.210
173	A	7	4	1.00	19	0.210
174	A	10	4	1.00	19	0.210
175	A	13	4	1.00	19	0.210
176	A	12	3	1.00	19	0.158
177	A	9	3	1.00	17	0.176
178	A	6	3	1.00	15	0.200
179	A	3	2	1.00	19	0.105
180	A	4	3	1.00	19	0.158
181	A	5	4	1.00	19	0.210
182	A	8	4	1.00	19	0.210
183	A	11	4	1.00	19	0.210
184	A	14	4	1.00	19	0.210
185	A	14	3	1.00	19	0.158
186	A	11	3	1.00	19	0.158
187	A	8	3	1.00	19	0.158
188	A	5	3	1.00	17	0.176
189	A	2	2	1.00	15	0.133
190	A	3	3	1.00	19	0.158
191	A	6	3	1.00	19	0.158
192	A	9	3	1.00	19	0.158
193	A	12	3	1.00	19	0.158
194	A	12	4	1.00	19	0.210
195	A	9	4	1.00	19	0.210
196	A	6	4	1.00	19	0.210
197	A	3	3	1.00	17	0.176
198	A	3	3	1.00	15	0.200
199	A	6	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	9	4	1.00	19	0.210
201	A	12	4	1.00	19	0.210
202	A	15	4	1.00	19	0.210
203	A	2	1	1.00	15	0.067
204	A	2	1	1.00	13	0.077
205	A	1	0	1.00	11	0.000
206	A	2	1	1.00	15	0.067
207	A	2	1	1.00	15	0.067
208	A	3	2	1.00	17	0.118
209	A	3	2	1.00	15	0.133
210	A	3	2	1.00	13	0.154
211	A	3	2	1.00	17	0.118
212	A	3	2	1.00	17	0.118
213	A	3	2	1.00	17	0.118
214	A	3	2	1.00	17	0.118
215	A	3	2	1.00	17	0.118
216	A	3	2	1.00	17	0.118
217	A	2	2	1.00	17	0.118
218	A	4	4	1.00	15	0.267
219	A	3	2	1.00	13	0.154
220	A	3	2	1.00	17	0.118
221	A	3	2	1.00	17	0.118
222	A	3	2	1.00	17	0.118
223	A	3	2	1.00	17	0.118
224	A	3	2	1.00	17	0.118
225	A	3	2	1.00	17	0.118
226	A	2	2	1.00	17	0.118
227	A	3	2	1.00	17	0.118
228	A	3	2	1.00	17	0.118
229	A	3	2	1.00	15	0.133
230	A	3	2	1.00	13	0.154
231	A	3	2	1.00	17	0.118
232	A	4	3	1.00	19	0.158
233	A	3	3	1.00	17	0.176
234	A	2	2	1.00	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	1	1	1.00	19	0.053
236	A	3	3	1.00	19	0.158
237	A	3	3	1.00	19	0.158
238	A	4	4	1.00	19	0.210
239	A	5	4	1.00	19	0.210
240	A	6	3	1.00	19	0.158
241	A	5	3	1.00	17	0.176
242	A	4	3	1.00	15	0.200
243	A	3	2	1.00	19	0.105
244	A	2	2	1.00	19	0.105
245	A	1	1	1.00	19	0.053
246	A	4	3	1.00	19	0.158
247	A	4	4	1.00	19	0.210
248	A	4	3	1.00	19	0.158
249	A	5	4	1.00	19	0.210
250	A	6	4	1.00	19	0.210
251	A	7	4	1.00	19	0.210
252	A	4	2	1.00	19	0.105
253	A	3	2	1.00	19	0.105
254	A	2	2	1.00	19	0.105
255	A	1	1	1.00	17	0.059
256	A	2	2	1.00	15	0.133
257	A	3	3	1.00	19	0.158
258	A	4	3	1.00	19	0.158
259	A	5	3	1.00	19	0.158
260	A	4	3	1.00	19	0.158
261	A	3	3	1.00	19	0.158
262	A	2	2	1.00	19	0.105
263	A	1	1	1.00	19	0.053
264	A	3	3	1.00	19	0.158
265	A	4	4	1.00	17	0.235
266	A	5	4	1.00	15	0.267
267	A	6	4	1.00	19	0.210
268	A	7	4	1.00	19	0.210
269	A	5	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	4	3	1.00	21	0.143
271	A	3	3	1.00	21	0.143
272	A	2	2	1.00	21	0.095
273	A	1	1	1.00	21	0.048
274	A	2	2	1.00	21	0.095
275	A	3	2	1.00	21	0.095
276	A	4	2	1.00	21	0.095
277	A	3	3	1.00	21	0.143
278	A	3	3	1.27	19	0.158
279	A	3	3	1.00	21	0.143
280	A	1	1	1.00	21	0.048
281	A	2	2	1.00	21	0.095
282	A	3	2	1.00	21	0.095
283	A	2	2	1.00	17	0.118
284	A	3	2	1.00	19	0.105
285	A	2	2	1.00	19	0.105
286	A	1	1	1.00	19	0.053
287	A	2	2	1.00	15	0.133
288	A	3	3	1.00	19	0.158
289	A	3	3	1.00	19	0.158
290	A	2	2	1.00	17	0.118
291	A	3	3	1.00	19	0.158
292	A	5	5	1.00	19	0.263
293	A	4	4	1.00	19	0.210
294	A	5	5	1.00	19	0.263
295	A	5	4	1.00	21	0.190
296	A	6	6	1.00	21	0.286
297	A	3	3	1.00	21	0.143
298	A	4	4	1.00	21	0.190
299	A	5	5	1.00	21	0.238
300	A	2	2	1.00	21	0.095
301	A	3	3	1.00	21	0.143
302	A	6	6	1.00	21	0.286
303	A	1	1	1.00	21	0.048
304	A	4	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	7	6	1.00	21	0.286
306	A	2	2	1.00	21	0.095
307	A	5	4	1.00	21	0.190
308	A	3	2	1.00	15	0.133
309	A	3	2	1.00	13	0.154
310	A	5	3	1.00	19	0.158
311	A	4	3	1.00	19	0.158
312	A	3	3	1.00	19	0.158
313	A	2	2	1.00	17	0.118
314	A	1	1	1.00	15	0.067
315	A	2	2	1.00	19	0.105
316	A	3	2	1.00	19	0.105
317	A	4	2	1.00	19	0.105
318	A	5	2	1.00	19	0.105
319	A	4	3	1.00	11	0.273
320	A	4	3	1.00	13	0.231
321	A	3	3	1.00	9	0.333
322	A	3	3	1.00	9	0.333
323	A	3	3	1.00	9	0.333
324	A	3	3	1.00	13	0.231
325	A	3	3	1.00	13	0.231
326	A	3	3	1.00	13	0.231
327	A	3	3	1.00	13	0.231
328	A	2	2	1.00	11	0.182
329	A	2	2	1.00	15	0.133
330	A	2	2	1.00	15	0.133
331	A	2	2	1.00	23	0.087
332	A	2	2	1.00	11	0.182
333	A	2	2	1.00	13	0.154
334	A	2	2	1.00	13	0.154
335	A	2	2	1.00	17	0.118
336	A	2	2	1.00	17	0.118
337	A	2	2	1.00	17	0.118
338	A	2	2	1.00	11	0.182
339	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	2	2	1.00	11	0.182
341	A	2	2	1.00	11	0.182
342	A	2	2	1.00	13	0.154
343	A	2	2	1.00	13	0.154
344	A	3	2	1.00	11	0.182
345	A	4	3	1.00	11	0.273
346	A	3	2	1.00	11	0.182
347	A	7	7	1.00	9	0.778
348	A	2	2	1.00	22	0.091
349	A	1	1	1.00	13	0.077
350	A	2	2	1.00	15	0.133
351	A	2	2	1.00	17	0.118
352	A	1	1	1.00	13	0.077
353	A	2	2	1.00	15	0.133
354	A	1	1	1.00	13	0.077
355	A	2	2	1.00	11	0.182
356	A	5	5	1.00	13	0.385
357	A	2	2	1.00	15	0.133
358	A	5	5	1.00	13	0.385
359	A	2	2	1.00	15	0.133
360	A	2	2	1.00	11	0.182
361	A	2	2	1.00	11	0.182
362	A	2	2	1.00	9	0.222
363	A	5	5	1.00	11	0.454
364	A	3	3	1.00	25	0.120
365	A	4	4	1.00	27	0.148
366	A	4	4	1.00	23	0.174
367	A	4	4	1.00	22	0.182
368	A	4	4	1.00	21	0.190
369	A	5	5	1.00	18	0.278
370	A	4	4	1.00	23	0.174
371	A	3	3	1.00	15	0.200
372	A	4	4	1.00	23	0.174
373	A	5	4	1.00	27	0.148
374	A	5	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	5	4	1.00	22	0.182
376	A	5	4	1.00	21	0.190
377	A	6	5	1.00	18	0.278
378	A	5	4	1.00	23	0.174
379	A	5	5	1.00	22	0.227
380	A	5	4	1.00	23	0.174
381	A	5	4	1.00	22	0.182
382	A	5	5	1.00	13	0.385
383	A	5	5	1.00	15	0.333
384	A	4	4	1.00	15	0.267
385	A	4	4	1.00	15	0.267
386	A	5	5	1.00	15	0.333
387	A	5	5	1.00	17	0.294
388	A	4	4	1.00	17	0.235
389	A	4	4	1.00	17	0.235
390	A	3	3	1.00	27	0.111
391	A	3	3	1.00	23	0.130
392	A	2	2	1.00	15	0.133
393	A	3	3	1.00	21	0.143
394	A	4	4	1.00	18	0.222
395	A	3	3	1.00	23	0.130
396	A	3	3	1.00	22	0.136
397	A	3	3	1.00	23	0.130
398	A	4	4	1.00	27	0.148
399	A	4	4	1.00	23	0.174
400	A	4	4	1.00	22	0.182
401	A	4	4	1.00	21	0.190
402	A	5	5	1.00	18	0.278
403	A	4	4	1.00	23	0.174
404	A	4	4	1.00	22	0.182
405	A	4	4	1.00	23	0.174
406	A	4	4	1.00	22	0.182
407	A	3	3	1.00	15	0.200
408	A	3	3	1.00	15	0.200
409	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	3	3	1.00	19	0.158
411	A	3	3	1.00	16	0.188
412	A	3	3	1.00	16	0.188
413	A	3	3	1.00	16	0.188
414	A	3	3	1.00	20	0.150
415	A	3	3	1.00	19	0.158
416	A	3	3	1.00	17	0.176
417	A	3	3	1.00	17	0.176
418	A	3	3	1.00	20	0.150
419	A	3	3	1.00	19	0.158
420	A	3	3	1.00	18	0.167
421	A	3	3	1.00	21	0.143
422	A	3	3	1.00	21	0.143
423	A	3	3	1.00	21	0.143
424	A	3	3	1.00	21	0.143
425	A	3	3	1.00	21	0.143
426	A	3	3	1.00	15	0.200
427	A	3	3	1.00	15	0.200
428	A	3	3	1.00	15	0.200
429	A	3	3	1.00	15	0.200
430	A	3	3	1.00	15	0.200
431	A	2	2	1.00	11	0.182
432	A	1	1	1.00	11	0.091
433	A	3	3	1.00	13	0.231
434	A	1	1	1.00	17	0.059
435	A	1	1	1.00	17	0.059
436	A	2	2	1.00	15	0.133
437	A	11	10	1.00	19	0.526
438	A	9	8	1.00	19	0.421
439	A	3	3	1.03	17	0.176
440	A	3	3	1.00	22	0.136
441	A	3	3	1.12	22	0.136
442	A	3	3	1.00	27	0.111
443	A	3	3	1.00	27	0.111
444	A	1	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	2	2	1.00	17	0.118
446	A	2	2	1.00	22	0.091
447	A	1	1	1.00	23	0.043
448	A	2	2	1.00	19	0.105
449	A	2	2	1.00	19	0.105
450	A	2	2	1.00	19	0.105
451	A	2	2	1.00	19	0.105
452	A	2	2	1.00	19	0.105
453	A	1	1	1.00	25	0.040
454	A	2	2	1.00	28	0.071

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^2(ax+bx^3)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int x^2\left(ax+bx^3\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}$$

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\int x^2\left(ax+bx^3\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}$$

```
[In] Int[x^2*(a*x + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax^3+bx^5\right)\,dx\\&=\frac{ax^4}{4}+\frac{bx^6}{6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2\left(ax+bx^3\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}$$

```
[In] Integrate[x^2*(a*x + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14
norman	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14
risch	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14
parallelrisch	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}$	14
gosper	$\frac{x^{4}\left(2b\,x^{2}+3a\right)}{12}$	16

```
[In] int(x^2*(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(ax+bx^3\right)\,dx=\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2 \left(ax + bx^3\right)\,dx = \frac{ax^4}{4} + \frac{bx^6}{6}$$

```
[In] integrate(x**2*(b*x**3+a*x),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 \left(ax + bx^3\right)\,dx = \frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 \left(ax + bx^3\right)\,dx = \frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 \left(ax + bx^3\right)\,dx = \frac{b\,x^6}{6} + \frac{a\,x^4}{4}$$

```
[In] int(x^2*(a*x + b*x^3),x)
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

3.2 $\int x(ax+bx^3)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int x(ax+bx^3)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}$$

```
[Out] 1/3*a*x^3+1/5*b*x^5
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\int x(ax+bx^3)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}$$

```
[In] Int[x*(a*x + b*x^3),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int(ax^2+bx^4)\,dx\\&=\frac{ax^3}{3}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x(ax+bx^3)\ dx=\frac{ax^3}{3}+\frac{bx^5}{5}$$

```
[In] Integrate[x*(a*x + b*x^3),x]
[Out] (a*x^3)/3 + (b*x^5)/5
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
norman	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
risch	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
parallelrisch	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
gosper	$\frac{x^3(3b\,x^2+5a)}{15}$	16

```
[In] int(x*(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/3*a*x^3+1/5*b*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(ax+bx^3)\ dx=\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="fricas")
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x\left(ax+bx^{3}\right)\,dx=\frac{ax^{3}}{3}+\frac{bx^{5}}{5}$$

```
[In] integrate(x*(b*x**3+a*x),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(ax+bx^{3}\right)\,dx=\frac{1}{5}\,bx^{5}+\frac{1}{3}\,ax^{3}$$

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(ax+bx^{3}\right)\,dx=\frac{1}{5}\,bx^{5}+\frac{1}{3}\,ax^{3}$$

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(ax+bx^{3}\right)\,dx=\frac{b\,x^{5}}{5}+\frac{a\,x^{3}}{3}$$

```
[In] int(x*(a*x + b*x^3),x)
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

3.3 $\int (ax + bx^3)\, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \left(ax + bx^3\right) dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

```
[Out] 1/2*a*x^2+1/4*b*x^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(ax + bx^3\right) dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

```
[In] Int[a*x + b*x^3,x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

Rubi steps

$$\text{integral} = \frac{ax^2}{2} + \frac{bx^4}{4}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (ax+bx^3)\,dx=\frac{ax^2}{2}+\frac{bx^4}{4}$$

```
[In] Integrate[a*x + b*x^3,x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
norman	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4$	14
risch	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4$	14
parallelrisch	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4$	14
parts	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4$	14
gosper	$\frac{x^2(b\,x^2+2a)}{4}$	15
default	$\frac{(b\,x^2+a)^2}{4b}$	15

```
[In] int(b*x^3+a*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+1/4*b*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax+bx^3)\,dx=\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

```
[In] integrate(b*x^3+a*x,x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(ax+bx^3\right)\,dx=\frac{ax^2}{2}+\frac{bx^4}{4}$$

```
[In] integrate(b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^3\right)\,dx=\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

```
[In] integrate(b*x^3+a*x,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^3\right)\,dx=\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

```
[In] integrate(b*x^3+a*x,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax+bx^3\right)\,dx=\frac{b\,x^4}{4}+\frac{a\,x^2}{2}$$

```
[In] int(a*x + b*x^3,x)
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

3.4 $\int \frac{ax+bx^3}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{ax+bx^3}{x} \, dx = ax+\frac{bx^3}{3}$$

```
[Out] a*x+1/3*b*x^3
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\int \frac{ax+bx^3}{x} \, dx = ax+\frac{bx^3}{3}$$

```
[In] Int[(a*x + b*x^3)/x,x]
```

```
[Out] a*x + (b*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a+bx^2\right) \, dx \\ &= ax+\frac{bx^3}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3}{x}\,dx = ax+\frac{bx^3}{3}$$

```
[In] Integrate[(a*x + b*x^3)/x,x]
```

```
[Out] a*x + (b*x^3)/3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$ax+\frac{1}{3}b\,x^3$	11
norman	$ax+\frac{1}{3}b\,x^3$	11
risch	$ax+\frac{1}{3}b\,x^3$	11
parallelrisch	$ax+\frac{1}{3}b\,x^3$	11
parts	$ax+\frac{1}{3}b\,x^3$	11
gosper	$\frac{x\left(b\,x^2+3a\right)}{3}$	13

```
[In] int((b*x^3+a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*x+1/3*b*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax+bx^3}{x}\,dx = \frac{1}{3}\,bx^3+ax$$

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{ax + bx^3}{x} \, dx = ax + \frac{bx^3}{3}$$

```
[In] integrate((b*x**3+a*x)/x,x)
```

```
[Out] a*x + b*x**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax + bx^3}{x} \, dx = \frac{1}{3} \, bx^3 + ax$$

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax + bx^3}{x} \, dx = \frac{1}{3} \, bx^3 + ax$$

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax + bx^3}{x} \, dx = \frac{b\,x^3}{3} + a\,x$$

```
[In] int((a*x + b*x^3)/x,x)
```

```
[Out] a*x + (b*x^3)/3
```

3.5 $\int \frac{ax+bx^3}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{bx^2}{2} + a\log(x)$$

```
[Out] 1/2*b*x^2+a*ln(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\int \frac{ax+bx^3}{x^2}\,dx = a\log(x) + \frac{bx^2}{2}$$

```
[In] Int[(a*x + b*x^3)/x^2,x]
```

```
[Out] (b*x^2)/2 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{x} + bx\right) dx \\ &= \frac{bx^2}{2} + a\log(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{ax+bx^3}{x^2}\,dx=\frac{bx^2}{2}+a\log(x)$$

```
[In] Integrate[(a*x + b*x^3)/x^2,x]
[Out] (b*x^2)/2 + a*Log[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{b\,x^2}{2}+a\ln\left(x\right)$	12
norman	$\frac{b\,x^2}{2}+a\ln\left(x\right)$	12
risch	$\frac{b\,x^2}{2}+a\ln\left(x\right)$	12
parallelrisch	$\frac{b\,x^2}{2}+a\ln\left(x\right)$	12

```
[In] int((b*x^3+a*x)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/2*b*x^2+a*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax+bx^3}{x^2}\,dx=\frac{1}{2}\,bx^2+a\log\left(x\right)$$

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="fricas")
[Out] 1/2*b*x^2 + a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{ax+bx^3}{x^2}\,dx = a\log{(x)} + \frac{bx^2}{2}$$

```
[In] integrate((b*x**3+a*x)/x**2,x)
```

```
[Out] a*log(x) + b*x**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{1}{2}\,bx^2 + a\log{(x)}$$

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log{(x^2)}$$

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{ax+bx^3}{x^2}\,dx = \frac{b\,x^2}{2} + a\,\ln{(x)}$$

```
[In] int((a*x + b*x^3)/x^2,x)
```

```
[Out] (b*x^2)/2 + a*log(x)
```

3.6 $\int x^2(ax+bx^3)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int x^2\left(ax+bx^3\right)^2\,dx=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 276}

$$\int x^2\left(ax+bx^3\right)^2\,dx=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

```
[In] Int[x^2*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^4\left(a+bx^2\right)^2\,dx\\
&= \int \left(a^2x^4+2abx^6+b^2x^8\right)\,dx\\
&= \frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^2\left(ax+bx^3\right)^2\,dx=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

```
[In] Integrate[x^2*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{5}x^5a^2+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
norman	$\frac{1}{5}x^5a^2+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
risch	$\frac{1}{5}x^5a^2+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
parallelrisch	$\frac{1}{5}x^5a^2+\frac{2}{7}ab\,x^7+\frac{1}{9}b^2x^9$	25
gosper	$\frac{x^5\left(35b^2x^4+90ab\,x^2+63a^2\right)}{315}$	27

```
[In] int(x^2*(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*a^2+2/7*a*b*x^7+1/9*b^2*x^9
```

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2\left(ax+bx^3\right)^2\,dx=\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="fricas")
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^2\left(ax+bx^3\right)^2\,dx=\frac{a^2x^5}{5}+\frac{2abx^7}{7}+\frac{b^2x^9}{9}$$

```
[In] integrate(x**2*(b*x**3+a*x)**2,x)
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2\left(ax+bx^3\right)^2\,dx=\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="maxima")
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2\left(ax+bx^3\right)^2\,dx=\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2\left(ax+bx^3\right)^2\,dx=\frac{a^2\,x^5}{5}+\frac{2\,a\,b\,x^7}{7}+\frac{b^2\,x^9}{9}$$

```
[In] int(x^2*(a*x + b*x^3)^2,x)
```

```
[Out] (a^2*x^5)/5 + (b^2*x^9)/9 + (2*a*b*x^7)/7
```

3.7 $\int x(ax+bx^3)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x(ax+bx^3)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 272, 45}

$$\int x(ax+bx^3)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

```
[In] Int[x*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^3\left(a+bx^2\right)^2\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right)\,dx,x,x^2\right)\\ &= \frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x\left(ax+bx^3\right)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

```
[In] Integrate[x*(a*x + b*x^3)^2,x]
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
parallelrisch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
gosper	$\frac{x^4\left(3b^2x^4+8ab\,x^2+6a^2\right)}{24}$	27

```
[In] int(x*(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax+bx^3\right)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax+bx^3\right)^2\,dx=\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}$$

```
[In] integrate(x*(b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax+bx^3\right)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax+bx^3\right)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax+bx^3\right)^2\,dx=\frac{a^2\,x^4}{4}+\frac{a\,b\,x^6}{3}+\frac{b^2\,x^8}{8}$$

```
[In] int(x*(a*x + b*x^3)^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

3.8 $\int (ax+bx^3)^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \left(ax+bx^3\right)^2 \, dx = \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 276}

$$\int \left(ax+bx^3\right)^2 \, dx = \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{b^2x^7}{7}$$

```
[In] Int[(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^2\left(a+bx^2\right)^2\,dx \\
&= \int \left(a^2x^2+2abx^4+b^2x^6\right)\,dx \\
&= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(ax+bx^3\right)^2\,dx = \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

```
[In] Integrate[(a*x + b*x^3)^2,x]
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
norman	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
risch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
parallelrisch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
gosper	$\frac{x^3\left(15b^2x^4+42ab\,x^2+35a^2\right)}{105}$	27

```
[In] int((b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="fricas")
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \left(ax+bx^3\right)^2\,dx=\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}$$

```
[In] integrate((b*x**3+a*x)**2,x)
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="maxima")
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax+bx^3\right)^2\,dx=\frac{a^2\,x^3}{3}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^7}{7}$$

```
[In] int((a*x + b*x^3)^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

3.9 $\int \frac{(ax+bx^3)^2}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{(ax+bx^3)^2}{x} dx = \frac{(a+bx^2)^3}{6b}$$

```
[Out] 1/6*(b*x^2+a)^3/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 267}

$$\int \frac{(ax+bx^3)^2}{x} dx = \frac{(a+bx^2)^3}{6b}$$

```
[In] Int[(a*x + b*x^3)^2/x,x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x\left(a+bx^2\right)^2\,dx$$
$$= \frac{\left(a+bx^2\right)^3}{6b}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\left(ax+bx^3\right)^2}{x}\,dx = \frac{\left(a+bx^2\right)^3}{6b}$$

```
[In] Integrate[(a*x + b*x^3)^2/x,x]
[Out] (a + b*x^2)^3/(6*b)
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\left(b\,x^{2}+a\right)^{3}}{6b}$	15
norman	$\frac{1}{6}b^{2}x^{6}+\frac{1}{2}ab\,x^{4}+\frac{1}{2}a^{2}x^{2}$	25
parallelrisch	$\frac{1}{6}b^{2}x^{6}+\frac{1}{2}ab\,x^{4}+\frac{1}{2}a^{2}x^{2}$	25
gosper	$\frac{x^{2}\left(b^{2}x^{4}+3ab\,x^{2}+3a^{2}\right)}{6}$	26
risch	$\frac{b^{2}x^{6}}{6}+\frac{ab\,x^{4}}{2}+\frac{a^{2}x^{2}}{2}+\frac{a^{3}}{6b}$	33

```
[In] int((b*x^3+a*x)^2/x,x,method=_RETURNVERBOSE)
[Out] 1/6*(b*x^2+a)^3/b
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="fricas")
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

```
[In] integrate((b*x**3+a*x)**2/x,x)
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="maxima")
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="giac")
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{(ax+bx^3)^2}{x}\,dx = \frac{a^2\,x^2}{2} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^6}{6}$$

```
[In] int((a*x + b*x^3)^2/x,x)
[Out] (a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

3.10 $\int \frac{(ax+bx^3)^2}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx = a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 200}

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx = a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[In] Int[(a*x + b*x^3)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a+bx^2\right)^2 \, dx \\
&= \int \left(a^2+2abx^2+b^2x^4\right) \, dx \\
&= a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\left(ax+bx^3\right)^2}{x^2} \, dx = a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[In] Integrate[(a*x + b*x^3)^2/x^2,x]
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$a^{2} x +\frac{2}{3} a b \,x^{3}+\frac{1}{5} b^{2} x^{5}$	22
risch	$a^{2} x +\frac{2}{3} a b \,x^{3}+\frac{1}{5} b^{2} x^{5}$	22
parallelrisch	$a^{2} x +\frac{2}{3} a b \,x^{3}+\frac{1}{5} b^{2} x^{5}$	22
gosper	$\frac{x \left(3 b^{2} x^{4}+10 a b \,x^{2}+15 a^{2}\right)}{15}$	25
norman	$\frac{a^{2} x^{2}+\frac{1}{5} b^{2} x^{6}+\frac{2}{3} a b \,x^{4}}{x}$	28

```
[In] int((b*x^3+a*x)^2/x^2,x,method=_RETURNVERBOSE)
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax + bx^3)^2}{x^2} \, dx = \frac{1}{5} \, b^2 x^5 + \frac{2}{3} \, abx^3 + a^2 x$$

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="fricas")
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(ax + bx^3)^2}{x^2} \, dx = a^2 x + \frac{2abx^3}{3} + \frac{b^2 x^5}{5}$$

```
[In] integrate((b*x**3+a*x)**2/x**2,x)
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax + bx^3)^2}{x^2} \, dx = \frac{1}{5} \, b^2 x^5 + \frac{2}{3} \, abx^3 + a^2 x$$

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="maxima")
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax + bx^3)^2}{x^2} \, dx = \frac{1}{5} \, b^2 x^5 + \frac{2}{3} \, abx^3 + a^2 x$$

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="giac")
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(ax+bx^3)^2}{x^2}\,dx = a^2\,x+\frac{2\,a\,b\,x^3}{3}+\frac{b^2\,x^5}{5}$$

```
[In] int((a*x + b*x^3)^2/x^2,x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

3.11 $\int \left(-4x+3x^3\right)^6 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \left(-4x+3x^3\right)^6 \, dx=\frac{4096x^7}{7}-2048x^9+\frac{34560x^{11}}{11}-\frac{34560x^{13}}{13}+1296x^{15}-\frac{5832x^{17}}{17}+\frac{729x^{19}}{19}$$

```
[Out] 4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729/
19*x^19
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 276}

$$\int \left(-4x+3x^3\right)^6 \, dx=\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

```
[In] Int[(-4*x + 3*x^3)^6,x]
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^6\left(-4+3x^2\right)^6\,dx\\ &= \int \left(4096x^6-18432x^8+34560x^{10}-34560x^{12}+19440x^{14}-5832x^{16}+729x^{18}\right)\,dx\\ &= \frac{4096x^7}{7}-2048x^9+\frac{34560x^{11}}{11}-\frac{34560x^{13}}{13}+1296x^{15}-\frac{5832x^{17}}{17}+\frac{729x^{19}}{19}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \left(-4x+3x^3\right)^6\,dx=\frac{4096x^7}{7}-2048x^9+\frac{34560x^{11}}{11}-\frac{34560x^{13}}{13}+1296x^{15}-\frac{5832x^{17}}{17}+\frac{729x^{19}}{19}$$

```
[In] Integrate[(-4*x + 3*x^3)^6,x]
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{4096}{7}x^7-2048x^9+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
norman	$\frac{4096}{7}x^7-2048x^9+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
risch	$\frac{4096}{7}x^7-2048x^9+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
parallelrisch	$\frac{4096}{7}x^7-2048x^9+\frac{34560}{11}x^{11}-\frac{34560}{13}x^{13}+1296x^{15}-\frac{5832}{17}x^{17}+\frac{729}{19}x^{19}$	37
gosper	$\frac{x^7\left(12405393x^{12}-110918808x^{10}+419026608x^8-859541760x^6+1015822080x^4-662165504x^2+189190144\right)}{323323}$	38

```
[In] int((3*x^3-4*x)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729/
19*x^19
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^3\right)^6 \,dx = \frac{729}{19}\,x^{19} - \frac{5832}{17}\,x^{17} + 1296\,x^{15} - \frac{34560}{13}\,x^{13} + \frac{34560}{11}\,x^{11} - 2048\,x^{9} + \frac{4096}{7}\,x^{7}$$

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="fricas")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(-4x+3x^3\right)^6 \,dx = \frac{729x^{19}}{19} - \frac{5832x^{17}}{17} + 1296x^{15} - \frac{34560x^{13}}{13} + \frac{34560x^{11}}{11} - 2048x^{9} + \frac{4096x^{7}}{7}$$

```
[In] integrate((3*x**3-4*x)**6,x)
```

```
[Out] 729*x**19/19 - 5832*x**17/17 + 1296*x**15 - 34560*x**13/13 + 34560*x**11/11
 - 2048*x**9 + 4096*x**7/7
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^3\right)^6 \,dx = \frac{729}{19}\,x^{19} - \frac{5832}{17}\,x^{17} + 1296\,x^{15} - \frac{34560}{13}\,x^{13} + \frac{34560}{11}\,x^{11} - 2048\,x^{9} + \frac{4096}{7}\,x^{7}$$

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="maxima")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^3\right)^6\,dx=\frac{729}{19}\,x^{19}-\frac{5832}{17}\,x^{17}+1296\,x^{15}-\frac{34560}{13}\,x^{13}+\frac{34560}{11}\,x^{11}-2048\,x^9+\frac{4096}{7}\,x^7$$

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="giac")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(-4x+3x^3\right)^6\,dx=\frac{729\,x^{19}}{19}-\frac{5832\,x^{17}}{17}+1296\,x^{15}-\frac{34560\,x^{13}}{13}+\frac{34560\,x^{11}}{11}-2048\,x^9+\frac{4096\,x^7}{7}$$

```
[In] int((4*x - 3*x^3)^6,x)
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

3.12 $\int \frac{x^4}{ax+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{x^4}{ax+bx^3}\,dx=\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 272, 45}

$$\int \frac{x^4}{ax+bx^3}\,dx=\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

```
[In] Int[x^4/(a*x + b*x^3),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3}{a+bx^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\,dx, x, x^2\right) \\
&= \frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{ax+bx^3}\,dx = \frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

```
[In] Integrate[x^4/(a*x + b*x^3),x]
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{-b\,x^2+a\ln\left(b\,x^2+a\right)}{2b^2}$	23
default	$\frac{x^2}{2b}-\frac{a\ln\left(b\,x^2+a\right)}{2b^2}$	24
norman	$\frac{x^2}{2b}-\frac{a\ln\left(b\,x^2+a\right)}{2b^2}$	24
risch	$\frac{x^2}{2b}-\frac{a\ln\left(b\,x^2+a\right)}{2b^2}$	24

```
[In] int(x^4/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-b*x^2+a*ln(b*x^2+a))/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{ax+bx^3}\,dx = \frac{bx^2 - a\log\left(bx^2+a\right)}{2\,b^2}$$

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="fricas")
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{ax+bx^3}\,dx = -\frac{a\log\left(a+bx^2\right)}{2b^2} + \frac{x^2}{2b}$$

```
[In] integrate(x**4/(b*x**3+a*x),x)
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{ax+bx^3}\,dx = \frac{x^2}{2\,b} - \frac{a\log\left(bx^2+a\right)}{2\,b^2}$$

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="maxima")
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{ax+bx^3}\,dx=\frac{x^2}{2\,b}-\frac{a\log\left(|bx^2+a|\right)}{2\,b^2}$$

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="giac")
[Out] 1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{ax+bx^3}\,dx=-\frac{a\,\ln\left(b\,x^2+a\right)-b\,x^2}{2\,b^2}$$

```
[In] int(x^4/(a*x + b*x^3),x)
[Out] -(a*log(a + b*x^2) - b*x^2)/(2*b^2)
```

3.13 $\int \frac{x^3}{ax+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^3}{ax+bx^3}\,dx = \frac{x}{b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] x/b-arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1598, 327, 211}

$$\int \frac{x^3}{ax+bx^3}\,dx = \frac{x}{b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[In] Int[x^3/(a*x + b*x^3),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2}{a+bx^2}\,dx\\ &= \frac{x}{b}-\frac{a\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax+bx^3}\,dx=\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[In] Integrate[x^3/(a*x + b*x^3),x]
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	27
risch	$\frac{x}{b}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^2}-\frac{\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{2b^2}$	56

```
[In] int(x^3/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] x/b-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \frac{x^3}{ax+bx^3}\,dx = \left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},\ -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{x^3}{ax+bx^3}\,dx = \frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2} - \frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2} + \frac{x}{b}$$

```
[In] integrate(x**3/(b*x**3+a*x),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{ax+bx^3}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{x}{b}$$

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{ax+bx^3}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{x}{b}$$

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Mupad [B] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{ax+bx^3}\,dx = \frac{x}{b} - \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[In] int(x^3/(a*x + b*x^3),x)
```

```
[Out] x/b - (a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

3.14 $\int \frac{x^2}{ax+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^2}{ax+bx^3}\,dx=\frac{\log\left(a+bx^2\right)}{2b}$$

```
[Out] 1/2*ln(b*x^2+a)/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 266}

$$\int \frac{x^2}{ax+bx^3}\,dx=\frac{\log\left(a+bx^2\right)}{2b}$$

```
[In] Int[x^2/(a*x + b*x^3),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x}{a+bx^2}\,dx\\ &= \frac{\log\left(a+bx^2\right)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log\left(a+bx^2\right)}{2b}$$

```
[In] Integrate[x^2/(a*x + b*x^3),x]
[Out] Log[a + b*x^2]/(2*b)
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln\left(b\,x^{2}+a\right)}{2b}$	14
norman	$\frac{\ln\left(b\,x^{2}+a\right)}{2b}$	14
risch	$\frac{\ln\left(b\,x^{2}+a\right)}{2b}$	14
parallelrisch	$\frac{\ln\left(b\,x^{2}+a\right)}{2b}$	14

```
[In] int(x^2/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(b*x^2+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log(bx^2+a)}{2\,b}$$

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="fricas")
[Out] 1/2*log(b*x^2 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log(a+bx^2)}{2b}$$

```
[In] integrate(x**2/(b*x**3+a*x),x)
[Out] log(a + b*x**2)/(2*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log(bx^2+a)}{2\,b}$$

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="maxima")
[Out] 1/2*log(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{ax+bx^3}\,dx = \frac{\log(|bx^2+a|)}{2\,b}$$

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 10.69 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{ax+bx^3}\,dx=\frac{\ln\left(b\,x^2+a\right)}{2\,b}$$

```
[In] int(x^2/(a*x + b*x^3),x)
[Out] log(a + b*x^2)/(2*b)
```

3.15 $\int \frac{x}{ax+bx^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{x}{ax+bx^3} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1598, 211}

$$\int \frac{x}{ax+bx^3} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Int[x/(a*x + b*x^3),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{a+bx^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax+bx^3}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[In] Integrate[x/(a*x + b*x^3),x]
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

```
[In] int(x/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{x}{ax+bx^3}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="fricas")
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{x}{ax+bx^3}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

```
[In] integrate(x/(b*x**3+a*x),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{x}{ax+bx^3}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{x}{ax+bx^3}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{x}{ax+bx^3}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[In] int(x/(a*x + b*x^3),x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

3.16 $\int \frac{1}{ax+bx^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{1}{ax+bx^3}\,dx = \frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

```
[Out] ln(x)/a-1/2*ln(b*x^2+a)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {1607, 272, 36, 29, 31}

$$\int \frac{1}{ax+bx^3}\,dx = \frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

```
[In] Int[(a*x + b*x^3)^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x\left(a+bx^2\right)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^2\right)}{2a}\\ &= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx^3}\,dx = \frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

```
[In] Integrate[(a*x + b*x^3)^(-1),x]
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^2+a)}{2a}$	21
norman	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^2+a)}{2a}$	21
risch	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^2+a)}{2a}$	21
parallelrisch	$\frac{2\ln(x)-\ln(b\,x^2+a)}{2a}$	21

```
[In] int(1/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)/a-1/2*ln(b*x^2+a)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{ax+bx^3}\,dx=-\frac{\log\left(bx^2+a\right)-2\,\log\left(x\right)}{2\,a}$$

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{1}{ax+bx^3}\,dx=\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a}$$

```
[In] integrate(1/(b*x**3+a*x),x)
```

```
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{ax+bx^3}\,dx = -\frac{\log\left(bx^2+a\right)}{2\,a} + \frac{\log\left(x\right)}{a}$$

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 + a)/a + log(x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{ax+bx^3}\,dx = \frac{\log\left(x^2\right)}{2\,a} - \frac{\log\left(|bx^2+a|\right)}{2\,a}$$

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{ax+bx^3}\,dx = -\frac{\ln\left(b\,x^2+a\right)-2\,\ln\left(x\right)}{2\,a}$$

```
[In] int(1/(a*x + b*x^3),x)
```

```
[Out] -(log(a + b*x^2) - 2*log(x))/(2*a)
```

3.17 $\int \frac{1}{x(ax+bx^3)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx=-\frac{1}{ax}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -1/a/x-arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 331, 211}

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx=-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

```
[In] Int[1/(x*(a*x + b*x^3)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^2\left(a+bx^2\right)}\,dx\\ &= -\frac{1}{ax}-\frac{b\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx = -\frac{1}{ax}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] Integrate[1/(x*(a*x + b*x^3)),x]
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{ax}$	30
risch	$-\frac{1}{ax}+\frac{\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{2a^2}-\frac{\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{2a^2}$	58

```
[In] int(1/x/(b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] -b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/a/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx=\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax},-\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(29) = 58.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx=\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

```
[In] integrate(1/x/(b*x**3+a*x),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(ax+bx^3\right)}\,dx=-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{1}{ax}$$

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\,(ax+bx^3)}\,dx = -\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{1}{ax}$$

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="giac")
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 10.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\,(ax+bx^3)}\,dx = -\frac{1}{a\,x} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[In] int(1/(x*(a*x + b*x^3)),x)
[Out] - 1/(a*x) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

3.18 $\int \frac{1}{x^2(ax+bx^3)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx=-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}$$

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 272, 46}

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx=\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[In] Int[1/(x^2*(a*x + b*x^3)),x]
```

```
[Out] -1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^3\left(a+bx^2\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)\,dx, x, x^2\right) \\
&= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx = -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}$$

```
[In] Integrate[1/(x^2*(a*x + b*x^3)),x]
[Out] -1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(b\,x^2+a\right)}{2a^2}$	32
norman	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(b\,x^2+a\right)}{2a^2}$	32
parallelrisch	$-\frac{2b\ln(x)x^2-b\ln\left(b\,x^2+a\right)x^2+a}{2x^2a^2}$	33
risch	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(-b\,x^2-a\right)}{2a^2}$	35

```
[In] int(1/x^2/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx=\frac{bx^2\log\left(bx^2+a\right)-2\,bx^2\log\left(x\right)-a}{2\,a^2x^2}$$

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx=-\frac{1}{2ax^2}-\frac{b\log\left(x\right)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

```
[In] integrate(1/x**2/(b*x**3+a*x),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx=\frac{b\log\left(bx^2+a\right)}{2\,a^2}-\frac{b\log\left(x\right)}{a^2}-\frac{1}{2\,ax^2}$$

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/a^2 - b*log(x)/a^2 - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx = -\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{b\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2*
x^2)
```

Mupad [B] (verification not implemented)

Time = 10.66 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx = \frac{b\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{1}{2\,a\,x^2}-\frac{b\,\ln\left(x\right)}{a^2}$$

```
[In] int(1/(x^2*(a*x + b*x^3)),x)
```

```
[Out] (b*log(a + b*x^2))/(2*a^2) - 1/(2*a*x^2) - (b*log(x))/a^2
```

3.19 $\int \frac{1}{x^3(ax+bx^3)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx=-\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] -1/3/a/x^3+b/a^2/x+b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 331, 211}

$$\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx=\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

```
[In] Int[1/(x^3*(a*x + b*x^3)),x]
[Out] -1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^4\left(a+bx^2\right)}\,dx\\ &= -\frac{1}{3ax^3}-\frac{b\int \frac{1}{x^2(a+bx^2)}\,dx}{a}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{a^2}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx = -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[In] Integrate[1/(x^3*(a*x + b*x^3)),x]
[Out] -1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{3a\,x^3}+\frac{b}{x\,a^2}+\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$	39
risch	$\frac{\frac{b\,x^2}{a^2}-\frac{1}{3a}}{x^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^5\_Z^2+b^3\right)}\_R\ln\left(\left(3a^5\_R^2+2b^3\right)x-a^3b\_R\right)\right)}{2}$	64

```
[In] int(1/x^3/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/x^3+b/x/a^2+b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{1}{x^3\,(ax+bx^3)}\,dx = \left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3},\ \frac{3\,bx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(37) = 74.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int \frac{1}{x^3\,(ax+bx^3)}\,dx = -\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

```
[In] integrate(1/x**3/(b*x**3+a*x),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2-a}{3\,a^2x^3}$$

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="maxima")
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2-a}{3\,a^2x^3}$$

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="giac")
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx = \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{1}{3\,a}-\frac{b\,x^2}{a^2}}{x^3}$$

```
[In] int(1/(x^3*(a*x + b*x^3)),x)
[Out] (b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2) - (1/(3*a) - (b*x^2)/a^2)/x^3
```

3.20 $\int \frac{1}{x^4(ax+bx^3)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^4(ax+bx^3)}\,dx = -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{b^2\log(x)}{a^3} - \frac{b^2\log(a+bx^2)}{2a^3}$$

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 272, 46}

$$\int \frac{1}{x^4(ax+bx^3)}\,dx = -\frac{b^2\log(a+bx^2)}{2a^3} + \frac{b^2\log(x)}{a^3} + \frac{b}{2a^2x^2} - \frac{1}{4ax^4}$$

```
[In] Int[1/(x^4*(a*x + b*x^3)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2
*a^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^5\left(a+bx^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)\,dx, x, x^2\right)\\
&= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(ax+bx^3\right)}\,dx = -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^2\right)}{2a^3}$$

```
[In] Integrate[1/(x^4*(a*x + b*x^3)),x]
[Out] -1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2
*a^3)
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{1}{4a\,x^{4}}+\frac{b}{2a^{2}x^{2}}+\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln(b\,x^{2}+a)}{2a^{3}}$	44
norman	$\frac{-\frac{1}{4a}+\frac{b\,x^{2}}{2a^{2}}}{x^{4}}+\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln(b\,x^{2}+a)}{2a^{3}}$	46
risch	$\frac{-\frac{1}{4a}+\frac{b\,x^{2}}{2a^{2}}}{x^{4}}+\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln(b\,x^{2}+a)}{2a^{3}}$	46
parallelrisch	$\frac{4b^{2}\ln(x)x^{4}-2b^{2}\ln(b\,x^{2}+a)x^{4}+2ab\,x^{2}-a^{2}}{4a^{3}x^{4}}$	48

```
[In] int(1/x^4/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4\left(ax+bx^3\right)}\,dx=-\frac{2\,b^2x^4\log\left(bx^2+a\right)-4\,b^2x^4\log\left(x\right)-2\,abx^2+a^2}{4\,a^3x^4}$$

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*x
^4)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^4\left(ax+bx^3\right)}\,dx=\frac{-a+2bx^2}{4a^2x^4}+\frac{b^2\log\left(x\right)}{a^3}-\frac{b^2\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

```
[In] integrate(1/x**4/(b*x**3+a*x),x)
```

```
[Out] (-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2*
a**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx = -\frac{b^2 \log \left(bx^2 + a\right)}{2 a^3} + \frac{b^2 \log \left(x\right)}{a^3} + \frac{2 bx^2 - a}{4 a^2 x^4}$$

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="maxima")
[Out] -1/2*b^2*log(b*x^2 + a)/a^3 + b^2*log(x)/a^3 + 1/4*(2*b*x^2 - a)/(a^2*x^4)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx = \frac{b^2 \log \left(x^2\right)}{2 a^3} - \frac{b^2 \log \left(|bx^2 + a|\right)}{2 a^3} - \frac{3 b^2 x^4 - 2 abx^2 + a^2}{4 a^3 x^4}$$

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="giac")
[Out] 1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 - 2
*a*b*x^2 + a^2)/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 (ax + bx^3)} \, dx = \frac{b^2 \ln \left(x\right)}{a^3} - \frac{b^2 \ln \left(b x^2 + a\right)}{2 a^3} - \frac{\frac{1}{4 a} - \frac{b x^2}{2 a^2}}{x^4}$$

```
[In] int(1/(x^4*(a*x + b*x^3)),x)
[Out] (b^2*log(x))/a^3 - (b^2*log(a + b*x^2))/(2*a^3) - (1/(4*a) - (b*x^2)/(2*a^2
))/x^4
```

3.21 $\int \frac{x^2}{(ax+bx^3)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx=\frac{x}{2a(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

```
[Out] 1/2*x/a/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 205, 211}

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}$$

```
[In] Int[x^2/(a*x + b*x^3)^2,x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(a+bx^2)^2}\,dx\\ &= \frac{x}{2a(a+bx^2)}+\frac{\int \frac{1}{a+bx^2}\,dx}{2a}\\ &= \frac{x}{2a(a+bx^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{x}{2a(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

```
[In] Integrate[x^2/(a*x + b*x^3)^2,x]
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2a(b\,x^2+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	36
risch	$\frac{x}{2a(b\,x^2+a)}-\frac{\ln(bx+\sqrt{-ab})}{4\sqrt{-ab}\,a}+\frac{\ln(-bx+\sqrt{-ab})}{4\sqrt{-ab}\,a}$	62

```
[In] int(x^2/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{x^2}{\left(ax+bx^3\right)^2}\,dx = \left[\frac{2\,abx-\left(bx^2+a\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,\left(a^2b^2x^2+a^3b\right)}, \frac{abx+\left(bx^2+a\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,\left(a^2b^2x^2+a^3b\right)}\right]$$

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(36) = 72.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{\left(ax+bx^3\right)^2}\,dx = \frac{x}{2a^2+2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

```
[In] integrate(x**2/(b*x**3+a*x)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\left(ax+bx^3\right)^2}\,dx = \frac{x}{2\,\left(abx^2+a^2\right)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a}$$

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{x}{2\,(bx^2+a)a}$$

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 10.68 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(ax+bx^3)^2}\,dx = \frac{x}{2\,a\,\left(b\,x^2+a\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

```
[In] int(x^2/(a*x + b*x^3)^2,x)
[Out] x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

3.22 $\int \frac{x}{(ax+bx^3)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{1}{2a(a+bx^2)} + \frac{\log(x)}{a^2} - \frac{\log(a+bx^2)}{2a^2}$$

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 272, 46}

$$\int \frac{x}{(ax+bx^3)^2}\,dx = -\frac{\log(a+bx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{1}{2a(a+bx^2)}$$

```
[In] Int[x/(a*x + b*x^3)^2,x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x\left(a+bx^2\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)\,dx, x, x^2\right) \\
&= \frac{1}{2a\left(a+bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x}{\left(ax+bx^3\right)^2}\,dx = \frac{\frac{a}{a+bx^2}+2\log(x)-\log\left(a+bx^2\right)}{2a^2}$$

```
[In] Integrate[x/(a*x + b*x^3)^2,x]
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{1}{2a(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	35
norman	$-\frac{b\,x^2}{2a^2(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	39
default	$\frac{\ln(x)}{a^2}-\frac{b\left(\frac{\ln\left(b\,x^2+a\right)}{b}-\frac{a}{b\left(b\,x^2+a\right)}\right)}{2a^2}$	42
parallelrisch	$\frac{2b\ln(x)x^2-b\ln(b\,x^2+a)x^2-b\,x^2+2a\ln(x)-a\ln(b\,x^2+a)}{2a^2(b\,x^2+a)}$	60

```
[In] int(x/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{x}{(ax+bx^3)^2}\,dx = -\frac{(bx^2+a)\log(bx^2+a)-2(bx^2+a)\log(x)-a}{2(a^2bx^2+a^3)}$$

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a
^3)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{1}{2a^2+2abx^2}+\frac{\log(x)}{a^2}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

```
[In] integrate(x/(b*x**3+a*x)**2,x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{1}{2\,(abx^2+a^2)} - \frac{\log(bx^2+a)}{2\,a^2} + \frac{\log(x)}{a^2}$$

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="maxima")
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + log(x)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{\log(x^2)}{2\,a^2} - \frac{\log(|bx^2+a|)}{2\,a^2} + \frac{bx^2+2\,a}{2\,(bx^2+a)a^2}$$

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
+ a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x}{(ax+bx^3)^2}\,dx = \frac{\ln(x)}{a^2} + \frac{1}{2\,a\,\left(b\,x^2+a\right)} - \frac{\ln\left(b\,x^2+a\right)}{2\,a^2}$$

```
[In] int(x/(a*x + b*x^3)^2,x)
[Out] log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

3.23 $\int \frac{1}{(ax+bx^3)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 57

$$\int \frac{1}{(ax+bx^3)^2}\,dx = -\frac{3}{2a^2x} + \frac{1}{2ax(a+bx^2)} - \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

```
[Out] -3/2/a^2/x+1/2/a/x/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1607, 296, 331, 211}

$$\int \frac{1}{(ax+bx^3)^2}\,dx = -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{3}{2a^2x} + \frac{1}{2ax(a+bx^2)}$$

```
[In] Int[(a*x + b*x^3)^(-2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^2 (a+bx^2)^2} \, dx \\
&= \frac{1}{2ax (a+bx^2)} + \frac{3 \int \frac{1}{x^2(a+bx^2)} \, dx}{2a} \\
&= -\frac{3}{2a^2x} + \frac{1}{2ax (a+bx^2)} - \frac{(3b) \int \frac{1}{a+bx^2} \, dx}{2a^2} \\
&= -\frac{3}{2a^2x} + \frac{1}{2ax (a+bx^2)} - \frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{(ax+bx^3)^2} \, dx = -\frac{1}{a^2x} - \frac{bx}{2a^2 (a+bx^2)} - \frac{3\sqrt{b} \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

```
[In] Integrate[(a*x + b*x^3)^(-2),x]
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{a^{2} x}-\frac{b \left(\frac{x}{2 b \,x^{2}+2 a}+\frac{3 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{a^{2}}$	45
risch	$\frac{-\frac{3 b \,x^{2}}{2 a^{2}}-\frac{1}{a}}{x \left(b \,x^{2}+a \right)}+\frac{3 \left(\sum_{\_R =\operatorname{RootOf}\left(a^{5} \_Z^{2}+b \right)}\_R \ln \left(\left(3 a^{5} \_R^{2}+2 b \right) x +a^{3} \_R \right)\right)}{4}$	68

```
[In] int(1/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/x-b/a^2*(1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

$$\int \frac{1}{\left(ax+bx^3\right)^2}\,dx=\left[-\frac{6\,bx^{2}-3\,\left(bx^{3}+ax\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+4\,a}{4\,\left(a^{2}bx^{3}+a^{3}x\right)},\ -\frac{3\,bx^{2}+3\,\left(bx^{3}+ax\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,a}{2\,\left(a^{2}bx^{3}+a^{3}x\right)}\right]$$

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(ax+bx^3\right)^2}\,dx = \frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4} - \frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4} + \frac{-2a-3bx^2}{2a^3x+2a^2bx^3}$$

```
[In] integrate(1/(b*x**3+a*x)**2,x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(ax+bx^3\right)^2}\,dx = -\frac{3\,bx^2+2\,a}{2\,(a^2bx^3+a^3x)} - \frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}$$

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(ax+bx^3\right)^2}\,dx = -\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{3\,bx^2+2\,a}{2\,(bx^3+ax)a^2}$$

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(ax+bx^3\right)^2}\,dx=-\frac{\frac{1}{a}+\frac{3\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

```
[In] int(1/(a*x + b*x^3)^2,x)
```

```
[Out] - (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^(
1/2)))/(2*a^(5/2))
```

3.24 $\int \frac{1}{x(ax+bx^3)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x(ax+bx^3)^2}\,dx = -\frac{1}{2a^2x^2} - \frac{b}{2a^2(a+bx^2)} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx^2)}{a^3}$$

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1598, 272, 46}

$$\int \frac{1}{x(ax+bx^3)^2}\,dx = \frac{b\log(a+bx^2)}{a^3} - \frac{2b\log(x)}{a^3} - \frac{b}{2a^2(a+bx^2)} - \frac{1}{2a^2x^2}$$

```
[In] Int[1/(x*(a*x + b*x^3)^2),x]
```

```
[Out] -1/2*1/(a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*
x^2])/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^3\left(a+bx^2\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx, x, x^2\right) \\
&= -\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = -\frac{a\left(\frac{1}{x^2}+\frac{b}{a+bx^2}\right)+4b\log(x)-2b\log\left(a+bx^2\right)}{2a^3}$$

```
[In] Integrate[1/(x*(a*x + b*x^3)^2),x]
[Out] -1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result	size
norman	$\frac{\frac{b^{2}x^{4}}{a^{3}}-\frac{1}{2a}}{x^{2}\left(b\,x^{2}+a\right)}+\frac{b\ln\left(b\,x^{2}+a\right)}{a^{3}}-\frac{2b\ln\left(x\right)}{a^{3}}$	52
risch	$\frac{-\frac{b\,x^{2}}{a^{2}}-\frac{1}{2a}}{x^{2}\left(b\,x^{2}+a\right)}-\frac{2b\ln\left(x\right)}{a^{3}}+\frac{b\ln\left(-b\,x^{2}-a\right)}{a^{3}}$	54
default	$-\frac{1}{2a^{2}x^{2}}-\frac{2b\ln\left(x\right)}{a^{3}}+\frac{b^{2}\left(\frac{2\ln\left(b\,x^{2}+a\right)}{b}-\frac{a}{b\left(b\,x^{2}+a\right)}\right)}{2a^{3}}$	55
parallelrisch	$-\frac{4b^{2}\ln\left(x\right)x^{4}-2b^{2}\ln\left(b\,x^{2}+a\right)x^{4}-2b^{2}x^{4}+4\ln\left(x\right)x^{2}ab-2\ln\left(b\,x^{2}+a\right)x^{2}ab+a^{2}}{2a^{3}x^{2}\left(b\,x^{2}+a\right)}$	80

```
[In] int(1/x/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (b^2/a^3*x^4-1/2/a)/x^2/(b*x^2+a)+b*ln(b*x^2+a)/a^3-2*b*ln(x)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.80 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \frac{1}{x\left(ax+bx^{3}\right)^{2}}\,dx=-\frac{2\,abx^{2}+a^{2}-2\left(b^{2}x^{4}+abx^{2}\right)\log\left(bx^{2}+a\right)+4\left(b^{2}x^{4}+abx^{2}\right)\log\left(x\right)}{2\left(a^{3}bx^{4}+a^{4}x^{2}\right)}$$

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(ax+bx^{3}\right)^{2}}\,dx=\frac{-a-2bx^{2}}{2a^{3}x^{2}+2a^{2}bx^{4}}-\frac{2b\log\left(x\right)}{a^{3}}+\frac{b\log\left(\frac{a}{b}+x^{2}\right)}{a^{3}}$$

```
[In] integrate(1/x/(b*x**3+a*x)**2,x)
```

```
[Out] (-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = -\frac{2\,bx^2+a}{2\left(a^2bx^4+a^3x^2\right)}+\frac{b\log\left(bx^2+a\right)}{a^3}-\frac{2\,b\log\left(x\right)}{a^3}$$

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - 2*b*log(x
)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = -\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bx^2+a}{2\left(bx^4+ax^2\right)a^2}$$

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx = \frac{b\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{\frac{1}{2\,a}+\frac{b\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{2\,b\,\ln\left(x\right)}{a^3}$$

```
[In] int(1/(x*(a*x + b*x^3)^2),x)
```

```
[Out] (b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*log
(x))/a^3
```

3.25 $\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx = -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

```
[Out] -5/6/a^2/x^3+5/2*b/a^3/x+1/2/a/x^3/(b*x^2+a)+5/2*b^(3/2)*arctan(x*b^(1/2)/a
^(1/2))/a^(7/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1598, 296, 331, 211}

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx = \frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

```
[In] Int[1/(x^2*(a*x + b*x^3)^2),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^4\left(a+bx^2\right)^2}\,dx \\
&= \frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a} \\
&= -\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}-\frac{(5b)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^2} \\
&= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{\left(5b^2\right)\int \frac{1}{a+bx^2}\,dx}{2a^3} \\
&= -\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{1}{2ax^3\left(a+bx^2\right)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx=-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}+\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

```
[In] Integrate[1/(x^2*(a*x + b*x^3)^2),x]
```

```
[Out] -1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{1}{3a^2x^3}+\frac{2b}{a^3x}+\frac{b^2\left(\frac{x}{2b\,x^2+2a}+\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$	55
risch	$\frac{\frac{5b^2x^4}{2a^3}+\frac{5b\,x^2}{3a^2}-\frac{1}{3a}}{x^3(b\,x^2+a)}+\frac{5\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{4a^4}-\frac{5\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{4a^4}$	91

```
[In] int(1/x^2/(b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2/x^3+2*b/a^3/x+b^2/a^3*(1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.53

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx$$

$$=\left[\frac{30\,b^2x^4+20\,abx^2+15\,(b^2x^5+abx^3)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-4\,a^2}{12\,(a^3bx^5+a^4x^3)},\ \frac{15\,b^2x^4+10\,abx^2+15\,(b^2x^5+a}{6\,(a^3bx^5}\right.$$

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
```

```
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.68

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx = -\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4} + \frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4} + \frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

```
[In] integrate(1/x**2/(b*x**3+a*x)**2,x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx = \frac{15\,b^2x^4+10\,abx^2-2\,a^2}{6\left(a^3bx^5+a^4x^3\right)} + \frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3}$$

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx = \frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3} + \frac{b^2x}{2\left(bx^2+a\right)a^3} + \frac{6\,bx^2-a}{3\,a^3x^3}$$

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx=\frac{\frac{5\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{5\,b^2\,x^4}{2\,a^3}}{b\,x^5+a\,x^3}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{7/2}}$$

```
[In] int(1/(x^2*(a*x + b*x^3)^2),x)
```

```
[Out] ((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b^
(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

3.26 $\int \frac{x^5}{x-x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^5}{x-x^3} \, dx = -x-\frac{x^3}{3}+\operatorname{arctanh}(x)$$

```
[Out] -x-1/3*x^3+arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 308, 212}

$$\int \frac{x^5}{x-x^3} \, dx = \operatorname{arctanh}(x)-\frac{x^3}{3}-x$$

```
[In] Int[x^5/(x - x^3),x]
```

```
[Out] -x - x^3/3 + ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^4}{1-x^2}\,dx\\ &= \int \left(-1-x^2+\frac{1}{1-x^2}\right)dx\\ &= -x-\frac{x^3}{3}+\int \frac{1}{1-x^2}\,dx\\ &= -x-\frac{x^3}{3}+\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.23

$$\int \frac{x^5}{x-x^3}\,dx=-x-\frac{x^3}{3}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

```
[In] Integrate[x^5/(x - x^3),x]
[Out] -x - x^3/3 - Log[1 - x]/2 + Log[1 + x]/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

method	result	size
meijerg	$-\frac{i\left(-\frac{2ix\left(5x^{2}+15\right)}{15}+2i\,\mathrm{arctanh}\left(x\right)\right)}{2}$	21
default	$-\frac{x^{3}}{3}-x-\frac{\ln\left(-1+x\right)}{2}+\frac{\ln\left(1+x\right)}{2}$	22
norman	$-\frac{x^{3}}{3}-x-\frac{\ln\left(-1+x\right)}{2}+\frac{\ln\left(1+x\right)}{2}$	22
risch	$-\frac{x^{3}}{3}-x-\frac{\ln\left(-1+x\right)}{2}+\frac{\ln\left(1+x\right)}{2}$	22
parallelrisch	$-\frac{x^{3}}{3}-x-\frac{\ln\left(-1+x\right)}{2}+\frac{\ln\left(1+x\right)}{2}$	22

```
[In] int(x^5/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] -1/2*I*(-2/15*I*x*(5*x^2+15)+2*I*arctanh(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{x^5}{x-x^3}\,dx = -\frac{1}{3}\,x^{3} - x + \frac{1}{2}\,\log\left(x + 1\right) - \frac{1}{2}\,\log\left(x - 1\right)$$

```
[In] integrate(x^5/(-x^3+x),x, algorithm="fricas")
[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{x^5}{x-x^3}\,dx = -\frac{x^{3}}{3} - x - \frac{\log\left(x - 1\right)}{2} + \frac{\log\left(x + 1\right)}{2}$$

```
[In] integrate(x**5/(-x**3+x),x)
[Out] -x**3/3 - x - log(x - 1)/2 + log(x + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{x^5}{x-x^3}\,dx = -\frac{1}{3}\,x^3 - x + \frac{1}{2}\,\log\left(x+1\right) - \frac{1}{2}\,\log\left(x-1\right)$$

```
[In] integrate(x^5/(-x^3+x),x, algorithm="maxima")
[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{x^5}{x-x^3}\,dx = -\frac{1}{3}\,x^3 - x + \frac{1}{2}\,\log\left(|x+1|\right) - \frac{1}{2}\,\log\left(|x-1|\right)$$

```
[In] integrate(x^5/(-x^3+x),x, algorithm="giac")
[Out] -1/3*x^3 - x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{x-x^3}\,dx = \operatorname{atanh}(x) - x - \frac{x^3}{3}$$

```
[In] int(x^5/(x - x^3),x)
[Out] atanh(x) - x - x^3/3
```

3.27 $\int \frac{x^4}{x-x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] -1/2*x^2-1/2*ln(-x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 272, 45}

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Int[x^4/(x - x^3),x]
```

```
[Out] -1/2*x^2 - Log[1 - x^2]/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^3}{1-x^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{1-x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-1+\frac{1}{1-x}\right)\,dx, x, x^2\right)\\ &= -\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{x^2}{2}-\frac{1}{2}\log\left(-1+x^2\right)$$

```
[In] Integrate[x^4/(x - x^3),x]
[Out] -1/2*x^2 - Log[-1 + x^2]/2
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{x^2}{2}-\frac{\ln(x^2-1)}{2}$	15
meijerg	$-\frac{x^2}{2}-\frac{\ln(-x^2+1)}{2}$	17
default	$-\frac{x^2}{2}-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	19
norman	$-\frac{x^2}{2}-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	19
parallelrisch	$-\frac{x^2}{2}-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	19

```
[In] int(x^4/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2-1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2-1\right)$$

```
[In] integrate(x^4/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 - 1/2*log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{x^2}{2} - \frac{\log\left(x^2-1\right)}{2}$$

```
[In] integrate(x**4/(-x**3+x),x)
```

```
[Out] -x**2/2 - log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x+1\right) - \frac{1}{2}\,\log\left(x-1\right)$$

```
[In] integrate(x^4/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(x^4/(-x^3+x),x, algorithm="giac")
[Out] -1/2*x^2 - 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{x-x^3}\,dx = -\frac{\ln\left(x^2-1\right)}{2} - \frac{x^2}{2}$$

```
[In] int(x^4/(x - x^3),x)
[Out] - log(x^2 - 1)/2 - x^2/2
```

3.28 $\int \frac{x^3}{x-x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{x^3}{x-x^3} \, dx = -x + \operatorname{arctanh}(x)$$

```
[Out] -x+arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1598, 327, 212}

$$\int \frac{x^3}{x-x^3} \, dx = \operatorname{arctanh}(x) - x$$

```
[In] Int[x^3/(x - x^3),x]
```

```
[Out] -x + ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2}{1-x^2}\,dx\\ &= -x+\int \frac{1}{1-x^2}\,dx\\ &= -x+\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 22 vs. $2(6) = 12$.

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 3.67

$$\int \frac{x^3}{x-x^3}\,dx = -x-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

```
[In] Integrate[x^3/(x - x^3),x]
[Out] -x - Log[1 - x]/2 + Log[1 + x]/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

method	result	size
meijerg	$\frac{i(2ix-2i\operatorname{arctanh}(x))}{2}$	14
default	$-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	17
norman	$-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	17
risch	$-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	17
parallelrisch	$-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	17

```
[In] int(x^3/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(2*I*x-2*I*arctanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{x^3}{x-x^3}\,dx = -x + \frac{1}{2}\log{(x+1)} - \frac{1}{2}\log{(x-1)}$$

```
[In] integrate(x^3/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(3) = 6$.

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{x^3}{x-x^3}\,dx = -x - \frac{\log{(x-1)}}{2} + \frac{\log{(x+1)}}{2}$$

```
[In] integrate(x**3/(-x**3+x),x)
```

```
[Out] -x - log(x - 1)/2 + log(x + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{x^3}{x-x^3}\,dx = -x + \frac{1}{2}\log{(x+1)} - \frac{1}{2}\log{(x-1)}$$

```
[In] integrate(x^3/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 3.00

$$\int \frac{x^3}{x - x^3}\,dx = -x + \frac{1}{2}\log\left(|x + 1|\right) - \frac{1}{2}\log\left(|x - 1|\right)$$

```
[In] integrate(x^3/(-x^3+x),x, algorithm="giac")
[Out] -x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{x - x^3}\,dx = \operatorname{atanh}(x) - x$$

```
[In] int(x^3/(x - x^3),x)
[Out] atanh(x) - x
```

3.29 $\int \frac{x^2}{x-x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{x^2}{x-x^3} \, dx = -\frac{1}{2} \log \left(1-x^2\right)$$

```
[Out] -1/2*ln(-x^2+1)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1598, 266}

$$\int \frac{x^2}{x-x^3} \, dx = -\frac{1}{2} \log \left(1-x^2\right)$$

```
[In] Int[x^2/(x - x^3),x]
```

```
[Out] -1/2*Log[1 - x^2]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \frac{x}{1-x^2}\,dx\\&=-\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{x-x^3}\,dx=-\frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Integrate[x^2/(x - x^3),x]
[Out] -1/2*Log[1 - x^2]
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\ln\left(x^{2}-1\right)}{2}$	9
meijerg	$-\frac{\ln\left(-x^{2}+1\right)}{2}$	11
default	$-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	14
norman	$-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	14
parallelrisch	$-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	14

```
[In] int(x^2/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{x-x^3}\,dx=-\frac{1}{2}\,\log\left(x^2-1\right)$$

```
[In] integrate(x^2/(-x^3+x),x, algorithm="fricas")
[Out] -1/2*log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{x-x^3}\,dx=-\frac{\log\left(x^2-1\right)}{2}$$

```
[In] integrate(x**2/(-x**3+x),x)
[Out] -log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{x-x^3}\,dx=-\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)$$

```
[In] integrate(x^2/(-x^3+x),x, algorithm="maxima")
[Out] -1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{x-x^3}\,dx=-\frac{1}{2}\,\log\left(|x+1|\right)-\frac{1}{2}\,\log\left(|x-1|\right)$$

```
[In] integrate(x^2/(-x^3+x),x, algorithm="giac")
[Out] -1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{x-x^3}\,dx = -\frac{\ln\left(x^2-1\right)}{2}$$

```
[In] int(x^2/(x - x^3),x)
```

```
[Out] -log(x^2 - 1)/2
```

3.30 $\int \frac{x}{x-x^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 2

$$\int \frac{x}{x-x^3} \, dx = \operatorname{arctanh}(x)$$

```
[Out] arctanh(x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1598, 212}

$$\int \frac{x}{x-x^3} \, dx = \operatorname{arctanh}(x)$$

```
[In] Int[x/(x - x^3),x]
```

```
[Out] ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{1-x^2}\,dx\\ &= \tanh^{-1}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(2) = 4$.

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 9.50

$$\int \frac{x}{x-x^3}\,dx = -\frac{1}{2}\log(1-x) + \frac{1}{2}\log(1+x)$$

```
[In] Integrate[x/(x - x^3),x]
[Out] -1/2*Log[1 - x] + Log[1 + x]/2
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\operatorname{arctanh}(x)$	3
meijerg	$\operatorname{arctanh}(x)$	3
norman	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
risch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
parallelrisch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14

```
[In] int(x/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] arctanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.

Time = 0.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{x}{x - x^3}\,dx = \frac{1}{2}\,\log\left(x + 1\right) - \frac{1}{2}\,\log\left(x - 1\right)$$

```
[In] integrate(x/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(2) = 4$.

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \frac{x}{x - x^3}\,dx = -\frac{\log\left(x - 1\right)}{2} + \frac{\log\left(x + 1\right)}{2}$$

```
[In] integrate(x/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 6.50

$$\int \frac{x}{x - x^3}\,dx = \frac{1}{2}\,\log\left(x + 1\right) - \frac{1}{2}\,\log\left(x - 1\right)$$

```
[In] integrate(x/(-x^3+x),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(2) = 4$.

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 7.50

$$\int \frac{x}{x-x^3}\,dx = \frac{1}{2}\log\left(|x+1|\right) - \frac{1}{2}\log\left(|x-1|\right)$$

```
[In] integrate(x/(-x^3+x),x, algorithm="giac")
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{x}{x-x^3}\,dx = \operatorname{atanh}(x)$$

```
[In] int(x/(x - x^3),x)
[Out] atanh(x)
```

3.31 $\int \frac{1}{x-x^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \frac{1}{x-x^3}\,dx = \log(x) - \frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] ln(x)-1/2*ln(-x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1607, 272, 36, 31, 29}

$$\int \frac{1}{x-x^3}\,dx = \log(x) - \frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Int[(x - x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 - x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x\left(1-x^2\right)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^2\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)\\ &= \log(x) - \frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x-x^3}\,dx = \log(x) - \frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Integrate[(x - x^3)^(-1),x]
[Out] Log[x] - Log[1 - x^2]/2
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^{2}-1\right)}{2}$	12
default	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	16
norman	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	16
parallelrisch	$\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	16
meijerg	$\ln\left(x\right)+\frac{i\pi}{2}-\frac{\ln\left(-x^{2}+1\right)}{2}$	18

```
[In] int(1/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{x-x^3}\,dx=-\frac{1}{2}\,\log\left(x^{2}-1\right)+\log\left(x\right)$$

```
[In] integrate(1/(-x^3+x),x, algorithm="fricas")
[Out] -1/2*log(x^2 - 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{x-x^3}\,dx=\log\left(x\right)-\frac{\log\left(x^{2}-1\right)}{2}$$

```
[In] integrate(1/(-x**3+x),x)
[Out] log(x) - log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x-x^3}\,dx = -\frac{1}{2}\log\left(x+1\right) - \frac{1}{2}\log\left(x-1\right) + \log\left(x\right)$$

```
[In] integrate(1/(-x^3+x),x, algorithm="maxima")
[Out] -1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1}{x-x^3}\,dx = \frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(1/(-x^3+x),x, algorithm="giac")
[Out] 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{x-x^3}\,dx = \ln\left(x\right) - \frac{\ln\left(x^2-1\right)}{2}$$

```
[In] int(1/(x - x^3),x)
[Out] log(x) - log(x^2 - 1)/2
```

3.32 $\int \frac{1}{x(x-x^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{1}{x(x-x^3)}\,dx = -\frac{1}{x} + \operatorname{arctanh}(x)$$

```
[Out] -1/x+arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 331, 212}

$$\int \frac{1}{x(x-x^3)}\,dx = \operatorname{arctanh}(x) - \frac{1}{x}$$

```
[In] Int[1/(x*(x - x^3)),x]
```

```
[Out] -x^(-1) + ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^2\,(1-x^2)}\,dx\\ &= -\frac{1}{x}+\int \frac{1}{1-x^2}\,dx\\ &= -\frac{1}{x}+\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 24 vs. 2(8) = 16.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 3.00

$$\int \frac{1}{x\,(x-x^3)}\,dx = -\frac{1}{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

```
[In] Integrate[1/(x*(x - x^3)),x]
[Out] -x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

method	result	size
meijerg	$\frac{i\left(\frac{2i}{x}-2i\,\mathrm{arctanh}(x)\right)}{2}$	16
default	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}-\frac{1}{x}$	19
norman	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}-\frac{1}{x}$	19
risch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}-\frac{1}{x}$	19
parallelrisch	$\frac{\ln(1+x)x-\ln(-1+x)x-2}{2x}$	21

```
[In] int(1/x/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] 1/2*I*(2*I/x-2*I*arctanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\,(x-x^3)}\,dx = \frac{x\log\left(x+1\right) - x\log\left(x-1\right) - 2}{2\,x}$$

```
[In] integrate(1/x/(-x^3+x),x, algorithm="fricas")
[Out] 1/2*(x*log(x + 1) - x*log(x - 1) - 2)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1}{x\,(x-x^3)}\,dx = -\frac{\log\left(x-1\right)}{2} + \frac{\log\left(x+1\right)}{2} - \frac{1}{x}$$

```
[In] integrate(1/x/(-x**3+x),x)
[Out] -log(x - 1)/2 + log(x + 1)/2 - 1/x
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{x\left(x-x^3\right)}\,dx = -\frac{1}{x} + \frac{1}{2}\,\log\left(x+1\right) - \frac{1}{2}\,\log\left(x-1\right)$$

```
[In] integrate(1/x/(-x^3+x),x, algorithm="maxima")
[Out] -1/x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\left(x-x^3\right)}\,dx = -\frac{1}{x} + \frac{1}{2}\,\log\left(|x+1|\right) - \frac{1}{2}\,\log\left(|x-1|\right)$$

```
[In] integrate(1/x/(-x^3+x),x, algorithm="giac")
[Out] -1/x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(x-x^3\right)}\,dx = \operatorname{atanh}(x) - \frac{1}{x}$$

```
[In] int(1/(x*(x - x^3)),x)
[Out] atanh(x) - 1/x
```

3.33 $\int \frac{1}{x^2(x-x^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x^2\left(x-x^3\right)}\,dx=-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] -1/2/x^2+ln(x)-1/2*ln(-x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 272, 46}

$$\int \frac{1}{x^2\left(x-x^3\right)}\,dx=-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

```
[In] Int[1/(x^2*(x - x^3)),x]
```

```
[Out] -1/2*1/x^2 + Log[x] - Log[1 - x^2]/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^3(1-x^2)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^2}+\frac{1}{x}\right)\,dx, x, x^2\right)\\ &= -\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(x-x^3)}\,dx = -\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Integrate[1/(x^2*(x - x^3)),x]
[Out] -1/2*1/x^2 + Log[x] - Log[1 - x^2]/2
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{1}{2x^2}+\ln(x)-\frac{\ln(x^2-1)}{2}$	17
default	$-\frac{1}{2x^2}+\ln(x)-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	21
norman	$-\frac{1}{2x^2}+\ln(x)-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$	21
meijerg	$-\frac{1}{2x^2}+\ln(x)+\frac{i\pi}{2}-\frac{\ln(-x^2+1)}{2}$	23
parallelrisch	$\frac{2\ln(x)x^2-\ln(1+x)x^2-\ln(-1+x)x^2-1}{2x^2}$	33

```
[In] int(1/x^2/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] -1/2/x^2+ln(x)-1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(x-x^3\right)}\,dx=-\frac{x^2\log\left(x^2-1\right)-2\,x^2\log\left(x\right)+1}{2\,x^2}$$

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="fricas")
[Out] -1/2*(x^2*log(x^2 - 1) - 2*x^2*log(x) + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2\left(x-x^3\right)}\,dx=\log\left(x\right)-\frac{\log\left(x^2-1\right)}{2}-\frac{1}{2x^2}$$

```
[In] integrate(1/x**2/(-x**3+x),x)
[Out] log(x) - log(x**2 - 1)/2 - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(x-x^3\right)}\,dx=-\frac{1}{2\,x^2}-\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)+\log\left(x\right)$$

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="maxima")
[Out] -1/2/x^2 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2(x-x^3)}\,dx = -\frac{x^2+1}{2\,x^2} + \frac{1}{2}\,\log\left(x^2\right) - \frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="giac")
[Out] -1/2*(x^2 + 1)/x^2 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2(x-x^3)}\,dx = \ln(x) - \frac{\ln(x^2-1)}{2} - \frac{1}{2\,x^2}$$

```
[In] int(1/(x^2*(x - x^3)),x)
[Out] log(x) - log(x^2 - 1)/2 - 1/(2*x^2)
```

3.34 $\int \frac{1}{x^3(x-x^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{1}{x^3\left(x-x^3\right)}\,dx=-\frac{1}{3x^3}-\frac{1}{x}+\operatorname{arctanh}(x)$$

```
[Out] -1/3/x^3-1/x+arctanh(x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 331, 212}

$$\int \frac{1}{x^3\left(x-x^3\right)}\,dx=\operatorname{arctanh}(x)-\frac{1}{3x^3}-\frac{1}{x}$$

```
[In] Int[1/(x^3*(x - x^3)),x]
```

```
[Out] -1/3*1/x^3 - x^(-1) + ArcTanh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^4(1-x^2)}\,dx\\ &= -\frac{1}{3x^3}+\int \frac{1}{x^2(1-x^2)}\,dx\\ &= -\frac{1}{3x^3}-\frac{1}{x}+\int \frac{1}{1-x^2}\,dx\\ &= -\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{1}{x^3(x-x^3)}\,dx = -\frac{1}{3x^3}-\frac{1}{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

```
[In] Integrate[1/(x^3*(x - x^3)),x]
[Out] -1/3*1/x^3 - x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

method	result	size
meijerg	$-\frac{i\left(-\frac{2i}{x}-\frac{2i}{3x^{3}}+2i\,\mathrm{arctanh}(x)\right)}{2}$	22
default	$-\frac{1}{3x^{3}}-\frac{1}{x}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	24
norman	$\frac{-\frac{1}{3}-x^{2}}{x^{3}}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	25
risch	$\frac{-\frac{1}{3}-x^{2}}{x^{3}}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	25
parallelrisch	$\frac{3\ln(1+x)x^{3}-3\ln(-1+x)x^{3}-2-6x^{2}}{6x^{3}}$	31

```
[In] int(1/x^3/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] -1/2*I*(-2*I/x-2/3*I/x^3+2*I*arctanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^3\left(x-x^3\right)}\,dx=\frac{3\,x^3\log\left(x+1\right)-3\,x^3\log\left(x-1\right)-6\,x^2-2}{6\,x^3}$$

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="fricas")
[Out] 1/6*(3*x^3*log(x + 1) - 3*x^3*log(x - 1) - 6*x^2 - 2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{1}{x^3\left(x-x^3\right)}\,dx=-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}-\frac{3x^2+1}{3x^3}$$

```
[In] integrate(1/x**3/(-x**3+x),x)
[Out] -log(x - 1)/2 + log(x + 1)/2 - (3*x**2 + 1)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^3 (x - x^3)}\, dx = -\frac{3\, x^2 + 1}{3\, x^3} + \frac{1}{2}\, \log\left(x + 1\right) - \frac{1}{2}\, \log\left(x - 1\right)$$

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="maxima")
[Out] -1/3*(3*x^2 + 1)/x^3 + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \frac{1}{x^3 (x - x^3)}\, dx = -\frac{3\, x^2 + 1}{3\, x^3} + \frac{1}{2}\, \log\left(|x + 1|\right) - \frac{1}{2}\, \log\left(|x - 1|\right)$$

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="giac")
[Out] -1/3*(3*x^2 + 1)/x^3 + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (x - x^3)}\, dx = \operatorname{atanh}(x) - \frac{x^2 + \frac{1}{3}}{x^3}$$

```
[In] int(1/(x^3*(x - x^3)),x)
[Out] atanh(x) - (x^2 + 1/3)/x^3
```

3.35 $\int \frac{1}{x^4(x-x^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{1}{x^4(x-x^3)}\,dx = -\frac{1}{4x^4}-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] -1/4/x^4-1/2/x^2+ln(x)-1/2*ln(-x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 272, 46}

$$\int \frac{1}{x^4(x-x^3)}\,dx = -\frac{1}{4x^4}-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

```
[In] Int[1/(x^4*(x - x^3)),x]
```

```
[Out] -1/4*1/x^4 - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^5\left(1-x^2\right)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x^3}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right)\,dx, x, x^2\right)\\ &= -\frac{1}{4x^4}-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(x-x^3\right)}\,dx = -\frac{1}{4x^4}-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

```
[In] Integrate[1/(x^4*(x - x^3)),x]
[Out] -1/4*1/x^4 - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{-\frac{1}{4}-\frac{x^2}{2}}{x^4}+\ln\left(x\right)-\frac{\ln\left(x^2-1\right)}{2}$	23
default	$-\frac{1}{4x^4}-\frac{1}{2x^2}+\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	26
norman	$\frac{-\frac{1}{4}-\frac{x^2}{2}}{x^4}+\ln\left(x\right)-\frac{\ln\left(-1+x\right)}{2}-\frac{\ln\left(1+x\right)}{2}$	27
meijerg	$-\frac{1}{4x^4}-\frac{1}{2x^2}+\ln\left(x\right)+\frac{i\pi}{2}-\frac{\ln\left(-x^2+1\right)}{2}$	28
parallelrisch	$\frac{4\ln\left(x\right)x^4-2\ln\left(1+x\right)x^4-2\ln\left(-1+x\right)x^4-1-2x^2}{4x^4}$	38

```
[In] int(1/x^4/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4-1/2*x^2)/x^4+ln(x)-1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4\left(x-x^3\right)}\,dx = -\frac{2\,x^4\log\left(x^2-1\right)-4\,x^4\log\left(x\right)+2\,x^2+1}{4\,x^4}$$

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^4*log(x^2 - 1) - 4*x^4*log(x) + 2*x^2 + 1)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4\left(x-x^3\right)}\,dx = \log\left(x\right)-\frac{\log\left(x^2-1\right)}{2}-\frac{2x^2+1}{4x^4}$$

```
[In] integrate(1/x**4/(-x**3+x),x)
```

```
[Out] log(x) - log(x**2 - 1)/2 - (2*x**2 + 1)/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4\left(x-x^3\right)}\,dx = -\frac{2\,x^2+1}{4\,x^4}-\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)+\log\left(x\right)$$

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 + 1)/x^4 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 (x - x^3)}\, dx = -\frac{3\, x^4 + 2\, x^2 + 1}{4\, x^4} + \frac{1}{2}\, \log\left(x^2\right) - \frac{1}{2}\, \log\left(\left|x^2 - 1\right|\right)$$

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/4*(3*x^4 + 2*x^2 + 1)/x^4 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^4 (x - x^3)}\, dx = \ln(x) - \frac{\ln(x^2 - 1)}{2} - \frac{\frac{x^2}{2} + \frac{1}{4}}{x^4}$$

```
[In] int(1/(x^4*(x - x^3)),x)
```

```
[Out] log(x) - log(x^2 - 1)/2 - (x^2/2 + 1/4)/x^4
```

3.36 $\int \frac{1}{x+bx^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int \frac{1}{x+bx^3}\,dx=\log(x)-\frac{1}{2}\log\left(1+bx^2\right)$$

```
[Out] ln(x)-1/2*ln(b*x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1607, 272, 36, 29, 31}

$$\int \frac{1}{x+bx^3}\,dx=\log(x)-\frac{1}{2}\log\left(bx^2+1\right)$$

```
[In] Int[(x + b*x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x\left(1+bx^2\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+bx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}b\text{Subst}\left(\int \frac{1}{1+bx}\,dx, x, x^2\right) \\
&= \log(x) - \frac{1}{2}\log\left(1+bx^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+bx^3}\,dx = \log(x) - \frac{1}{2}\log\left(1+bx^2\right)$$

```
[In] Integrate[(x + b*x^3)^(-1),x]
[Out] Log[x] - Log[1 + b*x^2]/2
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
norman	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
risch	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
parallelrisch	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
meijerg	$\ln\left(x\right)+\frac{\ln\left(b\right)}{2}-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	18

```
[In] int(1/(b*x^3+x),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/2*ln(b*x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x+bx^3}\,dx=-\frac{1}{2}\,\log\left(bx^2+1\right)+\log\left(x\right)$$

```
[In] integrate(1/(b*x^3+x),x, algorithm="fricas")
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{1}{x+bx^3}\,dx=\log\left(x\right)-\frac{\log\left(x^2+\frac{1}{b}\right)}{2}$$

```
[In] integrate(1/(b*x**3+x),x)
[Out] log(x) - log(x**2 + 1/b)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x+bx^3}\,dx = -\frac{1}{2}\,\log\left(bx^2+1\right)+\log\left(x\right)$$

```
[In] integrate(1/(b*x^3+x),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{x+bx^3}\,dx = \frac{1}{2}\,\log\left(x^2\right)-\frac{1}{2}\,\log\left(\left|bx^2+1\right|\right)$$

```
[In] integrate(1/(b*x^3+x),x, algorithm="giac")
[Out] 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 10.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{x+bx^3}\,dx = \ln\left(x\right)-\frac{\ln\left(\frac{3\,b\,x^2}{2}+\frac{3}{2}\right)}{2}$$

```
[In] int(1/(x + b*x^3),x)
[Out] log(x) - log((3*b*x^2)/2 + 3/2)/2
```

3.37 $\int \frac{1}{-x+bx^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{-x+bx^3}\,dx = -\log(x)+\frac{1}{2}\log\left(1-bx^2\right)$$

```
[Out] -ln(x)+1/2*ln(-b*x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {1607, 272, 36, 29, 31}

$$\int \frac{1}{-x+bx^3}\,dx = \frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

```
[In] Int[(-x + b*x^3)^(-1),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x\left(-1+bx^2\right)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(-1+bx)}\,dx,x,x^2\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)\right)+\frac{1}{2}b\text{Subst}\left(\int \frac{1}{-1+bx}\,dx,x,x^2\right)\\ &= -\log(x)+\frac{1}{2}\log\left(1-bx^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+bx^3}\,dx = -\log(x)+\frac{1}{2}\log\left(1-bx^2\right)$$

```
[In] Integrate[(-x + b*x^3)^(-1),x]
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
default	$-\ln\left(x\right)+\frac{\ln\left(b\,x^{2}-1\right)}{2}$	16
norman	$-\ln\left(x\right)+\frac{\ln\left(b\,x^{2}-1\right)}{2}$	16
parallelrisch	$-\ln\left(x\right)+\frac{\ln\left(b\,x^{2}-1\right)}{2}$	16
risch	$-\ln\left(x\right)+\frac{\ln\left(-b\,x^{2}+1\right)}{2}$	17
meijerg	$-\ln\left(x\right)-\frac{\ln\left(-b\right)}{2}+\frac{\ln\left(-b\,x^{2}+1\right)}{2}$	23

```
[In] int(1/(b*x^3-x),x,method=_RETURNVERBOSE)
[Out] -ln(x)+1/2*ln(b*x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{-x+bx^3}\,dx=\frac{1}{2}\log\left(bx^2-1\right)-\log\left(x\right)$$

```
[In] integrate(1/(b*x^3-x),x, algorithm="fricas")
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{-x+bx^3}\,dx=-\log\left(x\right)+\frac{\log\left(x^2-\frac{1}{b}\right)}{2}$$

```
[In] integrate(1/(b*x**3-x),x)
[Out] -log(x) + log(x**2 - 1/b)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{-x+bx^3}\,dx = \frac{1}{2}\log\left(bx^2-1\right) - \log\left(x\right)$$

```
[In] integrate(1/(b*x^3-x),x, algorithm="maxima")
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x+bx^3}\,dx = -\frac{1}{2}\log\left(x^2\right) + \frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

```
[In] integrate(1/(b*x^3-x),x, algorithm="giac")
[Out] -1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{-x+bx^3}\,dx = \frac{\ln\left(\frac{3}{2}-\frac{3\,b\,x^2}{2}\right)}{2} - \ln\left(x\right)$$

```
[In] int(-1/(x - b*x^3),x)
[Out] log(3/2 - (3*b*x^2)/2)/2 - log(x)
```

3.38 $\int x^3\sqrt{ax+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 163

$$\begin{aligned}\int x^3\sqrt{ax+bx^3}\,dx &= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x^4\sqrt{ax+bx^3}\\ &\quad+\frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}\end{aligned}$$

```
[Out] -20/231*a^2*(b*x^3+a*x)^(1/2)/b^2+4/77*a*x^2*(b*x^3+a*x)^(1/2)/b+2/11*x^4*(
b*x^3+a*x)^(1/2)+10/231*a^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^
(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2046, 2049, 2036, 335, 226}

$$\begin{aligned}\int x^3\sqrt{ax+bx^3}\,dx &= \frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}\\ &\quad-\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}\end{aligned}$$

```
[In] Int[x^3*Sqrt[a*x + b*x^3],x]
```

```
[Out] (-20*a^2*Sqrt[a*x + b*x^3])/(231*b^2) + (4*a*x^2*Sqrt[a*x + b*x^3])/(77*b)
+ (2*x^4*Sqrt[a*x + b*x^3])/11 + (10*a^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[x])/a^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{1}{11}(2a)\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$$

$$
\begin{aligned}
&= \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} - \frac{(10a^2)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{77b} \\
&= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{(10a^3)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{231b^2} \\
&= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{\left(10a^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{231b^2\sqrt{ax+bx^3}} \\
&= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} \\
&\quad + \frac{\left(20a^3\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{231b^2\sqrt{ax+bx^3}} \\
&= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} \\
&\quad + \frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$
\int x^3\sqrt{ax+bx^3}\,dx
$$

$$
= \frac{2\sqrt{x\left(a+bx^2\right)}\left(\sqrt{1+\frac{bx^2}{a}}\left(-5a^2+2abx^2+7b^2x^4\right)+5a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt{1+\frac{bx^2}{a}}}
$$

```
[In] Integrate[x^3*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*(Sqrt[1 + (b*x^2)/a]*(-5*a^2 + 2*a*b*x^2 + 7*b^2*x^4
) + 5*a^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(77*b^2*Sqrt[1
+ (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{2\left(-21 b^{2} x^{4}-6 a b\,x^{2}+10 a^{2}\right) x\left(b\,x^{2}+a\right)}{231 b^{2} \sqrt{x\left(b\,x^{2}+a\right)}}+\frac{10 a^{3} \sqrt{-a b}\, \sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{x b}{\sqrt{-a b}}}\, F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right)}{231 b^{3} \sqrt{b\,x^{3}+a x}}$
default	$\frac{2 x^{4} \sqrt{b\,x^{3}+a x}}{11}+\frac{4 a\,x^{2} \sqrt{b\,x^{3}+a x}}{77 b}-\frac{20 a^{2} \sqrt{b\,x^{3}+a x}}{231 b^{2}}+\frac{10 a^{3} \sqrt{-a b}\, \sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{x b}{\sqrt{-a b}}}\, F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right.}{\sqrt{-a b}}}\right.}{231 b^{3} \sqrt{b\,x^{3}+a x}}$
elliptic	$\frac{2 x^{4} \sqrt{b\,x^{3}+a x}}{11}+\frac{4 a\,x^{2} \sqrt{b\,x^{3}+a x}}{77 b}-\frac{20 a^{2} \sqrt{b\,x^{3}+a x}}{231 b^{2}}+\frac{10 a^{3} \sqrt{-a b}\, \sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\, \sqrt{-\frac{x b}{\sqrt{-a b}}}\, F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right.}{\sqrt{-a b}}}\right.}{231 b^{3} \sqrt{b\,x^{3}+a x}}$

```
[In] int(x^3*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/231*(-21*b^2*x^4-6*a*b*x^2+10*a^2)/b^2*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+1
0/231*a^3/b^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x
-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x
)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int x^3\sqrt{ax+bx^3}\,dx$$
$$=\frac{2\left(10\,a^3\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(21\,b^3x^4+6\,ab^2x^2-10\,a^2b\right)\sqrt{bx^3+ax}\right)}{231\,b^3}$$

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(10*a^3*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (21*b^3*x^4 + 6*a
*b^2*x^2 - 10*a^2*b)*sqrt(b*x^3 + a*x))/b^3
```

Sympy [F]

$$\int x^3\sqrt{ax+bx^3}\,dx=\int x^3\sqrt{x\left(a+bx^2\right)}\,dx$$

```
[In] integrate(x**3*(b*x**3+a*x)**(1/2),x)
[Out] Integral(x**3*sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int x^3\sqrt{ax+bx^3}\,dx=\int \sqrt{bx^3+ax}x^3\,dx$$

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{ax+bx^3}\,dx=\int \sqrt{bx^3+ax}x^3\,dx$$

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{ax+bx^3}\,dx=\int x^3\,\sqrt{b\,x^3+a\,x}\,dx$$

```
[In] int(x^3*(a*x + b*x^3)^(1/2),x)
[Out] int(x^3*(a*x + b*x^3)^(1/2), x)
```

3.39 $\int x^2\sqrt{ax+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 281

$$\begin{aligned}\int x^2\sqrt{ax+bx^3}\,dx &= -\frac{4a^2x(a+bx^2)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}\\ &\quad+\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}\\ &\quad-\frac{2a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}\end{aligned}$$

```
[Out] -4/15*a^2*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+4/45*a*
x*(b*x^3+a*x)^(1/2)/b+2/9*x^3*(b*x^3+a*x)^(1/2)+4/15*a^(9/4)*(cos(2*arctan(
b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*E
llipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(
1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(
1/2)-2/15*a^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2)
)^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2046, 2049, 2057, 335, 311, 226, 1210}

$$\begin{aligned}\int x^2\sqrt{ax+bx^3}\,dx = &-\frac{2a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}\\ &+\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}\\ &-\frac{4a^2x(a+bx^2)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{2}{9}x^3\sqrt{ax+bx^3}+\frac{4ax\sqrt{ax+bx^3}}{45b}\end{aligned}$$

```
[In] Int[x^2*Sqrt[a*x + b*x^3],x]
```

```
[Out] (-4*a^2*x*(a + b*x^2))/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
 + (4*a*x*Sqrt[a*x + b*x^3])/(45*b) + (2*x^3*Sqrt[a*x + b*x^3])/9 + (4*a^(9
/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]
*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a*x
+ b*x^3]) - (2*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt
[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(1
5*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{9}x^3\sqrt{ax+bx^3} + \frac{1}{9}(2a)\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx \\
&= \frac{4ax\sqrt{ax+bx^3}}{45b} + \frac{2}{9}x^3\sqrt{ax+bx^3} - \frac{(2a^2)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{15b} \\
&= \frac{4ax\sqrt{ax+bx^3}}{45b} + \frac{2}{9}x^3\sqrt{ax+bx^3} - \frac{\left(2a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{15b\sqrt{ax+bx^3}} \\
&= \frac{4ax\sqrt{ax+bx^3}}{45b} + \frac{2}{9}x^3\sqrt{ax+bx^3} - \frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{15b\sqrt{ax+bx^3}}
\end{aligned}$$

$$= \frac{4ax\sqrt{ax+bx^3}}{45b} + \frac{2}{9}x^3\sqrt{ax+bx^3} - \frac{\left(4a^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15b^{3/2}\sqrt{ax+bx^3}} + \frac{\left(4a^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15b^{3/2}\sqrt{ax+bx^3}}$$

$$= -\frac{4a^2x(a+bx^2)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{4ax\sqrt{ax+bx^3}}{45b} + \frac{2}{9}x^3\sqrt{ax+bx^3} + \frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}} - \frac{2a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.28

$$\int x^2\sqrt{ax+bx^3}\,dx = \frac{2x\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{9b\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[x^2*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometri
c2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.70

method	result
default	$\frac{2x^{3}\sqrt{b\,x^{3}+ax}}{9}+\frac{4ax\sqrt{b\,x^{3}+ax}}{45b}-\frac{2a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15b^{2}\sqrt{b\,x^{3}+ax}}$
elliptic	$\frac{2x^{3}\sqrt{b\,x^{3}+ax}}{9}+\frac{4ax\sqrt{b\,x^{3}+ax}}{45b}-\frac{2a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15b^{2}\sqrt{b\,x^{3}+ax}}$
risch	$\frac{2x^{2}\left(5b\,x^{2}+2a\right)\left(b\,x^{2}+a\right)}{45b\sqrt{x\left(b\,x^{2}+a\right)}}-\frac{2a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15b^{2}\sqrt{b\,x^{3}+ax}}$

```
[In] int(x^2*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x^3*(b*x^3+a*x)^(1/2)+4/45*a*x*(b*x^3+a*x)^(1/2)/b-2/15*a^2/b^2*(-a*b)^
(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*
b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1
/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*
b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.20

$$\int x^{2}\sqrt{ax+bx^{3}}\,dx$$
$$=\frac{2\left(6\,a^{2}\sqrt{b}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(5\,b^{2}x^{3}+2\,abx\right)\sqrt{bx^{3}+ax}\right)}{45\,b^{2}}$$

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(6*a^2*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
0, x)) + (5*b^2*x^3 + 2*a*b*x)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]

$$\int x^2\sqrt{ax+bx^3}\,dx=\int x^2\sqrt{x\,(a+bx^2)}\,dx$$

```
[In] integrate(x**2*(b*x**3+a*x)**(1/2),x)
[Out] Integral(x**2*sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int x^2\sqrt{ax+bx^3}\,dx=\int \sqrt{bx^3+ax}x^2\,dx$$

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{ax+bx^3}\,dx=\int \sqrt{bx^3+ax}x^2\,dx$$

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{ax+bx^3}\,dx=\int x^2\,\sqrt{b\,x^3+a\,x}\,dx$$

```
[In] int(x^2*(a*x + b*x^3)^(1/2),x)
[Out] int(x^2*(a*x + b*x^3)^(1/2), x)
```

3.40 $\int x\sqrt{ax+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 137

$$\int x\sqrt{ax+bx^3}\,dx=\frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}-\frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] 4/21*a*(b*x^3+a*x)^(1/2)/b+2/7*x^2*(b*x^3+a*x)^(1/2)-2/21*a^(7/4)*(cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^3+a
*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2046, 2049, 2036, 335, 226}

$$\int x\sqrt{ax+bx^3}\,dx=-\frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}+\frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

```
[In] Int[x*Sqrt[a*x + b*x^3],x]
```

```
[Out] (4*a*Sqrt[a*x + b*x^3])/(21*b) + (2*x^2*Sqrt[a*x + b*x^3])/7 - (2*a^(7/4)*S
qrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[a*x + b*x
^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{2}{7}x^2\sqrt{ax+bx^3} + \frac{1}{7}(2a)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx$$

$$
\begin{aligned}
&= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{(2a^2)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{21b} \\
&= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{\left(2a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{21b\sqrt{ax+bx^3}} \\
&= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{21b\sqrt{ax+bx^3}} \\
&= \frac{4a\sqrt{ax+bx^3}}{21b} + \frac{2}{7}x^2\sqrt{ax+bx^3} - \frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.58

$$
\int x\sqrt{ax+bx^3}\,dx = \frac{2\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}} - a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{7b\sqrt{1+\frac{bx^2}{a}}}
$$

```
[In] Integrate[x*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometric2
F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{2x^2\sqrt{b\,x^3+ax}}{7}+\frac{4a\sqrt{b\,x^3+ax}}{21b}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^2\sqrt{b\,x^3+ax}}$	146
elliptic	$\frac{2x^2\sqrt{b\,x^3+ax}}{7}+\frac{4a\sqrt{b\,x^3+ax}}{21b}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^2\sqrt{b\,x^3+ax}}$	146
risch	$\frac{2(3b\,x^2+2a)\,x\,(b\,x^2+a)}{21b\sqrt{x\,(b\,x^2+a)}}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^2\sqrt{b\,x^3+ax}}$	147

```
[In] int(x*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*x^2*(b*x^3+a*x)^(1/2)+4/21*a*(b*x^3+a*x)^(1/2)/b-2/21*a^2/b^2*(-a*b)^(1
/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(
-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.36

$$\int x\sqrt{ax+bx^3}\,dx=-\frac{2\left(2\,a^2\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\left(3\,b^2x^2+2\,ab\right)\sqrt{bx^3+ax}\right)}{21\,b^2}$$

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/21*(2*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (3*b^2*x^2 + 2*a*b
)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]

$$\int x\sqrt{ax+bx^3}\,dx=\int x\sqrt{x\left(a+bx^2\right)}\,dx$$

```
[In] integrate(x*(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int x\sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}x\,dx$$

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x)*x, x)
```

Giac [F]

$$\int x\sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}x\,dx$$

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{ax+bx^3}\,dx = \int x\,\sqrt{b\,x^3+a\,x}\,dx$$

```
[In] int(x*(a*x + b*x^3)^(1/2),x)
[Out] int(x*(a*x + b*x^3)^(1/2), x)
```

3.41 $\int \sqrt{ax+bx^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 255

$$
\begin{aligned}
\int \sqrt{ax+bx^3}\,dx = {} & \frac{4ax(a+bx^2)}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3} \\
& - \frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}} \\
& + \frac{2a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

```
[Out] 4/5*a*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+2/5*x*(b*x^
3+a*x)^(1/2)-4/5*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*
b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)+2/5*a^(5/4)*(cos(2*arctan(b^(1/
4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ellipt
icF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))
*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2029, 2057, 335, 311, 226, 1210}

$$\int \sqrt{ax+bx^3}\,dx = \frac{2a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}} - \frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}} + \frac{2}{5}x\sqrt{ax+bx^3} + \frac{4ax(a+bx^2)}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}$$

```
[In] Int[Sqrt[a*x + b*x^3],x]
```

```
[Out] (4*a*x*(a + b*x^2))/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) + (
2*x*Sqrt[a*x + b*x^3])/5 - (4*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(5*b^(3/4)*Sqrt[a*x + b*x^3]) + (2*a^(5/4)*Sqrt[x]*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^
(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{5}x\sqrt{ax+bx^3}+\frac{1}{5}(2a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{2}{5}x\sqrt{ax+bx^3}+\frac{\left(2a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}}\\
&= \frac{2}{5}x\sqrt{ax+bx^3}+\frac{\left(4a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}\\
&= \frac{2}{5}x\sqrt{ax+bx^3}+\frac{\left(4a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{b}\sqrt{ax+bx^3}}\\
&\quad -\frac{\left(4a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{b}\sqrt{ax+bx^3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{4ax(a+bx^2)}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3} \\
&\quad - \frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}} \\
&\quad + \frac{2a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.20

$$
\int \sqrt{ax+bx^3}\,dx = \frac{2x\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}
$$

```
[In] Integrate[Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)])/(
3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.69

method	result
default	$\frac{2x\sqrt{b\,x^{3}+ax}}{5}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{5b\sqrt{b\,x^{3}+ax}}$
elliptic	$\frac{2x\sqrt{b\,x^{3}+ax}}{5}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{5b\sqrt{b\,x^{3}+ax}}$
risch	$\frac{2x^{2}\left(b\,x^{2}+a\right)}{5\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{5b\sqrt{b\,x^{3}+ax}}$

```
[In] int((b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x*(b*x^3+a*x)^(1/2)+2/5*a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/
2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b
)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/
(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2
)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \sqrt{ax+bx^3}\,dx$$
$$=\frac{2\left(\sqrt{bx^3+ax}bx-2\,a\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)\right)}{5\,b}$$

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(sqrt(b*x^3 + a*x)*b*x - 2*a*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierst
rassPInverse(-4*a/b, 0, x)))/b
```

Sympy [F]

$$\int \sqrt{ax+bx^3}\,dx = \int \sqrt{ax+bx^3}\,dx$$

```
[In] integrate((b*x**3+a*x)**(1/2),x)
[Out] Integral(sqrt(a*x + b*x**3), x)
```

Maxima [F]

$$\int \sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}\,dx$$

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \sqrt{ax+bx^3}\,dx = \int \sqrt{bx^3+ax}\,dx$$

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x), x)
```

Mupad [B] (verification not implemented)

Time = 11.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.16

$$\int \sqrt{ax+bx^3}\,dx = \frac{2\,x\,\sqrt{b\,x^3+a\,x}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{b\,x^2}{a}\right)}{3\,\sqrt{\frac{b\,x^2}{a}+1}}$$

```
[In] int((a*x + b*x^3)^(1/2),x)
[Out] (2*x*(a*x + b*x^3)^(1/2)*hypergeom([-1/2, 3/4], 7/4, -(b*x^2)/a))/(3*((b*x^
2)/a + 1)^(1/2))
```

3.42 $\int \frac{\sqrt{ax+bx^3}}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \frac{2}{3}\sqrt{ax+bx^3} + \frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}$$

```
[Out] 2/3*(b*x^3+a*x)^(1/2)+2/3*a^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/
4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a
^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2046, 2036, 335, 226}

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}} + \frac{2}{3}\sqrt{ax+bx^3}$$

```
[In] Int[Sqrt[a*x + b*x^3]/x,x]
```

```
[Out] (2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{3}\sqrt{ax+bx^3}+\frac{1}{3}(2a)\int\frac{1}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{2}{3}\sqrt{ax+bx^3}+\frac{\left(2a\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}}\\
&= \frac{2}{3}\sqrt{ax+bx^3}+\frac{\left(4a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{ax+bx^3}}\\
&= \frac{2}{3}\sqrt{ax+bx^3}+\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx=\frac{2\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[Sqrt[a*x + b*x^3]/x,x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/Sqr
t[1 + (b*x^2)/a]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{2\sqrt{b\,x^{3}+ax}}{3}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b\sqrt{b\,x^{3}+ax}}$	124
elliptic	$\frac{2\sqrt{b\,x^{3}+ax}}{3}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b\sqrt{b\,x^{3}+ax}}$	124
risch	$\frac{2x\left(b\,x^{2}+a\right)}{3\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b\sqrt{b\,x^{3}+ax}}$	132

```
[In] int((b*x^3+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*x^3+a*x)^(1/2)+2/3*a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^
(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)
,1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \frac{2\left(2\,a\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^3+ax}b\right)}{3\,b}$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="fricas")
[Out] 2/3*(2*a*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)*b)/b
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \int \frac{\sqrt{x\left(a+bx^2\right)}}{x}\,dx$$

```
[In] integrate((b*x**3+a*x)**(1/2)/x,x)
[Out] Integral(sqrt(x*(a + b*x**2))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \int \frac{\sqrt{bx^3+ax}}{x}\,dx$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \int \frac{\sqrt{bx^3+ax}}{x}\,dx$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x)/x, x)
```

Mupad [**F(-1)**]

Timed out.

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx=\int \frac{\sqrt{b\,x^3+a\,x}}{x}\,dx$$

```
[In] int((a*x + b*x^3)^(1/2)/x,x)
[Out] int((a*x + b*x^3)^(1/2)/x, x)
```

3.43 $\int \frac{\sqrt{ax+bx^3}}{x^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 248

$$
\begin{aligned}
\int \frac{\sqrt{ax+bx^3}}{x^2} \, dx
&= \frac{4\sqrt{b}x(a+bx^2)}{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{x} \\
&\quad - \frac{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{ax+bx^3}} \\
&\quad + \frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt{ax+bx^3}}
\end{aligned}
$$

```
[Out] 4*x*(b*x^2+a)*b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*(b*x^3+a*x)^(
1/2)/x-4*a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*
b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)+2*a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)
*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Elliptic
F(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x
^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2045, 2057, 335, 311, 226, 1210}

$$\begin{aligned}\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = {} & \frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt{ax+bx^3}} \\ & - \frac{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{ax+bx^3}} \\ & - \frac{2\sqrt{ax+bx^3}}{x} + \frac{4\sqrt{b}x(a+bx^2)}{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}\end{aligned}$$

```
[In] Int[Sqrt[a*x + b*x^3]/x^2,x]
```

```
[Out] (4*Sqrt[b]*x*(a + b*x^2))/((Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*Sq
rt[a*x + b*x^3])/x - (4*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a
^(1/4)], 1/2])/Sqrt[a*x + b*x^3] + (2*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sq
rt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1
/4)*Sqrt[x])/a^(1/4)], 1/2])/Sqrt[a*x + b*x^3]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax+bx^3}}{x} + (2b)\int \frac{x}{\sqrt{ax+bx^3}}\,dx \\
&= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{\left(2b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}} \\
&= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{\left(4b\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}} \\
&= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{\left(4\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}} \\
&\quad - \frac{\left(4\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}}
\end{aligned}
$$

$$\begin{aligned}&= \frac{4\sqrt{b}x(a+bx^2)}{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{x}\\&- \frac{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{ax+bx^3}}\\&+ \frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{ax+bx^3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = -\frac{2\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^2}{a}\right)}{x\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^2,x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)])/(
x*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.71

method	result
default	$-\frac{2(b x^2+a)}{\sqrt{x(b x^2+a)}}+\frac{2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{b}\right.}{\sqrt{b x^3+ax}}$
risch	$-\frac{2(b x^2+a)}{\sqrt{x(b x^2+a)}}+\frac{2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{b}\right.}{\sqrt{b x^3+ax}}$
elliptic	$-\frac{2(b x^2+a)}{\sqrt{x(b x^2+a)}}+\frac{2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{b}\right.}{\sqrt{b x^3+ax}}$

```
[In] int((b*x^3+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2
)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/
b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(
1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = -\frac{2\left(2\sqrt{b}x\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^3+ax}\right)}{x}$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -2*(2*sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x
)) + sqrt(b*x^3 + a*x))/x
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \int \frac{\sqrt{x\,(a+bx^2)}}{x^2}\,dx$$

```
[In] integrate((b*x**3+a*x)**(1/2)/x**2,x)
[Out] Integral(sqrt(x*(a + b*x**2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^2}\,dx$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^2}\,dx$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = \int \frac{\sqrt{b\,x^3+a\,x}}{x^2}\,dx$$

```
[In] int((a*x + b*x^3)^(1/2)/x^2,x)
[Out] int((a*x + b*x^3)^(1/2)/x^2, x)
```

3.44 $\int \frac{\sqrt{ax+bx^3}}{x^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 116

$$\int \frac{\sqrt{ax+bx^3}}{x^3} \, dx = -\frac{2\sqrt{ax+bx^3}}{3x^2} + \frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}}$$

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/x^2+2/3*b^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(
b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+
a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2045, 2036, 335, 226}

$$\int \frac{\sqrt{ax+bx^3}}{x^3} \, dx = \frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{3x^2}$$

```
[In] Int[Sqrt[a*x + b*x^3]/x^3,x]
```

```
[Out] (-2*Sqrt[a*x + b*x^3])/(3*x^2) + (2*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{1}{3}(2b)\int\frac{1}{\sqrt{ax+bx^3}}\,dx\\
&= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{\left(2b\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{\left(4b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx = -\frac{2\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3x^2\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^3,x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^2)/a)])/(
3*x^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2\sqrt{b\,x^{3}+ax}}{3x^{2}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+ax}}$	123
elliptic	$-\frac{2\sqrt{b\,x^{3}+ax}}{3x^{2}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+ax}}$	123
risch	$-\frac{2\left(b\,x^{2}+a\right)}{3x\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+ax}}$	130

```
[In] int(1/x^3*(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/x^2+2/3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)
^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2
),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx = \frac{2\left(2\sqrt{b}x^2\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^3+ax}\right)}{3x^2}$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="fricas")
[Out] 2/3*(2*sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x))/x
^2
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx = \int \frac{\sqrt{x(a+bx^2)}}{x^3}\,dx$$

```
[In] integrate((b*x**3+a*x)**(1/2)/x**3,x)
[Out] Integral(sqrt(x*(a + b*x**2))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^3}\,dx$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx = \int \frac{\sqrt{bx^3+ax}}{x^3}\,dx$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx=\int \frac{\sqrt{b\,x^3+a\,x}}{x^3}\,dx$$

```
[In] int((a*x + b*x^3)^(1/2)/x^3,x)
[Out] int((a*x + b*x^3)^(1/2)/x^3, x)
```

3.45 $\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 283

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \frac{4b^{3/2}x(a+bx^2)}{5a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{5x^3} - \frac{4b\sqrt{ax+bx^3}}{5ax} - \frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}} + \frac{2b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] 4/5*b^(3/2)*x*(b*x^2+a)/a/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2/5*(b*x^3+
a*x)^(1/2)/x^3-4/5*b*(b*x^3+a*x)^(1/2)/a/x-4/5*b^(5/4)*(cos(2*arctan(b^(1/4
)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ellipti
cE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*
x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)+2
/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/
2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2045, 2050, 2057, 335, 311, 226, 1210}

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = \frac{2b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}} - \frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}} + \frac{4b^{3/2}x(a+bx^2)}{5a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{4b\sqrt{ax+bx^3}}{5ax} - \frac{2\sqrt{ax+bx^3}}{5x^3}$$

```
[In] Int[Sqrt[a*x + b*x^3]/x^4,x]
```

```
[Out] (4*b^(3/2)*x*(a + b*x^2))/(5*a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (
2*Sqrt[a*x + b*x^3])/(5*x^3) - (4*b*Sqrt[a*x + b*x^3])/(5*a*x) - (4*b^(5/4)
*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a*x + b*
x^3]) + (2*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(
3/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax+bx^3}}{5x^3}+\frac{1}{5}(2b)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx\\
&= -\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}+\frac{(2b^2)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}+\frac{\left(2b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}+\frac{\left(4b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5a\sqrt{ax+bx^3}}
\end{aligned}
$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3} - \frac{4b\sqrt{ax+bx^3}}{5ax} + \frac{\left(4b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{a}\sqrt{ax+bx^3}}$$
$$- \frac{\left(4b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{a}\sqrt{ax+bx^3}}$$
$$= \frac{4b^{3/2}x(a+bx^2)}{5a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{5x^3} - \frac{4b\sqrt{ax+bx^3}}{5ax}$$
$$- \frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}$$
$$+ \frac{2b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = -\frac{2\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^3\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^4,x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])/
(5*x^3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2(bx^2+a)(2bx^2+a)}{5x^2\sqrt{x(bx^2+a)}\,a}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5a\sqrt{bx^3+ax}}$
default	$-\frac{2\sqrt{bx^3+ax}}{5x^3}-\frac{4(bx^2+a)b}{5a\sqrt{x(bx^2+a)}}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5a\sqrt{bx^3+ax}}$
elliptic	$-\frac{2\sqrt{bx^3+ax}}{5x^3}-\frac{4(bx^2+a)b}{5a\sqrt{x(bx^2+a)}}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5a\sqrt{bx^3+ax}}$

```
[In] int((b*x^3+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x^2+a)*(2*b*x^2+a)/x^2/(x*(b*x^2+a))^(1/2)/a+2/5/a*b*(-a*b)^(1/2)*(
(x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*E
llipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2
)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = -\frac{2\left(2\,b^{\frac{3}{2}}x^3\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}(2\,bx^2+a)\right)}{5\,ax^3}$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -2/5*(2*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
0, x)) + sqrt(b*x^3 + a*x)*(2*b*x^2 + a))/(a*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx=\int \frac{\sqrt{x\,(a+bx^2)}}{x^4}\,dx$$

```
[In] integrate((b*x**3+a*x)**(1/2)/x**4,x)
[Out] Integral(sqrt(x*(a + b*x**2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx=\int \frac{\sqrt{bx^3+ax}}{x^4}\,dx$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx=\int \frac{\sqrt{bx^3+ax}}{x^4}\,dx$$

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx=\int \frac{\sqrt{b\,x^3+a\,x}}{x^4}\,dx$$

```
[In] int((a*x + b*x^3)^(1/2)/x^4,x)
[Out] int((a*x + b*x^3)^(1/2)/x^4, x)
```

3.46 $\int x^2(ax+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\begin{aligned}\int x^2\left(ax+bx^3\right)^{3/2}\,dx =& -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b} \\ &+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2} \\ &+\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}\end{aligned}$$

```
[Out] 2/15*x^3*(b*x^3+a*x)^(3/2)-8/231*a^3*(b*x^3+a*x)^(1/2)/b^2+8/385*a^2*x^2*(b
*x^3+a*x)^(1/2)/b+4/55*a*x^4*(b*x^3+a*x)^(1/2)+4/231*a^(15/4)*(cos(2*arctan
(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*
EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^
(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+a*x)^
(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used

$= \{2046, 2049, 2036, 335, 226\}$

$$\int x^2\left(ax+bx^3\right)^{3/2}dx=\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}-\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{4}{55}ax^4\sqrt{ax+bx^3}$$

```
[In] Int[x^2*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-8*a^3*Sqrt[a*x + b*x^3])/(231*b^2) + (8*a^2*x^2*Sqrt[a*x + b*x^3])/(385*b
) + (4*a*x^4*Sqrt[a*x + b*x^3])/55 + (2*x^3*(a*x + b*x^3)^(3/2))/15 + (4*a^
(15/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[
a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{1}{5}(2a)\int x^3\sqrt{ax+bx^3}\,dx\\
&= \frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{1}{55}\left(4a^2\right)\int\frac{x^4}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\right)\int\frac{x^2}{\sqrt{ax+bx^3}}\,dx}{77b}\\
&= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}\\
&\quad+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{\left(4a^4\right)\int\frac{1}{\sqrt{ax+bx^3}}\,dx}{231b^2}\\
&= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}\\
&\quad+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{\left(4a^4\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{231b^2\sqrt{ax+bx^3}}\\
&= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}\\
&\quad+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{\left(8a^4\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{231b^2\sqrt{ax+bx^3}}\\
&= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}\\
&\quad+\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51

$$\int x^2\left(ax+bx^3\right)^{3/2}\,dx=\frac{2\sqrt{x\left(a+bx^2\right)}\left(-\left(\left(5a-11bx^2\right)\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}}\right)+5a^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\right.\right.}{165b^2\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[x^2*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*(-((5*a - 11*b*x^2)*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a
]) + 5*a^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(165*b^2*Sqrt[
1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{2\left(-77b^3x^6-119a\,b^2x^4-12a^2b\,x^2+20a^3\right)x\left(b\,x^2+a\right)}{1155b^2\sqrt{x\left(b\,x^2+a\right)}}+\frac{4a^4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{231b^3\sqrt{b\,x^3+ax}}$
default	$\frac{2b\,x^6\sqrt{b\,x^3+ax}}{15}+\frac{34a\,x^4\sqrt{b\,x^3+ax}}{165}+\frac{8a^2x^2\sqrt{b\,x^3+ax}}{385b}-\frac{8a^3\sqrt{b\,x^3+ax}}{231b^2}+\frac{4a^4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-}}{231b^3\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2b\,x^6\sqrt{b\,x^3+ax}}{15}+\frac{34a\,x^4\sqrt{b\,x^3+ax}}{165}+\frac{8a^2x^2\sqrt{b\,x^3+ax}}{385b}-\frac{8a^3\sqrt{b\,x^3+ax}}{231b^2}+\frac{4a^4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-}}{231b^3\sqrt{b\,x^3+ax}}$

```
[In] int(x^2*(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/1155*(-77*b^3*x^6-119*a*b^2*x^4-12*a^2*b*x^2+20*a^3)/b^2*x*(b*x^2+a)/(x*
(b*x^2+a))^(1/2)+4/231*a^4/b^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)
^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2
),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int x^2(ax+bx^3)^{3/2}\,dx=\frac{2\left(20\,a^4\sqrt{b}\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(77\,b^4x^6+119\,ab^3x^4+12\,a^2b^2x^2-20\,a^3b\right)\sqrt{bx^3+}\right.}{1155\,b^3}$$

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(20*a^4*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (77*b^4*x^6 + 11
9*a*b^3*x^4 + 12*a^2*b^2*x^2 - 20*a^3*b)*sqrt(b*x^3 + a*x))/b^3
```

Sympy [F]

$$\int x^2\left(ax+bx^3\right)^{3/2}\,dx=\int x^2\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**2*(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int x^2\left(ax+bx^3\right)^{3/2}\,dx=\int \left(bx^3+ax\right)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2\left(ax+bx^3\right)^{3/2}\,dx=\int \left(bx^3+ax\right)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(ax+bx^3\right)^{3/2}\,dx=\int x^2\,{\left(b\,x^3+a\,x\right)}^{3/2}\,dx$$

```
[In] int(x^2*(a*x + b*x^3)^(3/2),x)
[Out] int(x^2*(a*x + b*x^3)^(3/2), x)
```

3.47 $\int x(ax+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 304

$$\int x\left(ax+bx^3\right)^{3/2}\,dx=-\frac{8a^3x(a+bx^2)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{8a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}-\frac{4a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] 2/13*x^2*(b*x^3+a*x)^(3/2)-8/65*a^3*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))
/(b*x^3+a*x)^(1/2)+8/195*a^2*x*(b*x^3+a*x)^(1/2)/b+4/39*a*x^3*(b*x^3+a*x)^(
1/2)+8/65*a^(13/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2
))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)-4/65*a^(13/4)*(cos(2*arctan(b^(1/4)*x
^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(
sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(
1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2046, 2049, 2057, 335, 311, 226, 1210}

$$\begin{aligned}
&\int x\left(ax+bx^3\right)^{3/2}\,dx = \\
&-\frac{4a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}} \\
&+\frac{8a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}} \\
&-\frac{8a^3x(a+bx^2)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{8a^2x\sqrt{ax+bx^3}}{195b} \\
&+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}
\end{aligned}$$

```
[In] Int[x*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-8*a^3*x*(a + b*x^2))/(65*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
 + (8*a^2*x*Sqrt[a*x + b*x^3])/(195*b) + (4*a*x^3*Sqrt[a*x + b*x^3])/39 + (
2*x^2*(a*x + b*x^3)^(3/2))/13 + (8*a^(13/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a*x + b*x^3]) - (4*a^(13/4)*Sqrt[x]*(Sq
rt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{1}{13}(6a)\int x^2\sqrt{ax+bx^3}\,dx\\
&= \frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{1}{39}\left(4a^2\right)\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{65b}
\end{aligned}
$$

$$= \frac{8a^2x\sqrt{ax+bx^3}}{195b} + \frac{4}{39}ax^3\sqrt{ax+bx^3} + \frac{2}{13}x^2\left(ax+bx^3\right)^{3/2} - \frac{\left(4a^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{65b\sqrt{ax+bx^3}}$$

$$= \frac{8a^2x\sqrt{ax+bx^3}}{195b} + \frac{4}{39}ax^3\sqrt{ax+bx^3} + \frac{2}{13}x^2\left(ax+bx^3\right)^{3/2} - \frac{\left(8a^3\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{65b\sqrt{ax+bx^3}}$$

$$= \frac{8a^2x\sqrt{ax+bx^3}}{195b} + \frac{4}{39}ax^3\sqrt{ax+bx^3} + \frac{2}{13}x^2\left(ax+bx^3\right)^{3/2} - \frac{\left(8a^{7/2}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{65b^{3/2}\sqrt{ax+bx^3}} + \frac{\left(8a^{7/2}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{65b^{3/2}\sqrt{ax+bx^3}}$$

$$= -\frac{8a^3x(a+bx^2)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{8a^2x\sqrt{ax+bx^3}}{195b} + \frac{4}{39}ax^3\sqrt{ax+bx^3} + \frac{2}{13}x^2\left(ax+bx^3\right)^{3/2} + \frac{8a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}} - \frac{4a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.28

$$\int x\left(ax+bx^3\right)^{3/2}dx = \frac{2x\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}} - a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{13b\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[x*(a*x + b*x^3)^(3/2),x]
[Out] (2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeom
etric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(13*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.69

method	result
risch	$\frac{2x^2(15b^2x^4+25ab\,x^2+4a^2)(b\,x^2+a)}{195b\sqrt{x(b\,x^2+a)}} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{2(x-\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,(-\frac{2\sqrt{-ab}\,E(\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2})}{b}+}{65b^2\sqrt{b\,x^3+ax}}$
default	$\frac{2b\,x^5\sqrt{b\,x^3+ax}}{13}+\frac{10a\,x^3\sqrt{b\,x^3+ax}}{39}+\frac{8a^2x\sqrt{b\,x^3+ax}}{195b} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{2(x-\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,(-\frac{2\sqrt{-ab}\,E(}{65b^2\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2b\,x^5\sqrt{b\,x^3+ax}}{13}+\frac{10a\,x^3\sqrt{b\,x^3+ax}}{39}+\frac{8a^2x\sqrt{b\,x^3+ax}}{195b} - \frac{4a^3\sqrt{-ab}\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{2(x-\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,(-\frac{2\sqrt{-ab}\,E(}{65b^2\sqrt{b\,x^3+ax}}$

```
[In] int(x*(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/195*x^2*(15*b^2*x^4+25*a*b*x^2+4*a^2)/b*(b*x^2+a)/(x*(b*x^2+a))^(1/2)-4/6
5/b^2*a^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a
*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1
/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),
1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(
1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.22

$$\int x(ax+bx^3)^{3/2}\,dx = \frac{2\left(12\,a^3\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+(15\,b^3x^5+25\,ab^2x^3+4\right.}{195\,b^2}$$

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/195*(12*a^3*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b
, 0, x)) + (15*b^3*x^5 + 25*a*b^2*x^3 + 4*a^2*b*x)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]

$$\int x \left(a x+b x^3\right)^{3/2} \, dx=\int x \left(x \left(a+b x^2\right)\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x*(b*x**3+a*x)**(3/2),x)
[Out] Integral(x*(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int x \left(a x+b x^3\right)^{3/2} \, dx=\int { \left (b x^{3} + a x\right)}^{\frac{3}{2}} x \,d x$$

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)*x, x)
```

Giac [F]

$$\int x \left(a x+b x^3\right)^{3/2} \, dx=\int { \left (b x^{3} + a x\right)}^{\frac{3}{2}} x \,d x$$

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \left(a x+b x^3\right)^{3/2} \, dx=\int x\,{\left (b\,x^3+a\,x\right)}^{3/2} \,d x$$

```
[In] int(x*(a*x + b*x^3)^(3/2),x)
[Out] int(x*(a*x + b*x^3)^(3/2), x)
```

3.48 $\int \left(ax+bx^3\right)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 158

$$\int \left(ax+bx^3\right)^{3/2}\,dx=\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] 2/11*x*(b*x^3+a*x)^(3/2)+8/77*a^2*(b*x^3+a*x)^(1/2)/b+12/77*a*x^2*(b*x^3+a*
x)^(1/2)-4/77*a^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^
(1/2))^2)^(1/2)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2029, 2046, 2049, 2036, 335, 226}

$$\int \left(ax+bx^3\right)^{3/2}\,dx=-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}}+\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}+\frac{12}{77}ax^2\sqrt{ax+bx^3}$$

```
[In] Int[(a*x + b*x^3)^(3/2),x]
```

```
[Out] (8*a^2*Sqrt[a*x + b*x^3])/(77*b) + (12*a*x^2*Sqrt[a*x + b*x^3])/77 + (2*x*(
a*x + b*x^3)^(3/2))/11 - (4*a^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(77*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
```

```
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{11}x\left(ax+bx^3\right)^{3/2}+\frac{1}{11}(6a)\int x\sqrt{ax+bx^3}\,dx\\
&= \frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}+\frac{1}{77}\left(12a^2\right)\int\frac{x^2}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\right)\int\frac{1}{\sqrt{ax+bx^3}}\,dx}{77b}\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{77b\sqrt{ax+bx^3}}\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}\\
&\quad-\frac{\left(8a^3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{77b\sqrt{ax+bx^3}}\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}\\
&\quad-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.53

$$\int\left(ax+bx^3\right)^{3/2}\,dx=\frac{2\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}}-a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{11b\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeomet
ric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

method	result
risch	$\frac{2\left(7b^{2}x^{4}+13ab\,x^{2}+4a^{2}\right)x\left(b\,x^{2}+a\right)}{77b\sqrt{x\left(b\,x^{2}+a\right)}}-\frac{4a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{77b^{2}\sqrt{b\,x^{3}+ax}}$
default	$\frac{2b\,x^{4}\sqrt{b\,x^{3}+ax}}{11}+\frac{26a\,x^{2}\sqrt{b\,x^{3}+ax}}{77}+\frac{8a^{2}\sqrt{b\,x^{3}+ax}}{77b}-\frac{4a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{77b^{2}\sqrt{b\,x^{3}+ax}}$
elliptic	$\frac{2b\,x^{4}\sqrt{b\,x^{3}+ax}}{11}+\frac{26a\,x^{2}\sqrt{b\,x^{3}+ax}}{77}+\frac{8a^{2}\sqrt{b\,x^{3}+ax}}{77b}-\frac{4a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{77b^{2}\sqrt{b\,x^{3}+ax}}$

```
[In] int((b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77*(7*b^2*x^4+13*a*b*x^2+4*a^2)/b*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)-4/77/b^
2*a^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*
EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.38

$$\int \left(ax+bx^{3}\right)^{3/2}\,dx = -\frac{2\left(4\,a^{3}\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\left(7\,b^{3}x^{4}+13\,ab^{2}x^{2}+4\,a^{2}b\right)\sqrt{bx^{3}+ax}\right)}{77\,b^{2}}$$

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*a^3*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (7*b^3*x^4 + 13*a*
b^2*x^2 + 4*a^2*b)*sqrt(b*x^3 + a*x))/b^2
```

Sympy [F]

$$\int \left(ax+bx^3\right)^{3/2}\,dx=\int \left(ax+bx^3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x**3+a*x)**(3/2),x)
[Out] Integral((a*x + b*x**3)**(3/2), x)
```

Maxima [F]

$$\int \left(ax+bx^3\right)^{3/2}\,dx=\int \left(bx^3+ax\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \left(ax+bx^3\right)^{3/2}\,dx=\int \left(bx^3+ax\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.25

$$\int \left(ax+bx^3\right)^{3/2}\,dx=\frac{2\,x\,\left(b\,x^3+a\,x\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{5}{4};\ \frac{9}{4};\ -\frac{b\,x^2}{a}\right)}{5\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

```
[In] int((a*x + b*x^3)^(3/2),x)
[Out] (2*x*(a*x + b*x^3)^(3/2)*hypergeom([-3/2, 5/4], 9/4, -(b*x^2)/a))/(5*((b*x^
2)/a + 1)^(3/2))
```

3.49 $\int \frac{(ax+bx^3)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 275

$$\begin{aligned}\int \frac{(ax+bx^3)^{3/2}}{x}\,dx &= \frac{8a^2x(a+bx^2)}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{4}{15}ax\sqrt{ax+bx^3} \\ &\quad +\frac{2}{9}\left(ax+bx^3\right)^{3/2}-\frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}} \\ &\quad +\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}\end{aligned}$$

```
[Out] 2/9*(b*x^3+a*x)^(3/2)+8/15*a^2*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x
^3+a*x)^(1/2)+4/15*a*x*(b*x^3+a*x)^(1/2)-8/15*a^(9/4)*(cos(2*arctan(b^(1/4)
*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Elliptic
E(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x
^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)+4/
15*a^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/
2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2046, 2029, 2057, 335, 311, 226, 1210}

$$\int \frac{(ax+bx^3)^{3/2}}{x}\,dx = \frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}$$
$$-\frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}$$
$$+\frac{8a^2x(a+bx^2)}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}$$

```
[In] Int[(a*x + b*x^3)^(3/2)/x,x]
```

```
[Out] (8*a^2*x*(a + b*x^2))/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
+ (4*a*x*Sqrt[a*x + b*x^3])/15 + (2*(a*x + b*x^3)^(3/2))/9 - (8*a^(9/4)*Sqr
t[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a*x + b*x^3
]) + (4*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/
4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{1}{3}(2a)\int\sqrt{ax+bx^3}\,dx \\
&= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{1}{15}\left(4a^2\right)\int\frac{x}{\sqrt{ax+bx^3}}\,dx \\
&= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{15\sqrt{ax+bx^3}} \\
&= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{\left(8a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{ax+bx^3}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{\left(8a^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{b}\sqrt{ax+bx^3}} \\
&\quad-\frac{\left(8a^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{b}\sqrt{ax+bx^3}} \\
&= \frac{8a^2x(a+bx^2)}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2} \\
&\quad-\frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}} \\
&\quad+\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.19

$$
\int\frac{\left(ax+bx^3\right)^{3/2}}{x}\,dx=\frac{2ax\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}
$$

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x,x]
```

```
[Out] (2*a*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)])
/(3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.71

method	result
default	$\frac{2b\,x^3\sqrt{b\,x^3+ax}}{9}+\frac{22ax\sqrt{b\,x^3+ax}}{45}+\frac{4a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15b\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2b\,x^3\sqrt{b\,x^3+ax}}{9}+\frac{22ax\sqrt{b\,x^3+ax}}{45}+\frac{4a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15b\sqrt{b\,x^3+ax}}$
risch	$\frac{2x^2\left(5b\,x^2+11a\right)\left(b\,x^2+a\right)}{45\sqrt{x\left(b\,x^2+a\right)}}+\frac{4a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\sqrt{-ab}\,F\left(\right.\right.}{15b\sqrt{b\,x^3+ax}}$

```
[In] int((b*x^3+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*b*x^3*(b*x^3+a*x)^(1/2)+22/45*a*x*(b*x^3+a*x)^(1/2)+4/15*a^2*(-a*b)^(1/
2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b
)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/
2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b
)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.21

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x}\,dx =$$
$$-\frac{2\left(12\,a^2\sqrt{b}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-\left(5\,b^2x^3+11\,abx\right)\sqrt{bx^3+ax}\right)}{45\,b}$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -2/45*(12*a^2*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b
, 0, x)) - (5*b^2*x^3 + 11*a*b*x)*sqrt(b*x^3 + a*x))/b
```

Sympy [F]

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x}\,dx=\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x**3+a*x)**(3/2)/x,x)
[Out] Integral((x*(a + b*x**2))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x}\,dx=\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x}\,dx=\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x}\,dx=\int \frac{{\left(b\,x^3+a\,x\right)}^{3/2}}{x}\,dx$$

```
[In] int((a*x + b*x^3)^(3/2)/x,x)
[Out] int((a*x + b*x^3)^(3/2)/x, x)
```

3.50 $\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2(ax+bx^3)^{3/2}}{7x} + \frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{ax+bx^3}}$$

```
[Out] 2/7*(b*x^3+a*x)^(3/2)/x+4/7*a*(b*x^3+a*x)^(1/2)+4/7*a^(7/4)*(cos(2*arctan(b
^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*El
lipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1
/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^3+a*x)^(1
/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2046, 2036, 335, 226}

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{ax+bx^3}} + \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2(ax+bx^3)^{3/2}}{7x}$$

```
[In] Int[(a*x + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (4*a*Sqrt[a*x + b*x^3])/7 + (2*(a*x + b*x^3)^(3/2))/(7*x) + (4*a^(7/4)*Sqrt
[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{1}{7}(6a)\int \frac{\sqrt{ax+bx^3}}{x}\,dx\\
&= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{1}{7}\left(4a^2\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{7\sqrt{ax+bx^3}}\\
&= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{\left(8a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{7\sqrt{ax+bx^3}}
\end{aligned}$$

$$=\frac{4}{7}a\sqrt{ax+bx^3}+\frac{2(ax+bx^3)^{3/2}}{7x}+\frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.37

$$\int\frac{(ax+bx^3)^{3/2}}{x^2}\,dx=\frac{2a\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/S
qrt[1 + (b*x^2)/a]
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{2\left(b\,x^{2}+3a\right)x\left(b\,x^{2}+a\right)}{7\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{4a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7b\sqrt{b\,x^{3}+ax}}$	143
default	$\frac{2b\,x^{2}\sqrt{b\,x^{3}+ax}}{7}+\frac{6a\sqrt{b\,x^{3}+ax}}{7}+\frac{4a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7b\sqrt{b\,x^{3}+ax}}$	144
elliptic	$\frac{2b\,x^{2}\sqrt{b\,x^{3}+ax}}{7}+\frac{6a\sqrt{b\,x^{3}+ax}}{7}+\frac{4a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7b\sqrt{b\,x^{3}+ax}}$	144

```
[In] int((b*x^3+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*(b*x^2+3*a)*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+4/7*a^2*(-a*b)^(1/2)/b*((x+
(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b
)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1
/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.35

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \frac{2\left(4\,a^2\sqrt{b}\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(b^2x^2+3\,ab\right)\sqrt{bx^3+ax}\right)}{7\,b}$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 2/7*(4*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (b^2*x^2 + 3*a*b)*sq
rt(b*x^3 + a*x))/b
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x**3+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx=\int \frac{{\left(b\,x^3+a\,x\right)}^{3/2}}{x^2}\,dx$$

```
[In] int((a*x + b*x^3)^(3/2)/x^2,x)
[Out] int((a*x + b*x^3)^(3/2)/x^2, x)
```

3.51 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 274

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x^3}\,dx = \frac{24a\sqrt{b}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{12}{5}bx\sqrt{ax+bx^3}$$
$$-\frac{2(ax+bx^3)^{3/2}}{x^2}-\frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}$$
$$+\frac{12a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}$$

```
[Out] -2*(b*x^3+a*x)^(3/2)/x^2+24/5*a*x*(b*x^2+a)*b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*
x^3+a*x)^(1/2)+12/5*b*x*(b*x^3+a*x)^(1/2)-24/5*a^(5/4)*b^(1/4)*(cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)+1
2/5*a^(5/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/
2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2045, 2029, 2057, 335, 311, 226, 1210}

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \frac{12a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}$$
$$-\frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}$$
$$+\frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2(ax+bx^3)^{3/2}}{x^2}+\frac{24a\sqrt{b}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}$$

```
[In] Int[(a*x + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (24*a*Sqrt[b]*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) +
(12*b*x*Sqrt[a*x + b*x^3])/5 - (2*(a*x + b*x^3)^(3/2))/x^2 - (24*a^(5/4)*b^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3
]) + (12*a^(5/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(5*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+(6b)\int\sqrt{ax+bx^3}\,dx\\
&= \frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+\frac{1}{5}(12ab)\int\frac{x}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+\frac{\left(12ab\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}}\\
&= \frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+\frac{\left(24ab\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{12}{5}bx\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{x^2} \\
&\quad + \frac{\left(24a^{3/2}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{ax+bx^3}} \\
&\quad - \frac{\left(24a^{3/2}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5\sqrt{ax+bx^3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{24a\sqrt{b}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{12}{5}bx\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{x^2} \\
&\quad - \frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}} \\
&\quad + \frac{12a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.19

$$
\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = -\frac{2a\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^2}{a}\right)}{x\sqrt{1+\frac{bx^2}{a}}}
$$

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])
/(x*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2\left(b\,x^{2}+a\right)\left(-b\,x^{2}+5a\right)}{5\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{12a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5\sqrt{b\,x^{3}+ax}}$
default	$-\frac{2\left(b\,x^{2}+a\right)a}{\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{2bx\sqrt{b\,x^{3}+ax}}{5}+\frac{12a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5\sqrt{b\,x^{3}+ax}}$
elliptic	$-\frac{2\left(b\,x^{2}+a\right)a}{\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{2bx\sqrt{b\,x^{3}+ax}}{5}+\frac{12a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5\sqrt{b\,x^{3}+ax}}$

```
[In] int((b*x^3+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x^2+a)*(-b*x^2+5*a)/(x*(b*x^2+a))^(1/2)+12/5*a*(-a*b)^(1/2)*((x+(-a
*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(
1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*Ellipti
cE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*El
lipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.19

$$\int \frac{\left(ax+bx^{3}\right)^{3/2}}{x^{3}}\,dx = -\frac{2\left(12\,a\sqrt{b}x\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-\sqrt{bx^{3}+ax}\left(bx^{2}-5\,a\right)\right)}{5\,x}$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -2/5*(12*a*sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
 0, x)) - sqrt(b*x^3 + a*x)*(b*x^2 - 5*a))/x
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x**3+a*x)**(3/2)/x**3,x)
[Out] Integral((x*(a + b*x**2))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^3}\,dx$$

```
[In] int((a*x + b*x^3)^(3/2)/x^3,x)
[Out] int((a*x + b*x^3)^(3/2)/x^3, x)
```

3.52 $\int \frac{(ax+bx^3)^{3/2}}{x^4} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int \frac{(ax+bx^3)^{3/2}}{x^4} \, dx = \frac{4}{3} b\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt{ax+bx^3}}$$

```
[Out] -2/3*(b*x^3+a*x)^(3/2)/x^3+4/3*b*(b*x^3+a*x)^(1/2)+4/3*a^(3/4)*b^(3/4)*(cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a
^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)
^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2045, 2046, 2036, 335, 226}

$$\int \frac{(ax+bx^3)^{3/2}}{x^4} \, dx = \frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt{ax+bx^3}} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{4}{3} b\sqrt{ax+bx^3}$$

```
[In] Int[(a*x + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (4*b*Sqrt[a*x + b*x^3])/3 - (2*(a*x + b*x^3)^(3/2))/(3*x^3) + (4*a^(3/4)*b^
(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*Sqrt[a*x + b*x^3
])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(ax+bx^3)^{3/2}}{3x^3} + (2b)\int \frac{\sqrt{ax+bx^3}}{x}\,dx$$

$$= \frac{4}{3}b\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{1}{3}(4ab)\int \frac{1}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{4}{3}b\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{\left(4ab\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}}$$

$$= \frac{4}{3}b\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{\left(8ab\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{3\sqrt{ax+bx^3}}$$

$$= \frac{4}{3}b\sqrt{ax+bx^3} - \frac{2(ax+bx^3)^{3/2}}{3x^3} + \frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.40

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx = -\frac{2a\sqrt{x\,(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{bx^2}{a}\right)}{3x^2\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^2)/a)])
/(3*x^2*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{2a\sqrt{b\,x^3+ax}}{3x^2}+\frac{2b\sqrt{b\,x^3+ax}}{3}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^3+ax}}$	139
risch	$-\frac{2(b\,x^2+a)\,(-b\,x^2+a)}{3x\sqrt{x(b\,x^2+a)}}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^3+ax}}$	139
elliptic	$-\frac{2a\sqrt{b\,x^3+ax}}{3x^2}+\frac{2b\sqrt{b\,x^3+ax}}{3}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^3+ax}}$	139

```
[In] int((b*x^3+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a*(b*x^3+a*x)^(1/2)/x^2+2/3*b*(b*x^3+a*x)^(1/2)+4/3*a*(-a*b)^(1/2)*((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(
1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.34

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx=\frac{2\left(4\,a\sqrt{b}x^2\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^3+ax}(bx^2-a)\right)}{3\,x^2}$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 2/3*(4*a*sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)*
(b*x^2 - a))/x^2
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx=\int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x**3+a*x)**(3/2)/x**4,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx=\int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx=\int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx=\int \frac{(b\,x^3+a\,x)^{3/2}}{x^4}\,dx$$

```
[In] int((a*x + b*x^3)^(3/2)/x^4,x)
[Out] int((a*x + b*x^3)^(3/2)/x^4, x)
```

3.53 $\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 277

$$\begin{aligned}\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx &= \frac{24b^{3/2}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{12b\sqrt{ax+bx^3}}{5x} - \frac{2(ax+bx^3)^{3/2}}{5x^4} \\ &\quad - \frac{24\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}} \\ &\quad + \frac{12\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}\end{aligned}$$

```
[Out] -2/5*(b*x^3+a*x)^(3/2)/x^4+24/5*b^(3/2)*x*(b*x^2+a)/(a^(1/2)+x*b^(1/2))/(b*
x^3+a*x)^(1/2)-12/5*b*(b*x^3+a*x)^(1/2)/x-24/5*a^(1/4)*b^(5/4)*(cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)+1
2/5*a^(1/4)*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/
2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2045, 2057, 335, 311, 226, 1210}

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \frac{12\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}$$
$$-\frac{24\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}$$
$$+\frac{24b^{3/2}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2(ax+bx^3)^{3/2}}{5x^4}$$

```
[In] Int[(a*x + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (24*b^(3/2)*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (1
2*b*Sqrt[a*x + b*x^3])/(5*x) - (2*(a*x + b*x^3)^(3/2))/(5*x^4) - (24*a^(1/4
)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b
*x^3]) + (12*a^(1/4)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/
2])/(5*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{1}{5}(6b)\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx\\
&= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{1}{5}\left(12b^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx\\
&= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{\left(12b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}}\\
&= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{\left(24b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}\\
&= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}\\
&\quad+\frac{\left(24\sqrt{a}b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}\\
&\quad-\frac{\left(24\sqrt{a}b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{24b^{3/2}x(a+bx^2)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{12b\sqrt{ax+bx^3}}{5x} - \frac{2(ax+bx^3)^{3/2}}{5x^4} \\
&\quad - \frac{24\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}} \\
&\quad + \frac{12\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$
\int \frac{\left(ax+bx^3\right)^{3/2}}{x^5}\,dx = -\frac{2a\sqrt{x\left(a+bx^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^3\sqrt{1+\frac{bx^2}{a}}}
$$

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^2)/a)]
)/(5*x^3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{2(bx^2+a)(7bx^2+a)}{5x^2\sqrt{x(bx^2+a)}}+\frac{12b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\right.}\right.}{}\right.}{5\sqrt{bx^3+ax}}$
default	$-\frac{2a\sqrt{bx^3+ax}}{5x^3}-\frac{14(bx^2+a)b}{5\sqrt{x(bx^2+a)}}+\frac{12b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5\sqrt{bx^3+ax}}$
elliptic	$-\frac{2a\sqrt{bx^3+ax}}{5x^3}-\frac{14(bx^2+a)b}{5\sqrt{x(bx^2+a)}}+\frac{12b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5\sqrt{bx^3+ax}}$

```
[In] int((b*x^3+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x^2+a)*(7*b*x^2+a)/x^2/(x*(b*x^2+a))^(1/2)+12/5*b*(-a*b)^(1/2)*((x+
(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b
)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*Elli
pticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b
*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.18

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx =$$
$$-\frac{2\left(12\,b^{\frac{3}{2}}x^3\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}(7\,bx^2+a)\right)}{5\,x^3}$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -2/5*(12*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b,
 0, x)) + sqrt(b*x^3 + a*x)*(7*b*x^2 + a))/x^3
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x**3+a*x)**(3/2)/x**5,x)
[Out] Integral((x*(a + b*x**2))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^5}\,dx$$

```
[In] int((a*x + b*x^3)^(3/2)/x^5,x)
[Out] int((a*x + b*x^3)^(3/2)/x^5, x)
```

3.54 $\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 137

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = -\frac{4b\sqrt{ax+bx^3}}{7x^2} - \frac{2(ax+bx^3)^{3/2}}{7x^5} + \frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}}$$

```
[Out] -2/7*(b*x^3+a*x)^(3/2)/x^5-4/7*b*(b*x^3+a*x)^(1/2)/x^2+4/7*b^(7/4)*(cos(2*a
rctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)
+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^3+
a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2045, 2036, 335, 226}

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}} - \frac{2(ax+bx^3)^{3/2}}{7x^5} - \frac{4b\sqrt{ax+bx^3}}{7x^2}$$

```
[In] Int[(a*x + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-4*b*Sqrt[a*x + b*x^3])/(7*x^2) - (2*(a*x + b*x^3)^(3/2))/(7*x^5) + (4*b^(
7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2
]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*a^(1/4)*Sqrt[a*x
+ b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(ax+bx^3)^{3/2}}{7x^5} + \frac{1}{7}(6b)\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx \\
&= -\frac{4b\sqrt{ax+bx^3}}{7x^2} - \frac{2(ax+bx^3)^{3/2}}{7x^5} + \frac{1}{7}\left(4b^2\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx \\
&= -\frac{4b\sqrt{ax+bx^3}}{7x^2} - \frac{2(ax+bx^3)^{3/2}}{7x^5} + \frac{\left(4b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{7\sqrt{ax+bx^3}} \\
&= -\frac{4b\sqrt{ax+bx^3}}{7x^2} - \frac{2(ax+bx^3)^{3/2}}{7x^5} + \frac{\left(8b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{7\sqrt{ax+bx^3}}
\end{aligned}$$

$$= -\frac{4b\sqrt{ax+bx^3}}{7x^2} - \frac{2(ax+bx^3)^{3/2}}{7x^5} + \frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.39

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = -\frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{bx^2}{a}\right)}{7x^4\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^2)/a)]
)/(7*x^4*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

method	result	size
risch	$-\frac{2(b\,x^2+a)(3b\,x^2+a)}{7x^3\sqrt{x(b\,x^2+a)}}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7\sqrt{b\,x^3+ax}}$	139
default	$-\frac{2a\sqrt{b\,x^3+ax}}{7x^4}-\frac{6b\sqrt{b\,x^3+ax}}{7x^2}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7\sqrt{b\,x^3+ax}}$	142
elliptic	$-\frac{2a\sqrt{b\,x^3+ax}}{7x^4}-\frac{6b\sqrt{b\,x^3+ax}}{7x^2}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7\sqrt{b\,x^3+ax}}$	142

```
[In] int((b*x^3+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(b*x^2+a)*(3*b*x^2+a)/x^3/(x*(b*x^2+a))^(1/2)+4/7*b*(-a*b)^(1/2)*((x+(
-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)
^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/
2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.32

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \frac{2\left(4\,b^{\frac{3}{2}}x^4\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\sqrt{bx^3+ax}(3\,bx^2+a)\right)}{7\,x^4}$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="fricas")
[Out] 2/7*(4*b^(3/2)*x^4*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)*(3
*b*x^2 + a))/x^4
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b*x**3+a*x)**(3/2)/x**6,x)
[Out] Integral((x*(a + b*x**2))**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx=\int \frac{{\left(b\,x^3+a\,x\right)}^{3/2}}{x^6}\,dx$$

```
[In] int((a*x + b*x^3)^(3/2)/x^6,x)
[Out] int((a*x + b*x^3)^(3/2)/x^6, x)
```

3.55 $\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 306

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = \frac{8b^{5/2}x(a+bx^2)}{15a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax}$$
$$- \frac{2(ax+bx^3)^{3/2}}{9x^6} - \frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}$$
$$+ \frac{4b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] -2/9*(b*x^3+a*x)^(3/2)/x^6+8/15*b^(5/2)*x*(b*x^2+a)/a/(a^(1/2)+x*b^(1/2))/(
b*x^3+a*x)^(1/2)-4/15*b*(b*x^3+a*x)^(1/2)/x^3-8/15*b^2*(b*x^3+a*x)^(1/2)/a/
x-8/15*b^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2
)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)+4/15*b^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2
*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*
((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2045, 2050, 2057, 335, 311, 226, 1210}

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = \frac{4b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}$$
$$-\frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}$$
$$+\frac{8b^{5/2}x(a+bx^2)}{15a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{8b^2\sqrt{ax+bx^3}}{15ax}-\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{2(ax+bx^3)^{3/2}}{9x^6}$$

```
[In] Int[(a*x + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (8*b^(5/2)*x*(a + b*x^2))/(15*a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) -
(4*b*Sqrt[a*x + b*x^3])/(15*x^3) - (8*b^2*Sqrt[a*x + b*x^3])/(15*a*x) - (2*
(a*x + b*x^3)^(3/2))/(9*x^6) - (8*b^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a*x + b*x^3]) + (4*b^(9/4)*Sqrt[x]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}+\frac{1}{3}(2b)\int\frac{\sqrt{ax+bx^3}}{x^4}\,dx\\
&= -\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}+\frac{1}{15}\left(4b^2\right)\int\frac{1}{x\sqrt{ax+bx^3}}\,dx\\
&= -\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{8b^2\sqrt{ax+bx^3}}{15ax}-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}+\frac{\left(4b^3\right)\int\frac{x}{\sqrt{ax+bx^3}}\,dx}{15a}\\
&= -\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{8b^2\sqrt{ax+bx^3}}{15ax}-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}+\frac{\left(4b^3\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{15a\sqrt{ax+bx^3}}
\end{aligned}
$$

$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2(ax+bx^3)^{3/2}}{9x^6} + \frac{\left(8b^3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{15a\sqrt{ax+bx^3}}$$

$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2(ax+bx^3)^{3/2}}{9x^6} + \frac{\left(8b^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{15\sqrt{a}\sqrt{ax+bx^3}} - \frac{\left(8b^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{15\sqrt{a}\sqrt{ax+bx^3}}$$

$$= \frac{8b^{5/2}x(a+bx^2)}{15a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2(ax+bx^3)^{3/2}}{9x^6} - \frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}} + \frac{4b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.18

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx = -\frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{9}{4}, -\frac{3}{2}, -\frac{5}{4}, -\frac{bx^2}{a}\right)}{9x^5\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^2)/a)]
)/(9*x^5*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2(bx^2+a)(12b^2x^4+11abx^2+5a^2)}{45x^4\sqrt{x(bx^2+a)}\,a}+\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15a\sqrt{bx^3+ax}}$
default	$-\frac{2a\sqrt{bx^3+ax}}{9x^5}-\frac{22b\sqrt{bx^3+ax}}{45x^3}-\frac{8(bx^2+a)b^2}{15a\sqrt{x(bx^2+a)}}+\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{(x+}\right.}{b}\right.}{15a\sqrt{bx^3+ax}}$
elliptic	$-\frac{2a\sqrt{bx^3+ax}}{9x^5}-\frac{22b\sqrt{bx^3+ax}}{45x^3}-\frac{8(bx^2+a)b^2}{15a\sqrt{x(bx^2+a)}}+\frac{4b^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{(x+}\right.}{b}\right.}{15a\sqrt{bx^3+ax}}$

```
[In] int((b*x^3+a*x)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(b*x^2+a)*(12*b^2*x^4+11*a*b*x^2+5*a^2)/x^4/(x*(b*x^2+a))^(1/2)/a+4/1
5*b^2/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2
)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/
2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/
2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx =$$
$$-\frac{2\left(12\,b^{\frac{5}{2}}x^5\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(12\,b^2x^4+11\,abx^2+5\,a^2\right)\sqrt{bx^3+ax}\right.}{45\,ax^5}$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] -2/45*(12*b^(5/2)*x^5*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b
, 0, x)) + (12*b^2*x^4 + 11*a*b*x^2 + 5*a^2)*sqrt(b*x^3 + a*x))/(a*x^5)
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx=\int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((b*x**3+a*x)**(3/2)/x**7,x)
[Out] Integral((x*(a + b*x**2))**(3/2)/x**7, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx=\int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^7, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx=\int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx=\int \frac{(b\,x^3+a\,x)^{3/2}}{x^7}\,dx$$

```
[In] int((a*x + b*x^3)^(3/2)/x^7,x)
[Out] int((a*x + b*x^3)^(3/2)/x^7, x)
```

3.56 $\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 163

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = -\frac{12b\sqrt{ax+bx^3}}{77x^4} - \frac{8b^2\sqrt{ax+bx^3}}{77ax^2} - \frac{2(ax+bx^3)^{3/2}}{11x^7} - \frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] -2/11*(b*x^3+a*x)^(3/2)/x^7-12/77*b*(b*x^3+a*x)^(1/2)/x^4-8/77*b^2*(b*x^3+a
*x)^(1/2)/a/x^2-4/77*b^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x
^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/
2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2045, 2050, 2036, 335, 226}

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = -\frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}} - \frac{8b^2\sqrt{ax+bx^3}}{77ax^2} - \frac{2(ax+bx^3)^{3/2}}{11x^7} - \frac{12b\sqrt{ax+bx^3}}{77x^4}$$

```
[In] Int[(a*x + b*x^3)^(3/2)/x^8,x]
```

```
[Out] (-12*b*Sqrt[a*x + b*x^3])/(77*x^4) - (8*b^2*Sqrt[a*x + b*x^3])/(77*a*x^2) -
 (2*(a*x + b*x^3)^(3/2))/(11*x^7) - (4*b^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[x])/a^(1/4)], 1/2])/(77*a^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule steps

$$\text{integral} = -\frac{2(ax+bx^3)^{3/2}}{11x^7} + \frac{1}{11}(6b)\int \frac{\sqrt{ax+bx^3}}{x^5}\,dx$$

$$
\begin{aligned}
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4} - \frac{2(ax+bx^3)^{3/2}}{11x^7} + \frac{1}{77}(12b^2)\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx \\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4} - \frac{8b^2\sqrt{ax+bx^3}}{77ax^2} - \frac{2(ax+bx^3)^{3/2}}{11x^7} - \frac{(4b^3)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{77a} \\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4} - \frac{8b^2\sqrt{ax+bx^3}}{77ax^2} - \frac{2(ax+bx^3)^{3/2}}{11x^7} - \frac{\left(4b^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{77a\sqrt{ax+bx^3}} \\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4} - \frac{8b^2\sqrt{ax+bx^3}}{77ax^2} - \frac{2(ax+bx^3)^{3/2}}{11x^7} \\
&\quad - \frac{\left(8b^3\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{77a\sqrt{ax+bx^3}} \\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4} - \frac{8b^2\sqrt{ax+bx^3}}{77ax^2} - \frac{2(ax+bx^3)^{3/2}}{11x^7} \\
&\quad - \frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$
\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = -\frac{2a\sqrt{x(a+bx^2)}\,\text{Hypergeometric2F1}\left(-\frac{11}{4}, -\frac{3}{2}, -\frac{7}{4}, -\frac{bx^2}{a}\right)}{11x^6\sqrt{1+\frac{bx^2}{a}}}
$$

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^8,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^2)/a)
])/(11*x^6*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{2\left(b\,x^{2}+a\right)\left(4b^{2}x^{4}+13ab\,x^{2}+7a^{2}\right)}{77x^{5}\sqrt{x\left(b\,x^{2}+a\right)}\,a}-\frac{4b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{77a\sqrt{b\,x^{3}+ax}}$
default	$-\frac{2a\sqrt{b\,x^{3}+ax}}{11x^{6}}-\frac{26b\sqrt{b\,x^{3}+ax}}{77x^{4}}-\frac{8b^{2}\sqrt{b\,x^{3}+ax}}{77a\,x^{2}}-\frac{4b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\Bigg(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},}{77a\sqrt{b\,x^{3}+ax}}$
elliptic	$-\frac{2a\sqrt{b\,x^{3}+ax}}{11x^{6}}-\frac{26b\sqrt{b\,x^{3}+ax}}{77x^{4}}-\frac{8b^{2}\sqrt{b\,x^{3}+ax}}{77a\,x^{2}}-\frac{4b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\Bigg(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},}{77a\sqrt{b\,x^{3}+ax}}$

```
[In] int((b*x^3+a*x)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*(b*x^2+a)*(4*b^2*x^4+13*a*b*x^2+7*a^2)/x^5/(x*(b*x^2+a))^(1/2)/a-4/77
*b^2/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)
^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)
*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = -\frac{2\left(4\,b^{\frac{5}{2}}x^{6}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(4\,b^{2}x^{4}+13\,abx^{2}+7\,a^{2}\right)\sqrt{bx^{3}+ax}\right)}{77\,ax^{6}}$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] -2/77*(4*b^(5/2)*x^6*weierstrassPInverse(-4*a/b, 0, x) + (4*b^2*x^4 + 13*a*
b*x^2 + 7*a^2)*sqrt(b*x^3 + a*x))/(a*x^6)
```

Sympy [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(x(a+bx^2))^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((b*x**3+a*x)**(3/2)/x**8,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(bx^3+ax)^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(b\,x^3+a\,x)^{3/2}}{x^8}\,dx$$

```
[In] int((a*x + b*x^3)^(3/2)/x^8,x)
[Out] int((a*x + b*x^3)^(3/2)/x^8, x)
```

3.57 $\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 140

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = -\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}}$$

```
[Out] -10/21*a*(b*x^3+a*x)^(1/2)/b^2+2/7*x^2*(b*x^3+a*x)^(1/2)/b+5/21*a^(7/4)*(co
s(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^
(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b
*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2049, 2036, 335, 226}

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}} - \frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b}$$

```
[In] Int[x^4/Sqrt[a*x + b*x^3],x]
```

```
[Out] (-10*a*Sqrt[a*x + b*x^3])/(21*b^2) + (2*x^2*Sqrt[a*x + b*x^3])/(7*b) + (5*a
^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[a
*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x^2\sqrt{ax+bx^3}}{7b} - \frac{(5a)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{7b} \\
&= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{(5a^2)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{21b^2} \\
&= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{\left(5a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{21b^2\sqrt{ax+bx^3}} \\
&= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{\left(10a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{21b^2\sqrt{ax+bx^3}}
\end{aligned}
$$

$$= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.57

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \frac{2x\left(-5a^2-2abx^2+3b^2x^4+5a^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{21b^2\sqrt{x\left(a+bx^2\right)}}$$

```
[In] Integrate[x^4/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(21*b^2*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

method	result	size
risch	$-\frac{2\left(-3b\,x^{2}+5a\right)x\left(b\,x^{2}+a\right)}{21b^{2}\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{5a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^{3}\sqrt{b\,x^{3}+ax}}$	147
default	$\frac{2x^{2}\sqrt{b\,x^{3}+ax}}{7b}-\frac{10a\sqrt{b\,x^{3}+ax}}{21b^{2}}+\frac{5a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^{3}\sqrt{b\,x^{3}+ax}}$	149
elliptic	$\frac{2x^{2}\sqrt{b\,x^{3}+ax}}{7b}-\frac{10a\sqrt{b\,x^{3}+ax}}{21b^{2}}+\frac{5a^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21b^{3}\sqrt{b\,x^{3}+ax}}$	149

```
[In] int(x^4/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(-3*b*x^2+5*a)/b^2*x*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+5/21*a^2/b^3*(-a*b
)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF((
(x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.34

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \frac{2\left(5\,a^2\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(3\,b^2x^2-5\,ab\right)\sqrt{bx^3+ax}\right)}{21\,b^3}$$

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*a^2*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (3*b^2*x^2 - 5*a*b)
*sqrt(b*x^3 + a*x))/b^3
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^4}{\sqrt{x\left(a+bx^2\right)}}\,dx$$

```
[In] integrate(x**4/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx=\int \frac{x^4}{\sqrt{b\,x^3+a\,x}}\,dx$$

```
[In] int(x^4/(a*x + b*x^3)^(1/2),x)
[Out] int(x^4/(a*x + b*x^3)^(1/2), x)
```

3.58 $\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 258

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = &-\frac{6ax(a+bx^2)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{2x\sqrt{ax+bx^3}}{5b}\\ &+\frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}\\ &-\frac{3a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}\end{aligned}$$

```
[Out] -6/5*a*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+2/5*x*(b*x
^3+a*x)^(1/2)/b+6/5*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(
1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)
+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)-3/5*a^(5/4)*(cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ell
ipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/
2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/
2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2049, 2057, 335, 311, 226, 1210}

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = &-\frac{3a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}\\ &+\frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}\\ &-\frac{6ax(a+bx^2)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{2x\sqrt{ax+bx^3}}{5b}\end{aligned}$$

```
[In] Int[x^3/Sqrt[a*x + b*x^3],x]
```

```
[Out] (-6*a*x*(a + b*x^2))/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) +
(2*x*Sqrt[a*x + b*x^3])/(5*b) + (6*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sq
rt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x]
)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3]) - (3*a^(5/4)*Sqrt[x]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{(3a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5b}\\
&= \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{\left(3a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5b\sqrt{ax+bx^3}}\\
&= \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{\left(6a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5b\sqrt{ax+bx^3}}\\
&= \frac{2x\sqrt{ax+bx^3}}{5b} - \frac{\left(6a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5b^{3/2}\sqrt{ax+bx^3}}\\
&\quad + \frac{\left(6a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5b^{3/2}\sqrt{ax+bx^3}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{6ax(a+bx^2)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{2x\sqrt{ax+bx^3}}{5b} \\
&+ \frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}} \\
&- \frac{3a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26

$$
\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \frac{2x^2\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{5b\sqrt{x\left(a+bx^2\right)}}
$$

```
[In] Integrate[x^3/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x^2*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4,
-((b*x^2)/a)]))/(5*b*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.69

method	result
default	$\frac{2x\sqrt{b\,x^3+ax}}{5b}-\frac{3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{5b^2\sqrt{b\,x^3+ax}}$
elliptic	$\frac{2x\sqrt{b\,x^3+ax}}{5b}-\frac{3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{5b^2\sqrt{b\,x^3+ax}}$
risch	$\frac{2x^2\left(b\,x^2+a\right)}{5b\sqrt{x\left(b\,x^2+a\right)}}-\frac{3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b}\right.}{5b^2\sqrt{b\,x^3+ax}}$

```
[In] int(x^3/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x*(b*x^3+a*x)^(1/2)/b-3/5*a/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/
2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)
/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.17

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \frac{2\left(\sqrt{bx^3+ax}bx+3\,a\sqrt{b}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)\right)}{5\,b^2}$$

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(sqrt(b*x^3 + a*x)*b*x + 3*a*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierst
rassPInverse(-4*a/b, 0, x)))/b^2
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^3}{\sqrt{x\left(a+bx^2\right)}}\,dx$$

```
[In] integrate(x**3/(b*x**3+a*x)**(1/2),x)
[Out] Integral(x**3/sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(b*x^3 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^3}{\sqrt{b\,x^3+a\,x}}\,dx$$

```
[In] int(x^3/(a*x + b*x^3)^(1/2),x)
[Out] int(x^3/(a*x + b*x^3)^(1/2), x)
```

3.59 $\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 116

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx=\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] 2/3*(b*x^3+a*x)^(1/2)/b-1/3*a^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(
1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/
(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2049, 2036, 335, 226}

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx=\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

```
[In] Int[x^2/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int\frac{1}{\sqrt{ax+bx^3}}\,dx}{3b}\\
&= \frac{2\sqrt{ax+bx^3}}{3b}-\frac{\left(a\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3b\sqrt{ax+bx^3}}\\
&= \frac{2\sqrt{ax+bx^3}}{3b}-\frac{\left(2a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3b\sqrt{ax+bx^3}}\\
&= \frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \frac{2x\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3b\sqrt{x\left(a+bx^2\right)}}$$

```
[In] Integrate[x^2/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(
(b*x^2)/a)]))/(3*b*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{2\sqrt{b\,x^3+ax}}{3b}-\frac{a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b^2\sqrt{b\,x^3+ax}}$	127
elliptic	$\frac{2\sqrt{b\,x^3+ax}}{3b}-\frac{a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b^2\sqrt{b\,x^3+ax}}$	127
risch	$\frac{2x\left(b\,x^2+a\right)}{3b\sqrt{x\left(b\,x^2+a\right)}}-\frac{a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b^2\sqrt{b\,x^3+ax}}$	135

```
[In] int(x^2/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*x^3+a*x)^(1/2)/b-1/3*a/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(
1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)
*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(
1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = -\frac{2\left(a\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\sqrt{bx^3+ax}b\right)}{3\,b^2}$$

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] -2/3*(a*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)*b)/b^
2
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^2}{\sqrt{x\,(a+bx^2)}}\,dx$$

```
[In] integrate(x**2/(b*x**3+a*x)**(1/2),x)
[Out] Integral(x**2/sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^2}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx = \int \frac{x^2}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(b*x^3 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx=\int \frac{x^2}{\sqrt{b\,x^3+a\,x}} \,dx$$

```
[In] int(x^2/(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x^2/(a*x + b*x^3)^(1/2), x)
```

3.60 $\int \frac{x}{\sqrt{ax+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 229

$$\begin{aligned}\int \frac{x}{\sqrt{ax+bx^3}}\,dx &= \frac{2x(a+bx^2)}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}\\ &\quad - \frac{2\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}\\ &\quad + \frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}\end{aligned}$$

```
[Out] 2*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*a^(1/4)*(cos(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^
(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1
/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x
^3+a*x)^(1/2)+a^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a
^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(
1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2057, 335, 311, 226, 1210}

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}} - \frac{2\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}} + \frac{2x(a+bx^2)}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}$$

```
[In] Int[x/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*(a + b*x^2))/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*a^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a*x +
 b*x^3]) + (a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(3
/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}} \\
&= \frac{\left(2\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}} \\
&= \frac{\left(2\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{ax+bx^3}} \\
&\quad - \frac{\left(2\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{ax+bx^3}} \\
&= \frac{2x(a+bx^2)}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} \\
&\quad - \frac{2\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}} \\
&\quad + \frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.23

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \frac{2x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{x\left(a+bx^2\right)}}$$

```
[In] Integrate[x/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)])/
(3*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right)}{b\sqrt{b\,x^3+ax}}$	158
elliptic	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right)}{b\sqrt{b\,x^3+ax}}$	158

```
[In] int(x/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2
)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*
(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1
/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2
*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.10

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = -\frac{2\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)}{\sqrt{b}}$$

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] -2*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x))/sqrt(b)
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \int \frac{x}{\sqrt{x\left(a+bx^2\right)}}\,dx$$

```
[In] integrate(x/(b*x**3+a*x)**(1/2),x)
[Out] Integral(x/sqrt(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(b*x^3 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \int \frac{x}{\sqrt{b\,x^3+a\,x}}\,dx$$

```
[In] int(x/(a*x + b*x^3)^(1/2),x)
[Out] int(x/(a*x + b*x^3)^(1/2), x)
```

3.61 $\int \frac{1}{\sqrt{ax+bx^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

```
[Out] (cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*
(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)
/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2036, 335, 226}

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

```
[In] Int[1/Sqrt[a*x + b*x^3],x]
```

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[a*
x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}}\\
&= \frac{\left(2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}}\\
&= \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{x\left(a+bx^2\right)}}$$

```
[In] Integrate[1/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/Sq
rt[x*(a + b*x^2)]
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^{3}+ax}}$	108
elliptic	$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^{3}+ax}}$	108

```
[In] int(1/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2
)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*Elli
pticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.15

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \frac{2\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)}{\sqrt{b}}$$

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*weierstrassPInverse(-4*a/b, 0, x)/sqrt(b)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{ax+bx^3}}\,dx$$

```
[In] integrate(1/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*x^3 + a*x), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}}\,dx$$

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*x^3 + a*x), x)
```

Mupad [B] (verification not implemented)

Time = 10.60 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \frac{2\,x\,\sqrt{\frac{b\,x^2}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{b\,x^2}{a}\right)}{\sqrt{b\,x^3+a\,x}}$$

```
[In] int(1/(a*x + b*x^3)^(1/2),x)
[Out] (2*x*((b*x^2)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(b*x^2)/a))/(a*x + b
*x^3)^(1/2)
```

3.62 $\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 253

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = \frac{2\sqrt{b}x(a+bx^2)}{a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{ax} - \frac{2\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}} + \frac{\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] 2*x*(b*x^2+a)*b^(1/2)/a/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*(b*x^3+a*x)
^(1/2)/a/x-2*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1
/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)+b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)
*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2050, 2057, 335, 311, 226, 1210}

$$\begin{aligned}\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = {} & \frac{\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}} \\ & - \frac{2\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}} \\ & - \frac{2\sqrt{ax+bx^3}}{ax} + \frac{2\sqrt{b}x(a+bx^2)}{a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}\end{aligned}$$

```
[In] Int[1/(x*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (2*Sqrt[b]*x*(a + b*x^2))/(a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*
Sqrt[a*x + b*x^3])/(a*x) - (2*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3]) + (b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[x])/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{b\int\frac{x}{\sqrt{ax+bx^3}}\,dx}{a}\\
&= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(b\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(2b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(2\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a}\sqrt{ax+bx^3}}\\
&\quad -\frac{\left(2\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a}\sqrt{ax+bx^3}}
\end{aligned}
$$

$$= \frac{2\sqrt{b}x(a+bx^2)}{a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{ax} - \frac{2\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}} + \frac{\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^2}{a}\right)}{\sqrt{x\left(a+bx^2\right)}}$$

```
[In] Integrate[1/(x*Sqrt[a*x + b*x^3]),x]
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)])/Sq
rt[x*(a + b*x^2)]
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.72

method	result
default	$-\frac{2(b x^2+a)}{a\sqrt{x(b x^2+a)}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{b}\right.}{a\sqrt{b x^3+ax}}$
risch	$-\frac{2(b x^2+a)}{a\sqrt{x(b x^2+a)}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{b}\right.}{a\sqrt{b x^3+ax}}$
elliptic	$-\frac{2(b x^2+a)}{a\sqrt{x(b x^2+a)}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{b}\right.}{a\sqrt{b x^3+ax}}$

```
[In] int(1/x/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x^2+a)/a/(x*(b*x^2+a))^(1/2)+1/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a
*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^
(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1
/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*
b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.17

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = -\frac{2\left(\sqrt{b}x\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^3+ax}\right)}{ax}$$

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(sqrt(b)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x))
 + sqrt(b*x^3 + a*x))/(a*x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx=\int \frac{1}{x\sqrt{x\,(a+bx^2)}}\,dx$$

```
[In] integrate(1/x/(b*x**3+a*x)**(1/2),x)
[Out] Integral(1/(x*sqrt(x*(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+ax}x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+ax}x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx=\int \frac{1}{x\,\sqrt{b\,x^3+a\,x}}\,dx$$

```
[In] int(1/(x*(a*x + b*x^3)^(1/2)),x)
[Out] int(1/(x*(a*x + b*x^3)^(1/2)), x)
```

3.63 $\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 119

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{ax+bx^3}}{3ax^2} - \frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/a/x^2-1/3*b^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1
/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^
2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2050, 2036, 335, 226}

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = -\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{3ax^2}$$

```
[In] Int[1/(x^2*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^3])/(3*a*x^2) - (b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{b\int\frac{1}{\sqrt{ax+bx^3}}\,dx}{3a}\\
&= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{\left(b\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{\left(2b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3x\sqrt{x\left(a+bx^2\right)}}$$

```
[In] Integrate[1/(x^2*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^2)/a)])/(3
*x*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2\sqrt{b\,x^3+ax}}{3a\,x^2}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3a\sqrt{b\,x^3+ax}}$	129
elliptic	$-\frac{2\sqrt{b\,x^3+ax}}{3a\,x^2}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3a\sqrt{b\,x^3+ax}}$	129
risch	$-\frac{2\left(b\,x^2+a\right)}{3ax\sqrt{x\left(b\,x^2+a\right)}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3a\sqrt{b\,x^3+ax}}$	136

```
[In] int(1/x^2/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/a/x^2-1/3/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2
)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^
(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = -\frac{2\left(\sqrt{b}x^2\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\sqrt{bx^3+ax}\right)}{3\,ax^2}$$

```
[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] -2/3*(sqrt(b)*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x))/(a
*x^2)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = \int \frac{1}{x^2\sqrt{x\left(a+bx^2\right)}}\,dx$$

```
[In] integrate(1/x**2/(b*x**3+a*x)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(x*(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx=\int \frac{1}{x^2\,\sqrt{b\,x^3+a\,x}}\,dx$$

```
[In] int(1/(x^2*(a*x + b*x^3)^(1/2)),x)
[Out] int(1/(x^2*(a*x + b*x^3)^(1/2)), x)
```

3.64 $\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 286

$$\begin{aligned}\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx &= -\frac{6b^{3/2}x(a+bx^2)}{5a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} \\ &\quad + \frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}} \\ &\quad - \frac{3b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}}\end{aligned}$$

```
[Out] -6/5*b^(3/2)*x*(b*x^2+a)/a^2/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2/5*(b*x
^3+a*x)^(1/2)/a/x^3+6/5*b*(b*x^3+a*x)^(1/2)/a^2/x+6/5*b^(5/4)*(cos(2*arctan
(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*
EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^
(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^3+a*x)^
(1/2)-3/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2)
)^2)^(1/2)/a^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2050, 2057, 335, 311, 226, 1210}

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = -\frac{3b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}} + \frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}} - \frac{6b^{3/2}x(a+bx^2)}{5a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} - \frac{2\sqrt{ax+bx^3}}{5ax^3}$$

```
[In] Int[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-6*b^(3/2)*x*(a + b*x^2))/(5*a^2*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
- (2*Sqrt[a*x + b*x^3])/(5*a*x^3) + (6*b*Sqrt[a*x + b*x^3])/(5*a^2*x) + (6*
b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a
*x + b*x^3]) - (3*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])
/(5*a^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax+bx^3}}{5ax^3} - \frac{(3b)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} - \frac{\left(3b^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5a^2}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} - \frac{\left(3b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5a^2\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} - \frac{\left(6b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5a^2\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} - \frac{\left(6b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5a^{3/2}\sqrt{ax+bx^3}}\\
&\quad + \frac{\left(6b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5a^{3/2}\sqrt{ax+bx^3}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{6b^{3/2}x(a+bx^2)}{5a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{5ax^3} + \frac{6b\sqrt{ax+bx^3}}{5a^2x} \\
&\quad + \frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}} \\
&\quad - \frac{3b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$
\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^2\sqrt{x\left(a+bx^2\right)}}
$$

```
[In] Integrate[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^2)/a)])/(
5*x^2*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{2(bx^2+a)(-3bx^2+a)}{5a^2x^2\sqrt{x(bx^2+a)}}-\frac{3b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5a^2\sqrt{bx^3+ax}}$
default	$-\frac{2\sqrt{bx^3+ax}}{5ax^3}+\frac{6(bx^2+a)b}{5a^2\sqrt{x(bx^2+a)}}-\frac{3b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5a^2\sqrt{bx^3+ax}}$
elliptic	$-\frac{2\sqrt{bx^3+ax}}{5ax^3}+\frac{6(bx^2+a)b}{5a^2\sqrt{x(bx^2+a)}}-\frac{3b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{5a^2\sqrt{bx^3+ax}}$

```
[In] int(1/x^3/(b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x^2+a)*(-3*b*x^2+a)/a^2/x^2/(x*(b*x^2+a))^(1/2)-3/5*b/a^2*(-a*b)^(1
/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2
)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)
^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx$$
$$=\frac{2\left(3\,b^{\frac{3}{2}}x^3\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}(3\,bx^2-a)\right)}{5\,a^2x^3}$$

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*b^(3/2)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0
, x)) + sqrt(b*x^3 + a*x)*(3*b*x^2 - a))/(a^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx=\int \frac{1}{x^3\sqrt{x\left(a+bx^2\right)}}\,dx$$

```
[In] integrate(1/x**3/(b*x**3+a*x)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(x*(a + b*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+ax}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+ax}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx=\int \frac{1}{x^3\,\sqrt{b\,x^3+a\,x}}\,dx$$

```
[In] int(1/(x^3*(a*x + b*x^3)^(1/2)),x)
[Out] int(1/(x^3*(a*x + b*x^3)^(1/2)), x)
```

3.65 $\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 161

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = -\frac{x^5}{b\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} + \frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}}$$

```
[Out] -x^5/b/(b*x^3+a*x)^(1/2)-15/7*a*(b*x^3+a*x)^(1/2)/b^3+9/7*x^2*(b*x^3+a*x)^(
1/2)/b^2+15/14*a^(7/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^
(1/2))^2)^(1/2)/b^(13/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2047, 2049, 2036, 335, 226}

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} - \frac{x^5}{b\sqrt{ax+bx^3}}$$

```
[In] Int[x^7/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^5/(b*Sqrt[a*x + b*x^3])) - (15*a*Sqrt[a*x + b*x^3])/(7*b^3) + (9*x^2*Sq
rt[a*x + b*x^3])/(7*b^2) + (15*a^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(14*b^(13/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{x^5}{b\sqrt{ax+bx^3}} + \frac{9\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx}{2b}$$

$$= -\frac{x^5}{b\sqrt{ax+bx^3}} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} - \frac{(45a)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{14b^2}$$

$$= -\frac{x^5}{b\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} + \frac{(15a^2)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{14b^3}$$

$$= -\frac{x^5}{b\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} + \frac{\left(15a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{14b^3\sqrt{ax+bx^3}}$$

$$= -\frac{x^5}{b\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} + \frac{\left(15a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{7b^3\sqrt{ax+bx^3}}$$

$$= -\frac{x^5}{b\sqrt{ax+bx^3}} - \frac{15a\sqrt{ax+bx^3}}{7b^3} + \frac{9x^2\sqrt{ax+bx^3}}{7b^2} + \frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx = \frac{x\left(-15a^2-6abx^2+2b^2x^4+15a^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{7b^3\sqrt{x\left(a+bx^2\right)}}$$

```
[In] Integrate[x^7/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x*(-15*a^2 - 6*a*b*x^2 + 2*b^2*x^4 + 15*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(7*b^3*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.07

method	result
default	$-\frac{x\,a^2}{b^3\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x^2\sqrt{b\,x^3+ax}}{7b^2}-\frac{8a\sqrt{b\,x^3+ax}}{7b^3}+\frac{15a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{14b^4\sqrt{b\,x^3+ax}}$
elliptic	$-\frac{x\,a^2}{b^3\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x^2\sqrt{b\,x^3+ax}}{7b^2}-\frac{8a\sqrt{b\,x^3+ax}}{7b^3}+\frac{15a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{14b^4\sqrt{b\,x^3+ax}}$
risch	$-\frac{2\left(-b\,x^2+4a\right)\left(b\,x^2+a\right)x}{7b^3\sqrt{x\left(b\,x^2+a\right)}}+\frac{a^2\left(\frac{11\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^3+ax}}-7a\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\right.\right.}{7b^3}$

```
[In] int(x^7/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*x*a^2/((x^2+a/b)*b*x)^(1/2)+2/7*x^2*(b*x^3+a*x)^(1/2)/b^2-8/7*a*(b*x
^3+a*x)^(1/2)/b^3+15/14*a^2/b^4*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/
2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b
)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/
2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int \frac{x^7}{\left(ax+bx^3\right)^{3/2}}\,dx=\frac{15\left(a^2bx^2+a^3\right)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(2b^3x^4-6ab^2x^2-15a^2b\right)\sqrt{bx^3}}{7\left(b^5x^2+ab^4\right)}$$

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/7*(15*(a^2*b*x^2 + a^3)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + (2*b^
3*x^4 - 6*a*b^2*x^2 - 15*a^2*b)*sqrt(b*x^3 + a*x))/(b^5*x^2 + a*b^4)
```

Sympy [F]

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx=\int \frac{x^7}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**7/(b*x**3+a*x)**(3/2),x)
[Out] Integral(x**7/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx=\int \frac{x^7}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx=\int \frac{x^7}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(ax+bx^3)^{3/2}}\,dx=\int \frac{x^7}{(b\,x^3+a\,x)^{3/2}}\,dx$$

```
[In] int(x^7/(a*x + b*x^3)^(3/2),x)
[Out] int(x^7/(a*x + b*x^3)^(3/2), x)
```

3.66 $\int \frac{x^6}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 279

$$\int \frac{x^6}{\left(ax+bx^3\right)^{3/2}}\,dx=-\frac{x^4}{b\sqrt{ax+bx^3}}-\frac{21ax\left(a+bx^2\right)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{7x\sqrt{ax+bx^3}}{5b^2}+\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{ax+bx^3}}-\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}}$$

```
[Out] -x^4/b/(b*x^3+a*x)^(1/2)-21/5*a*x*(b*x^2+a)/b^(5/2)/(a^(1/2)+x*b^(1/2))/(b*
x^3+a*x)^(1/2)+7/5*x*(b*x^3+a*x)^(1/2)/b^2+21/5*a^(5/4)*(cos(2*arctan(b^(1/
4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ellipt
icE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))
*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(11/4)/(b*x^3+a*x)^(1/2)
-21/10*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2
)^(1/2)/b^(11/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2047, 2049, 2057, 335, 311, 226, 1210}

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = -\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}} + \frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{ax+bx^3}} - \frac{21ax(a+bx^2)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} - \frac{x^4}{b\sqrt{ax+bx^3}}$$

```
[In] Int[x^6/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^4/(b*Sqrt[a*x + b*x^3])) - (21*a*x*(a + b*x^2))/(5*b^(5/2)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[a*x + b*x^3]) + (7*x*Sqrt[a*x + b*x^3])/(5*b^2) + (21*a^(5/4
)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[a*x +
b*x^3]) - (21*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[
a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10
*b^(11/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^4}{b\sqrt{ax+bx^3}}+\frac{7\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x^4}{b\sqrt{ax+bx^3}}+\frac{7x\sqrt{ax+bx^3}}{5b^2}-\frac{(21a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{10b^2}\\
&= -\frac{x^4}{b\sqrt{ax+bx^3}}+\frac{7x\sqrt{ax+bx^3}}{5b^2}-\frac{\left(21a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{10b^2\sqrt{ax+bx^3}}
\end{aligned}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} - \frac{\left(21a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5b^2\sqrt{ax+bx^3}}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} - \frac{\left(21a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5b^{5/2}\sqrt{ax+bx^3}} + \frac{\left(21a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5b^{5/2}\sqrt{ax+bx^3}}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} - \frac{21ax(a+bx^2)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} + \frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{ax+bx^3}} - \frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.24

$$\int \frac{x^6}{\left(ax+bx^3\right)^{3/2}}\,dx = \frac{2x^2\left(-7a+bx^2+7a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{5b^2\sqrt{x\left(a+bx^2\right)}}$$

```
[In] Integrate[x^6/(a*x + b*x^3)^(3/2),x]
[Out] (2*x^2*(-7*a + b*x^2 + 7*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2,
7/4, -((b*x^2)/a)]))/(5*b^2*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.72

method	result
default	$\frac{x^2 a}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x\sqrt{b\,x^3+ax}}{5b^2}-\frac{21a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{10b^3\sqrt{b\,x^3+ax}}$
elliptic	$\frac{x^2 a}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2x\sqrt{b\,x^3+ax}}{5b^2}-\frac{21a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{10b^3\sqrt{b\,x^3+ax}}$
risch	$\frac{2x^2\left(b\,x^2+a\right)}{5b^2\sqrt{x\left(b\,x^2+a\right)}}-\frac{a\left(\frac{8\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{b}\right.}{b\sqrt{b\,x^3+ax}}\right.}{}$

```
[In] int(x^6/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*x^2*a/((x^2+a/b)*b*x)^(1/2)+2/5*x*(b*x^3+a*x)^(1/2)/b^2-21/10*a/b^3*(
-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b
)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a
*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)
)+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^
(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.27

$$\int \frac{x^6}{\left(ax+bx^3\right)^{3/2}}\,dx=\frac{21\left(abx^2+a^2\right)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(2\,b^2x^3-\right.}{5\left(b^4x^2+ab^3\right)}$$

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(21*(a*b*x^2 + a^2)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInve
rse(-4*a/b, 0, x)) + (2*b^2*x^3 + 7*a*b*x)*sqrt(b*x^3 + a*x))/(b^4*x^2 + a*
b^3)
```

Sympy [F]

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**6/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**6/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(b\,x^3+a\,x)^{3/2}}\,dx$$

```
[In] int(x^6/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^6/(a*x + b*x^3)^(3/2), x)
```

3.67 $\int \frac{x^5}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 137

$$\int \frac{x^5}{\left(ax+bx^3\right)^{3/2}}\,dx = -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2} - \frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}}$$

```
[Out] -x^3/b/(b*x^3+a*x)^(1/2)+5/3*(b*x^3+a*x)^(1/2)/b^2-5/6*a^(3/4)*(cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+a*x)
^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2047, 2049, 2036, 335, 226}

$$\int \frac{x^5}{\left(ax+bx^3\right)^{3/2}}\,dx = -\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}} + \frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

```
[In] Int[x^5/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^3/(b*Sqrt[a*x + b*x^3])) + (5*Sqrt[a*x + b*x^3])/(3*b^2) - (5*a^(3/4)*S
qrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*b^(9/4)*Sqrt[a*x + b*x^
3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{(5a)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{6b^2}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{\left(5a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{6b^2\sqrt{ax+bx^3}}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{\left(5a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3b^2\sqrt{ax+bx^3}}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.49

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx=\frac{x\left(5a+2bx^2-5a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3b^2\sqrt{x\left(a+bx^2\right)}}$$

```
[In] Integrate[x^5/(a*x + b*x^3)^(3/2),x]
[Out] (x*(5*a + 2*b*x^2 - 5*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4
, -((b*x^2)/a)]))/(3*b^2*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

method	result
default	$\frac{xa}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2\sqrt{b\,x^3+ax}}{3b^2}-\frac{5a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6b^3\sqrt{b\,x^3+ax}}$
elliptic	$\frac{xa}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{2\sqrt{b\,x^3+ax}}{3b^2}-\frac{5a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6b^3\sqrt{b\,x^3+ax}}$
risch	$\frac{2\left(b\,x^2+a\right)x}{3b^2\sqrt{x\left(b\,x^2+a\right)}}-\frac{a\left(\frac{4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^3+ax}}-3a\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\sqrt{-ab}\,\sqrt{\left(x+\right.}\right.\right.}{3b^2}$

```
[In] int(x^5/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*x*a/((x^2+a/b)*b*x)^(1/2)+2/3*(b*x^3+a*x)^(1/2)/b^2-5/6*a/b^3*(-a*b)^
(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*
b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

$$\int \frac{x^5}{\left(ax+bx^3\right)^{3/2}}\,dx = -\frac{5\left(abx^2+a^2\right)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\left(2b^2x^2+5ab\right)\sqrt{bx^3+ax}}{3\left(b^4x^2+ab^3\right)}$$

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(5*(a*b*x^2 + a^2)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - (2*b^2*
x^2 + 5*a*b)*sqrt(b*x^3 + a*x))/(b^4*x^2 + a*b^3)
```

Sympy [F]

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5/(b*x**3+a*x)**(3/2),x)
[Out] Integral(x**5/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^5}{(b\,x^3+a\,x)^{3/2}}\,dx$$

```
[In] int(x^5/(a*x + b*x^3)^(3/2),x)
[Out] int(x^5/(a*x + b*x^3)^(3/2), x)
```

3.68 $\int \frac{x^4}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 253

$$\begin{aligned}\int \frac{x^4}{\left(ax+bx^3\right)^{3/2}}\,dx &= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{3x(a+bx^2)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}\\ &\quad-\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{ax+bx^3}}\\ &\quad+\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}}\end{aligned}$$

```
[Out] -x^2/b/(b*x^3+a*x)^(1/2)+3*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a
*x)^(1/2)-3*a^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/
2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)+3/2*a^(1/4)*(cos(2*arctan(b^(1/4)*x^
(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(s
in(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1
/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2047, 2057, 335, 311, 226, 1210}

$$\int \frac{x^4}{\left(ax+bx^3\right)^{3/2}}\,dx=\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{ax+bx^3}}+\frac{3x(a+bx^2)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{x^2}{b\sqrt{ax+bx^3}}$$

```
[In] Int[x^4/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^2/(b*Sqrt[a*x + b*x^3])) + (3*x*(a + b*x^2))/(b^(3/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[a*x + b*x^3]) - (3*a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(b^(7/4)*Sqrt[a*x + b*x^3]) + (3*a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqr
t[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/
4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{3\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{\left(3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2b\sqrt{ax+bx^3}}\\
&= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{\left(3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{b\sqrt{ax+bx^3}}\\
&= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{\left(3\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{b^{3/2}\sqrt{ax+bx^3}}\\
&\quad -\frac{\left(3\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{b^{3/2}\sqrt{ax+bx^3}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{x^2}{b\sqrt{ax+bx^3}} + \frac{3x(a+bx^2)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} \\
&\quad - \frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{ax+bx^3}} \\
&\quad + \frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.23

$$
\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = -\frac{2x^2\left(-1+\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{b\sqrt{x\,(a+bx^2)}}
$$

```
[In] Integrate[x^4/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^2*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2
)/a)]))/(b*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.72

method	result
default	$-\frac{x^2}{b\sqrt{(x^2+\frac{a}{b})bx}}+\frac{3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\Bigg(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\Bigg(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}}{b}}{2b^2\sqrt{b\,x^3+ax}}$
elliptic	$-\frac{x^2}{b\sqrt{(x^2+\frac{a}{b})bx}}+\frac{3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\Bigg(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\Bigg(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}}{b}}{2b^2\sqrt{b\,x^3+ax}}$

```
[In] int(x^4/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*x^2/((x^2+a/b)*b*x)^(1/2)+3/2/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a
*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^
(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1
/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*
b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.24

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = -\frac{\sqrt{bx^3+ax}bx+3\,(bx^2+a)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{b^3x^2+ab^2}$$

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*x^3 + a*x)*b*x + 3*(b*x^2 + a)*sqrt(b)*weierstrassZeta(-4*a/b, 0,
weierstrassPInverse(-4*a/b, 0, x)))/(b^3*x^2 + a*b^2)
```

Sympy [F]

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(x\,(a+bx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**4/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(b\,x^3+a\,x)^{3/2}}\,dx$$

```
[In] int(x^4/(a*x + b*x^3)^(3/2),x)
[Out] int(x^4/(a*x + b*x^3)^(3/2), x)
```

3.69 $\int \frac{x^3}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int \frac{x^3}{\left(ax+bx^3\right)^{3/2}}\,dx = -\frac{x}{b\sqrt{ax+bx^3}} + \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] -x/b/(b*x^3+a*x)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1
/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+
x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2047, 2036, 335, 226}

$$\int \frac{x^3}{\left(ax+bx^3\right)^{3/2}}\,dx = \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{ax+bx^3}} - \frac{x}{b\sqrt{ax+bx^3}}$$

```
[In] Int[x^3/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x/(b*Sqrt[a*x + b*x^3])) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2
)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1
/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{2b\sqrt{ax+bx^3}}\\
&= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{b\sqrt{ax+bx^3}}\\
&= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \frac{x\left(-1+\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{b\sqrt{x\left(a+bx^2\right)}}$$

```
[In] Integrate[x^3/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]
))/(b*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13

method	result	size
default	$-\frac{x}{b\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2b^2\sqrt{b\,x^3+ax}}$	130
elliptic	$-\frac{x}{b\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2b^2\sqrt{b\,x^3+ax}}$	130

```
[In] int(x^3/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*x/((x^2+a/b)*b*x)^(1/2)+1/2/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b
)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1
/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b
)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \frac{(bx^2+a)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^3+ax}b}{b^3x^2+ab^2}$$

```
[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b*x^2 + a)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^3 + a*x)*
b)/(b^3*x^2 + a*b^2)
```

Sympy [F]

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(x(a+bx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(b\,x^3+a\,x)^{3/2}}\,dx$$

```
[In] int(x^3/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^3/(a*x + b*x^3)^(3/2), x)
```

3.70 $\int \frac{x^2}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 254

$$
\begin{aligned}
\int \frac{x^2}{\left(ax+bx^3\right)^{3/2}}\,dx &= \frac{x^2}{a\sqrt{ax+bx^3}}-\frac{x\left(a+bx^2\right)}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} \\
&+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}} \\
&-\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

```
[Out] x^2/a/(b*x^3+a*x)^(1/2)-x*(b*x^2+a)/a/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*
x)^(1/2)+(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*
2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2
)/a^(3/4)/b^(3/4)/(b*x^3+a*x)^(1/2)-1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan
(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2
+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2048, 2057, 335, 311, 226, 1210}

$$\begin{aligned}\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = &-\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}}\\ &+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}}\\ &+\frac{x^2}{a\sqrt{ax+bx^3}}-\frac{x(a+bx^2)}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}\end{aligned}$$

```
[In] Int[x^2/(a*x + b*x^3)^(3/2),x]
```

```
[Out] x^2/(a*Sqrt[a*x + b*x^3]) - (x*(a + b*x^2))/(a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(
a^(3/4)*b^(3/4)*Sqrt[a*x + b*x^3]) - (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2}{a\sqrt{ax+bx^3}} - \frac{\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2a} \\
&= \frac{x^2}{a\sqrt{ax+bx^3}} - \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2a\sqrt{ax+bx^3}} \\
&= \frac{x^2}{a\sqrt{ax+bx^3}} - \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{a\sqrt{ax+bx^3}} \\
&= \frac{x^2}{a\sqrt{ax+bx^3}} - \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{ax+bx^3}} \\
&\quad + \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{ax+bx^3}}
\end{aligned}
$$

$$= \frac{x^2}{a\sqrt{ax+bx^3}} - \frac{x(a+bx^2)}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}} - \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.22

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx = \frac{2x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)}{3a\sqrt{x\,(a+bx^2)}}$$

```
[In] Integrate[x^2/(a*x + b*x^3)^(3/2),x]
[Out] (2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2)/a)])/
(3*a*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.72

method	result
default	$\frac{x^2}{a\sqrt{(x^2+\frac{a}{b})bx}} - \frac{\sqrt{-ab}\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{2(x-\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}},\right.}{b}\right.}{2ab\sqrt{b\,x^3+ax}}$
elliptic	$\frac{x^2}{a\sqrt{(x^2+\frac{a}{b})bx}} - \frac{\sqrt{-ab}\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{2(x-\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{(x+\frac{\sqrt{-ab}}{b})b}{\sqrt{-ab}}},\right.}{b}\right.}{2ab\sqrt{b\,x^3+ax}}$

```
[In] int(x^2/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^2/a/((x^2+a/b)*b*x)^(1/2)-1/2/a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/
2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)
/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.24

$$\int \frac{x^2}{\left(ax+bx^3\right)^{3/2}}\,dx = \frac{\sqrt{bx^3+ax}bx+\left(bx^2+a\right)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{ab^2x^2+a^2b}$$

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*x^3 + a*x)*b*x + (b*x^2 + a)*sqrt(b)*weierstrassZeta(-4*a/b, 0, wei
erstrassPInverse(-4*a/b, 0, x)))/(a*b^2*x^2 + a^2*b)
```

Sympy [F]

$$\int \frac{x^2}{\left(ax+bx^3\right)^{3/2}}\,dx = \int \frac{x^2}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(ax+bx^3\right)^{3/2}}\,dx = \int \frac{x^2}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx=\int \frac{x^2}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax+bx^3)^{3/2}}\,dx=\int \frac{x^2}{(b\,x^3+a\,x)^{3/2}}\,dx$$

```
[In] int(x^2/(a*x + b*x^3)^(3/2),x)
[Out] int(x^2/(a*x + b*x^3)^(3/2), x)
```

3.71 $\int \frac{x}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 114

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \frac{x}{a\sqrt{ax+bx^3}} + \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

```
[Out] x/a/(b*x^3+a*x)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x
*b^(1/2))^2)^(1/2)/a^(5/4)/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2048, 2036, 335, 226}

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}} + \frac{x}{a\sqrt{ax+bx^3}}$$

```
[In] Int[x/(a*x + b*x^3)^(3/2),x]
```

```
[Out] x/(a*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2]
)/(2*a^(5/4)*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{2a}\\
&= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{2a\sqrt{ax+bx^3}}\\
&= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^3}}\\
&= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.47

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx=\frac{x+x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)}{a\sqrt{x\,(a+bx^2)}}$$

```
[In] Integrate[x/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x + x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/
(a*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{x}{a\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2ab\sqrt{b\,x^{3}+ax}}$	132
elliptic	$\frac{x}{a\sqrt{\left(x^{2}+\frac{a}{b}\right)bx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2ab\sqrt{b\,x^{3}+ax}}$	132

```
[In] int(x/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/a/((x^2+a/b)*b*x)^(1/2)+1/2/a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(
1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)
*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(
1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.45

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx=\frac{(bx^2+a)\sqrt{b}\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^3+ax}b}{ab^2x^2+a^2b}$$

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b*x^2 + a)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^3 + a*x)*
b)/(a*b^2*x^2 + a^2*b)
```

Sympy [F]

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x**3+a*x)**(3/2),x)
[Out] Integral(x/(x*(a + b*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x/(b*x^3 + a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x/(b*x^3 + a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax+bx^3)^{3/2}}\,dx = \int \frac{x}{\left(b\,x^3+a\,x\right)^{3/2}}\,dx$$

```
[In] int(x/(a*x + b*x^3)^(3/2),x)
[Out] int(x/(a*x + b*x^3)^(3/2), x)
```

3.72 $\int \frac{1}{(ax+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 273

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = \frac{1}{a\sqrt{ax+bx^3}} + \frac{3\sqrt{b}x(a+bx^2)}{a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} - \frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{a^{7/4}\sqrt{ax+bx^3}} + \frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] 1/a/(b*x^3+a*x)^(1/2)+3*x*(b*x^2+a)*b^(1/2)/a^2/(a^(1/2)+x*b^(1/2))/(b*x^3+
a*x)^(1/2)-3*(b*x^3+a*x)^(1/2)/a^2/x-3*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2
*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*
((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^3+a*x)^(1/2)+3/2*b^(1/
4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^
(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7
/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2031, 2050, 2057, 335, 311, 226, 1210}

$$\int \frac{1}{\left(ax+bx^3\right)^{3/2}}\,dx = \frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}}$$
$$-\frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{7/4}\sqrt{ax+bx^3}}$$
$$-\frac{3\sqrt{ax+bx^3}}{a^2x}+\frac{3\sqrt{b}x(a+bx^2)}{a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{1}{a\sqrt{ax+bx^3}}$$

```
[In] Int[(a*x + b*x^3)^(-3/2),x]
```

```
[Out] 1/(a*Sqrt[a*x + b*x^3]) + (3*Sqrt[b]*x*(a + b*x^2))/(a^2*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[a*x + b*x^3]) - (3*Sqrt[a*x + b*x^3])/(a^2*x) - (3*b^(1/4)*Sqrt[x]
*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[
2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(7/4)*Sqrt[a*x + b*x^3]) + (3
*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*
x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[
a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{a\sqrt{ax+bx^3}} + \frac{3\int \frac{1}{x\sqrt{ax+bx^3}}\,dx}{2a}\\
&= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{(3b)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2a^2}\\
&= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{\left(3b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2a^2\sqrt{ax+bx^3}}\\
&= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{\left(3b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{a^2\sqrt{ax+bx^3}}
\end{aligned}
$$

$$= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{\left(3\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{a^{3/2}\sqrt{ax+bx^3}}$$
$$- \frac{\left(3\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{a^{3/2}\sqrt{ax+bx^3}}$$
$$= \frac{1}{a\sqrt{ax+bx^3}} + \frac{3\sqrt{b}x(a+bx^2)}{a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x}$$
$$- \frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{7/4}\sqrt{ax+bx^3}}$$
$$+ \frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.19

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{bx^2}{a}}\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{bx^2}{a}\right)}{a\sqrt{x(a+bx^2)}}$$

```
[In] Integrate[(a*x + b*x^3)^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^2)/a)])/(a
*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.75

method	result
default	$-\frac{2(b x^2+a)}{a^2\sqrt{x(b x^2+a)}}-\frac{b x^2}{a^2\sqrt{(x^2+\frac{a}{b})bx}}+\frac{3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{2a^2\sqrt{b x^3+ax}}$
elliptic	$-\frac{2(b x^2+a)}{a^2\sqrt{x(b x^2+a)}}-\frac{b x^2}{a^2\sqrt{(x^2+\frac{a}{b})bx}}+\frac{3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{2a^2\sqrt{b x^3+ax}}$
risch	$-\frac{2(b x^2+a)}{a^2\sqrt{x(b x^2+a)}}+\frac{b^2\left(\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a}}{b}\right.}{\sqrt{-ab}}}\right.}{b}\right.}{b^2\sqrt{b x^3+ax}}\right.}{}$

```
[In] int(1/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x^2+a)/a^2/(x*(b*x^2+a))^(1/2)-b*x^2/a^2/((x^2+a/b)*b*x)^(1/2)+3/2/a^
2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2
)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*
(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1
/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2
*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.26

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = -\frac{3\,(bx^3+ax)\sqrt{b}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\sqrt{bx^3+ax}(3\,bx^2+2\,a)}{a^2bx^3+a^3x}$$

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
[Out] -(3*(b*x^3 + a*x)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4
*a/b, 0, x)) + sqrt(b*x^3 + a*x)*(3*b*x^2 + 2*a))/(a^2*b*x^3 + a^3*x)
```

Sympy [F]

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = \int \frac{1}{(ax+bx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**3+a*x)**(3/2),x)
[Out] Integral((a*x + b*x**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{1}{(ax+bx^3)^{3/2}}\,dx = -\frac{2\,x\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{2};\ \frac{3}{4};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^3+a\,x\right)^{3/2}}$$

```
[In] int(1/(a*x + b*x^3)^(3/2),x)
```

```
[Out] -(2*x*((b*x^2)/a + 1)^(3/2)*hypergeom([-1/4, 3/2], 3/4, -(b*x^2)/a))/(a*x +
 b*x^3)^(3/2)
```

3.73 $\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\, dx$

Optimal result

Integrand size = 17, antiderivative size = 139

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\, dx = \frac{1}{ax\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} - \frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}$$

```
[Out] 1/a/x/(b*x^3+a*x)^(1/2)-5/3*(b*x^3+a*x)^(1/2)/a^2/x^2-5/6*b^(3/4)*(cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)/(b*x^3+a
*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2048, 2050, 2036, 335, 226}

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\, dx = -\frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}} - \frac{5\sqrt{ax+bx^3}}{3a^2x^2} + \frac{1}{ax\sqrt{ax+bx^3}}$$

```
[In] Int[1/(x*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] 1/(a*x*Sqrt[a*x + b*x^3]) - (5*Sqrt[a*x + b*x^3])/(3*a^2*x^2) - (5*b^(3/4)*
Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*a^(9/4)*Sqrt[a*x + b*x
^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{ax\sqrt{ax+bx^3}}+\frac{5\int\frac{1}{x^2\sqrt{ax+bx^3}}\,dx}{2a}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{(5b)\int\frac{1}{\sqrt{ax+bx^3}}\,dx}{6a^2}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{\left(5b\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{6a^2\sqrt{ax+bx^3}}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{\left(5b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3a^2\sqrt{ax+bx^3}}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.40

$$
\int\frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx=-\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3ax\sqrt{x\left(a+bx^2\right)}}
$$

```
[In] Integrate[1/(x*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^2)/a)])/(3
*a*x*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08

method	result
default	$-\frac{bx}{a^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}-\frac{2\sqrt{b\,x^3+ax}}{3a^2x^2}-\frac{5\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6a^2\sqrt{b\,x^3+ax}}$
elliptic	$-\frac{bx}{a^2\sqrt{\left(x^2+\frac{a}{b}\right)bx}}-\frac{2\sqrt{b\,x^3+ax}}{3a^2x^2}-\frac{5\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6a^2\sqrt{b\,x^3+ax}}$
risch	$-\frac{2\left(b\,x^2+a\right)}{3a^2x\sqrt{x\left(b\,x^2+a\right)}}-\frac{b\left(\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{b\,x^3+ax}}\right)+3a\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bx}}+\right.}{3a^2}$

```
[In] int(1/x/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -b*x/a^2/((x^2+a/b)*b*x)^(1/2)-2/3*(b*x^3+a*x)^(1/2)/a^2/x^2-5/6/a^2*(-a*b)
^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a
*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((
x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.49

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx = -\frac{5\left(bx^4+ax^2\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^3+ax}\left(5\,bx^2+2\,a\right)}{3\left(a^2bx^4+a^3x^2\right)}$$

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(5*(b*x^4 + a*x^2)*sqrt(b)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*
x^3 + a*x)*(5*b*x^2 + 2*a))/(a^2*b*x^4 + a^3*x^2)
```

Sympy [F]

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx=\int \frac{1}{x\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(b*x**3+a*x)**(3/2),x)
[Out] Integral(1/(x*(x*(a + b*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx=\int \frac{1}{x\left(b\,x^3+a\,x\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(a*x + b*x^3)^(3/2)),x)
[Out] int(1/(x*(a*x + b*x^3)^(3/2)), x)
```

3.74 $\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 306

$$\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx = \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{21b^{3/2}x(a+bx^2)}{5a^3\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} + \frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+bx^3}} - \frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}}$$

```
[Out] 1/a/x^2/(b*x^3+a*x)^(1/2)-21/5*b^(3/2)*x*(b*x^2+a)/a^3/(a^(1/2)+x*b^(1/2))/
(b*x^3+a*x)^(1/2)-7/5*(b*x^3+a*x)^(1/2)/a^2/x^3+21/5*b*(b*x^3+a*x)^(1/2)/a^
3/x+21/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))
^2)^(1/2)/a^(11/4)/(b*x^3+a*x)^(1/2)-21/10*b^(5/4)*(cos(2*arctan(b^(1/4)*x^
(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(s
in(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1
/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2048, 2050, 2057, 335, 311, 226, 1210}

$$\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx = -\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}} + \frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+bx^3}} - \frac{21b^{3/2}x(a+bx^2)}{5a^3\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{1}{ax^2\sqrt{ax+bx^3}}$$

```
[In] Int[1/(x^2*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] 1/(a*x^2*Sqrt[a*x + b*x^3]) - (21*b^(3/2)*x*(a + b*x^2))/(5*a^3*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (7*Sqrt[a*x + b*x^3])/(5*a^2*x^3) + (21*b*S
qrt[a*x + b*x^3])/(5*a^3*x) + (21*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[a*x + b*x^3]) - (21*b^(5/4)*Sqrt[x]*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10*a^(11/4)*Sqrt[a*x + b*x^3])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{ax^2\sqrt{ax+bx^3}} + \frac{7\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx}{2a} \\
&= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} - \frac{(21b)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx}{10a^2} \\
&= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{(21b^2)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{10a^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{ax^2\sqrt{ax+bx^3}}-\frac{7\sqrt{ax+bx^3}}{5a^2x^3}+\frac{21b\sqrt{ax+bx^3}}{5a^3x}-\frac{\left(21b^2\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{10a^3\sqrt{ax+bx^3}}\\
&= \frac{1}{ax^2\sqrt{ax+bx^3}}-\frac{7\sqrt{ax+bx^3}}{5a^2x^3}+\frac{21b\sqrt{ax+bx^3}}{5a^3x}\\
&\quad-\frac{\left(21b^2\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int\frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5a^3\sqrt{ax+bx^3}}\\
&= \frac{1}{ax^2\sqrt{ax+bx^3}}-\frac{7\sqrt{ax+bx^3}}{5a^2x^3}+\frac{21b\sqrt{ax+bx^3}}{5a^3x}\\
&\quad-\frac{\left(21b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5a^{5/2}\sqrt{ax+bx^3}}\\
&\quad+\frac{\left(21b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int\frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5a^{5/2}\sqrt{ax+bx^3}}\\
&= \frac{1}{ax^2\sqrt{ax+bx^3}}-\frac{21b^{3/2}x(a+bx^2)}{5a^3\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{7\sqrt{ax+bx^3}}{5a^2x^3}+\frac{21b\sqrt{ax+bx^3}}{5a^3x}\\
&\quad+\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+bx^3}}\\
&\quad-\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.18

$$
\int\frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx=-\frac{2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5ax^2\sqrt{x\left(a+bx^2\right)}}
$$

```
[In] Integrate[1/(x^2*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^2)/a)])/(
5*a*x^2*Sqrt[x*(a + b*x^2)])
```

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.75

method	result
default	$-\frac{2\sqrt{b\,x^3+ax}}{5a^2x^3}+\frac{16(b\,x^2+a)b}{5a^3\sqrt{x(b\,x^2+a)}}+\frac{b^2x^2}{a^3\sqrt{(x^2+\frac{a}{b})bx}}-\frac{21b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\left(\right.}\right.}{}\right.}{10a^3\sqrt{b\,x^3+ax}}$
elliptic	$-\frac{2\sqrt{b\,x^3+ax}}{5a^2x^3}+\frac{16(b\,x^2+a)b}{5a^3\sqrt{x(b\,x^2+a)}}+\frac{b^2x^2}{a^3\sqrt{(x^2+\frac{a}{b})bx}}-\frac{21b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\left(\right.}\right.}{}\right.}{10a^3\sqrt{b\,x^3+ax}}$
risch	$-\frac{2(b\,x^2+a)(-8b\,x^2+a)}{5a^3x^2\sqrt{x(b\,x^2+a)}}-\frac{b^2\left(\frac{8\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\right.}\right.}{}\right.}{b\sqrt{b\,x^3+ax}}\right.}{}$

```
[In] int(1/x^2/(b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x^3+a*x)^(1/2)/a^2/x^3+16/5*(b*x^2+a)/a^3*b/(x*(b*x^2+a))^(1/2)+b^2
*x^2/a^3/((x^2+a/b)*b*x)^(1/2)-21/10*b/a^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)
/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a
*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+
(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx=\frac{21\left(b^2x^5+abx^3\right)\sqrt{b}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+(21\,b}{5\left(a^3bx^5+a^4x^3\right)}$$

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(21*(b^2*x^5 + a*b*x^3)*sqrt(b)*weierstrassZeta(-4*a/b, 0, weierstrassP
Inverse(-4*a/b, 0, x)) + (21*b^2*x^4 + 14*a*b*x^2 - 2*a^2)*sqrt(b*x^3 + a*x
))/(a^3*b*x^5 + a^4*x^3)
```

Sympy [F]

$$\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x*(a + b*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(ax+bx^3\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(b\,x^3+a\,x\right)}^{3/2}} \,dx$$

```
[In] int(1/(x^2*(a*x + b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^3)^(3/2)), x)
```

3.75 $\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}} \, dx = -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} - \frac{9a\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}}$$

```
[Out] -1/7*x^(25/2)/b/(b*x^3+a*x)^(7/2)-9/35*x^(19/2)/b^2/(b*x^3+a*x)^(5/2)-3/5*x
^(13/2)/b^3/(b*x^3+a*x)^(3/2)-9/2*a*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x)^(1/
2))/b^(11/2)-3*x^(7/2)/b^4/(b*x^3+a*x)^(1/2)+9/2*x^(1/2)*(b*x^3+a*x)^(1/2)/
b^5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2047, 2049, 2054, 212}

$$\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}} \, dx = -\frac{9a\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[In] Int[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(25/2)/(b*(a*x + b*x^3)^(7/2)) - (9*x^(19/2))/(35*b^2*(a*x + b*x^3)^
(5/2)) - (3*x^(13/2))/(5*b^3*(a*x + b*x^3)^(3/2)) - (3*x^(7/2))/(b^4*Sqrt[a
```

```
*x + b*x^3]) + (9*Sqrt[x]*Sqrt[a*x + b*x^3])/(2*b^5) - (9*a*ArcTanh[(Sqrt[b
]*x^(3/2))/Sqrt[a*x + b*x^3]])/(2*b^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{9\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\
&= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}}+\frac{9\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{5b^2}\\
&= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}}+\frac{3\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{b^3}\\
&= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}+\frac{9\int \frac{x^{5/2}}{\sqrt{ax+bx^3}}\,dx}{b^4}
\end{aligned}
$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} - \frac{(9a)\int \frac{\sqrt{x}}{\sqrt{ax+bx^3}}\,dx}{2b^5}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} - \frac{(9a)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^5}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} - \frac{9a\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{9/2}\left(\sqrt{b}(a+bx^2)\left(315a^4x+1050a^3bx^3+1218a^2b^2x^5+528ab^3x^7+35b^4x^9\right)-630a(a\right.}{70b^{11/2}\left(x\left(a+bx^2\right)\right)^{9/2}}$$

```
[In] Integrate[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(Sqrt[b]*(a + b*x^2)*(315*a^4*x + 1050*a^3*b*x^3 + 1218*a^2*b^2*x^
5 + 528*a*b^3*x^7 + 35*b^4*x^9) - 630*a*(a + b*x^2)^(9/2)*ArcTanh[(Sqrt[b]*
x)/(-Sqrt[a] + Sqrt[a + b*x^2])]))/(70*b^(11/2)*(x*(a + b*x^2))^(9/2))
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.33

method	result
default	$-\frac{\sqrt{x(b\,x^2+a)}\left(-35x^9b^{\frac{9}{2}}+315\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)a\,b^3x^6\sqrt{b\,x^2+a}-528b^{\frac{7}{2}}a\,x^7+945\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)a^2b^2x^4\sqrt{b\,x^2+a}-1218b^{\frac{5}{2}}a^2x\right.}{70b^{\frac{11}{2}}\sqrt{x}\,(b\,x^2+a)^4}$
risch	$\frac{x^{\frac{3}{2}}\left(b\,x^2+a\right)}{2b^5\sqrt{x(b\,x^2+a)}}+\frac{\left(-\frac{9a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{11}{2}}}-\frac{a^3\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}}{112b^7\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)^4}-\frac{53a^2\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}}{560b^7\left(x+\frac{\sqrt{-ab}}{b}\right)^3}+\frac{571a^2}{}\right.}{}$

```
[In] int(x^(29/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/70*(x*(b*x^2+a))^(1/2)/b^(11/2)*(-35*x^9*b^(9/2)+315*ln(x*b^(1/2)+(b*x^2
+a)^(1/2))*a*b^3*x^6*(b*x^2+a)^(1/2)-528*b^(7/2)*a*x^7+945*ln(x*b^(1/2)+(b*
x^2+a)^(1/2))*a^2*b^2*x^4*(b*x^2+a)^(1/2)-1218*b^(5/2)*a^2*x^5+945*ln(x*b^(
1/2)+(b*x^2+a)^(1/2))*a^3*b*x^2*(b*x^2+a)^(1/2)-1050*b^(3/2)*a^3*x^3+315*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))*a^4*(b*x^2+a)^(1/2)-315*b^(1/2)*a^4*x)/x^(1/2)/
(b*x^2+a)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.36

$$\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \left[\frac{315\,\left(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5\right)\sqrt{b}\log\Big(2\,bx^2-2\,\sqrt{bx^3+ax}\sqrt{b}\sqrt{x}}{140\,\big(b^{10}x^8+4\,ab^9x^6+6}\right.$$

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(b)*log(2*b*x^2 - 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) + 2*(35*b^5
*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*b)*sqr
t(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7
*x^2 + a^4*b^6), 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a
^4*b*x^2 + a^5)*sqrt(-b)*arctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2))) + (
35*b^5*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*
b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a
^3*b^7*x^2 + a^4*b^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(29/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{29}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(29/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.63

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(\left(\left(x^2\left(\frac{35\,x^2}{b}+\frac{528\,a}{b^2}\right)+\frac{1218\,a^2}{b^3}\right)x^2+\frac{1050\,a^3}{b^4}\right)x^2+\frac{315\,a^4}{b^5}\right)x}{70\,(bx^2+a)^{\frac{7}{2}}} + \frac{9\,a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}} - \frac{9\,a\log\left(|a|\right)}{4\,b^{\frac{11}{2}}}$$

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
+ 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(11/2) - 9/4*a*log(abs(a))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{29/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{29/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

```
[In] int(x^(29/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(29/2)/(a*x + b*x^3)^(9/2), x)
```

3.76 $\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{23/2}}{7b(ax+bx^3)^{7/2}} - \frac{8x^{17/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{16x^{11/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} + \frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}$$

```
[Out] -1/7*x^(23/2)/b/(b*x^3+a*x)^(7/2)-8/35*x^(17/2)/b^2/(b*x^3+a*x)^(5/2)-16/35
*x^(11/2)/b^3/(b*x^3+a*x)^(3/2)-64/35*x^(5/2)/b^4/(b*x^3+a*x)^(1/2)+128/35*
(b*x^3+a*x)^(1/2)/b^5/x^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2040, 2039}

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} - \frac{16x^{11/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{8x^{17/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{x^{23/2}}{7b(ax+bx^3)^{7/2}}$$

```
[In] Int[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(23/2)/(b*(a*x + b*x^3)^(7/2)) - (8*x^(17/2))/(35*b^2*(a*x + b*x^3)^
(5/2)) - (16*x^(11/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (64*x^(5/2))/(35*b^4*
Sqrt[a*x + b*x^3]) + (128*Sqrt[a*x + b*x^3])/(35*b^5*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{8\int\frac{x^{21/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\
&= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}+\frac{48\int\frac{x^{15/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2}\\
&= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}+\frac{64\int\frac{x^{9/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35b^3}\\
&= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}\\
&\quad-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}+\frac{128\int\frac{x^{3/2}}{\sqrt{ax+bx^3}}\,dx}{35b^4}\\
&= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}\\
&\quad-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}+\frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx=\frac{x^{7/2}(128a^4+448a^3bx^2+560a^2b^2x^4+280ab^3x^6+35b^4x^8)}{35b^5\left(x\left(a+bx^2\right)\right)^{7/2}}$$

```
[In] Integrate[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(7/2)*(128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^
4*x^8))/(35*b^5*(x*(a + b*x^2))^(7/2))
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)\left(35x^{8}b^{4}+280a\,b^{3}x^{6}+560a^{2}x^{4}b^{2}+448a^{3}b\,x^{2}+128a^{4}\right)x^{\frac{9}{2}}}{35b^{5}\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	70
default	$\frac{\sqrt{x\left(b\,x^{2}+a\right)}\,\left(35x^{8}b^{4}+280a\,b^{3}x^{6}+560a^{2}x^{4}b^{2}+448a^{3}b\,x^{2}+128a^{4}\right)}{35\sqrt{x}\,\left(b\,x^{2}+a\right)^{4}b^{5}}$	72
risch	$\frac{\left(b\,x^{2}+a\right)\sqrt{x}}{b^{5}\sqrt{x\left(b\,x^{2}+a\right)}}+\frac{\left(b\,x^{2}+a\right)\left(140b^{3}x^{6}+350a\,b^{2}x^{4}+308a^{2}b\,x^{2}+93a^{3}\right)a\sqrt{x}}{35b^{5}\left(x^{8}b^{4}+4a\,b^{3}x^{6}+6a^{2}x^{4}b^{2}+4a^{3}b\,x^{2}+a^{4}\right)\sqrt{x\left(b\,x^{2}+a\right)}}$	128

```
[In] int(x^(27/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(b*x^2+a)*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+448*a^3*b*x^2+128*
a^4)*x^(9/2)/b^5/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx=\frac{\left(35\,b^4x^8+280\,ab^3x^6+560\,a^2b^2x^4+448\,a^3bx^2+128\,a^4\right)\sqrt{bx^3+ax}\sqrt{x}}{35\,\left(b^9x^9+4\,ab^8x^7+6\,a^2b^7x^5+4\,a^3b^6x^3+a^4b^5x\right)}$$

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a^
4)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^9*x^9 + 4*a*b^8*x^7 + 6*a^2*b^7*x^5 + 4*a^3
*b^6*x^3 + a^4*b^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(27/2)/(b*x**3+a*x)**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{27}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(27/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{x^{27/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{bx^2+a}}{b^5} - \frac{128\,\sqrt{a}}{35\,b^5} + \frac{140\,(bx^2+a)^3a - 70\,(bx^2+a)^2a^2 + 28\,(bx^2+a)a^3 - 5\,a^4}{35\,(bx^2+a)^{\frac{7}{2}}b^5}$$

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] sqrt(b*x^2 + a)/b^5 - 128/35*sqrt(a)/b^5 + 1/35*(140*(b*x^2 + a)^3*a - 70*(
b*x^2 + a)^2*a^2 + 28*(b*x^2 + a)*a^3 - 5*a^4)/((b*x^2 + a)^(7/2)*b^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{27/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\int \frac{x^{27/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}} \,d x$$

```
[In] int(x^(27/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(27/2)/(a*x + b*x^3)^(9/2), x)
```

3.77 $\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}} \, dx = -\frac{x^{21/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{x^{15/2}}{5b^2\,(ax+bx^3)^{5/2}} - \frac{x^{9/2}}{3b^3\,(ax+bx^3)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}$$

```
[Out] -1/7*x^(21/2)/b/(b*x^3+a*x)^(7/2)-1/5*x^(15/2)/b^2/(b*x^3+a*x)^(5/2)-1/3*x^
(9/2)/b^3/(b*x^3+a*x)^(3/2)+arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x)^(1/2))/b^(9
/2)-x^(3/2)/b^4/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2047, 2054, 212}

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} - \frac{x^{9/2}}{3b^3\,(ax+bx^3)^{3/2}} - \frac{x^{15/2}}{5b^2\,(ax+bx^3)^{5/2}} - \frac{x^{21/2}}{7b\,(ax+bx^3)^{7/2}}$$

```
[In] Int[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(21/2)/(b*(a*x + b*x^3)^(7/2)) - x^(15/2)/(5*b^2*(a*x + b*x^3)^(5/2)
) - x^(9/2)/(3*b^3*(a*x + b*x^3)^(3/2)) - x^(3/2)/(b^4*Sqrt[a*x + b*x^3]) +
 ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x + b*x^3]]/b^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{b}\\
&= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}+\frac{\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{b^2}\\
&= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}+\frac{\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{b^3}\\
&= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{3/2}}{b^4\sqrt{ax+bx^3}}+\frac{\int \frac{\sqrt{x}}{\sqrt{ax+bx^3}}\,dx}{b^4}\\
&= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}\\
&\quad -\frac{x^{3/2}}{b^4\sqrt{ax+bx^3}}+\frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^4}\\
&= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}\\
&\quad -\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{3/2}}{b^4\sqrt{ax+bx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{x^{25/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{9/2}\Big(-\sqrt{b}x(a+bx^2)\left(105a^3+350a^2bx^2+406ab^2x^4+176b^3x^6\right)+210(a+bx^2)^{9/2}\,\mathrm{arct}}{105b^{9/2}\left(x\left(a+bx^2\right)\right)^{9/2}}$$

```
[In] Integrate[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(-(Sqrt[b]*x*(a + b*x^2)*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4
+ 176*b^3*x^6)) + 210*(a + b*x^2)^(9/2)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqr
t[a + b*x^2])]))/(105*b^(9/2)*(x*(a + b*x^2))^(9/2))
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.52

method	result
default	$\frac{\sqrt{x(b\,x^2+a)}\,\Big(105\ln\Big(x\sqrt{b}+\sqrt{b\,x^2+a}\Big)b^3x^6\sqrt{b\,x^2+a}-176x^7b^{\frac{7}{2}}+315\ln\Big(x\sqrt{b}+\sqrt{b\,x^2+a}\Big)a\,b^2x^4\sqrt{b\,x^2+a}-406b^{\frac{5}{2}}a\,x^5+315\ln\Big(x\sqrt{b}+\sqrt{b}}{105b^{\frac{9}{2}}\sqrt{x}\,(b\,x^2+a)^4}$

```
[In] int(x^(25/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(x*(b*x^2+a))^(1/2)/b^(9/2)*(105*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*b^3*x^
6*(b*x^2+a)^(1/2)-176*x^7*b^(7/2)+315*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*a*b^2*x
^4*(b*x^2+a)^(1/2)-406*b^(5/2)*a*x^5+315*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*a^2*
b*x^2*(b*x^2+a)^(1/2)-350*b^(3/2)*a^2*x^3+105*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
*a^3*(b*x^2+a)^(1/2)-105*b^(1/2)*a^3*x)/x^(1/2)/(b*x^2+a)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.68

$$\int \frac{x^{25/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \Bigg[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{b}\log\Big(2\,bx^2+2\,\sqrt{bx^3+ax}\sqrt{b}\sqrt{x}+a}{210\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\right.}$$

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(b)*log(2*b*x^2 + 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) - 2*(176*b^4*x^
6 + 406*a*b^3*x^4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))
/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*
```

```
(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-b)*a
rctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2))) + (176*b^4*x^6 + 406*a*b^3*x^
4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^9*x^8 + 4*a*
b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(25/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{25}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(25/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.66

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{\left(2\left(x^2\left(\frac{88x^2}{b}+\frac{203a}{b^2}\right)+\frac{175a^2}{b^3}\right)x^2+\frac{105a^3}{b^4}\right)x}{105\,(bx^2+a)^{\frac{7}{2}}} - \frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}} + \frac{\log\left(|a|\right)}{2\,b^{\frac{9}{2}}}$$

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/(
b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2) + 1/2*log
(abs(a))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{25/2}}{(ax+bx^3)^{9/2}}\,dx=\int \frac{x^{25/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

```
[In] int(x^(25/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(25/2)/(a*x + b*x^3)^(9/2), x)
```

3.78 $\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{19/2}}{7b(ax+bx^3)^{7/2}} - \frac{6x^{13/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{8x^{7/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}$$

```
[Out] -1/7*x^(19/2)/b/(b*x^3+a*x)^(7/2)-6/35*x^(13/2)/b^2/(b*x^3+a*x)^(5/2)-8/35*
x^(7/2)/b^3/(b*x^3+a*x)^(3/2)-16/35*x^(1/2)/b^4/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2040, 2039}

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}} - \frac{8x^{7/2}}{35b^3(ax+bx^3)^{3/2}} - \frac{6x^{13/2}}{35b^2(ax+bx^3)^{5/2}} - \frac{x^{19/2}}{7b(ax+bx^3)^{7/2}}$$

```
[In] Int[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(19/2)/(b*(a*x + b*x^3)^(7/2)) - (6*x^(13/2))/(35*b^2*(a*x + b*x^3)^
(5/2)) - (8*x^(7/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (16*Sqrt[x])/(35*b^4*Sq
rt[a*x + b*x^3])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{6\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\
&= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}+\frac{24\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2}\\
&= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}+\frac{16\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35b^3}\\
&= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$
\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{9/2}(a+bx^2)\left(-16a^3-56a^2bx^2-70ab^2x^4-35b^3x^6\right)}{35b^4\left(x\left(a+bx^2\right)\right)^{9/2}}
$$

```
[In] Integrate[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(a + b*x^2)*(-16*a^3 - 56*a^2*b*x^2 - 70*a*b^2*x^4 - 35*b^3*x^6))/
(35*b^4*(x*(a + b*x^2))^(9/2))
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{(b\,x^{2}+a)\,(35b^{3}x^{6}+70a\,b^{2}x^{4}+56a^{2}b\,x^{2}+16a^{3})\,x^{\frac{9}{2}}}{35b^{4}(b\,x^{3}+ax)^{\frac{9}{2}}}$	59
default	$-\frac{\sqrt{x(b\,x^{2}+a)}\,(35b^{3}x^{6}+70a\,b^{2}x^{4}+56a^{2}b\,x^{2}+16a^{3})}{35\sqrt{x}\,(b\,x^{2}+a)^{4}b^{4}}$	61

```
[In] int(x^(23/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(b*x^2+a)*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)*x^(9/2)/b^4/(
b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{(35\,b^3x^6+70\,ab^2x^4+56\,a^2bx^2+16\,a^3)\sqrt{bx^3+ax}\sqrt{x}}{35\,(b^8x^9+4\,ab^7x^7+6\,a^2b^6x^5+4\,a^3b^5x^3+a^4b^4x)}$$

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^3 + a*x)
*sqrt(x)/(b^8*x^9 + 4*a*b^7*x^7 + 6*a^2*b^6*x^5 + 4*a^3*b^5*x^3 + a^4*b^4*x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(23/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\int \frac{x^{\frac{23}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(23/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{16}{35\,\sqrt{a}b^4}-\frac{35\,\left(bx^2+a\right)^3-35\,\left(bx^2+a\right)^2a+21\,\left(bx^2+a\right)a^2-5\,a^3}{35\,\left(bx^2+a\right)^{\frac{7}{2}}b^4}$$

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 16/35/(sqrt(a)*b^4) - 1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x
^2 + a)*a^2 - 5*a^3)/((b*x^2 + a)^(7/2)*b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\int \frac{x^{23/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

```
[In] int(x^(23/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(23/2)/(a*x + b*x^3)^(9/2), x)
```

3.79 $\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] 1/7*x^(21/2)/a/(b*x^3+a*x)^(7/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2039}

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[In] Int[x^(21/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(21/2)/(7*a*(a*x + b*x^3)^(7/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral}=\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{21/2}}{7a\left(x\left(a+bx^2\right)\right)^{7/2}}$$

```
[In] Integrate[x^(21/2)/(a*x + b*x^3)^(9/2),x]
[Out] x^(21/2)/(7*a*(x*(a + b*x^2))^(7/2))
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)x^{\frac{23}{2}}}{7a\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	27
default	$\frac{x^{\frac{13}{2}}\sqrt{x\left(b\,x^{2}+a\right)}}{7a\left(b\,x^{2}+a\right)^{4}}$	29

```
[In] int(x^(21/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
[Out] 1/7*(b*x^2+a)/a*x^(23/2)/(b*x^3+a*x)^(9/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.

Time = 0.48 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.44

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{\sqrt{bx^3+ax}\,x^{\frac{13}{2}}}{7\left(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5\right)}$$

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] 1/7*sqrt(b*x^3 + a*x)*x^(13/2)/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 +
 4*a^4*b*x^2 + a^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\text{Timed out}$$

```
[In] integrate(x**(21/2)/(b*x**3+a*x)**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\int \frac{x^{\frac{21}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(21/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^7}{7\left(bx^2+a\right)^{\frac{7}{2}}a}$$

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] 1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\int \frac{x^{21/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

```
[In] int(x^(21/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(21/2)/(a*x + b*x^3)^(9/2), x)
```

3.80 $\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx=-\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}}-\frac{4x^{9/2}}{35b^2\,(ax+bx^3)^{5/2}}-\frac{8x^{3/2}}{105b^3\,(ax+bx^3)^{3/2}}$$

```
[Out] -1/7*x^(15/2)/b/(b*x^3+a*x)^(7/2)-4/35*x^(9/2)/b^2/(b*x^3+a*x)^(5/2)-8/105*
x^(3/2)/b^3/(b*x^3+a*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2040, 2039}

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx=-\frac{8x^{3/2}}{105b^3\,(ax+bx^3)^{3/2}}-\frac{4x^{9/2}}{35b^2\,(ax+bx^3)^{5/2}}-\frac{x^{15/2}}{7b\,(ax+bx^3)^{7/2}}$$

```
[In] Int[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(15/2)/(b*(a*x + b*x^3)^(7/2)) - (4*x^(9/2))/(35*b^2*(a*x + b*x^3)^(
5/2)) - (8*x^(3/2))/(105*b^3*(a*x + b*x^3)^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{4\int\frac{x^{13/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\
&= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}+\frac{8\int\frac{x^{7/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2}\\
&= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int\frac{x^{19/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{7/2}(-8a^2-28abx^2-35b^2x^4)}{105b^3\left(x\left(a+bx^2\right)\right)^{7/2}}$$

```
[In] Integrate[x^(19/2)/(a*x + b*x^3)^(9/2),x]
[Out] (x^(7/2)*(-8*a^2 - 28*a*b*x^2 - 35*b^2*x^4))/(105*b^3*(x*(a + b*x^2))^(7/2)
)
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)\left(35b^{2}x^{4}+28ab\,x^{2}+8a^{2}\right)x^{\frac{9}{2}}}{105b^{3}\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	48
default	$-\frac{\sqrt{x\left(b\,x^{2}+a\right)}\,\left(35b^{2}x^{4}+28ab\,x^{2}+8a^{2}\right)}{105\sqrt{x}\,\left(b\,x^{2}+a\right)^{4}b^{3}}$	50

```
[In] int(x^(19/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(b*x^2+a)*(35*b^2*x^4+28*a*b*x^2+8*a^2)*x^(9/2)/b^3/(b*x^3+a*x)^(9/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{(35\,b^2x^4+28\,abx^2+8\,a^2)\sqrt{bx^3+ax}\sqrt{x}}{105\,(b^7x^9+4\,ab^6x^7+6\,a^2b^5x^5+4\,a^3b^4x^3+a^4b^3x)}$$

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^7*x^9
 + 4*a*b^6*x^7 + 6*a^2*b^5*x^5 + 4*a^3*b^4*x^3 + a^4*b^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(19/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{19}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(19/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.66

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{8}{105\,a^{\frac{3}{2}}b^3} - \frac{35\,(bx^2+a)^2 - 42\,(bx^2+a)a + 15\,a^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] 8/105/(a^(3/2)*b^3) - 1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/
((b*x^2 + a)^(7/2)*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{19/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{19/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

```
[In] int(x^(19/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(19/2)/(a*x + b*x^3)^(9/2), x)
```

3.81 $\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}$$

```
[Out] 1/7*x^(17/2)/a/(b*x^3+a*x)^(7/2)+2/35*x^(15/2)/a^2/(b*x^3+a*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2040, 2039}

$$\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[In] Int[x^(17/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(17/2)/(7*a*(a*x + b*x^3)^(7/2)) + (2*x^(15/2))/(35*a^2*(a*x + b*x^3)^(5/
2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\ &= \frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{7/2}(7ax^5+2bx^7)}{35a^2\left(x\left(a+bx^2\right)\right)^{7/2}}$$

```
[In] Integrate[x^(17/2)/(a*x + b*x^3)^(9/2),x]
[Out] (x^(7/2)*(7*a*x^5 + 2*b*x^7))/(35*a^2*(x*(a + b*x^2))^(7/2))
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)x^{\frac{19}{2}}\left(2b\,x^{2}+7a\right)}{35a^{2}\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	37
default	$\frac{x^{\frac{9}{2}}\sqrt{x\left(b\,x^{2}+a\right)}\,\left(2b\,x^{2}+7a\right)}{35a^{2}\left(b\,x^{2}+a\right)^{4}}$	39

```
[In] int(x^(17/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
[Out] 1/35*(b*x^2+a)*x^(19/2)*(2*b*x^2+7*a)/a^2/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{(2\,bx^6+7\,ax^4)\sqrt{bx^3+ax}\sqrt{x}}{35\,(a^2b^4x^8+4\,a^3b^3x^6+6\,a^4b^2x^4+4\,a^5bx^2+a^6)}$$

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] 1/35*(2*b*x^6 + 7*a*x^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^2*b^4*x^8 + 4*a^3*b^3
*x^6 + 6*a^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(17/2)/(b*x**3+a*x)**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{17}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(17/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int \frac{x^{17/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^5\left(\frac{2\,bx^2}{a^2}+\frac{7}{a}\right)}{35\,(bx^2+a)^{\frac{7}{2}}}$$

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] 1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\int \frac{x^{17/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}} \,d x$$

```
[In] int(x^(17/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(17/2)/(a*x + b*x^3)^(9/2), x)
```

3.82 $\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{11/2}}{7b\,(ax+bx^3)^{7/2}} - \frac{2x^{5/2}}{35b^2\,(ax+bx^3)^{5/2}}$$

```
[Out] -1/7*x^(11/2)/b/(b*x^3+a*x)^(7/2)-2/35*x^(5/2)/b^2/(b*x^3+a*x)^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2040, 2039}

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{2x^{5/2}}{35b^2\,(ax+bx^3)^{5/2}} - \frac{x^{11/2}}{7b\,(ax+bx^3)^{7/2}}$$

```
[In] Int[x^(15/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(11/2)/(b*(a*x + b*x^3)^(7/2)) - (2*x^(5/2))/(35*b^2*(a*x + b*x^3)^(
5/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{2\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\ &= -\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{7/2}(-2a-7bx^2)}{35b^2\left(x\left(a+bx^2\right)\right)^{7/2}}$$

```
[In] Integrate[x^(15/2)/(a*x + b*x^3)^(9/2),x]
[Out] (x^(7/2)*(-2*a - 7*b*x^2))/(35*b^2*(x*(a + b*x^2))^(7/2))
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)\left(7b\,x^{2}+2a\right)x^{\frac{9}{2}}}{35b^{2}\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	37
default	$-\frac{\sqrt{x\left(b\,x^{2}+a\right)}\,\left(7b\,x^{2}+2a\right)}{35\sqrt{x}\,\left(b\,x^{2}+a\right)^{4}b^{2}}$	39

```
[In] int(x^(15/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
[Out] -1/35*(b*x^2+a)*(7*b*x^2+2*a)*x^(9/2)/b^2/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{\sqrt{bx^3+ax}(7\,bx^2+2\,a)\sqrt{x}}{35\,(b^6x^9+4\,ab^5x^7+6\,a^2b^4x^5+4\,a^3b^3x^3+a^4b^2x)}$$

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*sqrt(b*x^3 + a*x)*(7*b*x^2 + 2*a)*sqrt(x)/(b^6*x^9 + 4*a*b^5*x^7 + 6*
a^2*b^4*x^5 + 4*a^3*b^3*x^3 + a^4*b^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(15/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{15}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(15/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

$$\int \frac{x^{15/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{7\,bx^2+2\,a}{35\,(bx^2+a)^{\frac{7}{2}}b^2}+\frac{2}{35\,a^{\frac{5}{2}}b^2}$$

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2) + 2/35/(a^(5/2)*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\int \frac{x^{15/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}} \,d x$$

```
[In] int(x^(15/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(15/2)/(a*x + b*x^3)^(9/2), x)
```

3.83 $\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx=\frac{x^{13/2}}{7a\,(ax+bx^3)^{7/2}}+\frac{4x^{11/2}}{35a^2\,(ax+bx^3)^{5/2}}+\frac{8x^{9/2}}{105a^3\,(ax+bx^3)^{3/2}}$$

```
[Out] 1/7*x^(13/2)/a/(b*x^3+a*x)^(7/2)+4/35*x^(11/2)/a^2/(b*x^3+a*x)^(5/2)+8/105*
x^(9/2)/a^3/(b*x^3+a*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2040, 2039}

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx=\frac{8x^{9/2}}{105a^3\,(ax+bx^3)^{3/2}}+\frac{4x^{11/2}}{35a^2\,(ax+bx^3)^{5/2}}+\frac{x^{13/2}}{7a\,(ax+bx^3)^{7/2}}$$

```
[In] Int[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(13/2)/(7*a*(a*x + b*x^3)^(7/2)) + (4*x^(11/2))/(35*a^2*(a*x + b*x^3)^(5/
2)) + (8*x^(9/2))/(105*a^3*(a*x + b*x^3)^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4\int\frac{x^{11/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\ &= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8\int\frac{x^{9/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}\\ &= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int\frac{x^{13/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{9/2}(a+bx^2)\left(35a^2x^3+28abx^5+8b^2x^7\right)}{105a^3\left(x\left(a+bx^2\right)\right)^{9/2}}$$

```
[In] Integrate[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*(a + b*x^2)*(35*a^2*x^3 + 28*a*b*x^5 + 8*b^2*x^7))/(105*a^3*(x*(a
+ b*x^2))^(9/2))
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)x^{\frac{15}{2}}\left(8b^{2}x^{4}+28ab\,x^{2}+35a^{2}\right)}{105a^{3}\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	48
default	$\frac{x^{\frac{5}{2}}\sqrt{x\left(b\,x^{2}+a\right)}\left(8b^{2}x^{4}+28ab\,x^{2}+35a^{2}\right)}{105a^{3}\left(b\,x^{2}+a\right)^{4}}$	50

```
[In] int(x^(13/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105*(b*x^2+a)*x^(15/2)*(8*b^2*x^4+28*a*b*x^2+35*a^2)/a^3/(b*x^3+a*x)^(9/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.14

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{(8\,b^2x^6+28\,abx^4+35\,a^2x^2)\sqrt{bx^3+ax}\sqrt{x}}{105\,(a^3b^4x^8+4\,a^4b^3x^6+6\,a^5b^2x^4+4\,a^6bx^2+a^7)}$$

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*b^2*x^6 + 28*a*b*x^4 + 35*a^2*x^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^3*
b^4*x^8 + 4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x**(13/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{13}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(13/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3}+\frac{7\,b}{a^2}\right)+\frac{35}{a}\right)x^3}{105\,(bx^2+a)^{\frac{7}{2}}}$$

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{13/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}} \,dx$$

```
[In] int(x^(13/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(13/2)/(a*x + b*x^3)^(9/2), x)
```

3.84 $\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{7/2}}{7b\,(ax+bx^3)^{7/2}}$$

```
[Out] -1/7*x^(7/2)/b/(b*x^3+a*x)^(7/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2039}

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{x^{7/2}}{7b\,(ax+bx^3)^{7/2}}$$

```
[In] Int[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*x^(7/2)/(b*(a*x + b*x^3)^(7/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{x^{7/2}}{7b\,(ax+bx^3)^{7/2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=-\frac{x^{7/2}}{7b\left(x\left(a+bx^2\right)\right)^{7/2}}$$

```
[In] Integrate[x^(11/2)/(a*x + b*x^3)^(9/2),x]
[Out] -1/7*x^(7/2)/(b*(x*(a + b*x^2))^(7/2))
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)x^{\frac{9}{2}}}{7b\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	27
default	$-\frac{\sqrt{x\left(b\,x^{2}+a\right)}}{7\sqrt{x}\,\left(b\,x^{2}+a\right)^{4}b}$	29

```
[In] int(x^(11/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
[Out] -1/7*(b*x^2+a)/b*x^(9/2)/(b*x^3+a*x)^(9/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=-\frac{\sqrt{bx^3+ax}\sqrt{x}}{7\left(b^5x^9+4\,ab^4x^7+6\,a^2b^3x^5+4\,a^3b^2x^3+a^4bx\right)}$$

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] -1/7*sqrt(b*x^3 + a*x)*sqrt(x)/(b^5*x^9 + 4*a*b^4*x^7 + 6*a^2*b^3*x^5 + 4*a
^3*b^2*x^3 + a^4*b*x)
```

Sympy [F]

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{11}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x**(11/2)/(b*x**3+a*x)**(9/2),x)
[Out] Integral(x**(11/2)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{11}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(11/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}b} + \frac{1}{7\,a^{\frac{7}{2}}b}$$

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] -1/7/((b*x^2 + a)^(7/2)*b) + 1/7/(a^(7/2)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{11/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

```
[In] int(x^(11/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(11/2)/(a*x + b*x^3)^(9/2), x)
```

3.85 $\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}$$

```
[Out] 1/7*x^(9/2)/a/(b*x^3+a*x)^(7/2)+6/35*x^(7/2)/a^2/(b*x^3+a*x)^(5/2)+8/35*x^(
5/2)/a^3/(b*x^3+a*x)^(3/2)+16/35*x^(3/2)/a^4/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2040, 2039}

$$\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[In] Int[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(9/2)/(7*a*(a*x + b*x^3)^(7/2)) + (6*x^(7/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (8*x^(5/2))/(35*a^3*(a*x + b*x^3)^(3/2)) + (16*x^(3/2))/(35*a^4*Sqrt[a*
x + b*x^3])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{6\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\
&= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{24\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}\\
&= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{16\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35a^3}\\
&= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{7/2}(35a^3x+70a^2bx^3+56ab^2x^5+16b^3x^7)}{35a^4\left(x\left(a+bx^2\right)\right)^{7/2}}$$

```
[In] Integrate[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(7/2)*(35*a^3*x + 70*a^2*b*x^3 + 56*a*b^2*x^5 + 16*b^3*x^7))/(35*a^4*(x*
(a + b*x^2))^(7/2))
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)x^{\frac{11}{2}}\left(16b^{3}x^{6}+56a\,b^{2}x^{4}+70a^{2}b\,x^{2}+35a^{3}\right)}{35a^{4}\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	59
default	$\frac{\sqrt{x}\,\sqrt{x\left(b\,x^{2}+a\right)}\,\left(16b^{3}x^{6}+56a\,b^{2}x^{4}+70a^{2}b\,x^{2}+35a^{3}\right)}{35\left(b\,x^{2}+a\right)^{4}a^{4}}$	61

```
[In] int(x^(9/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*(b*x^2+a)*x^(11/2)*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/(
b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{\left(16\,b^3x^6+56\,ab^2x^4+70\,a^2bx^2+35\,a^3\right)\sqrt{bx^3+ax}\sqrt{x}}{35\,\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}$$

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^6 + 56*a*b^2*x^4 + 70*a^2*b*x^2 + 35*a^3)*sqrt(b*x^3 + a*x)*
sqrt(x)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [F]

$$\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\int \frac{x^{\frac{9}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x**(9/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(9/2)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx=\int \frac{x^{\frac{9}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.54

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx=\frac{\left(2\left(4\,x^2\left(\frac{2\,b^3x^2}{a^4}+\frac{7\,b^2}{a^3}\right)+\frac{35\,b}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}}$$

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^2
 + a)^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(ax+bx^3)^{9/2}}\,dx=\int \frac{x^{9/2}}{{(b\,x^3+a\,x)}^{9/2}}\,dx$$

```
[In] int(x^(9/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(9/2)/(a*x + b*x^3)^(9/2), x)
```

3.86 $\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}$$

```
[Out] 1/7*x^(7/2)/a/(b*x^3+a*x)^(7/2)+1/5*x^(5/2)/a^2/(b*x^3+a*x)^(5/2)+1/3*x^(3/
2)/a^3/(b*x^3+a*x)^(3/2)-arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(1/2))/a^(9/2)
+x^(1/2)/a^4/(b*x^3+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2048, 2054, 212}

$$\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[In] Int[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(7/2)/(7*a*(a*x + b*x^3)^(7/2)) + x^(5/2)/(5*a^2*(a*x + b*x^3)^(5/2)) + x
^(3/2)/(3*a^3*(a*x + b*x^3)^(3/2)) + Sqrt[x]/(a^4*Sqrt[a*x + b*x^3]) - ArcT
anh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]]/a^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{a}\\
&= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{a^2}\\
&= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{3/2}}\,dx}{a^3}\\
&= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3}}\,dx}{a^4}\\
&= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}\\
&\quad+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}-\frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^4}\\
&= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}\\
&\quad+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.82

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\sqrt{x}\left(\sqrt{a}(176a^3+406a^2bx^2+350ab^2x^4+105b^3x^6)-105(a+bx^2)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right.\right.}{105a^{9/2}\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

```
[In] Integrate[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(Sqrt[a]*(176*a^3 + 406*a^2*b*x^2 + 350*a*b^2*x^4 + 105*b^3*x^6) -
 105*(a + b*x^2)^(7/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(105*a^(9/2)*(a +
 b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(100) = 200.

Time = 2.16 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.67

method	result
default	$-\frac{\sqrt{x(b\,x^2+a)}\left(105\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)b^3x^6\sqrt{b\,x^2+a}-105\sqrt{a}\,b^3x^6+315\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)a\,b^2x^4\sqrt{b\,x^2+a}-350a^{\frac{3}{2}}b^2x^4+315\right.}{105a^{\frac{9}{2}}\sqrt{x}\,(b\,x^2+a)^4}$

```
[In] int(x^(7/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/105*(x*(b*x^2+a))^(1/2)/a^(9/2)*(105*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)
*b^3*x^6*(b*x^2+a)^(1/2)-105*a^(1/2)*b^3*x^6+315*ln(2*(a^(1/2)*(b*x^2+a)^(1
/2)+a)/x)*a*b^2*x^4*(b*x^2+a)^(1/2)-350*a^(3/2)*b^2*x^4+315*ln(2*(a^(1/2)*(
b*x^2+a)^(1/2)+a)/x)*a^2*b*x^2*(b*x^2+a)^(1/2)-406*a^(5/2)*b*x^2+105*ln(2*(
a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*(b*x^2+a)^(1/2)-176*a^(7/2))/x^(1/2)/(b*x
^2+a)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.77

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \left[\frac{105\left(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x\right)\sqrt{a}\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+ax}\sqrt{a}\sqrt{x}}{x^3}\right)+2}{210\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\right.}\right.$$

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*s
qrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3) + 2*(
```

```
105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x
)*sqrt(x))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9
*x), 1/105*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*
x)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (105*a*b^3*x^6
 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x)*sqrt(x))/(a
^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

Sympy [F]

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x**(7/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(7/2)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{7}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} - \frac{105\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+176\sqrt{-a}}{105\sqrt{-a}a^{\frac{9}{2}}} + \frac{105\left(bx^2+a\right)^3+35\left(bx^2+a\right)^2a+21\left(bx^2+a\right)a^2+15\,a^3}{105\left(bx^2+a\right)^{\frac{7}{2}}a^4}$$

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) - 1/105*(105*sqrt(a)*arctan
(sqrt(a)/sqrt(-a)) + 176*sqrt(-a))/(sqrt(-a)*a^(9/2)) + 1/105*(105*(b*x^2 +
 a)^3 + 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2
)*a^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{7/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

```
[In] int(x^(7/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(7/2)/(a*x + b*x^3)^(9/2), x)
```

3.87 $\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}} +\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}$$

```
[Out] 1/7*x^(5/2)/a/(b*x^3+a*x)^(7/2)+8/35*x^(3/2)/a^2/(b*x^3+a*x)^(5/2)+16/35*x^
(1/2)/a^3/(b*x^3+a*x)^(3/2)+64/35/a^4/x^(1/2)/(b*x^3+a*x)^(1/2)-128/35*(b*x
^3+a*x)^(1/2)/a^5/x^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2040, 2039}

$$\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}+\frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}} +\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[In] Int[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(5/2)/(7*a*(a*x + b*x^3)^(7/2)) + (8*x^(3/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (16*Sqrt[x])/(35*a^3*(a*x + b*x^3)^(3/2)) + 64/(35*a^4*Sqrt[x]*Sqrt[a*x
 + b*x^3]) - (128*Sqrt[a*x + b*x^3])/(35*a^5*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\
&= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{48\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}\\
&= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{64\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{3/2}}\,dx}{35a^3}\\
&= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}\\
&\quad+\frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}}+\frac{128\int \frac{1}{x^{3/2}\sqrt{ax+bx^3}}\,dx}{35a^4}\\
&= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}\\
&\quad+\frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \frac{x^{5/2}(-35a^4-280a^3bx^2-560a^2b^2x^4-448ab^3x^6-128b^4x^8)}{35a^5\,(x\,(a+bx^2))^{7/2}}$$

```
[In] Integrate[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(5/2)*(-35*a^4 - 280*a^3*b*x^2 - 560*a^2*b^2*x^4 - 448*a*b^3*x^6 - 128*b
^4*x^8))/(35*a^5*(x*(a + b*x^2))^(7/2))
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{x^{\frac{7}{2}}\left(b\,x^{2}+a\right)\left(128x^{8}b^{4}+448a\,b^{3}x^{6}+560a^{2}x^{4}b^{2}+280a^{3}b\,x^{2}+35a^{4}\right)}{35a^{5}\left(b\,x^{3}+ax\right)^{\frac{9}{2}}}$	70
default	$-\frac{\sqrt{x(b\,x^{2}+a)}\,\left(128x^{8}b^{4}+448a\,b^{3}x^{6}+560a^{2}x^{4}b^{2}+280a^{3}b\,x^{2}+35a^{4}\right)}{35x^{\frac{3}{2}}\left(b\,x^{2}+a\right)^{4}a^{5}}$	72
risch	$-\frac{b\,x^{2}+a}{a^{5}\sqrt{x}\,\sqrt{x(b\,x^{2}+a)}}-\frac{\left(b\,x^{2}+a\right)x^{\frac{3}{2}}\left(93b^{3}x^{6}+308a\,b^{2}x^{4}+350a^{2}b\,x^{2}+140a^{3}\right)b}{35(x^{8}b^{4}+4a\,b^{3}x^{6}+6a^{2}x^{4}b^{2}+4a^{3}b\,x^{2}+a^{4})a^{5}\sqrt{x(b\,x^{2}+a)}}$	129

```
[In] int(x^(5/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*x^(7/2)*(b*x^2+a)*(128*b^4*x^8+448*a*b^3*x^6+560*a^2*b^2*x^4+280*a^3*
b*x^2+35*a^4)/a^5/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{(128\,b^4x^8+448\,ab^3x^6+560\,a^2b^2x^4+280\,a^3bx^2+35\,a^4)\sqrt{bx^3+ax}\sqrt{x}}{35\,(a^5b^4x^{10}+4\,a^6b^3x^8+6\,a^7b^2x^6+4\,a^8bx^4+a^9x^2)}$$

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*a
^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6
 + 4*a^8*b*x^4 + a^9*x^2)
```

Sympy [F]

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{(x\,(a+bx^2))^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x**(5/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(5/2)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{5}{2}}}{(bx^3+ax)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{\left(\left(x^2\left(\frac{93\,b^4x^2}{a^5}+\frac{308\,b^3}{a^4}\right)+\frac{350\,b^2}{a^3}\right)x^2+\frac{140\,b}{a^2}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}} + \frac{2\,\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)*
x/(b*x^2 + a)^(7/2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{5/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}} \,d x$$

```
[In] int(x^(5/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(5/2)/(a*x + b*x^3)^(9/2), x)
```

3.88 $\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{9/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{9/2}} \, dx = \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{9b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}$$

```
[Out] 1/7*x^(3/2)/a/(b*x^3+a*x)^(7/2)+9/2*b*arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(
1/2))/a^(11/2)+3/5/a^3/(b*x^3+a*x)^(3/2)/x^(1/2)+9/35*x^(1/2)/a^2/(b*x^3+a*
x)^(5/2)+3/a^4/x^(3/2)/(b*x^3+a*x)^(1/2)-9/2*(b*x^3+a*x)^(1/2)/a^5/x^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2048, 2050, 2054, 212}

$$\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{9/2}} \, dx = \frac{9b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}+\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[In] Int[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(3/2)/(7*a*(a*x + b*x^3)^(7/2)) + (9*Sqrt[x])/(35*a^2*(a*x + b*x^3)^(5/2)
) + 3/(5*a^3*Sqrt[x]*(a*x + b*x^3)^(3/2)) + 3/(a^4*x^(3/2)*Sqrt[a*x + b*x^3
```

```
]) - (9*Sqrt[a*x + b*x^3])/(2*a^5*x^(5/2)) + (9*b*ArcTanh[(Sqrt[a]*Sqrt[x])
/Sqrt[a*x + b*x^3]])/(2*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{9\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\
&= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{9\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{5/2}}\,dx}{5a^2}\\
&= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}+\frac{3\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{3/2}}\,dx}{a^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}\\
&\quad+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}+\frac{9\int\frac{1}{x^{5/2}\sqrt{ax+bx^3}}\,dx}{a^4}\\
&= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}\\
&\quad+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}-\frac{(9b)\int\frac{1}{\sqrt{x}\sqrt{ax+bx^3}}\,dx}{2a^5}\\
&= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}\\
&\quad+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{(9b)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^5}\\
&= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}\\
&\quad+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{9b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.77

$$
\int\frac{x^{3/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{\sqrt{x\left(a+bx^2\right)}\Big(-\sqrt{a}(35a^4+528a^3bx^2+1218a^2b^2x^4+1050ab^3x^6+315b^4x^8)+315bx}{70a^{11/2}x^{5/2}\left(a+bx^2\right)^4}
$$

```
[In] Integrate[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x*(a + b*x^2)]*(-(Sqrt[a]*(35*a^4 + 528*a^3*b*x^2 + 1218*a^2*b^2*x^4
+ 1050*a*b^3*x^6 + 315*b^4*x^8)) + 315*b*x^2*(a + b*x^2)^(7/2)*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]]))/(70*a^(11/2)*x^(5/2)*(a + b*x^2)^4)
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.47

method	result
default	$\frac{\sqrt{x(b\,x^2+a)}\left(315\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)b^4x^8\sqrt{b\,x^2+a}-315\sqrt{a}\,b^4x^8+945\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)a\,b^3x^6\sqrt{b\,x^2+a}-1050a^{\frac{3}{2}}b^3x^6+945\ln\left(\right.\right.}{70a^{\frac{11}{2}}x^{\frac{5}{2}}(b\,x^2+a)^4}$
risch	$-\frac{b\,x^2+a}{2a^5x^{\frac{3}{2}}\sqrt{x(b\,x^2+a)}}+\frac{\left(\frac{9b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{11}{2}}}+\frac{2629b\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}}{1120a^5\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}-\frac{2629b\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\left(x-\frac{\sqrt{-}}{b}\right.}}{1120a^5\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}\right.}{}$

```
[In] int(x^(3/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/70*(x*(b*x^2+a))^(1/2)/a^(11/2)*(315*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*
b^4*x^8*(b*x^2+a)^(1/2)-315*a^(1/2)*b^4*x^8+945*ln(2*(a^(1/2)*(b*x^2+a)^(1/
2)+a)/x)*a*b^3*x^6*(b*x^2+a)^(1/2)-1050*a^(3/2)*b^3*x^6+945*ln(2*(a^(1/2)*(
b*x^2+a)^(1/2)+a)/x)*a^2*b^2*x^4*(b*x^2+a)^(1/2)-1218*a^(5/2)*b^2*x^4+315*l
n(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*b*x^2*(b*x^2+a)^(1/2)-528*a^(7/2)*b*
x^2-35*a^(9/2))/x^(5/2)/(b*x^2+a)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.49

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \left[\frac{315\,(b^5x^{11}+4\,ab^4x^9+6\,a^2b^3x^7+4\,a^3b^2x^5+a^4bx^3)\sqrt{a}\log\left(\frac{bx^3+2\,ax+2\,\sqrt{bx^3+ax}\sqrt{a}\sqrt{x}}{x^3}\right)-}{140\,(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b}\right.$$

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b
*x^3)*sqrt(a)*log((b*x^3 + 2*a*x + 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3
) - 2*(315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2
+ 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*
b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3), -1/70*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a
^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*s
qrt(-a)/(a*sqrt(x))) + (315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4
 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7
*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

Sympy [F]

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x**(3/2)/(b*x**3+a*x)**(9/2),x)
[Out] Integral(x**(3/2)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(3/2)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = -\frac{9\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}a^5} - \frac{\sqrt{bx^2+a}}{2\,a^5x^2} - \frac{140\,(bx^2+a)^3b + 35\,(bx^2+a)^2ab + 14\,(bx^2+a)a^2b + 5\,a^3b}{35\,(bx^2+a)^{\frac{7}{2}}a^5}$$

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] -9/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/2*sqrt(b*x^2 + a
)/(a^5*x^2) - 1/35*(140*(b*x^2 + a)^3*b + 35*(b*x^2 + a)^2*a*b + 14*(b*x^2
+ a)*a^2*b + 5*a^3*b)/((b*x^2 + a)^(7/2)*a^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{x^{3/2}}{(b\,x^3+a\,x)^{9/2}}\,dx$$

```
[In] int(x^(3/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(3/2)/(a*x + b*x^3)^(9/2), x)
```

3.89 $\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 152

$$\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}}+\frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}$$

```
[Out] 16/21/a^3/x^(3/2)/(b*x^3+a*x)^(3/2)+2/7/a^2/(b*x^3+a*x)^(5/2)/x^(1/2)+1/7*x
^(1/2)/a/(b*x^3+a*x)^(7/2)+32/7/a^4/x^(5/2)/(b*x^3+a*x)^(1/2)-128/21*(b*x^3
+a*x)^(1/2)/a^5/x^(7/2)+256/21*b*(b*x^3+a*x)^(1/2)/a^6/x^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2040, 2041, 2039}

$$\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}-\frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}+\frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[In] Int[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

```
[Out] Sqrt[x]/(7*a*(a*x + b*x^3)^(7/2)) + 2/(7*a^2*Sqrt[x]*(a*x + b*x^3)^(5/2)) +
 16/(21*a^3*x^(3/2)*(a*x + b*x^3)^(3/2)) + 32/(7*a^4*x^(5/2)*Sqrt[a*x + b*x
^3]) - (128*Sqrt[a*x + b*x^3])/(21*a^5*x^(7/2)) + (256*b*Sqrt[a*x + b*x^3])
/(21*a^6*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{10\int\frac{1}{\sqrt{x}\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\
&= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{16\int\frac{1}{x^{3/2}\left(ax+bx^3\right)^{5/2}}\,dx}{7a^2}\\
&= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}}+\frac{32\int\frac{1}{x^{5/2}\left(ax+bx^3\right)^{3/2}}\,dx}{7a^3}\\
&= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}}\\
&\quad+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}+\frac{128\int\frac{1}{x^{7/2}\sqrt{ax+bx^3}}\,dx}{7a^4}\\
&= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}}\\
&\quad+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}}-\frac{(256b)\int\frac{1}{x^{3/2}\sqrt{ax+bx^3}}\,dx}{21a^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} \\
&\quad + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} - \frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}} + \frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.52

$$
\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{\sqrt{x}(-7a^5+70a^4bx^2+560a^3b^2x^4+1120a^2b^3x^6+896ab^4x^8+256b^5x^{10})}{21a^6\left(x\left(a+bx^2\right)\right)^{7/2}}
$$

```
[In] Integrate[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896*
a*b^4*x^8 + 256*b^5*x^10))/(21*a^6*(x*(a + b*x^2))^(7/2))
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{x^{\frac{3}{2}} \left(b\,x^{2}+a \right) \left(-256 b^{5} x^{10}-896 a\,b^{4} x^{8}-1120 a^{2} b^{3} x^{6}-560 a^{3} b^{2} x^{4}-70 a^{4} b\,x^{2}+7 a^{5}\right)}{21 a^{6} \left(b\,x^{3}+a x \right)^{\frac{9}{2}}}$	81
default	$-\frac{\sqrt{x \left(b\,x^{2}+a \right)}\, \left(-256 b^{5} x^{10}-896 a\,b^{4} x^{8}-1120 a^{2} b^{3} x^{6}-560 a^{3} b^{2} x^{4}-70 a^{4} b\,x^{2}+7 a^{5}\right)}{21 x^{\frac{7}{2}} \left(b\,x^{2}+a \right)^{4} a^{6}}$	83
risch	$-\frac{\left(b\,x^{2}+a \right) \left(-14 b\,x^{2}+a \right)}{3 a^{6} x^{\frac{5}{2}} \sqrt{x \left(b\,x^{2}+a \right)}}+\frac{\left(b\,x^{2}+a \right) x^{\frac{3}{2}} \left(158 b^{3} x^{6}+511 a\,b^{2} x^{4}+560 a^{2} b\,x^{2}+210 a^{3}\right) b^{2}}{21 a^{6} \left(x^{8} b^{4}+4 a\,b^{3} x^{6}+6 a^{2} x^{4} b^{2}+4 a^{3} b\,x^{2}+a^{4}\right) \sqrt{x \left(b\,x^{2}+a \right)}}$	139

```
[In] int(x^(1/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21*x^(3/2)*(b*x^2+a)*(-256*b^5*x^10-896*a*b^4*x^8-1120*a^2*b^3*x^6-560*a
^3*b^2*x^4-70*a^4*b*x^2+7*a^5)/a^6/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.67 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(256\,b^5x^{10}+896\,ab^4x^8+1120\,a^2b^3x^6+560\,a^3b^2x^4+70\,a^4bx^2-7\,a^5\right)\sqrt{bx^3+ax}\sqrt{x}}{21\left(a^6b^4x^{12}+4\,a^7b^3x^{10}+6\,a^8b^2x^8+4\,a^9bx^6+a^{10}x^4\right)}$$

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] 1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 + 7
0*a^4*b*x^2 - 7*a^5)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^6*b^4*x^12 + 4*a^7*b^3*x^
10 + 6*a^8*b^2*x^8 + 4*a^9*b*x^6 + a^10*x^4)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{\sqrt{x}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x**(1/2)/(b*x**3+a*x)**(9/2),x)
[Out] Integral(sqrt(x)/(x*(a + b*x**2))**(9/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \int \frac{\sqrt{x}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)/(b*x^3 + a*x)^(9/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{158\,b^5x^2}{a^6}+\frac{511\,b^4}{a^5}\right)+\frac{560\,b^3}{a^4}\right)x^2+\frac{210\,b^2}{a^3}\right)x}{21\left(bx^2+a\right)^{\frac{7}{2}}} - \frac{4\left(6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+7\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^3
)*x/(b*x^2 + a)^(7/2) - 4/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) - 15
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 7*a^2*b^(3/2))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(ax+bx^3)^{9/2}}\,dx=\int \frac{\sqrt{x}}{{\left(b\,x^3+a\,x\right)}^{9/2}} \,d x$$

```
[In] int(x^(1/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^3)^(9/2), x)
```

3.90 $\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 189

$$\begin{aligned}\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx &= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}+\frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} \\ &+\frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} \\ &-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}+\frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}}-\frac{99b^2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}\end{aligned}$$

```
[Out] 11/35/a^2/x^(3/2)/(b*x^3+a*x)^(5/2)+33/35/a^3/x^(5/2)/(b*x^3+a*x)^(3/2)-99/
8*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(1/2))/a^(13/2)+1/7/a/(b*x^3+a*x)
^(7/2)/x^(1/2)+33/5/a^4/x^(7/2)/(b*x^3+a*x)^(1/2)-33/4*(b*x^3+a*x)^(1/2)/a^
5/x^(9/2)+99/8*b*(b*x^3+a*x)^(1/2)/a^6/x^(5/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2048, 2050, 2054, 212}

$$\begin{aligned}\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx &= -\frac{99b^2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}+\frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}} \\ &-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}}+\frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} \\ &+\frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}}+\frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}\end{aligned}$$

```
[In] Int[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] 1/(7*a*Sqrt[x]*(a*x + b*x^3)^(7/2)) + 11/(35*a^2*x^(3/2)*(a*x + b*x^3)^(5/2
)) + 33/(35*a^3*x^(5/2)*(a*x + b*x^3)^(3/2)) + 33/(5*a^4*x^(7/2)*Sqrt[a*x +
 b*x^3]) - (33*Sqrt[a*x + b*x^3])/(4*a^5*x^(9/2)) + (99*b*Sqrt[a*x + b*x^3]
)/(8*a^6*x^(5/2)) - (99*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]])/(
8*a^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}+\frac{11\int\frac{1}{x^{3/2}(ax+bx^3)^{7/2}}\,dx}{7a}\\
&= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}}+\frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}}+\frac{99\int\frac{1}{x^{5/2}(ax+bx^3)^{5/2}}\,dx}{35a^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} \\
&\quad + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33\int \frac{1}{x^{7/2}\left(ax+bx^3\right)^{3/2}}\,dx}{5a^3} \\
&= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} \\
&\quad + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} + \frac{33\int \frac{1}{x^{9/2}\sqrt{ax+bx^3}}\,dx}{a^4} \\
&= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} \\
&\quad + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} - \frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}} - \frac{(99b)\int \frac{1}{x^{5/2}\sqrt{ax+bx^3}}\,dx}{4a^5} \\
&= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} \\
&\quad + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} - \frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}} + \frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}} + \frac{\left(99b^2\right)\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3}}\,dx}{8a^6} \\
&= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} \\
&\quad + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} - \frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}} + \frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}} \\
&\quad - \frac{\left(99b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^6} \\
&= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} \\
&\quad + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} - \frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}} + \frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}} - \frac{99b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$
\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{\sqrt{x\left(a+bx^2\right)}\Bigl(\sqrt{a}(-70a^5+385a^4bx^2+5808a^3b^2x^4+13398a^2b^3x^6+11550ab^4x^8+}{280a^{13/2}x^{9/2}\left(a+bx^2\right)^4}
$$

```
[In] Integrate[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] (Sqrt[x*(a + b*x^2)]*(Sqrt[a]*(-70*a^5 + 385*a^4*b*x^2 + 5808*a^3*b^2*x^4 +
 13398*a^2*b^3*x^6 + 11550*a*b^4*x^8 + 3465*b^5*x^10) - 3465*b^2*x^4*(a + b
```

```
*x^2)^(7/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(280*a^(13/2)*x^(9/2)*(a + b
*x^2)^4)
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.31

method	result
default	$-\frac{\sqrt{x(b\,x^2+a)}\,\left(3465\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)b^5x^{10}\sqrt{b\,x^2+a}-3465\sqrt{a}\,b^5x^{10}+10395\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)a\,b^4x^8\sqrt{b\,x^2+a}-11550a^{\frac{3}{2}}b\right.}{28}$
risch	$-\frac{(b\,x^2+a)\,(-19b\,x^2+2a)}{8a^6x^{\frac{7}{2}}\sqrt{x(b\,x^2+a)}}+\frac{\left(-\frac{99b^2\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{8a^{\frac{13}{2}}}-\frac{6311b^2\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)}}{1120a^6\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{6311b^2\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2}}{1120a^6\sqrt{-ab}\,\left(x-\right.}\right.}{}$

```
[In] int(1/x^(1/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/280*(x*(b*x^2+a))^(1/2)/a^(13/2)*(3465*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/
x)*b^5*x^10*(b*x^2+a)^(1/2)-3465*a^(1/2)*b^5*x^10+10395*ln(2*(a^(1/2)*(b*x^
2+a)^(1/2)+a)/x)*a*b^4*x^8*(b*x^2+a)^(1/2)-11550*a^(3/2)*b^4*x^8+10395*ln(2
*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^2*b^3*x^6*(b*x^2+a)^(1/2)-13398*a^(5/2)*b
^3*x^6+3465*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*b^2*x^4*(b*x^2+a)^(1/2)
-5808*a^(7/2)*b^2*x^4-385*a^(9/2)*b*x^2+70*a^(11/2))/x^(9/2)/(b*x^2+a)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.95 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \left[\frac{3465\,(b^6x^{13}+4\,ab^5x^{11}+6\,a^2b^4x^9+4\,a^3b^3x^7+a^4b^2x^5)\sqrt{a}\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+ax}}{x^3}\right.}{560\,(a^7b^4x^{13}+}\right.$$

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/560*(3465*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4
*b^2*x^5)*sqrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))
/x^3) + 2*(3465*a*b^5*x^10 + 11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a
^4*b^2*x^4 + 385*a^5*b*x^2 - 70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^
13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5), 1/280*(3465
*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4*b^2*x^5)*sq
rt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (3465*a*b^5*x^10 +
11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a^4*b^2*x^4 + 385*a^5*b*x^2 -
70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b
^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \int \frac{1}{\sqrt{x}\,(x\,(a+bx^2))^{\frac{9}{2}}}\,dx$$

```
[In] integrate(1/x**(1/2)/(b*x**3+a*x)**(9/2),x)
[Out] Integral(1/(sqrt(x)*(x*(a + b*x**2))**(9/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \int \frac{1}{(bx^3+ax)^{\frac{9}{2}}\sqrt{x}}\,dx$$

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a*x)^(9/2)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx = \frac{99\,b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^6} + \frac{350\,(bx^2+a)^3b^2+70\,(bx^2+a)^2ab^2+21\,(bx^2+a)a^2b^2+5\,a^3b^2}{35\,(bx^2+a)^{\frac{7}{2}}a^6} + \frac{19\,(bx^2+a)^{\frac{3}{2}}b^2-21\,\sqrt{bx^2+a}ab^2}{8\,a^6b^2x^4}$$

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] 99/8*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^6) + 1/35*(350*(b*x^2
 + a)^3*b^2 + 70*(b*x^2 + a)^2*a*b^2 + 21*(b*x^2 + a)*a^2*b^2 + 5*a^3*b^2)/
((b*x^2 + a)^(7/2)*a^6) + 1/8*(19*(b*x^2 + a)^(3/2)*b^2 - 21*sqrt(b*x^2 + a
)*a*b^2)/(a^6*b^2*x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3)^{9/2}}\,dx=\int \frac{1}{\sqrt{x}\,\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

```
[In] int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)),x)
[Out] int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)), x)
```

3.91 $\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 180

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx=\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}+\frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}+\frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}}-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}$$

```
[Out] 1/7/a/x^(3/2)/(b*x^3+a*x)^(7/2)+12/35/a^2/x^(5/2)/(b*x^3+a*x)^(5/2)+8/7/a^3
/x^(7/2)/(b*x^3+a*x)^(3/2)+64/7/a^4/x^(9/2)/(b*x^3+a*x)^(1/2)-384/35*(b*x^3
+a*x)^(1/2)/a^5/x^(11/2)+512/35*b*(b*x^3+a*x)^(1/2)/a^6/x^(7/2)-1024/35*b^2
*(b*x^3+a*x)^(1/2)/a^7/x^(3/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2040, 2041, 2039}

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx=-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}+\frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}}+\frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}$$

```
[In] Int[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] 1/(7*a*x^(3/2)*(a*x + b*x^3)^(7/2)) + 12/(35*a^2*x^(5/2)*(a*x + b*x^3)^(5/2
)) + 8/(7*a^3*x^(7/2)*(a*x + b*x^3)^(3/2)) + 64/(7*a^4*x^(9/2)*Sqrt[a*x + b
*x^3]) - (384*Sqrt[a*x + b*x^3])/(35*a^5*x^(11/2)) + (512*b*Sqrt[a*x + b*x^
3])/(35*a^6*x^(7/2)) - (1024*b^2*Sqrt[a*x + b*x^3])/(35*a^7*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}+\frac{12\int \frac{1}{x^{5/2}(ax+bx^3)^{7/2}}\,dx}{7a}\\
&= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}+\frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}}+\frac{24\int \frac{1}{x^{7/2}(ax+bx^3)^{5/2}}\,dx}{7a^2}\\
&= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}+\frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{64\int \frac{1}{x^{9/2}(ax+bx^3)^{3/2}}\,dx}{7a^3}\\
&= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}+\frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}}\\
&\quad+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}+\frac{384\int \frac{1}{x^{11/2}\sqrt{ax+bx^3}}\,dx}{7a^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}+\frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} \\
&\quad +\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}-\frac{(1536b)\int \frac{1}{x^{7/2}\sqrt{ax+bx^3}}\,dx}{35a^5} \\
&= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}+\frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} \\
&\quad +\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}} \\
&\quad +\frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}}+\frac{(1024b^2)\int \frac{1}{x^{3/2}\sqrt{ax+bx^3}}\,dx}{35a^6} \\
&= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}}+\frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} \\
&\quad +\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}+\frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}}-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.50

$$
\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{-7a^6+28a^5bx^2-280a^4b^2x^4-2240a^3b^3x^6-4480a^2b^4x^8-3584ab^5x^{10}-1024b^6x}{35a^7x^{3/2}\left(x\left(a+bx^2\right)\right)^{7/2}}
$$

```
[In] Integrate[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] (-7*a^6 + 28*a^5*b*x^2 - 280*a^4*b^2*x^4 - 2240*a^3*b^3*x^6 - 4480*a^2*b^4*
x^8 - 3584*a*b^5*x^10 - 1024*b^6*x^12)/(35*a^7*x^(3/2)*(x*(a + b*x^2))^(7/2
))
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{\left(b\,x^2+a\right)\left(1024b^6x^{12}+3584a\,b^5x^{10}+4480a^2b^4x^8+2240a^3b^3x^6+280a^4b^2x^4-28a^5b\,x^2+7a^6\right)}{35\sqrt{x}\,a^7\left(b\,x^3+ax\right)^{\frac{9}{2}}}$	92
default	$-\frac{\sqrt{x\left(b\,x^2+a\right)}\,\left(1024b^6x^{12}+3584a\,b^5x^{10}+4480a^2b^4x^8+2240a^3b^3x^6+280a^4b^2x^4-28a^5b\,x^2+7a^6\right)}{35x^{\frac{11}{2}}\left(b\,x^2+a\right)^4a^7}$	94
risch	$-\frac{\left(b\,x^2+a\right)\left(66b^2x^4-8ab\,x^2+a^2\right)}{5a^7x^{\frac{9}{2}}\sqrt{x\left(b\,x^2+a\right)}}-\frac{\left(b\,x^2+a\right)x^{\frac{3}{2}}\left(562b^3x^6+1792a\,b^2x^4+1925a^2b\,x^2+700a^3\right)b^3}{35a^7\left(x^8b^4+4a\,b^3x^6+6a^2x^4b^2+4a^3b\,x^2+a^4\right)\sqrt{x\left(b\,x^2+a\right)}}$	150

```
[In] int(1/x^(3/2)/(b*x^3+a*x)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35*(b*x^2+a)*(1024*b^6*x^12+3584*a*b^5*x^10+4480*a^2*b^4*x^8+2240*a^3*b^
3*x^6+280*a^4*b^2*x^4-28*a^5*b*x^2+7*a^6)/x^(1/2)/a^7/(b*x^3+a*x)^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{\left(1024\,b^6x^{12}+3584\,ab^5x^{10}+4480\,a^2b^4x^8+2240\,a^3b^3x^6+280\,a^4b^2x^4-28\,a^5bx^2+7\,a^6\right)\sqrt{bx^3+ax}\sqrt{x}}{35\left(a^7b^4x^{14}+4\,a^8b^3x^{12}+6\,a^9b^2x^{10}+4\,a^{10}bx^8+a^{11}x^6\right)}$$

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(1024*b^6*x^12 + 3584*a*b^5*x^10 + 4480*a^2*b^4*x^8 + 2240*a^3*b^3*x^
6 + 280*a^4*b^2*x^4 - 28*a^5*b*x^2 + 7*a^6)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^7*
b^4*x^14 + 4*a^8*b^3*x^12 + 6*a^9*b^2*x^10 + 4*a^10*b*x^8 + a^11*x^6)
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(1/x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x*(a + b*x**2))**(9/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \int \frac{1}{\left(bx^3+ax\right)^{\frac{9}{2}}x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(9/2)*x^(3/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{\left(\left(2\,x^2\left(\frac{281\,b^6x^2}{a^7}+\frac{896\,b^5}{a^6}\right)+\frac{1925\,b^4}{a^5}\right)x^2+\frac{700\,b^3}{a^4}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}$$

$$+\frac{4\bigg(25\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8b^{\frac{5}{2}}-120\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{5}{2}}+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{5}{2}}-140\,\big(\sqrt{b}x-}{5\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5a^6}$$

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((2*x^2*(281*b^6*x^2/a^7 + 896*b^5/a^6) + 1925*b^4/a^5)*x^2 + 700*b^3
/a^4)*x/(b*x^2 + a)^(7/2) + 4/5*(25*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2)
 - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2) + 210*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*a^2*b^(5/2) - 140*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/2) +
 33*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \int \frac{1}{x^{3/2}\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

```
[In] int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)), x)
```

3.92 $\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

```
[Out] -1/3*a*arctanh(x^2*b^(1/2)/(b*x^4+a*x)^(1/2))/b^(3/2)+1/3*x*(b*x^4+a*x)^(1/
2)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2049, 2054, 212}

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

```
[In] Int[x^4/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*Sqrt[a*x + b*x^4])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/
(3*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\int\frac{x}{\sqrt{ax+bx^4}}\,dx}{2b}\\ &= \frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{ax+bx^4}}\right)}{3b}\\ &= \frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int\frac{x^4}{\sqrt{ax+bx^4}}\,dx=\frac{\sqrt{b}x^2(a+bx^3)-a\sqrt{x}\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3b^{3/2}\sqrt{x\left(a+bx^3\right)}}$$

```
[In] Integrate[x^4/Sqrt[a*x + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(a + b*x^3) - a*Sqrt[x]*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) +
Sqrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x*(a + b*x^3)])
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{-x\sqrt{b}\,\sqrt{x(b\,x^3+a)}+\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)a}{3b^{\frac{3}{2}}}$	45
pseudoelliptic	$-\frac{-x\sqrt{b}\,\sqrt{x(b\,x^3+a)}+\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)a}{3b^{\frac{3}{2}}}$	45
risch	$\frac{x^2(b\,x^3+a)}{3b\sqrt{x(b\,x^3+a)}}-\frac{a\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^3+a\right)}}{x^2\sqrt{b}}\right)}{3b^{\frac{3}{2}}}$	53
elliptic	Expression too large to display	997

```
[In] int(x^4/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b^(3/2)*(-x*b^(1/2)*(x*(b*x^3+a))^(1/2)+arctanh(1/x^2*(x*(b*x^3+a))^(1
/2)/b^(1/2))*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.42

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx = \left[\frac{4\,\sqrt{bx^4+ax}bx+a\sqrt{b}\log\left(-8\,b^2x^6-8\,abx^3-a^2+4\,(2\,bx^4+ax)\sqrt{bx^4+ax}\sqrt{b}\right)}{12\,b^2}, 2\,\sqrt{bx^4+ax}bx+a\surd\right.$$

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^4 + a*x)*b*x + a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2
 + 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)*sqrt(b)))/b^2, 1/6*(2*sqrt(b*x^4 + a
*x)*b*x + a*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b*x^3 + a)))/
b^2]
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^4}{\sqrt{x\,(a+bx^3)}}\,dx$$

```
[In] integrate(x**4/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^4}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^4 + a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx=\frac{\sqrt{bx^4+ax}\,x}{3\,b}+\frac{a\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}b}$$

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^4 + a*x)*x/b + 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/(sqrt(-b
)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^4}{\sqrt{b\,x^4+a\,x}}\,dx$$

```
[In] int(x^4/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a*x + b*x^4)^(1/2), x)
```

3.93 $\int \frac{x}{\sqrt{ax+bx^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{x}{\sqrt{ax+bx^4}} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

```
[Out] 2/3*arctanh(x^2*b^(1/2)/(b*x^4+a*x)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2054, 212}

$$\int \frac{x}{\sqrt{ax+bx^4}} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

```
[In] Int[x/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax+bx^4}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = \frac{2\sqrt{x}\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3\sqrt{b}\sqrt{x\left(a+bx^3\right)}}$$

```
[In] Integrate[x/Sqrt[a*x + b*x^4],x]
[Out] (2*Sqrt[x]*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(3*Sqrt[
b]*Sqrt[x*(a + b*x^3)])
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{b}}$	25
pseudoelliptic	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{b}}$	25
elliptic	Expression too large to display	979

```
[In] int(x/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/b^(1/2)*arctanh(1/x^2*(x*(b*x^3+a))^(1/2)/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.94

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = \left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-a^2-4\,(2\,bx^4+ax)\sqrt{bx^4+ax}\sqrt{b}\right)}{6\,\sqrt{b}},\right.$$
$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{bx^4+ax}\sqrt{-b}x}{2\,bx^3+a}\right)}{3\,b}\right]$$

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)
*sqrt(b))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b
*x^3 + a))/b]
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = \int \frac{x}{\sqrt{x\,(a+bx^3)}}\,dx$$

```
[In] integrate(x/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = \int \frac{x}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^4 + a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}}$$

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] -2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax+bx^4}}\,dx = \int \frac{x}{\sqrt{b\,x^4+a\,x}}\,dx$$

```
[In] int(x/(a*x + b*x^4)^(1/2),x)
[Out] int(x/(a*x + b*x^4)^(1/2), x)
```

3.94 $\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

```
[Out] -2/3*(b*x^4+a*x)^(1/2)/a/x^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2039}

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

```
[In] Int[1/(x^2*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(3*a*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{x\left(a+bx^3\right)}}{3ax^2}$$

```
[In] Integrate[1/(x^2*Sqrt[a*x + b*x^4]),x]
[Out] (-2*Sqrt[x*(a + b*x^3)])/(3*a*x^2)
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{2\sqrt{b\,x^4+ax}}{3a\,x^2}$	20
trager	$-\frac{2\sqrt{b\,x^4+ax}}{3a\,x^2}$	20
elliptic	$-\frac{2\sqrt{b\,x^4+ax}}{3a\,x^2}$	20
pseudoelliptic	$-\frac{2\sqrt{x(b\,x^3+a)}}{3a\,x^2}$	20
gosper	$-\frac{2(b\,x^3+a)}{3xa\sqrt{b\,x^4+ax}}$	27
risch	$-\frac{2(b\,x^3+a)}{3ax\sqrt{x(b\,x^3+a)}}$	27

```
[In] int(1/x^2/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(b*x^4+a*x)^(1/2)/a/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\,\sqrt{bx^4+ax}}{3\,ax^2}$$

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(b*x^4 + a*x)/(a*x^2)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^2\sqrt{x\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(1/x**2/(b*x**4+a*x)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(x*(a + b*x**3))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\left(bx^4+ax\right)}{3\sqrt{bx^3+a}ax^{\frac{5}{2}}}$$

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{b+\frac{a}{x^3}}}{3\,a}$$

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] -2/3*sqrt(b + a/x^3)/a
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{b\,x^4+a\,x}}{3\,a\,x^2}$$

```
[In] int(1/(x^2*(a*x + b*x^4)^(1/2)),x)
[Out] -(2*(a*x + b*x^4)^(1/2))/(3*a*x^2)
```

3.95 $\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{ax+bx^4}}{9ax^5}+\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}$$

```
[Out] -2/9*(b*x^4+a*x)^(1/2)/a/x^5+4/9*b*(b*x^4+a*x)^(1/2)/a^2/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2041, 2039}

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx=\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$

```
[In] Int[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(9*a*x^5) + (4*b*Sqrt[a*x + b*x^4])/(9*a^2*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\sqrt{ax+bx^4}}{9ax^5}-\frac{(2b)\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax+bx^4}}{9ax^5}+\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx=-\frac{2(a-2bx^3)\sqrt{x(a+bx^3)}}{9a^2x^5}$$

```
[In] Integrate[1/(x^5*Sqrt[a*x + b*x^4]),x]
[Out] (-2*(a - 2*b*x^3)*Sqrt[x*(a + b*x^3)])/(9*a^2*x^5)
```

Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.58

method	result	size
trager	$-\frac{2\left(-2b\,x^{3}+a\right)\sqrt{b\,x^{4}+ax}}{9x^{5}a^{2}}$	28
pseudoelliptic	$-\frac{2\left(-2b\,x^{3}+a\right)\sqrt{x\left(b\,x^{3}+a\right)}}{9x^{5}a^{2}}$	28
gosper	$-\frac{2\left(b\,x^{3}+a\right)\left(-2b\,x^{3}+a\right)}{9x^{4}a^{2}\sqrt{b\,x^{4}+ax}}$	35
risch	$-\frac{2\left(b\,x^{3}+a\right)\left(-2b\,x^{3}+a\right)}{9a^{2}x^{4}\sqrt{x\left(b\,x^{3}+a\right)}}$	35
default	$-\frac{2\sqrt{b\,x^{4}+ax}}{9a\,x^{5}}+\frac{4b\sqrt{b\,x^{4}+ax}}{9a^{2}x^{2}}$	41
elliptic	$-\frac{2\sqrt{b\,x^{4}+ax}}{9a\,x^{5}}+\frac{4b\sqrt{b\,x^{4}+ax}}{9a^{2}x^{2}}$	41

```
[In] int(1/x^5/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/9*(-2*b*x^3+a)/x^5/a^2*(b*x^4+a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = \frac{2\,\sqrt{bx^4+ax}(2\,bx^3-a)}{9\,a^2x^5}$$

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^4 + a*x)*(2*b*x^3 - a)/(a^2*x^5)
```

Sympy [F]

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^5\sqrt{x\,(a+bx^3)}}\,dx$$

```
[In] integrate(1/x**5/(b*x**4+a*x)**(1/2),x)
[Out] Integral(1/(x**5*sqrt(x*(a + b*x**3))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = \frac{2\,(2\,b^2x^7+abx^4-a^2x)}{9\,\sqrt{bx^3+a}a^2x^{\frac{11}{2}}}$$

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] 2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = -\frac{2\,\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}}{9\,a^2}+\frac{2\,\sqrt{b+\frac{a}{x^3}}b}{3\,a^2}$$

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] -2/9*(b + a/x^3)^(3/2)/a^2 + 2/3*sqrt(b + a/x^3)*b/a^2
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx = -\frac{2\,\sqrt{b\,x^4+a\,x}\,\left(a-2\,b\,x^3\right)}{9\,a^2\,x^5}$$

```
[In] int(1/(x^5*(a*x + b*x^4)^(1/2)),x)
[Out] -(2*(a*x + b*x^4)^(1/2)*(a - 2*b*x^3))/(9*a^2*x^5)
```

3.96 $\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{ax+bx^4}}{15ax^8}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}$$

```
[Out] -2/15*(b*x^4+a*x)^(1/2)/a/x^8+8/45*b*(b*x^4+a*x)^(1/2)/a^2/x^5-16/45*b^2*(b
*x^4+a*x)^(1/2)/a^3/x^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2041, 2039}

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx=-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

```
[In] Int[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(15*a*x^8) + (8*b*Sqrt[a*x + b*x^4])/(45*a^2*x^5) -
(16*b^2*Sqrt[a*x + b*x^4])/(45*a^3*x^2)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\sqrt{ax+bx^4}}{15ax^8}-\frac{(4b)\int\frac{1}{x^5\sqrt{ax+bx^4}}\,dx}{5a}\\ &= -\frac{2\sqrt{ax+bx^4}}{15ax^8}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}+\frac{(8b^2)\int\frac{1}{x^2\sqrt{ax+bx^4}}\,dx}{15a^2}\\ &= -\frac{2\sqrt{ax+bx^4}}{15ax^8}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int\frac{1}{x^8\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{x(a+bx^3)}(3a^2-4abx^3+8b^2x^6)}{45a^3x^8}$$

```
[In] Integrate[1/(x^8*Sqrt[a*x + b*x^4]),x]
[Out] (-2*Sqrt[x*(a + b*x^3)]*(3*a^2 - 4*a*b*x^3 + 8*b^2*x^6))/(45*a^3*x^8)
```

Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
trager	$-\frac{2(8b^2x^6-4ab\,x^3+3a^2)\sqrt{b\,x^4+ax}}{45x^8a^3}$	41
pseudoelliptic	$-\frac{2(8b^2x^6-4ab\,x^3+3a^2)\sqrt{x(b\,x^3+a)}}{45x^8a^3}$	41
gosper	$-\frac{2(b\,x^3+a)(8b^2x^6-4ab\,x^3+3a^2)}{45x^7a^3\sqrt{b\,x^4+ax}}$	48
risch	$-\frac{2(b\,x^3+a)(8b^2x^6-4ab\,x^3+3a^2)}{45a^3x^7\sqrt{x(b\,x^3+a)}}$	48
default	$-\frac{2\sqrt{b\,x^4+ax}}{15a\,x^8}+\frac{8b\sqrt{b\,x^4+ax}}{45a^2x^5}-\frac{16b^2\sqrt{b\,x^4+ax}}{45a^3x^2}$	63
elliptic	$-\frac{2\sqrt{b\,x^4+ax}}{15a\,x^8}+\frac{8b\sqrt{b\,x^4+ax}}{45a^2x^5}-\frac{16b^2\sqrt{b\,x^4+ax}}{45a^3x^2}$	63

```
[In] int(1/x^8/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/45*(8*b^2*x^6-4*a*b*x^3+3*a^2)/x^8/a^3*(b*x^4+a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\,(8\,b^2x^6-4\,abx^3+3\,a^2)\sqrt{bx^4+ax}}{45\,a^3x^8}$$

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] -2/45*(8*b^2*x^6 - 4*a*b*x^3 + 3*a^2)*sqrt(b*x^4 + a*x)/(a^3*x^8)
```

Sympy [F]

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^8\sqrt{x\,(a+bx^3)}}\,dx$$

```
[In] integrate(1/x**8/(b*x**4+a*x)**(1/2),x)
[Out] Integral(1/(x**8*sqrt(x*(a + b*x**3))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\,(8\,b^3x^{10}+4\,ab^2x^7-a^2bx^4+3\,a^3x)}{45\,\sqrt{bx^3+a}a^3x^{\frac{17}{2}}}$$

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] -2/45*(8*b^3*x^10 + 4*a*b^2*x^7 - a^2*b*x^4 + 3*a^3*x)/(sqrt(b*x^3 + a)*a^3
*x^(17/2))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\,\sqrt{b+\frac{a}{x^3}}b^2}{3\,a^3} - \frac{2\left(3\,\left(b+\frac{a}{x^3}\right)^{\frac{5}{2}} - 10\,\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}b\right)}{45\,a^3}$$

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] -2/3*sqrt(b + a/x^3)*b^2/a^3 - 2/45*(3*(b + a/x^3)^(5/2) - 10*(b + a/x^3)^(
3/2)*b)/a^3
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx = -\frac{2\,\sqrt{b\,x^4+a\,x}\,\left(3\,a^2-4\,a\,b\,x^3+8\,b^2\,x^6\right)}{45\,a^3\,x^8}$$

```
[In] int(1/(x^8*(a*x + b*x^4)^(1/2)),x)
[Out] -(2*(a*x + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^6 - 4*a*b*x^3))/(45*a^3*x^8)
```

3.97 $\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 224

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \frac{\sqrt{ax+bx^4}}{2b} - \frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

```
[Out] 1/2*(b*x^4+a*x)^(1/2)/b-1/12*a^(2/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1
/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/b/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*
x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used

= {2049, 2036, 335, 231}

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \frac{\sqrt{ax+bx^4}}{2b} - \frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

```
[In] Int[x^3/Sqrt[a*x + b*x^4],x]
```

```
[Out] Sqrt[a*x + b*x^4]/(2*b) - (a^(2/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ell
ipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
```

```
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax+bx^4}}{2b} - \frac{a\int \frac{1}{\sqrt{ax+bx^4}}\,dx}{4b}\\
&= \frac{\sqrt{ax+bx^4}}{2b} - \frac{\left(a\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{4b\sqrt{ax+bx^4}}\\
&= \frac{\sqrt{ax+bx^4}}{2b} - \frac{\left(a\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{2b\sqrt{ax+bx^4}}\\
&= \frac{\sqrt{ax+bx^4}}{2b}\\
&\quad - \frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.29

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \frac{x\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{2b\sqrt{x\left(a+bx^3\right)}}$$

```
[In] Integrate[x^3/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b
*x^3)/a)]))/(2*b*Sqrt[x*(a + b*x^3)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.34 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.07

method	result
default	$\frac{\sqrt{b x^{4}+a x}}{2 b}-\frac{a \left(\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{\frac{\left(-\frac{3 \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\, \left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2} \sqrt{\frac{\left(-a b^{2}\right)^{\frac{1}{3}} \left(x\right.}{b \left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}-\right.}}}{2 \left(-\frac{3 \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-\right.}{}\right.}$
elliptic	$\frac{\sqrt{b x^{4}+a x}}{2 b}-\frac{a \left(\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{\frac{\left(-\frac{3 \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\, \left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2} \sqrt{\frac{\left(-a b^{2}\right)^{\frac{1}{3}} \left(x\right.}{b \left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}-\right.}}}{2 \left(-\frac{3 \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-\right.}{}\right.}$
risch	$\frac{x \left(b x^{3}+a \right)}{2 b \sqrt{x \left(b x^{3}+a \right)}}-\frac{a \left(\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{\frac{\left(-\frac{3 \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\, \left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2} \sqrt{\frac{\left(-a b^{2}\right.}{b \left(-\frac{\left(-a b^{2}\right.}{2 b}\right.}}}{2 \left(-\frac{3 \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\right.}$

```
[In] int(x^3/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x^4+a*x)^(1/2)/b-1/2*a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2
)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3
)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
```

```
))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^3}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a*x)*x^2/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^3}{\sqrt{x\,(a+bx^3)}}\,dx$$

```
[In] integrate(x**3/(b*x**4+a*x)**(1/2),x)
[Out] Integral(x**3/sqrt(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^3}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(b*x^4 + a*x), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^3}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(b*x^4 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^3}{\sqrt{b\,x^4+a\,x}}\,dx$$

```
[In] int(x^3/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x^3/(a*x + b*x^4)^(1/2), x)
```

3.98 $\int \frac{1}{\sqrt{ax+bx^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 197

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

```
[Out] 1/3*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3
)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used

$= \{2036,\ 335,\ 231\}$

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx$$

$$= \frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

```
[In] Int[1/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt
[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^
(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{\sqrt{ax+bx^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a+bx^3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^4}}$$

$$= \frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.25

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \frac{2x\sqrt{1+\frac{bx^3}{a}}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}$$

```
[In] Integrate[1/Sqrt[a*x + b*x^4],x]
[Out] (2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])/Sq
rt[x*(a + b*x^3)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.41

method	result
default	$\dfrac{2\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\right.}{b\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}}}{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)(-a\,b}$
elliptic	$\dfrac{2\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\right.}{b\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}}}{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)(-a\,b}$

```
[In] int(1/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(
1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3
)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*b/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = -\frac{2\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)}{\sqrt{a}}$$

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*weierstrassPInverse(0, -4*b/a, 1/x)/sqrt(a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{ax+bx^4}}\,dx$$

```
[In] integrate(1/(b*x**4+a*x)**(1/2),x)
[Out] Integral(1/sqrt(a*x + b*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*x^4 + a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.20

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \frac{1}{3}\sqrt{bx^4+ax}\,x - \frac{a\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}}$$

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(b*x^4 + a*x)*x - 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.20

$$\int \frac{1}{\sqrt{ax+bx^4}}\,dx = \frac{2\,x\,\sqrt{\frac{b\,x^3}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{7}{6};\ -\frac{b\,x^3}{a}\right)}{\sqrt{b\,x^4+a\,x}}$$

```
[In] int(1/(a*x + b*x^4)^(1/2),x)
[Out] (2*x*((b*x^3)/a + 1)^(1/2)*hypergeom([1/6, 1/2], 7/6, -(b*x^3)/a))/(a*x + b
*x^4)^(1/2)
```

3.99 $\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 225

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

```
[Out] -2/5*(b*x^4+a*x)^(1/2)/a/x^3-2/15*b*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(
1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used

$= \{2050, 2036, 335, 231\}$

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = -\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}} - \frac{2\sqrt{ax+bx^4}}{5ax^3}$$

```
[In] Int[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(5*a*x^3) - (2*b*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2
]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x*(a^(
1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]
)
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax+bx^4}}{5ax^3} - \frac{(2b)\int \frac{1}{\sqrt{ax+bx^4}}\,dx}{5a} \\
&= -\frac{2\sqrt{ax+bx^4}}{5ax^3} - \frac{\left(2b\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{5a\sqrt{ax+bx^4}} \\
&= -\frac{2\sqrt{ax+bx^4}}{5ax^3} - \frac{\left(4b\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{5a\sqrt{ax+bx^4}} \\
&= -\frac{2\sqrt{ax+bx^4}}{5ax^3} \\
&\quad - \frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.24

$$
\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{5x^2\sqrt{x\left(a+bx^3\right)}}
$$

```
[In] Integrate[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(5
*x^2*Sqrt[x*(a + b*x^3)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 696, normalized size of antiderivative = 3.09

method	result
default	$-\frac{2\sqrt{b\,x^{4}+a x}}{5 a\,x^{3}}-\frac{4 b^{2}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b\left(-\frac{\left(-a\,b^{2}\right.}{2 b}\right.}}}{5 a\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\surd}{}\right.}$
elliptic	$-\frac{2\sqrt{b\,x^{4}+a x}}{5 a\,x^{3}}-\frac{4 b^{2}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b\left(-\frac{\left(-a\,b^{2}\right.}{2 b}\right.}}}{5 a\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\surd}{}\right.}$
risch	$-\frac{2\left(b\,x^{3}+a\right)}{5 a\,x^{2}\sqrt{x\left(b\,x^{3}+a\right)}}-\frac{4 b^{2}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{(}{b\left(-\right.}}}{5 a\left(-\frac{3\left(-a\,b^{2}\right)}{2 b}\right.}$

```
[In] int(1/x^3/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x^4+a*x)^(1/2)/a/x^3-4/5*b^2/a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x
/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/
3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
```

```
b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = \frac{2\left(2\,\sqrt{a}bx^3\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)-\sqrt{bx^4+ax}a\right)}{5\,a^2x^3}$$

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] 2/5*(2*sqrt(a)*b*x^3*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^4 + a*x
)*a)/(a^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^3\sqrt{x\,(a+bx^3)}}\,dx$$

```
[In] integrate(1/x**3/(b*x**4+a*x)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(x*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^4+a\,x}}\,dx$$

```
[In] int(1/(x^3*(a*x + b*x^4)^(1/2)),x)
[Out] int(1/(x^3*(a*x + b*x^4)^(1/2)), x)
```

3.100 $\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 503

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = &-\frac{5\left(1+\sqrt{3}\right)ax(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}}+\frac{x^2\sqrt{ax+bx^4}}{4b}\\
&+\frac{5\sqrt[4]{3}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}\\
&+\frac{5\left(1-\sqrt{3}\right)a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}
\end{aligned}
$$

```
[Out] -5/8*a*x*(b*x^3+a)*(1+3^(1/2))/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x
^4+a*x)^(1/2)+1/4*x^2*(b*x^4+a*x)^(1/2)/b+5/8*3^(1/4)*a^(4/3)*x*(a^(1/3)+b^
(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*E
llipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^4+a*x)^(1/2)/(b^(1
/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+5/48*a^(
4/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3
```

```
)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3
^(3/4)/b^(5/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2049, 2057, 335, 314, 231, 1895}

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx$$

$$= \frac{5\left(1-\sqrt{3}\right)a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$
$$+\frac{5\sqrt[4]{3}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$
$$-\frac{5\left(1+\sqrt{3}\right)ax\left(a+bx^3\right)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}}+\frac{x^2\sqrt{ax+bx^4}}{4b}$$

```
[In] Int[x^5/Sqrt[a*x + b*x^4],x]
```

```
[Out] (-5*(1 + Sqrt[3])*a*x*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)*Sqrt[a*x + b*x^4]) + (x^2*Sqrt[a*x + b*x^4])/(4*b) + (5*3^(1/4)*a^(4
/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - S
qrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/
(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) + (5*(1 - Sqrt[3])*a^(4/3)*x*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*3^(1/4)*b^(5/3)
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a*x + b*x^4])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
```

```
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{(5a)\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx}{8b}\\
&= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{\left(5a\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{8b\sqrt{ax+bx^4}}\\
&= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{\left(5a\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{4b\sqrt{ax+bx^4}}\\
&= \frac{x^2\sqrt{ax+bx^4}}{4b} + \frac{\left(5a\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{8b^{5/3}\sqrt{ax+bx^4}}\\
&\quad + \frac{\left(5\left(1-\sqrt{3}\right)a^{5/3}\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{8b^{5/3}\sqrt{ax+bx^4}}\\
&= -\frac{5\left(1+\sqrt{3}\right)ax(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}} + \frac{x^2\sqrt{ax+bx^4}}{4b}\\
&\quad + \frac{5\sqrt[4]{3}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}\\
&\quad + \frac{5\left(1-\sqrt{3}\right)a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.13

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx=\frac{x^3\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{4b\sqrt{x\,(a+bx^3)}}$$

```
[In] Integrate[x^5/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -
((b*x^3)/a)]))/(4*b*Sqrt[x*(a + b*x^3)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.49 (sec) , antiderivative size = 1079, normalized size of antiderivative = 2.15

method	result	size
default	Expression too large to display	1079
elliptic	Expression too large to display	1079
risch	Expression too large to display	1086

```
[In] int(x^5/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^2*(b*x^4+a*x)^(1/2)/b-5/8*a/b*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1
/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a
*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
```

```
^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
```

Fricas [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^5}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a*x)*x^4/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^5}{\sqrt{x\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(x**5/(b*x**4+a*x)**(1/2),x)
[Out] Integral(x**5/sqrt(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^5}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/sqrt(b*x^4 + a*x), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^5}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt(b*x^4 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^5}{\sqrt{b\,x^4+a\,x}}\,dx$$

```
[In] int(x^5/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x^5/(a*x + b*x^4)^(1/2), x)
```

3.101 $\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 474

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx=\frac{\left(1+\sqrt{3}\right)x\left(a+bx^3\right)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}}$$
$$-\frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$
$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

```
[Out] x*(b*x^3+a)*(1+3^(1/2))/b^(2/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^4+a*x)
^(1/2)-3^(1/4)*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)/b^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b
```

```
^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2
)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3
)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2057, 335, 314, 231, 1895}

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = -\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}} - \frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}} + \frac{\left(1+\sqrt{3}\right)x\left(a+bx^3\right)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}}$$

```
[In] Int[x^2/Sqrt[a*x + b*x^4],x]
```

```
[Out] ((1 + Sqrt[3])*x*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*
Sqrt[a*x + b*x^4]) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*
EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/
3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sq
rt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/
3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sq
rt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax+bx^4}}$$

$$
\begin{aligned}
&= \frac{\left(2\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^4}} \\
&= -\frac{\left(\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax+bx^4}} \\
&\quad - \frac{\left(\left(1-\sqrt{3}\right)a^{2/3}\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax+bx^4}} \\
&= \frac{\left(1+\sqrt{3}\right)x(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}} \\
&\quad - \frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}} \\
&\quad - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$
\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \frac{2x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{5\sqrt{x\left(a+bx^3\right)}}
$$

```
[In] Integrate[x^2/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)])
/(5*Sqrt[x*(a + b*x^3)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 1054, normalized size of antiderivative = 2.22

method	result	size
default	Expression too large to display	1054
elliptic	Expression too large to display	1054

```
[In] int(x^2/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1
/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2
/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3
)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(
1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1
/3)))/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a*x)*x/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{x\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(x**2/(b*x**4+a*x)**(1/2),x)
[Out] Integral(x**2/sqrt(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(b*x^4 + a*x), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(b*x^4 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx=\int \frac{x^2}{\sqrt{b\,x^4+a\,x}}\,dx$$

```
[In] int(x^2/(a*x + b*x^4)^(1/2),x)
[Out] int(x^2/(a*x + b*x^4)^(1/2), x)
```

3.102 $\int \frac{1}{x\sqrt{ax+bx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 497

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{ax}$$
$$-\frac{2\sqrt[4]{3}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$
$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

```
[Out] 2*b^(1/3)*x*(b*x^3+a)*(1+3^(1/2))/a/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^4+
a*x)^(1/2)-2*(b*x^4+a*x)^(1/2)/a/x-2*3^(1/4)*b^(1/3)*x*(a^(1/3)+b^(1/3)*x)*
((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE(
(1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^
(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/3*b^(1/3)*x*(a^
(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(
```

```
1/2))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(
2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2050, 2057, 335, 314, 231, 1895}

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = -\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}} - \frac{2\sqrt[4]{3}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}} - \frac{2\sqrt{ax+bx^4}}{ax} + \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x\left(a+bx^3\right)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}}$$

```
[In] Int[1/(x*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (2*(1 + Sqrt[3])*b^(1/3)*x*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)*Sqrt[a*x + b*x^4]) - (2*Sqrt[a*x + b*x^4])/(a*x) - (2*3^(1/4)*b^(1/3)*x
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3
])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(a^(2
/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sqrt[3])*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x
 + b*x^4])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
```

```
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax+bx^4}}{ax}+\frac{(2b)\int\frac{x^2}{\sqrt{ax+bx^4}}\,dx}{a}\\
&= -\frac{2\sqrt{ax+bx^4}}{ax}+\frac{\left(2b\sqrt{x}\sqrt{a+bx^3}\right)\int\frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{a\sqrt{ax+bx^4}}\\
&= -\frac{2\sqrt{ax+bx^4}}{ax}+\frac{\left(4b\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int\frac{x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^4}}\\
&= -\frac{2\sqrt{ax+bx^4}}{ax}-\frac{\left(2\sqrt[3]{b}\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int\frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^4}}\\
&\quad-\frac{\left(2\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{\sqrt[3]{a}\sqrt{ax+bx^4}}\\
&= \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{ax}\\
&\quad-\frac{2\sqrt[4]{3}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}\\
&\quad-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.10

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}$$

```
[In] Integrate[1/(x*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3)/a)])/Sq
rt[x*(a + b*x^3)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 1083, normalized size of antiderivative = 2.18

method	result	size
default	Expression too large to display	1083
risch	Expression too large to display	1083
elliptic	Expression too large to display	1083

```
[In] int(1/x/(b*x^4+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x^3+a)/a/(x*(b*x^3+a))^(1/2)+2*b/a*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))
^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/
b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
```

```
(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*x*(x-1/b*(-a*b^2)^(1/3))*
(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \frac{2\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)}{\sqrt{a}}$$

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x))/sqrt(a)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \int \frac{1}{x\sqrt{x\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(1/x/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax}x}\,dx$$

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+ax}x}\,dx$$

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx=\int \frac{1}{x\,\sqrt{b\,x^4+a\,x}}\,dx$$

```
[In] int(1/(x*(a*x + b*x^4)^(1/2)),x)
[Out] int(1/(x*(a*x + b*x^4)^(1/2)), x)
```

3.103 $\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 174

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{63b^5\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{64a^{11/2}}$$

```
[Out] -63/64*b^5*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(11/2)+63/64*b^
4*(b*x^(1/2)+a*x)^(1/2)/a^5+21/40*b^2*x*(b*x^(1/2)+a*x)^(1/2)/a^3-9/20*b*x^
(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^2+2/5*x^2*(b*x^(1/2)+a*x)^(1/2)/a-21/32*b^3*x
^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2043, 684, 654, 634, 212}

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{63b^5\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{11/2}} + \frac{63b^4\sqrt{ax+b\sqrt{x}}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^4} + \frac{21b^2x\sqrt{ax+b\sqrt{x}}}{40a^3} - \frac{9bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^2} + \frac{2x^2\sqrt{ax+b\sqrt{x}}}{5a}$$

```
[In] Int[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (63*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^5) - (21*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] +
a*x])/(32*a^4) + (21*b^2*x*Sqrt[b*Sqrt[x] + a*x])/(40*a^3) - (9*b*x^(3/2)*S
```

```
qrt[b*Sqrt[x] + a*x])/(20*a^2) + (2*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a) - (63*
b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(64*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{(9b)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{5a}\\
&= -\frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} + \frac{(63b^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{40a^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} \\
&\quad + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{(21b^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{16a^3} \\
&= -\frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} \\
&\quad + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} + \frac{(63b^4)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{64a^4} \\
&= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} \\
&\quad - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{(63b^5)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{128a^5} \\
&= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} \\
&\quad - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{(63b^5)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{64a^5} \\
&= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} \\
&\quad - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{64a^{11/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.65

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx &= \frac{\sqrt{b\sqrt{x}+ax}\left(315b^4 - 210ab^3\sqrt{x} + 168a^2b^2x - 144a^3bx^{3/2} + 128a^4x^2\right)}{320a^5} \\
&\quad - \frac{63b^5\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{64a^{11/2}}
\end{aligned}
$$

```
[In] Integrate[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(315*b^4 - 210*a*b^3*Sqrt[x] + 168*a^2*b^2*x - 144*a
^3*b*x^(3/2) + 128*a^4*x^2))/(320*a^5) - (63*b^5*ArcTanh[(Sqrt[a]*Sqrt[b*Sq
rt[x] + a*x])/(b + a*Sqrt[x])])/(64*a^(11/2))
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{2x^{2}\sqrt{b\sqrt{x}+ax}}{5a}-\frac{9b\left(\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}{4a}-\frac{7b\left(\frac{x\sqrt{b\sqrt{x}+ax}}{3a}-\frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right.}{8a}\right.}{5a}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(544\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}\sqrt{x}\,a^{\frac{7}{2}}b-256x\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{9}{2}}-880\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{5}{2}}b^{2}+1300\sqrt{b\sqrt{x}+ax}\,\sqrt{x}\,a^{\frac{5}{2}}b^{3}+650\right.}{640}$

```
[In] int(x^2/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^2*(b*x^(1/2)+a*x)^(1/2)/a-9/5*b/a*(1/4*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/
a-7/8*b/a*(1/3*x*(b*x^(1/2)+a*x)^(1/2)/a-5/6*b/a*(1/2*x^(1/2)*(b*x^(1/2)+a*
x)^(1/2)/a-3/4*b/a*((b*x^(1/2)+a*x)^(1/2)/a-1/2*b/a^(3/2)*ln((1/2*b+a*x^(1/
2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2))))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx=\text{Timed out}$$

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$$

$$=2\left(\begin{cases}\sqrt{ax+b\sqrt{x}}\left(\frac{x^2}{5a}-\frac{9bx^{\frac{3}{2}}}{40a^2}+\frac{21b^2x}{80a^3}-\frac{21b^3\sqrt{x}}{64a^4}+\frac{63b^4}{128a^5}\right)-\frac{63b^5\left(\begin{cases}\frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for }\frac{b^2}{a}\neq 0\\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise}\end{cases}\right)}{256a^5} & \text{for}\\ \frac{2\left(b\sqrt{x}\right)^{\frac{11}{2}}}{11b^6} & \text{for}\\ \tilde{\infty}x^3 & \text{otl}\end{cases}\right.$$

```
[In] integrate(x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] 2*Piecewise((sqrt(a*x + b*sqrt(x))*(x**2/(5*a) - 9*b*x**(3/2)/(40*a**2) + 2
1*b**2*x/(80*a**3) - 21*b**3*sqrt(x)/(64*a**4) + 63*b**4/(128*a**5)) - 63*b
**5*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(
a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/sqrt(a*(sqr
t(x) + b/(2*a))**2), True))/(256*a**5), Ne(a, 0)), (2*(b*sqrt(x))**(11/2)/(
11*b**6), Ne(b, 0)), (zoo*x**3, True))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx=\int \frac{x^2}{\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{320}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\sqrt{x}\left(\frac{8\sqrt{x}}{a}-\frac{9b}{a^2}\right)+\frac{21b^2}{a^3}\right)\sqrt{x}-\frac{105b^3}{a^4}\right)\sqrt{x}+\frac{315b^4}{a^5}\right) + \frac{63b^5\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{128a^{\frac{11}{2}}}$$

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x)*(8*sqrt(x)/a - 9*b/a^2) + 21*b
^2/a^3)*sqrt(x) - 105*b^3/a^4)*sqrt(x) + 315*b^4/a^5) + 63/128*b^5*log(abs(
2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^2}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(x^2/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(1/2))^(1/2), x)
```

3.104 $\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 116

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{7/2}}$$

```
[Out] -5/4*b^3*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(7/2)+5/4*b^2*(b*
x^(1/2)+a*x)^(1/2)/a^3+2/3*x*(b*x^(1/2)+a*x)^(1/2)/a-5/6*b*x^(1/2)*(b*x^(1/
2)+a*x)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2043, 684, 654, 634, 212}

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{5b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{7/2}} + \frac{5b^2\sqrt{ax+b\sqrt{x}}}{4a^3} - \frac{5b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^2} + \frac{2x\sqrt{ax+b\sqrt{x}}}{3a}$$

```
[In] Int[x/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (5*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^3) - (5*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
/(6*a^2) + (2*x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt
[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{3a}\\
&= -\frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} + \frac{(5b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}\\
&= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{8a^3}
\end{aligned}
$$

$$= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^3}$$

$$= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(15b^2-10ab\sqrt{x}+8a^2x\right)}{12a^3} - \frac{5b^3\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{4a^{7/2}}$$

```
[In] Integrate[x/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(15*b^2 - 10*a*b*Sqrt[x] + 8*a^2*x))/(12*a^3) - (5*b
^3*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(4*a^(7/2))
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{2a} - \frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a} - \frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{3a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(16\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{5}{2}}-36\sqrt{b\sqrt{x}+ax}\,\sqrt{x}\,a^{\frac{5}{2}}b-18\sqrt{b\sqrt{x}+ax}\,a^{\frac{3}{2}}b^2+48\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}\,a^{\frac{3}{2}}b^2-24a\ln\left(\frac{2a\sqrt{x}+2\sqrt{}}{}\right.\right.}{24a^{\frac{9}{2}}\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}}$

```
[In] int(x/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x*(b*x^(1/2)+a*x)^(1/2)/a-5/3*b/a*(1/2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a-
3/4*b/a*((b*x^(1/2)+a*x)^(1/2)/a-1/2*b/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)
+(b*x^(1/2)+a*x)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.32

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \sqrt{ax+b\sqrt{x}}\left(\frac{x}{3a}-\frac{5b\sqrt{x}}{12a^2}+\frac{5b^2}{8a^3}\right)-\frac{5b^3\left(\begin{cases} \frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a}\neq 0 \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise} \end{cases}\right)}{16a^3} & \text{for } a\neq 0 \\ \frac{2\left(b\sqrt{x}\right)^{\frac{7}{2}}}{7b^4} & \text{for } b\neq 0 \\ \tilde{\infty}x^2 & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(x/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] 2*Piecewise((sqrt(a*x + b*sqrt(x))*(x/(3*a) - 5*b*sqrt(x)/(12*a**2) + 5*b**
2/(8*a**3)) - 5*b**3*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*s
qrt(x) + b)/sqrt(a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(
2*a))/sqrt(a*(sqrt(x) + b/(2*a))**2), True))/(16*a**3), Ne(a, 0)), (2*(b*sq
rt(x))**(7/2)/(7*b**4), Ne(b, 0)), (zoo*x**2, True))
```

Maxima [F]

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x}{\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{12}\sqrt{ax+b\sqrt{x}}\left(2\sqrt{x}\left(\frac{4\sqrt{x}}{a}-\frac{5b}{a^2}\right)+\frac{15b^2}{a^3}\right) + \frac{5b^3\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{8a^{\frac{7}{2}}}$$

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a - 5*b/a^2) + 15*b^2/a^3)
 + 5/8*b^3*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)
)/a^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(x/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^(1/2))^(1/2), x)
```

3.105 $\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

```
[Out] -2*b*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(3/2)+2*(b*x^(1/2)+a*
x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2035, 2038, 634, 212}

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

```
[In] Int[1/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x]
 + a*x]])/a^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2035

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Dist[a*((2*n - j - 2)/(b*(n -
2))), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx}{2a}\\ &= \frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{a}\\ &= \frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{(2b)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a}\\ &= \frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{a^{3/2}}$$

```
[In] Integrate[1/Sqrt[b*Sqrt[x] + a*x],x]
[Out] (2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/
(b + a*Sqrt[x])])/a^(3/2)
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{a^{\frac{3}{2}}}$	50
default	$-\frac{\sqrt{b\sqrt{x}+ax}\,\left(b\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}+b}{2\sqrt{a}}\right)-2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}\right)}{\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{3}{2}}}$	83

```
[In] int(1/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(b*x^(1/2)+a*x)^(1/2)/a-b/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)
+a*x)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} -\frac{b\left(\begin{cases}\frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for}\ \frac{b^2}{a}\neq 0\\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise}\end{cases}\right)}{2a}+\frac{\sqrt{ax+b\sqrt{x}}}{a} & \text{for}\ a\neq 0\\ \frac{2\left(b\sqrt{x}\right)^{\frac{3}{2}}}{3b^2} & \text{for}\ b\neq 0\\ \tilde{\infty}x & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(1/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] 2*Piecewise((-b*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x
) + b)/sqrt(a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))
/sqrt(a*(sqrt(x) + b/(2*a))**2), True))/(2*a) + sqrt(a*x + b*sqrt(x))/a, Ne
(a, 0)), (2*(b*sqrt(x))**(3/2)/(3*b**2), Ne(b, 0)), (zoo*x, True))
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{b\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{a^{\frac{3}{2}}} + \frac{2\sqrt{ax+b\sqrt{x}}}{a}$$

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(3/2)
 + 2*sqrt(a*x + b*sqrt(x))/a
```

Mupad [B] (verification not implemented)

Time = 11.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\,x\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,\sqrt{x}}}{2\,a\,\sqrt{x}}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,x^{1/4}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,x^{3/4}}\right)\,\sqrt{\frac{a\,\sqrt{x}}{b}+1}}{3\,\sqrt{a\,x+b\,\sqrt{x}}}$$

```
[In] int(1/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] (4*x*((3*b^(1/2)*(b + a*x^(1/2))^(1/2))/(2*a*x^(1/2)) + (b^(3/2)*asin((a^(1
/2)*x^(1/4)*1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/4)))*((a*x^(1/2))/b + 1)^(1/2)
)/(3*(a*x + b*x^(1/2))^(1/2))
```

3.106 $\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

```
[Out] -4*(b*x^(1/2)+a*x)^(1/2)/b/x^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2039}

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

```
[In] Int[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

```
[In] Integrate[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(4\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}\sqrt{a}-2\sqrt{b\sqrt{x}+ax}\,a^{\frac{3}{2}}x-2\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{3}{2}}x-\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)abx+\ln\left(\frac{2a\sqrt{x}-}{}\right.\right.}{\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,b^{2}x\sqrt{a}}$

```
[In] int(1/x/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -4*(b*x^(1/2)+a*x)^(1/2)/b/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\,\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
[Out] -4*sqrt(a*x + b*sqrt(x))/(b*sqrt(x))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(1/x/(b*x**(1/2)+a*x)**(1/2),x)
[Out] Integral(1/(x*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x}\,dx$$

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4}{\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}}$$

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
[Out] 4/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(1/(x*(a*x + b*x^(1/2))^(1/2)),x)
[Out] int(1/(x*(a*x + b*x^(1/2))^(1/2)), x)
```

3.107 $\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}-\frac{32a^2\sqrt{b\sqrt{x}+ax}}{15b^3\sqrt{x}}$$

```
[Out] -4/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(3/2)+16/15*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x-32
/15*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2039}

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{15b^3\sqrt{x}}+\frac{16a\sqrt{ax+b\sqrt{x}}}{15b^2x}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

```
[In] Int[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) + (16*a*Sqrt[b*Sqrt[x] + a*x])/(15
*b^2*x) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(15*b^3*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}-\frac{(4a)\int\frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{5b}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}+\frac{(8a^2)\int\frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{15b^2}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}-\frac{32a^2\sqrt{b\sqrt{x}+ax}}{15b^3\sqrt{x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.57

$$\int\frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}\left(3b^2-4ab\sqrt{x}+8a^2x\right)}{15b^3x^{3/2}}$$

```
[In] Integrate[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(3*b^2 - 4*a*b*Sqrt[x] + 8*a^2*x))/(15*b^3*x^(3/2
))
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+ax}}{3b^{2}\sqrt{x}}\right)}{5b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\,(60\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{5}{2}}a^{\frac{5}{2}}-30\sqrt{b\sqrt{x}+ax}\,x^{\frac{7}{2}}a^{\frac{7}{2}}-30x^{\frac{7}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{7}{2}}-15x^{\frac{7}{2}}\ln(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+}{2\sqrt{a}}}{15\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,b^{4}}$

```
[In] int(1/x^2/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(3/2)-8/5*a/b*(-2/3*(b*x^(1/2)+a*x)^(1/2)/b/
x+4/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(4\,abx-\left(8\,a^2x+3\,b^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{15\,b^3x^2}$$

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/15*(4*a*b*x - (8*a^2*x + 3*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^3*x^2)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^2\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(20\,a\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+15\,\sqrt{a}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+3\,b^2\right)}{15\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5}$$

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(20*a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 15*sqrt(a)*b*(sqrt
(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 3*b^2)/(sqrt(a)*sqrt(x) - sqrt(a*x +
 b*sqrt(x)))^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^2\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(1/(x^2*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(1/2))^(1/2)), x)
```

3.108 $\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2}-\frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}}+\frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{315b^5\sqrt{x}}$$

```
[Out] -4/9*(b*x^(1/2)+a*x)^(1/2)/b/x^(5/2)+32/63*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^2-
64/105*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(3/2)+256/315*a^3*(b*x^(1/2)+a*x)^(1
/2)/b^4/x-512/315*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2039}

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{315b^5\sqrt{x}}+\frac{256a^3\sqrt{ax+b\sqrt{x}}}{315b^4x}-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{105b^3x^{3/2}}+\frac{32a\sqrt{ax+b\sqrt{x}}}{63b^2x^2}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

```
[In] Int[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) + (32*a*Sqrt[b*Sqrt[x] + a*x])/(63
*b^2*x^2) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(105*b^3*x^(3/2)) + (256*a^3*Sqr
t[b*Sqrt[x] + a*x])/(315*b^4*x) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(315*b^5*
Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}}-\frac{(8a)\int\frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{9b}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2}+\frac{(16a^2)\int\frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{21b^2}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2}-\frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}}-\frac{(64a^3)\int\frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{105b^3}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2}-\frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}}\\
&\quad+\frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x}+\frac{(128a^4)\int\frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{315b^4}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2}-\frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}}\\
&\quad+\frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{315b^5\sqrt{x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(35b^4-40ab^3\sqrt{x}+48a^2b^2x-64a^3bx^{3/2}+128a^4x^2\right)}{315b^5x^{5/2}}$$

```
[In] Integrate[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(35*b^4 - 40*a*b^3*Sqrt[x] + 48*a^2*b^2*x - 64*a^
3*b*x^(3/2) + 128*a^4*x^2))/(315*b^5*x^(5/2))
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{9b\,x^{\frac{5}{2}}}-\frac{16a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{7b\,x^{2}}-\frac{6a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+ax}}{3b^{2}\sqrt{x}}\right)}{5b}\right)}{7b}\right)}{9b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\,\left(1260\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{9}{2}}a^{\frac{9}{2}}-630\sqrt{b\sqrt{x}+ax}\,x^{\frac{11}{2}}a^{\frac{11}{2}}-315x^{\frac{11}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)a^{5}b-630x^{\frac{11}{2}}a^{\frac{11}{2}}\right.}{}$

```
[In] int(1/x^3/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/9*(b*x^(1/2)+a*x)^(1/2)/b/x^(5/2)-16/9*a/b*(-2/7*(b*x^(1/2)+a*x)^(1/2)/b
/x^2-6/7*a/b*(-2/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(3/2)-4/5*a/b*(-2/3*(b*x^(1/2)
+a*x)^(1/2)/b/x+4/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(64\,a^3bx^2+40\,ab^3x-\left(128\,a^4x^2+48\,a^2b^2x+35\,b^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{315\,b^5x^3}$$

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/315*(64*a^3*b*x^2 + 40*a*b^3*x - (128*a^4*x^2 + 48*a^2*b^2*x + 35*b^4)*sq
rt(x))*sqrt(a*x + b*sqrt(x))/(b^5*x^3)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^3\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx$$
$$= \frac{4\left(1008\,a^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+1680\,a^{\frac{3}{2}}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+1080\,ab^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\right.\right.}{315\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^9}$$

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/315*(1008*a^2*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 1680*a^(3/2)*
b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 1080*a*b^2*(sqrt(a)*sqrt(x)
 - sqrt(a*x + b*sqrt(x)))^2 + 315*sqrt(a)*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x +
 b*sqrt(x))) + 35*b^4)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^3\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(1/(x^3*(a*x + b*x^(1/2))^(1/2)),x)
[Out] int(1/(x^3*(a*x + b*x^(1/2))^(1/2)), x)
```

3.109 $\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 200

$$\begin{aligned}\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = &-\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}}+\frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3}-\frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}}\\&+\frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{1001b^5x^{3/2}}\\&+\frac{2048a^5\sqrt{b\sqrt{x}+ax}}{3003b^6x}-\frac{4096a^6\sqrt{b\sqrt{x}+ax}}{3003b^7\sqrt{x}}\end{aligned}$$

```
[Out] -4/13*(b*x^(1/2)+a*x)^(1/2)/b/x^(7/2)+48/143*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^
3-160/429*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(5/2)+1280/3003*a^3*(b*x^(1/2)+a*
x)^(1/2)/b^4/x^2-512/1001*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(3/2)+2048/3003*a
^5*(b*x^(1/2)+a*x)^(1/2)/b^6/x-4096/3003*a^6*(b*x^(1/2)+a*x)^(1/2)/b^7/x^(1
/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2039}

$$\begin{aligned}\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = &-\frac{4096a^6\sqrt{ax+b\sqrt{x}}}{3003b^7\sqrt{x}}+\frac{2048a^5\sqrt{ax+b\sqrt{x}}}{3003b^6x}\\&-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{1001b^5x^{3/2}}+\frac{1280a^3\sqrt{ax+b\sqrt{x}}}{3003b^4x^2}\\&-\frac{160a^2\sqrt{ax+b\sqrt{x}}}{429b^3x^{5/2}}+\frac{48a\sqrt{ax+b\sqrt{x}}}{143b^2x^3}-\frac{4\sqrt{ax+b\sqrt{x}}}{13bx^{7/2}}\end{aligned}$$

```
[In] Int[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(13*b*x^(7/2)) + (48*a*Sqrt[b*Sqrt[x] + a*x])/(1
43*b^2*x^3) - (160*a^2*Sqrt[b*Sqrt[x] + a*x])/(429*b^3*x^(5/2)) + (1280*a^3
*Sqrt[b*Sqrt[x] + a*x])/(3003*b^4*x^2) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(1
001*b^5*x^(3/2)) + (2048*a^5*Sqrt[b*Sqrt[x] + a*x])/(3003*b^6*x) - (4096*a^
6*Sqrt[b*Sqrt[x] + a*x])/(3003*b^7*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}}-\frac{(12a)\int\frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{13b}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}}+\frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3}+\frac{(120a^2)\int\frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{143b^2}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}}+\frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3}-\frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}}-\frac{(320a^3)\int\frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{429b^3}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}}+\frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3}-\frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}}\\
&\quad+\frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2}+\frac{(640a^4)\int\frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{1001b^4}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}}+\frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3}-\frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}}\\
&\quad+\frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{1001b^5x^{3/2}}-\frac{(512a^5)\int\frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{1001b^5}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}}+\frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3}-\frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}}+\frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2}\\
&\quad-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{1001b^5x^{3/2}}+\frac{2048a^5\sqrt{b\sqrt{x}+ax}}{3003b^6x}+\frac{(1024a^6)\int\frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{3003b^6}
\end{aligned}
$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} - \frac{512a^4\sqrt{b\sqrt{x}+ax}}{1001b^5x^{3/2}} + \frac{2048a^5\sqrt{b\sqrt{x}+ax}}{3003b^6x} - \frac{4096a^6\sqrt{b\sqrt{x}+ax}}{3003b^7\sqrt{x}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(231b^6 - 252ab^5\sqrt{x} + 280a^2b^4x - 320a^3b^3x^{3/2} + 384a^4b^2x^2 - 512a^5bx^{5/2} + 1024a^6x^3\right)}{3003b^7x^{7/2}}$$

```
[In] Integrate[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(231*b^6 - 252*a*b^5*Sqrt[x] + 280*a^2*b^4*x - 32
0*a^3*b^3*x^(3/2) + 384*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/(3
003*b^7*x^(7/2))
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{13b\,x^{\frac{7}{2}}}-\frac{24a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{11b\,x^{3}}-\frac{10a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{9b\,x^{\frac{5}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{7b\,x^{2}}-\frac{6a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{b\sqrt{}}}{3b^{2}}\right.}{5b}\right.}{7b}\right.}{9b}\right.}{11b}\right.}{13b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(12012\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{13}{2}}a^{\frac{13}{2}}-6006\sqrt{b\sqrt{x}+ax}\,x^{\frac{15}{2}}a^{\frac{15}{2}}-3003x^{\frac{15}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)a^{7}b-6006x\right.}{}$

```
[In] int(1/x^4/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/13*(b*x^(1/2)+a*x)^(1/2)/b/x^(7/2)-24/13*a/b*(-2/11*(b*x^(1/2)+a*x)^(1/2
)/b/x^3-10/11*a/b*(-2/9*(b*x^(1/2)+a*x)^(1/2)/b/x^(5/2)-8/9*a/b*(-2/7*(b*x^
(1/2)+a*x)^(1/2)/b/x^2-6/7*a/b*(-2/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(3/2)-4/5*a/
b*(-2/3*(b*x^(1/2)+a*x)^(1/2)/b/x+4/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2)))
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$$
$$=\frac{4\left(512\,a^{5}bx^{3}+320\,a^{3}b^{3}x^{2}+252\,ab^{5}x-\left(1024\,a^{6}x^{3}+384\,a^{4}b^{2}x^{2}+280\,a^{2}b^{4}x+231\,b^{6}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3003\,b^{7}x^{4}}$$

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3003*(512*a^5*b*x^3 + 320*a^3*b^3*x^2 + 252*a*b^5*x - (1024*a^6*x^3 + 384
*a^4*b^2*x^2 + 280*a^2*b^4*x + 231*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^7
*x^4)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx=\int \frac{1}{x^4\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(1/x**4/(b*x**(1/2)+a*x)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+b\sqrt{x}}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$$
$$=\frac{4\left(27456\,a^3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^6+72072\,a^{\frac{5}{2}}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5+80080\,a^2b^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax\right.\right.}{}$$

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
[Out] 4/3003*(27456*a^3*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^6 + 72072*a^(5/
2)*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 80080*a^2*b^2*(sqrt(a)*s
qrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 48048*a^(3/2)*b^3*(sqrt(a)*sqrt(x) - sq
rt(a*x + b*sqrt(x)))^3 + 16380*a*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x
)))^2 + 3003*sqrt(a)*b^5*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 231*b^
6)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^13
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx=\int \frac{1}{x^4\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(1/(x^4*(a*x + b*x^(1/2))^(1/2)),x)
[Out] int(1/(x^4*(a*x + b*x^(1/2))^(1/2)), x)
```

3.110 $\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 197

$$\begin{aligned}\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}}+\frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6}\\ &\quad-\frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5}+\frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}-\frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3}\\ &\quad+\frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2}-\frac{693b^5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{64a^{13/2}}\end{aligned}$$

```
[Out] -693/64*b^5*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(13/2)-4*x^3/a
/(b*x^(1/2)+a*x)^(1/2)+693/64*b^4*(b*x^(1/2)+a*x)^(1/2)/a^6+231/40*b^2*x*(b
*x^(1/2)+a*x)^(1/2)/a^4-99/20*b*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^3+22/5*x^2*
(b*x^(1/2)+a*x)^(1/2)/a^2-231/32*b^3*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^5
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2043, 682, 684, 654, 634, 212}

$$\begin{aligned}\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= -\frac{693b^5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{13/2}}\\ &\quad+\frac{693b^4\sqrt{ax+b\sqrt{x}}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{231b^2x\sqrt{ax+b\sqrt{x}}}{40a^4}\\ &\quad-\frac{99bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^3}+\frac{22x^2\sqrt{ax+b\sqrt{x}}}{5a^2}-\frac{4x^3}{a\sqrt{ax+b\sqrt{x}}}\end{aligned}$$

```
[In] Int[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^3)/(a*Sqrt[b*Sqrt[x] + a*x]) + (693*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^
6) - (231*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(32*a^5) + (231*b^2*x*Sqrt[b*S
qrt[x] + a*x])/(40*a^4) - (99*b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(20*a^3) + (
22*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a^2) - (693*b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/
Sqrt[b*Sqrt[x] + a*x]])/(64*a^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^7}{(bx+ax^2)^{3/2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{22\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} - \frac{(99b)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{5a^2}\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2}\\
&\quad + \frac{(693b^2)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{40a^3}\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3}\\
&\quad + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} - \frac{(231b^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{16a^4}\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}\\
&\quad - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} + \frac{(693b^4)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{64a^5}\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}\\
&\quad - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} - \frac{(693b^5)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{128a^6}\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}\\
&\quad - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} - \frac{(693b^5)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{64a^6}\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}\\
&\quad - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} - \frac{693b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{64a^{13/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.70

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}(3465b^5+1155ab^4\sqrt{x}-462a^2b^3x+264a^3b^2x^{3/2}-176a^4bx^2+128a^5x^{5/2}}{320a^6\left(b+a\sqrt{x}\right)}$$

$$-\frac{693b^5\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{64a^{13/2}}$$

```
[In] Integrate[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(3465*b^5 + 1155*a*b^4*Sqrt[x] - 462*a^2*b^3*x + 264
*a^3*b^2*x^(3/2) - 176*a^4*b*x^2 + 128*a^5*x^(5/2)))/(320*a^6*(b + a*Sqrt[x
])) - (693*b^5*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(6
4*a^(13/2))
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{2x^3}{5a\sqrt{b\sqrt{x}+ax}} - \frac{11b\left(\frac{x^{\frac{5}{2}}}{4a\sqrt{b\sqrt{x}+ax}} - \frac{9b\left(\frac{x^2}{3a\sqrt{b\sqrt{x}+ax}} - \frac{7b\left(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+ax}} - \frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+ax}} - \frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} - \frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}}}\right.}{}\right.}{}\right.}{6a}\right.}{8a}\right.}{5a}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(352\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{3}{2}}a^{\frac{11}{2}}b-256\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^2a^{\frac{13}{2}}-528\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x\,a^{\frac{9}{2}}b^2+4060\sqrt{b\sqrt{x}+ax}\,x^{\frac{3}{2}}a^{\frac{9}{2}}b^3-3\right.}{}$

```
[In] int(x^3/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^3/a/(b*x^(1/2)+a*x)^(1/2)-11/5*b/a*(1/4*x^(5/2)/a/(b*x^(1/2)+a*x)^(1/
2)-9/8*b/a*(1/3*x^2/a/(b*x^(1/2)+a*x)^(1/2)-7/6*b/a*(1/2*x^(3/2)/a/(b*x^(1/
2)+a*x)^(1/2)-5/4*b/a*(x/a/(b*x^(1/2)+a*x)^(1/2)-3/2*b/a*(-x^(1/2)/a/(b*x^(
1/2)+a*x)^(1/2)-1/2*b/a*(-1/a/(b*x^(1/2)+a*x)^(1/2)+1/b/a*(b+2*a*x^(1/2))/(
b*x^(1/2)+a*x)^(1/2))+1/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x
)^(1/2))))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{1}{320}\sqrt{ax+b\sqrt{x}}\Bigg(2\Bigg(4\Bigg(2\sqrt{x}\bigg(\frac{8\sqrt{x}}{a^2}-\frac{19\,b}{a^3}\bigg)+\frac{71\,b^2}{a^4}\Bigg)\sqrt{x}-\frac{515\,b^3}{a^5}\Bigg)\sqrt{x}+\frac{2185\,b^4}{a^6}$$

$$+\frac{693\,b^5\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{128\,a^{\frac{13}{2}}}+\frac{4\,b^6}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{13}{2}}}$$

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x)*(8*sqrt(x)/a^2 - 19*b/a^3) + 7
1*b^2/a^4)*sqrt(x) - 515*b^3/a^5)*sqrt(x) + 2185*b^4/a^6) + 693/128*b^5*log
(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(13/2) +
4*b^6/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*a^(13/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

```
[In] int(x^3/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(1/2))^(3/2), x)
```

3.111 $\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 139

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}}+\frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3}+\frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2}-\frac{35b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{9/2}}$$

```
[Out] -35/4*b^3*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(9/2)-4*x^2/a/(b
*x^(1/2)+a*x)^(1/2)+35/4*b^2*(b*x^(1/2)+a*x)^(1/2)/a^4+14/3*x*(b*x^(1/2)+a*
x)^(1/2)/a^2-35/6*b*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2043, 682, 684, 654, 634, 212}

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{35b^3\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{9/2}}+\frac{35b^2\sqrt{ax+b\sqrt{x}}}{4a^4} - \frac{35b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^3}+\frac{14x\sqrt{ax+b\sqrt{x}}}{3a^2}-\frac{4x^2}{a\sqrt{ax+b\sqrt{x}}}$$

```
[In] Int[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^2)/(a*Sqrt[b*Sqrt[x] + a*x]) + (35*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^4)
 - (35*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(6*a^3) + (14*x*Sqrt[b*Sqrt[x] + a*
```

```
x])/(3*a^2) - (35*b^3*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*
a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^5}{(bx+ax^2)^{3/2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{14\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{(35b)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{3a^2}\\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2}\\
&\quad + \frac{(35b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^3}\\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3}\\
&\quad + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{(35b^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{8a^4}\\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3}\\
&\quad + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{(35b^3)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^4}\\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3}\\
&\quad + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.81

$$
\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(105b^3+35ab^2\sqrt{x}-14a^2bx+8a^3x^{3/2}\right)}{12a^4\left(b+a\sqrt{x}\right)} - \frac{35b^3\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{4a^{9/2}}
$$

```
[In] Integrate[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(105*b^3 + 35*a*b^2*Sqrt[x] - 14*a^2*b*x + 8*a^3*x^(
3/2)))/(12*a^4*(b + a*Sqrt[x])) - (35*b^3*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] +
 a*x])/(b + a*Sqrt[x])])/(4*a^(9/2))
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{2x^2}{3a\sqrt{b\sqrt{x}+ax}}-\frac{7b\Bigg(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+ax}}-\frac{5b\Bigg(\frac{x}{a\sqrt{b\sqrt{x}+ax}}-\frac{3b\Bigg(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\right)}{2a}+\frac{\ln\Big(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{}}{a^{\frac{3}{2}}}}{2a}}{4a}}{3a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\,\Big(16x(b\sqrt{x}+ax)^{\frac{3}{2}}a^{\frac{9}{2}}-60\sqrt{b\sqrt{x}+ax}\,x^{\frac{3}{2}}a^{\frac{9}{2}}b+32(b\sqrt{x}+ax)^{\frac{3}{2}}\sqrt{x}\,a^{\frac{7}{2}}b-150\sqrt{b\sqrt{x}+ax}\,x\,a^{\frac{7}{2}}b^2+240x\sqrt{\sqrt{x}\,(a}}{}$

```
[In] int(x^2/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^2/a/(b*x^(1/2)+a*x)^(1/2)-7/3*b/a*(1/2*x^(3/2)/a/(b*x^(1/2)+a*x)^(1/2
)-5/4*b/a*(x/a/(b*x^(1/2)+a*x)^(1/2)-3/2*b/a*(-x^(1/2)/a/(b*x^(1/2)+a*x)^(1
/2)-1/2*b/a*(-1/a/(b*x^(1/2)+a*x)^(1/2)+1/b/a*(b+2*a*x^(1/2))/(b*x^(1/2)+a*
x)^(1/2))+1/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(x**2/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{1}{12}\sqrt{ax+b\sqrt{x}}\left(2\sqrt{x}\left(\frac{4\sqrt{x}}{a^2}-\frac{11\,b}{a^3}\right)+\frac{57\,b^2}{a^4}\right)+\frac{35\,b^3\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{8\,a^{\frac{9}{2}}}+\frac{4\,b^4}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{9}{2}}}$$

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] 1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a^2 - 11*b/a^3) + 57*b^2/a
^4) + 35/8*b^3*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
 - b))/a^(9/2) + 4*b^4/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
+ b)*a^(9/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

```
[In] int(x^2/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(1/2))^(3/2), x)
```

3.112 $\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{6b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{5/2}}$$

```
[Out] -6*b*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(5/2)-4*x/a/(b*x^(1/2
)+a*x)^(1/2)+6*(b*x^(1/2)+a*x)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2043, 682, 654, 634, 212}

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{6b\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{5/2}}+\frac{6\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{4x}{a\sqrt{ax+b\sqrt{x}}}$$

```
[In] Int[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x)/(a*Sqrt[b*Sqrt[x] + a*x]) + (6*Sqrt[b*Sqrt[x] + a*x])/a^2 - (6*b*Arc
Tanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^3}{(bx+ax^2)^{3/2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}} + \frac{6\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}} + \frac{6\sqrt{b\sqrt{x}+ax}}{a^2} - \frac{(3b)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}} + \frac{6\sqrt{b\sqrt{x}+ax}}{a^2} - \frac{(6b)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^2}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}} + \frac{6\sqrt{b\sqrt{x}+ax}}{a^2} - \frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{2\left(3b+a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{a^2\left(b+a\sqrt{x}\right)} - \frac{6b\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{a^{5/2}}$$

```
[In] Integrate[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (2*(3*b + a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(a^2*(b + a*Sqrt[x])) - (6*b*Ar
cTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/a^(5/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(59) = 118.

Time = 2.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{2x}{a\sqrt{b\sqrt{x}+ax}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\right)}{2a}+\frac{\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{a^{\frac{3}{2}}}\right)}{a}$
default	$\sqrt{b\sqrt{x}+ax}\left(6x\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{5}{2}}-3x\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{2}b+12\sqrt{x}\,\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{3}{2}}b-6\sqrt{x}\,\ln\right.$ $a^{\frac{5}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}$

```
[In] int(x/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x/a/(b*x^(1/2)+a*x)^(1/2)-3*b/a*(-x^(1/2)/a/(b*x^(1/2)+a*x)^(1/2)-1/2*b/a
*(-1/a/(b*x^(1/2)+a*x)^(1/2)+1/b/a*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))+1
/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= \frac{3\,b\log\left(\left|-2\,\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{a^{\frac{5}{2}}}\\&\quad+\frac{2\,\sqrt{ax+b\sqrt{x}}}{a^{2}}+\frac{4\,b^{2}}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{5}{2}}}\end{aligned}$$

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 3*b*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(5
/2) + 2*sqrt(a*x + b*sqrt(x))/a^2 + 4*b^2/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt
(a*x + b*sqrt(x))) + b)*a^(5/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

```
[In] int(x/(a*x + b*x^(1/2))^(3/2),x)
[Out] int(x/(a*x + b*x^(1/2))^(3/2), x)
```

3.113 $\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\sqrt{x}}{b\sqrt{b\sqrt{x}+ax}}$$

```
[Out] 4*x^(1/2)/b/(b*x^(1/2)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2025}

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

```
[In] Int[(b*Sqrt[x] + a*x)^(-3/2),x]
```

```
[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\text{integral}=\frac{4\sqrt{x}}{b\sqrt{b\sqrt{x}+ax}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\sqrt{b\sqrt{x}+ax}}{b\left(b+a\sqrt{x}\right)}$$

```
[In] Integrate[(b*Sqrt[x] + a*x)^(-3/2),x]
[Out] (4*Sqrt[b*Sqrt[x] + a*x])/(b*(b + a*Sqrt[x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(19) = 38.

Time = 2.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

method	result
derivativedivides	$-\frac{2}{a\sqrt{b\sqrt{x}+ax}}+\frac{2b+4a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}$
default	$\sqrt{b\sqrt{x}+ax}\,\Bigg(2\sqrt{b\sqrt{x}+ax}\,x\,a^{\frac{5}{2}}+x\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)a^{2}b+2x\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{5}{2}}-x\ln\Bigg(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}}{2\sqrt{a}}$

```
[In] int(1/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/a/(b*x^(1/2)+a*x)^(1/2)+2/b/a*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\,\sqrt{ax+b\sqrt{x}}\left(a\sqrt{x}-b\right)}{a^{2}bx-b^{3}}$$

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
[Out] 4*sqrt(a*x + b*sqrt(x))*(a*sqrt(x) - b)/(a^2*b*x - b^3)
```

Sympy [F]

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral((a*x + b*sqrt(x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*sqrt(x))^(-3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)\sqrt{a}}$$

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] 4/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*sqrt(a))
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\,x\,\left(\frac{b}{a\,\sqrt{x}}+1\right)}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}\,\left(\sqrt{\frac{b}{a\,\sqrt{x}}+1}+1\right)}$$

```
[In] int(1/(a*x + b*x^(1/2))^(3/2),x)
[Out] -(4*x*(b/(a*x^(1/2)) + 1))/((a*x + b*x^(1/2))^(3/2)*((b/(a*x^(1/2)) + 1)^(1
/2) + 1))
```

3.114 $\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}}-\frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x}+\frac{32a\sqrt{b\sqrt{x}+ax}}{3b^3\sqrt{x}}$$

```
[Out] 4/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)-16/3*(b*x^(1/2)+a*x)^(1/2)/b^2/x+32/3*a*(
b*x^(1/2)+a*x)^(1/2)/b^3/x^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2040, 2041, 2039}

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{32a\sqrt{ax+b\sqrt{x}}}{3b^3\sqrt{x}}-\frac{16\sqrt{ax+b\sqrt{x}}}{3b^2x}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

```
[In] Int[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]) - (16*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^3*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}}+\frac{4\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}\\ &= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}}-\frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x}-\frac{(8a)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{3b^2}\\ &= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}}-\frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x}+\frac{32a\sqrt{b\sqrt{x}+ax}}{3b^3\sqrt{x}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}\left(b^2-4ab\sqrt{x}-8a^2x\right)}{3b^3\left(b+a\sqrt{x}\right)x}$$

```
[In] Integrate[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(b^2 - 4*a*b*Sqrt[x] - 8*a^2*x))/(3*b^3*(b + a*Sq
rt[x])*x)
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

method	result
derivativedivides	$-\frac{4}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}+\frac{16a\left(b+2a\sqrt{x}\right)}{3b^{3}\sqrt{b\sqrt{x}+ax}}$
default	$\sqrt{b\sqrt{x}+ax}\,\left(24\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{5}{2}}a^{\frac{7}{2}}-6\sqrt{b\sqrt{x}+ax}\,x^{\frac{7}{2}}a^{\frac{9}{2}}-3x^{\frac{7}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)a^{4}b-6x^{\frac{7}{2}}a^{\frac{9}{2}}\sqrt{\sqrt{x}\,\left(a\sqrt{x}+b\right)}+$

```
[In] int(1/x/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)+16/3*a/b^3*(b+2*a*x^(1/2))/(b*x^(1/2)+
a*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.94 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\left(4\,a^{2}bx-b^{3}-\left(8\,a^{3}x-5\,ab^{2}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3\left(a^{2}b^{3}x^{2}-b^{5}x\right)}$$

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/3*(4*a^2*b*x - b^3 - (8*a^3*x - 5*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/
(a^2*b^3*x^2 - b^5*x)
```

Sympy [F]

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x*(a*x + b*x^(1/2))^(3/2)), x)
```

3.115 $\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2}+\frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}}-\frac{256a^2\sqrt{b\sqrt{x}+ax}}{35b^4x}+\frac{512a^3\sqrt{b\sqrt{x}+ax}}{35b^5\sqrt{x}}$$

```
[Out] 4/b/x^(3/2)/(b*x^(1/2)+a*x)^(1/2)-32/7*(b*x^(1/2)+a*x)^(1/2)/b^2/x^2+192/35
*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(3/2)-256/35*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x
+512/35*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2040, 2041, 2039}

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{512a^3\sqrt{ax+b\sqrt{x}}}{35b^5\sqrt{x}}-\frac{256a^2\sqrt{ax+b\sqrt{x}}}{35b^4x}+\frac{192a\sqrt{ax+b\sqrt{x}}}{35b^3x^{3/2}}-\frac{32\sqrt{ax+b\sqrt{x}}}{7b^2x^2}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

```
[In] Int[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x]) - (32*Sqrt[b*Sqrt[x] + a*x])/(7*b^2*x^2
) + (192*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x^(3/2)) - (256*a^2*Sqrt[b*Sqrt[x
] + a*x])/(35*b^4*x) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(35*b^5*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}+\frac{8\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}\\
&= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2}-\frac{(48a)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{7b^2}\\
&= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2}+\frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}}+\frac{(192a^2)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{35b^3}\\
&= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2}+\frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}}\\
&\quad -\frac{256a^2\sqrt{b\sqrt{x}+ax}}{35b^4x}-\frac{(128a^3)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{35b^4}\\
&= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}}-\frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2}+\frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}}\\
&\quad -\frac{256a^2\sqrt{b\sqrt{x}+ax}}{35b^4x}+\frac{512a^3\sqrt{b\sqrt{x}+ax}}{35b^5\sqrt{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(5b^4-8ab^3\sqrt{x}+16a^2b^2x-64a^3bx^{3/2}-128a^4x^2\right)}{35b^5\left(b+a\sqrt{x}\right)x^2}$$

```
[In] Integrate[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(5*b^4 - 8*a*b^3*Sqrt[x] + 16*a^2*b^2*x - 64*a^3*
b*x^(3/2) - 128*a^4*x^2))/(35*b^5*(b + a*Sqrt[x])*x^2)
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

method	result
derivativedivides	$-\frac{4}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{16a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{6a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}+\frac{8a\left(b+2a\sqrt{x}\right)}{3b^{3}\sqrt{b\sqrt{x}+ax}}\right)}{5b}\right)}{7b}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(560\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}x^{\frac{9}{2}}a^{\frac{11}{2}}-210\sqrt{b\sqrt{x}+ax}\,x^{\frac{11}{2}}a^{\frac{13}{2}}-210x^{\frac{11}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{13}{2}}-105x^{\frac{11}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+}{2\sqrt{a}}\right.\right.}{}$

```
[In] int(1/x^2/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/7/b/x^(3/2)/(b*x^(1/2)+a*x)^(1/2)-16/7*a/b*(-2/5/b/x/(b*x^(1/2)+a*x)^(1/
2)-6/5*a/b*(-2/3/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)+8/3*a/b^3*(b+2*a*x^(1/2))/
(b*x^(1/2)+a*x)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\left(64\,a^4bx^2-24\,a^2b^3x-5\,b^5-\left(128\,a^5x^2-80\,a^3b^2x-13\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\left(a^2b^5x^3-b^7x^2\right)}$$

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/35*(64*a^4*b*x^2 - 24*a^2*b^3*x - 5*b^5 - (128*a^5*x^2 - 80*a^3*b^2*x -
13*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^5*x^3 - b^7*x^2)
```

Sympy [F]

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(x**2*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^2*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^2*(a*x + b*x^(1/2))^(3/2)), x)
```

3.116 $\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 195

$$\begin{aligned}\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}\\&\quad-\frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2}+\frac{512a^3\sqrt{b\sqrt{x}+ax}}{77b^5x^{3/2}}-\frac{2048a^4\sqrt{b\sqrt{x}+ax}}{231b^6x}+\frac{4096a^5\sqrt{b\sqrt{x}+ax}}{231b^7\sqrt{x}}\end{aligned}$$

```
[Out] 4/b/x^(5/2)/(b*x^(1/2)+a*x)^(1/2)-48/11*(b*x^(1/2)+a*x)^(1/2)/b^2/x^3+160/3
3*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(5/2)-1280/231*a^2*(b*x^(1/2)+a*x)^(1/2)/b^
4/x^2+512/77*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(3/2)-2048/231*a^4*(b*x^(1/2)+
a*x)^(1/2)/b^6/x+4096/231*a^5*(b*x^(1/2)+a*x)^(1/2)/b^7/x^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2040, 2041, 2039}

$$\begin{aligned}\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= \frac{4096a^5\sqrt{ax+b\sqrt{x}}}{231b^7\sqrt{x}}-\frac{2048a^4\sqrt{ax+b\sqrt{x}}}{231b^6x}\\&\quad+\frac{512a^3\sqrt{ax+b\sqrt{x}}}{77b^5x^{3/2}}-\frac{1280a^2\sqrt{ax+b\sqrt{x}}}{231b^4x^2}+\frac{160a\sqrt{ax+b\sqrt{x}}}{33b^3x^{5/2}}-\frac{48\sqrt{ax+b\sqrt{x}}}{11b^2x^3}\\&\quad+\frac{4}{bx^{5/2}\sqrt{ax+b\sqrt{x}}}\end{aligned}$$

```
[In] Int[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^(5/2)*Sqrt[b*Sqrt[x] + a*x]) - (48*Sqrt[b*Sqrt[x] + a*x])/(11*b^2*x^
3) + (160*a*Sqrt[b*Sqrt[x] + a*x])/(33*b^3*x^(5/2)) - (1280*a^2*Sqrt[b*Sqrt
[x] + a*x])/(231*b^4*x^2) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(77*b^5*x^(3/2)
) - (2048*a^4*Sqrt[b*Sqrt[x] + a*x])/(231*b^6*x) + (4096*a^5*Sqrt[b*Sqrt[x]
 + a*x])/(231*b^7*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} + \frac{12\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} - \frac{(120a)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{11b^2}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} + \frac{(320a^2)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{33b^3}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2} - \frac{(640a^3)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{77b^4}$$

$$
\begin{aligned}
&= \frac{4}{b x^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} \\
&\quad -\frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2}+\frac{512a^3\sqrt{b\sqrt{x}+ax}}{77b^5x^{3/2}}+\frac{(512a^4)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{77b^5} \\
&= \frac{4}{b x^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2} \\
&\quad +\frac{512a^3\sqrt{b\sqrt{x}+ax}}{77b^5x^{3/2}}-\frac{2048a^4\sqrt{b\sqrt{x}+ax}}{231b^6x}-\frac{(1024a^5)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{231b^6} \\
&= \frac{4}{b x^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2} \\
&\quad +\frac{512a^3\sqrt{b\sqrt{x}+ax}}{77b^5x^{3/2}}-\frac{2048a^4\sqrt{b\sqrt{x}+ax}}{231b^6x}+\frac{4096a^5\sqrt{b\sqrt{x}+ax}}{231b^7\sqrt{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.54

$$
\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(21b^6-28ab^5\sqrt{x}+40a^2b^4x-64a^3b^3x^{3/2}+128a^4b^2x^2-512a^5bx^{5/2}-1024a^6x^3\right)}{231b^7\left(b+a\sqrt{x}\right)x^3}
$$

```
[In] Integrate[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(21*b^6 - 28*a*b^5*Sqrt[x] + 40*a^2*b^4*x - 64*a^
3*b^3*x^(3/2) + 128*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) - 1024*a^6*x^3))/(231*b
^7*(b + a*Sqrt[x])*x^3)
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{4}{11b\,x^{\frac{5}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{24a\left(-\frac{2}{9b\,x^{2}\sqrt{b\sqrt{x}+ax}}-\frac{10a\left(-\frac{2}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{8a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{6a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}+\frac{8a}{3b^{3}}\right.}{5b}\right.}{7b}\right.}{9b}\right.}{11b}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(8716(b\sqrt{x}+ax)^{\frac{3}{2}}x^{6}a^{\frac{13}{2}}b-4620\sqrt{b\sqrt{x}+ax}\,x^{7}a^{\frac{15}{2}}b-4620x^{7}a^{\frac{15}{2}}\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,b-512(b\sqrt{x}+ax)^{\frac{3}{2}}x^{5}a^{\frac{9}{2}}b\right.}{}$

```
[In] int(1/x^3/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/11/b/x^(5/2)/(b*x^(1/2)+a*x)^(1/2)-24/11*a/b*(-2/9/b/x^2/(b*x^(1/2)+a*x)
^(1/2)-10/9*a/b*(-2/7/b/x^(3/2)/(b*x^(1/2)+a*x)^(1/2)-8/7*a/b*(-2/5/b/x/(b*
x^(1/2)+a*x)^(1/2)-6/5*a/b*(-2/3/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)+8/3*a/b^3*
(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\left(512\,a^6bx^3-192\,a^4b^3x^2-68\,a^2b^5x-21\,b^7-\left(1024\,a^7x^3-640\,a^5b^2x^2-104\,a^3b^4x-49\,ab^6\right)\sqrt{x}\right)\sqrt{ax+}}{231\left(a^2b^7x^4-b^9x^3\right)}$$

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/231*(512*a^6*b*x^3 - 192*a^4*b^3*x^2 - 68*a^2*b^5*x - 21*b^7 - (1024*a^7
*x^3 - 640*a^5*b^2*x^2 - 104*a^3*b^4*x - 49*a*b^6)*sqrt(x))*sqrt(a*x + b*sq
rt(x))/(a^2*b^7*x^4 - b^9*x^3)
```

Sympy [F]

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(x**3*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^3*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^3*(a*x + b*x^(1/2))^(3/2)), x)
```

3.117 $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 204

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} -\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} +\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}+\frac{231b^6\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{256a^{13/2}}$$

```
[Out] 231/256*b^6*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(13/2)-231/256
*b^5*(b*x^(1/2)+a*x)^(1/2)/a^6-77/160*b^3*x*(b*x^(1/2)+a*x)^(1/2)/a^4+33/80
*b^2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^3-11/30*b*x^2*(b*x^(1/2)+a*x)^(1/2)/a^
2+1/3*x^(5/2)*(b*x^(1/2)+a*x)^(1/2)/a+77/128*b^4*x^(1/2)*(b*x^(1/2)+a*x)^(1
/2)/a^5
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2043, 684, 654, 634, 212}

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{231b^6\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{256a^{13/2}} -\frac{231b^5\sqrt{ax+b\sqrt{x}}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{ax+b\sqrt{x}}}{128a^5}-\frac{77b^3x\sqrt{ax+b\sqrt{x}}}{160a^4} +\frac{33b^2x^{3/2}\sqrt{ax+b\sqrt{x}}}{80a^3}-\frac{11bx^2\sqrt{ax+b\sqrt{x}}}{30a^2}+\frac{x^{5/2}\sqrt{ax+b\sqrt{x}}}{3a}$$

```
[In] Int[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-231*b^5*Sqrt[b*Sqrt[x] + a*x])/(256*a^6) + (77*b^4*Sqrt[x]*Sqrt[b*Sqrt[x]
 + a*x])/(128*a^5) - (77*b^3*x*Sqrt[b*Sqrt[x] + a*x])/(160*a^4) + (33*b^2*x
^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(80*a^3) - (11*b*x^2*Sqrt[b*Sqrt[x] + a*x])/(
30*a^2) + (x^(5/2)*Sqrt[b*Sqrt[x] + a*x])/(3*a) + (231*b^6*ArcTanh[(Sqrt[a]
*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(256*a^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$

$$
\begin{aligned}
&=\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}-\frac{(11b)\text{Subst}\left(\int\frac{x^5}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{6a}\\
&=-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2}+\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}+\frac{(33b^2)\,\text{Subst}\left(\int\frac{x^4}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{20a^2}\\
&=\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2}\\
&\quad+\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}-\frac{(231b^3)\,\text{Subst}\left(\int\frac{x^3}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{160a^3}\\
&=-\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2}\\
&\quad+\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}+\frac{(77b^4)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{64a^4}\\
&=\frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5}-\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}\\
&\quad-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2}+\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}-\frac{(231b^5)\,\text{Subst}\left(\int\frac{x}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{256a^5}\\
&=-\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5}-\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}\\
&\quad-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2}+\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}+\frac{(231b^6)\,\text{Subst}\left(\int\frac{1}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{512a^6}\\
&=-\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5}-\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}\\
&\quad-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2}+\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}+\frac{(231b^6)\,\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{256a^6}\\
&=-\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5}-\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}\\
&\quad+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2}\\
&\quad+\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}+\frac{231b^6\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{256a^{13/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.62

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(-3465b^5+2310ab^4\sqrt{x}-1848a^2b^3x+1584a^3b^2x^{3/2}-1408a^4bx^2+1280a^5\right.}{3840a^6}$$
$$+\frac{231b^6\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{256a^{13/2}}$$

```
[In] Integrate[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(-3465*b^5 + 2310*a*b^4*Sqrt[x] - 1848*a^2*b^3*x + 1
584*a^3*b^2*x^(3/2) - 1408*a^4*b*x^2 + 1280*a^5*x^(5/2)))/(3840*a^6) + (231
*b^6*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(256*a^(13/2
))
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}\sqrt{b\sqrt{x}+ax}}{3a}-\frac{11b\left(\frac{x^{2}\sqrt{b\sqrt{x}+ax}}{5a}-\frac{9b\left(\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}{4a}-\frac{7b\left(\frac{x\sqrt{b\sqrt{x}+ax}}{3a}-\frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{\ }}{\sqrt{a}}\right.}{}\right.}{4a}\right.}{6a}\right.}{8a}\right.}{10a}\right.}{6a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(2560x^{\frac{3}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{11}{2}}+8544a^{\frac{7}{2}}\sqrt{x}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}b^{2}-5376a^{\frac{9}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}bx+16860a^{\frac{5}{2}}\sqrt{x}\,\sqrt{b\sqrt{x}+ax}\,b^{4}\right.}{}$

```
[In] int(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(5/2)*(b*x^(1/2)+a*x)^(1/2)/a-11/6*b/a*(1/5*x^2*(b*x^(1/2)+a*x)^(1/2)
/a-9/10*b/a*(1/4*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a-7/8*b/a*(1/3*x*(b*x^(1/2)+
a*x)^(1/2)/a-5/6*b/a*(1/2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a-3/4*b/a*((b*x^(1/
2)+a*x)^(1/2)/a-1/2*b/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^
(1/2))))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.98

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \sqrt{ax+b\sqrt{x}}\left(\frac{x^{\frac{5}{2}}}{6a} - \frac{11bx^2}{60a^2} + \frac{33b^2x^{\frac{3}{2}}}{160a^3} - \frac{77b^3x}{320a^4} + \frac{77b^4\sqrt{x}}{256a^5} - \frac{231b^5}{512a^6}\right) + \frac{231b^6\left(\begin{cases} \frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{}}\right.}{\sqrt{a}} \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{}\right.}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right.}} \end{cases}\right.}{1} \\ \frac{2\left(b\sqrt{x}\right)^{\frac{13}{2}}}{13b^7} \\ \tilde{\infty}x^{\frac{7}{2}} \end{cases}\right.$$

```
[In] integrate(x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] 2*Piecewise((sqrt(a*x + b*sqrt(x))*(x**(5/2)/(6*a) - 11*b*x**2/(60*a**2) +
33*b**2*x**(3/2)/(160*a**3) - 77*b**3*x/(320*a**4) + 77*b**4*sqrt(x)/(256*a
**5) - 231*b**5/(512*a**6)) + 231*b**6*Piecewise((log(2*sqrt(a)*sqrt(a*x +
b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a), Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))
*log(sqrt(x) + b/(2*a))/sqrt(a*(sqrt(x) + b/(2*a))**2), True))/(1024*a**6),
 Ne(a, 0)), (2*(b*sqrt(x))**(13/2)/(13*b**7), Ne(b, 0)), (zoo*x**(7/2), Tru
e))
```

Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.60

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{3840}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\left(8\sqrt{x}\left(\frac{10\sqrt{x}}{a}-\frac{11\,b}{a^2}\right)+\frac{99\,b^2}{a^3}\right)\sqrt{x}-\frac{231\,b^3}{a^4}\right)\sqrt{x}+\frac{115}{a}\right.\right.\right.$$
$$-\frac{231\,b^6\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{512\,a^{\frac{13}{2}}}$$

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(a*x + b*sqrt(x))*(2*(4*(2*(8*sqrt(x)*(10*sqrt(x)/a - 11*b/a^2)
+ 99*b^2/a^3)*sqrt(x) - 231*b^3/a^4)*sqrt(x) + 1155*b^4/a^5)*sqrt(x) - 3465
*b^5/a^6) - 231/512*b^6*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*s
qrt(x))) + b))/a^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{5/2}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(x^(5/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(5/2)/(a*x + b*x^(1/2))^(1/2), x)
```

3.118 $\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} + \frac{35b^4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^{9/2}}$$

```
[Out] 35/32*b^4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(9/2)-35/32*b^3*
(b*x^(1/2)+a*x)^(1/2)/a^4-7/12*b*x*(b*x^(1/2)+a*x)^(1/2)/a^2+1/2*x^(3/2)*(b
*x^(1/2)+a*x)^(1/2)/a+35/48*b^2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2043, 684, 654, 634, 212}

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{35b^4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{9/2}} - \frac{35b^3\sqrt{ax+b\sqrt{x}}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{48a^3} - \frac{7bx\sqrt{ax+b\sqrt{x}}}{12a^2} + \frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a}$$

```
[In] Int[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-35*b^3*Sqrt[b*Sqrt[x] + a*x])/(32*a^4) + (35*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] +
 a*x])/(48*a^3) - (7*b*x*Sqrt[b*Sqrt[x] + a*x])/(12*a^2) + (x^(3/2)*Sqrt[b*
```

```
Sqrt[x] + a*x])/(2*a) + (35*b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] +
a*x]])/(32*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} - \frac{(7b)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a}\\
&= -\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} + \frac{(35b^2)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{24a^2}
\end{aligned}
$$

$$=\frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3}-\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}-\frac{(35b^3)\operatorname{Subst}\left(\int\frac{x}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{32a^3}$$

$$=-\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3}-\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}+\frac{(35b^4)\operatorname{Subst}\left(\int\frac{1}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{64a^4}$$

$$=-\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3}-\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}+\frac{(35b^4)\operatorname{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^4}$$

$$=-\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3}-\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}+\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^{9/2}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int\frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{\sqrt{b\sqrt{x}+ax}\left(-105b^3+70ab^2\sqrt{x}-56a^2bx+48a^3x^{3/2}\right)}{96a^4}+\frac{35b^4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{32a^{9/2}}$$

```
[In] Integrate[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(-105*b^3 + 70*a*b^2*Sqrt[x] - 56*a^2*b*x + 48*a^3*x
^(3/2)))/(96*a^4) + (35*b^4*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*
Sqrt[x])])/(32*a^(9/2))
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}{2a}-\frac{7b\left(\frac{x\sqrt{b\sqrt{x}+ax}}{3a}-\frac{5b\left(\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{2a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right)}{4a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(96(b\sqrt{x}+ax)^{\frac{3}{2}}\sqrt{x}\,a^{\frac{7}{2}}+348\sqrt{x}\,\sqrt{b\sqrt{x}+ax}\,a^{\frac{5}{2}}b^{2}-208(b\sqrt{x}+ax)^{\frac{3}{2}}a^{\frac{5}{2}}b+174\sqrt{b\sqrt{x}+ax}\,a^{\frac{3}{2}}b^{3}-384a^{\frac{3}{2}}\sqrt{\sqrt{x}}\right.}{192\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{11}{2}}}$

```
[In] int(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a-7/4*b/a*(1/3*x*(b*x^(1/2)+a*x)^(1/2)/a-
5/6*b/a*(1/2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a-3/4*b/a*((b*x^(1/2)+a*x)^(1/2)
/a-1/2*b/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \sqrt{ax+b\sqrt{x}}\left(\frac{x^{\frac{3}{2}}}{4a}-\frac{7bx}{24a^2}+\frac{35b^2\sqrt{x}}{96a^3}-\frac{35b^3}{64a^4}\right)+\frac{35b^4\left(\begin{cases}\frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a} \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherw} \end{cases}\right.}{128a^4} \\ \frac{2\left(b\sqrt{x}\right)^{\frac{9}{2}}}{9b^5} \\ \tilde{\infty}x^{\frac{5}{2}} \end{cases}\right.$$

```
[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] 2*Piecewise((sqrt(a*x + b*sqrt(x))*(x**(3/2)/(4*a) - 7*b*x/(24*a**2) + 35*b
**2*sqrt(x)/(96*a**3) - 35*b**3/(64*a**4)) + 35*b**4*Piecewise((log(2*sqrt(
a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a), Ne(b**2/a, 0)), ((sqrt
(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/sqrt(a*(sqrt(x) + b/(2*a))**2), True)
)/(128*a**4), Ne(a, 0)), (2*(b*sqrt(x))**(9/2)/(9*b**5), Ne(b, 0)), (zoo*x*
*(5/2), True))
```

Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{96}\sqrt{ax+b\sqrt{x}}\left(2\left(4\sqrt{x}\left(\frac{6\sqrt{x}}{a}-\frac{7b}{a^2}\right)+\frac{35b^2}{a^3}\right)\sqrt{x}-\frac{105b^3}{a^4}\right) - \frac{35b^4\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{64a^{\frac{9}{2}}}$$

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(6*sqrt(x)/a - 7*b/a^2) + 35*b^2/a
^3)*sqrt(x) - 105*b^3/a^4) - 35/64*b^4*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) -
 sqrt(a*x + b*sqrt(x))) + b))/a^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{x^{3/2}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(x^(3/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x + b*x^(1/2))^(1/2), x)
```

3.119 $\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2}+\frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a}+\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{5/2}}$$

```
[Out] 3/2*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(5/2)-3/2*b*(b*x^(
1/2)+a*x)^(1/2)/a^2+x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2043, 684, 654, 634, 212}

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx=\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{5/2}}-\frac{3b\sqrt{ax+b\sqrt{x}}}{2a^2}+\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a}$$

```
[In] Int[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-3*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a +
(3*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} - \frac{(3b)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{2a}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^2}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{\left(-3b+2a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{2a^2}+\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{2a^{5/2}}$$

```
[In] Integrate[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] ((-3*b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (3*b^2*ArcTanh[(Sqrt
[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(2*a^(5/2))
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}{a}-\frac{3b\left(\frac{\sqrt{b\sqrt{x}+ax}}{a}-\frac{b\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{2a^{\frac{3}{2}}}\right)}{2a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\,\left(4\sqrt{x}\,\sqrt{b\sqrt{x}+ax}\,a^{\frac{5}{2}}+2\sqrt{b\sqrt{x}+ax}\,a^{\frac{3}{2}}b+4\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}+b}{2\sqrt{a}}\right)a\,b^{2}-8\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{3}{2}}b-b^{2}\ln\right.}{4\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{7}{2}}}$

```
[In] int(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a-3/2*b/a*((b*x^(1/2)+a*x)^(1/2)/a-1/2*b/a^(3
/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \left(\frac{\sqrt{x}}{2a}-\frac{3b}{4a^2}\right)\sqrt{ax+b\sqrt{x}} + \frac{3b^2\left(\begin{cases}\frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } \frac{b^2}{a}\neq 0\\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{otherwise}\end{cases}\right)}{8a^2} & \text{for } a\neq 0\\ \frac{2(b\sqrt{x})^{\frac{5}{2}}}{5b^3} & \text{for } b\neq 0\\ \tilde{\infty}x^{\frac{3}{2}} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] 2*Piecewise(((sqrt(x)/(2*a) - 3*b/(4*a**2))*sqrt(a*x + b*sqrt(x)) + 3*b**2*
Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a),
Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/sqrt(a*(sqrt(x)
 + b/(2*a))**2), True))/(8*a**2), Ne(a, 0)), (2*(b*sqrt(x))**(5/2)/(5*b**3)
, Ne(b, 0)), (zoo*x**(3/2), True))
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1}{2}\sqrt{ax+b\sqrt{x}}\left(\frac{2\sqrt{x}}{a}-\frac{3b}{a^2}\right) - \frac{3b^2\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{4a^{\frac{5}{2}}}$$

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a - 3*b/a^2) - 3/4*b^2*log(abs(2*sqrt(
a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx=\int \frac{\sqrt{x}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(x^(1/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^(1/2))^(1/2), x)
```

3.120 $\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{\sqrt{a}}$$

```
[Out] 4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2038, 634, 212}

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

```
[In] Int[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\ &= 4\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)\\ &= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{\sqrt{a}}$$

```
[In] Integrate[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
[Out] (4*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/Sqrt[a]
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{\sqrt{a}}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)-2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}+b\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}+b}{2\sqrt{a}}\right)\right)}{\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,b\sqrt{a}}$

```
[In] int(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*ln((1/2*b+a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2))/a^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.91

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = 2\left(\begin{cases} \frac{\log\left(2\sqrt{a}\sqrt{ax+b\sqrt{x}}+2a\sqrt{x}+b\right)}{\sqrt{a}} & \text{for } a\neq 0 \wedge \frac{b^2}{a}\neq 0 \\ \frac{\left(\sqrt{x}+\frac{b}{2a}\right)\log\left(\sqrt{x}+\frac{b}{2a}\right)}{\sqrt{a\left(\sqrt{x}+\frac{b}{2a}\right)^2}} & \text{for } a\neq 0 \\ \frac{2\sqrt{b\sqrt{x}}}{b} & \text{for } b\neq 0 \\ \tilde{\infty}\sqrt{x} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] 2*Piecewise((log(2*sqrt(a)*sqrt(a*x + b*sqrt(x)) + 2*a*sqrt(x) + b)/sqrt(a)
, Ne(a, 0) & Ne(b**2/a, 0)), ((sqrt(x) + b/(2*a))*log(sqrt(x) + b/(2*a))/sq
rt(a*(sqrt(x) + b/(2*a))**2), Ne(a, 0)), (2*sqrt(b*sqrt(x))/b, Ne(b, 0)), (
zoo*sqrt(x), True))
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}\sqrt{x}}\,dx$$

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{2\log\left(\left|2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{\sqrt{a}}$$

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
[Out] -2*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/sqrt(a
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)),x)
[Out] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.121 $\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{8a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}$$

```
[Out] -4/3*(b*x^(1/2)+a*x)^(1/2)/b/x+8/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx=\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

```
[In] Int[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqr
t[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx} - \frac{(2a)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{3b}\\ &= -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx} + \frac{8a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\left(b-2a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{3b^2x}$$

```
[In] Integrate[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
[Out] (-4*(b - 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{8a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(6x^{\frac{5}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{5}{2}}+6x^{\frac{5}{2}}a^{\frac{5}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}+3x^{\frac{5}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+b}{2\sqrt{a}}\right)a^2b-3x^{\frac{5}{2}}\ln\left(\right.\right.}{3\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,b^3x^{\frac{5}{2}}\sqrt{a}}$

```
[In] int(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -4/3*(b*x^(1/2)+a*x)^(1/2)/b/x+8/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\,\sqrt{ax+b\sqrt{x}}\left(2\,a\sqrt{x}-b\right)}{3\,b^2x}$$

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
[Out] 4/3*sqrt(a*x + b*sqrt(x))*(2*a*sqrt(x) - b)/(b^2*x)
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
[Out] Integral(1/(x**(3/2)*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(3/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(3\,\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)}{3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3}$$

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
[Out] 4/3*(3*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)/(sqrt(a)*sqrt
(x) - sqrt(a*x + b*sqrt(x)))^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{3/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)),x)
[Out] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.122 $\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 112

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2}+\frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} -\frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x}+\frac{64a^3\sqrt{b\sqrt{x}+ax}}{35b^4\sqrt{x}}$$

```
[Out] -4/7*(b*x^(1/2)+a*x)^(1/2)/b/x^2+24/35*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(3/2)-
32/35*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x+64/35*a^3*(b*x^(1/2)+a*x)^(1/2)/b^4/x
^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{64a^3\sqrt{ax+b\sqrt{x}}}{35b^4\sqrt{x}} -\frac{32a^2\sqrt{ax+b\sqrt{x}}}{35b^3x}+\frac{24a\sqrt{ax+b\sqrt{x}}}{35b^2x^{3/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

```
[In] Int[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) + (24*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^2
*x^(3/2)) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x) + (64*a^3*Sqrt[b*Sqrt
[x] + a*x])/(35*b^4*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} - \frac{(6a)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{7b} \\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} + \frac{(24a^2)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{35b^2} \\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} - \frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x} - \frac{(16a^3)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{35b^3} \\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} - \frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x} + \frac{64a^3\sqrt{b\sqrt{x}+ax}}{35b^4\sqrt{x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(5b^3-6ab^2\sqrt{x}+8a^2bx-16a^3x^{3/2}\right)}{35b^4x^2}$$

```
[In] Integrate[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(5*b^3 - 6*a*b^2*Sqrt[x] + 8*a^2*b*x - 16*a^3*x^(
3/2)))/(35*b^4*x^2)
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{7b\,x^{2}}-\frac{12a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+ax}}{3b^{2}\sqrt{x}}\right)}{5b}\right)}{7b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\left(70x^{\frac{9}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{9}{2}}+70x^{\frac{9}{2}}a^{\frac{9}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}-140x^{\frac{7}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{7}{2}}+35x^{\frac{9}{2}}\ln\left(\frac{2\sqrt{b\sqrt{x}+ax}\,\sqrt{a}+2a\sqrt{x}+}{2\sqrt{a}}\right.\right.}{35\sqrt{\sqrt{x}}}$

```
[In] int(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/7*(b*x^(1/2)+a*x)^(1/2)/b/x^2-12/7*a/b*(-2/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(
3/2)-4/5*a/b*(-2/3*(b*x^(1/2)+a*x)^(1/2)/b/x+4/3*a*(b*x^(1/2)+a*x)^(1/2)/b^
2/x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx=-\frac{4\left(8\,a^{2}bx+5\,b^{3}-2\left(8\,a^{3}x+3\,ab^{2}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\,b^{4}x^{2}}$$

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/35*(8*a^2*b*x + 5*b^3 - 2*(8*a^3*x + 3*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt
(x))/(b^4*x^2)
```

Sympy [F]

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx=\int \frac{1}{x^{\frac{5}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}\,x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(5/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(70\,a^{\frac{3}{2}}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{3}+84\,ab\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{2}+35\,\sqrt{a}b^{2}\left(\sqrt{a}\right.\right.}{35\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{7}}$$

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/35*(70*a^(3/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 84*a*b*(sqrt
(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 35*sqrt(a)*b^2*(sqrt(a)*sqrt(x) -
sqrt(a*x + b*sqrt(x))) + 5*b^3)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{5/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.123 $\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3}+\frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}}-\frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2}+\frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{693b^5x}+\frac{1024a^5\sqrt{b\sqrt{x}+ax}}{693b^6\sqrt{x}}$$

```
[Out] -4/11*(b*x^(1/2)+a*x)^(1/2)/b/x^3+40/99*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(5/2)
-320/693*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^2+128/231*a^3*(b*x^(1/2)+a*x)^(1/2
)/b^4/x^(3/2)-512/693*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x+1024/693*a^5*(b*x^(1/
2)+a*x)^(1/2)/b^6/x^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{1024a^5\sqrt{ax+b\sqrt{x}}}{693b^6\sqrt{x}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{693b^5x}+\frac{128a^3\sqrt{ax+b\sqrt{x}}}{231b^4x^{3/2}}-\frac{320a^2\sqrt{ax+b\sqrt{x}}}{693b^3x^2}+\frac{40a\sqrt{ax+b\sqrt{x}}}{99b^2x^{5/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

```
[In] Int[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(11*b*x^3) + (40*a*Sqrt[b*Sqrt[x] + a*x])/(99*b^
2*x^(5/2)) - (320*a^2*Sqrt[b*Sqrt[x] + a*x])/(693*b^3*x^2) + (128*a^3*Sqrt[
```

```
b*Sqrt[x] + a*x])/(231*b^4*x^(3/2)) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(693*
b^5*x) + (1024*a^5*Sqrt[b*Sqrt[x] + a*x])/(693*b^6*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3}-\frac{(10a)\int\frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{11b}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3}+\frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}}+\frac{(80a^2)\int\frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{99b^2}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3}+\frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}}-\frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2}-\frac{(160a^3)\int\frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{231b^3}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3}+\frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}}-\frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2}\\
&\quad+\frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}}+\frac{(128a^4)\int\frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{231b^4}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3}+\frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}}-\frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2}\\
&\quad+\frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{693b^5x}-\frac{(256a^5)\int\frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{693b^5}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3}+\frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}}-\frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2}\\
&\quad+\frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}}-\frac{512a^4\sqrt{b\sqrt{x}+ax}}{693b^5x}+\frac{1024a^5\sqrt{b\sqrt{x}+ax}}{693b^6\sqrt{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(63b^5-70ab^4\sqrt{x}+80a^2b^3x-96a^3b^2x^{3/2}+128a^4bx^2-256a^5x^{5/2}\right)}{693b^6x^3}$$

```
[In] Integrate[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(63*b^5 - 70*a*b^4*Sqrt[x] + 80*a^2*b^3*x - 96*a^
3*b^2*x^(3/2) + 128*a^4*b*x^2 - 256*a^5*x^(5/2)))/(693*b^6*x^3)
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{4\sqrt{b\sqrt{x}+ax}}{11b\,x^{3}}-\frac{20a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{9b\,x^{\frac{5}{2}}}-\frac{8a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{7b\,x^{2}}-\frac{6a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{5b\,x^{\frac{3}{2}}}-\frac{4a\left(-\frac{2\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{4a\sqrt{b\sqrt{x}+ax}}{3b^{2}\sqrt{x}}\right)}{5b}\right)}{7b}\right)}{9b}\right)}{11b}$
default	$-\frac{\sqrt{b\sqrt{x}+ax}\,\Bigg(1386x^{\frac{13}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{13}{2}}+1386x^{\frac{13}{2}}a^{\frac{13}{2}}\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}-2772x^{\frac{11}{2}}(b\sqrt{x}+ax)^{\frac{3}{2}}a^{\frac{11}{2}}+693x^{\frac{13}{2}}\ln\Bigg(\frac{2\sqrt{b\sqrt{x}}}{}}{}$

```
[In] int(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/11*(b*x^(1/2)+a*x)^(1/2)/b/x^3-20/11*a/b*(-2/9*(b*x^(1/2)+a*x)^(1/2)/b/x
^(5/2)-8/9*a/b*(-2/7*(b*x^(1/2)+a*x)^(1/2)/b/x^2-6/7*a/b*(-2/5*(b*x^(1/2)+a
*x)^(1/2)/b/x^(3/2)-4/5*a/b*(-2/3*(b*x^(1/2)+a*x)^(1/2)/b/x+4/3*a*(b*x^(1/2
)+a*x)^(1/2)/b^2/x^(1/2)))))
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\left(128\,a^4bx^2+80\,a^2b^3x+63\,b^5-2\left(128\,a^5x^2+48\,a^3b^2x+35\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{693\,b^6x^3}$$

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/693*(128*a^4*b*x^2 + 80*a^2*b^3*x + 63*b^5 - 2*(128*a^5*x^2 + 48*a^3*b^2
*x + 35*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^6*x^3)
```

Sympy [F]

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

```
[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(a*x + b*sqrt(x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+b\sqrt{x}}\,x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(7/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \frac{4\left(3696\,a^{\frac{5}{2}}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5+7920\,a^2b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+6930\,a\right.}{6}$$

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/693*(3696*a^(5/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 7920*a^2*
b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 6930*a^(3/2)*b^2*(sqrt(a)*s
qrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 3080*a*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x
+ b*sqrt(x)))^2 + 693*sqrt(a)*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
 + 63*b^5)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^11
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = \int \frac{1}{x^{7/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

```
[In] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

3.124 $\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 171

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5}+\frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} -\frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3}+\frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2}+\frac{315b^4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^{11/2}}$$

```
[Out] 315/32*b^4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(11/2)-4*x^(5/2
)/a/(b*x^(1/2)+a*x)^(1/2)-315/32*b^3*(b*x^(1/2)+a*x)^(1/2)/a^5-21/4*b*x*(b*
x^(1/2)+a*x)^(1/2)/a^3+9/2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^2+105/16*b^2*x^(
1/2)*(b*x^(1/2)+a*x)^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2043, 682, 684, 654, 634, 212}

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{315b^4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{11/2}}-\frac{315b^3\sqrt{ax+b\sqrt{x}}}{32a^5} +\frac{105b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{16a^4}-\frac{21bx\sqrt{ax+b\sqrt{x}}}{4a^3}+\frac{9x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a^2}-\frac{4x^{5/2}}{a\sqrt{ax+b\sqrt{x}}}$$

```
[In] Int[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^(5/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (315*b^3*Sqrt[b*Sqrt[x] + a*x])/(3
2*a^5) + (105*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(16*a^4) - (21*b*x*Sqrt[b*
```

```
Sqrt[x] + a*x])/(4*a^3) + (9*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (315*
b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(32*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^6}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{18\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} - \frac{(63b)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\left(105b^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{8a^3}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} - \frac{\left(315b^3\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{32a^4}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\left(315b^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{64a^5}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\left(315b^4\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^5}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{32a^{11/2}}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{\sqrt{b\sqrt{x}+ax}\left(-315b^4-105ab^3\sqrt{x}+42a^2b^2x-24a^3bx^{3/2}+16a^4x^2\right)}{32a^5\left(b+a\sqrt{x}\right)} + \frac{315b^4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{32a^{11/2}}$$

```
[In] Integrate[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(-315*b^4 - 105*a*b^3*Sqrt[x] + 42*a^2*b^2*x - 24*a^
3*b*x^(3/2) + 16*a^4*x^2))/(32*a^5*(b + a*Sqrt[x])) + (315*b^4*ArcTanh[(Sqr
t[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/(32*a^(11/2))
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{x^{\frac{5}{2}}}{2a\sqrt{b\sqrt{x}+ax}} - \frac{9b\Bigg(\frac{x^{2}}{3a\sqrt{b\sqrt{x}+ax}} - \frac{7b\Bigg(\frac{x^{\frac{3}{2}}}{2a\sqrt{b\sqrt{x}+ax}} - \frac{5b\Bigg(\frac{x}{a\sqrt{b\sqrt{x}+ax}} - \frac{3b\Bigg(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} - \frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\right)}{2a} -}{2a}}{4a}}{6a}}{4a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\,\Big(32x^{\frac{3}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{11}{2}}+276x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{9}{2}}b^{2}-48a^{\frac{9}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}bx+690x\sqrt{b\sqrt{x}+ax}\,a^{\frac{7}{2}}b^{3}-768x\,a^{\frac{7}{2}}\sqrt{\sqrt{}}}{}$

```
[In] int(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(5/2)/a/(b*x^(1/2)+a*x)^(1/2)-9/4*b/a*(1/3*x^2/a/(b*x^(1/2)+a*x)^(1/2
)-7/6*b/a*(1/2*x^(3/2)/a/(b*x^(1/2)+a*x)^(1/2)-5/4*b/a*(x/a/(b*x^(1/2)+a*x)
^(1/2)-3/2*b/a*(-x^(1/2)/a/(b*x^(1/2)+a*x)^(1/2)-1/2*b/a*(-1/a/(b*x^(1/2)+a
*x)^(1/2)+1/b/a*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))+1/a^(3/2)*ln((1/2*b+
a*x^(1/2))/a^(1/2)+(b*x^(1/2)+a*x)^(1/2))))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^{\frac{5}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(x**(5/2)/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^{\frac{5}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^(5/2)/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.78

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{1}{32}\sqrt{ax+b\sqrt{x}}\left(2\left(4\sqrt{x}\left(\frac{2\sqrt{x}}{a^2}-\frac{5b}{a^3}\right)+\frac{41b^2}{a^4}\right)\sqrt{x}-\frac{187b^3}{a^5}\right)-\frac{315b^4\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{64a^{\frac{11}{2}}}-\frac{4b^5}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{11}{2}}}$$

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] 1/32*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(2*sqrt(x)/a^2 - 5*b/a^3) + 41*b^2
/a^4)*sqrt(x) - 187*b^3/a^5) - 315/64*b^4*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(
x) - sqrt(a*x + b*sqrt(x))) - b))/a^(11/2) - 4*b^5/((sqrt(a)*(sqrt(a)*sqrt(
x) - sqrt(a*x + b*sqrt(x))) + b)*a^(11/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{x^{5/2}}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

```
[In] int(x^(5/2)/(a*x + b*x^(1/2))^(3/2),x)
[Out] int(x^(5/2)/(a*x + b*x^(1/2))^(3/2), x)
```

3.125 $\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3} + \frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2} + \frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{7/2}}$$

```
[Out] 15/2*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(7/2)-4*x^(3/2)/a
/(b*x^(1/2)+a*x)^(1/2)-15/2*b*(b*x^(1/2)+a*x)^(1/2)/a^3+5*x^(1/2)*(b*x^(1/2
)+a*x)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2043, 682, 684, 654, 634, 212}

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{7/2}} - \frac{15b\sqrt{ax+b\sqrt{x}}}{2a^3} + \frac{5\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^2} - \frac{4x^{3/2}}{a\sqrt{ax+b\sqrt{x}}}$$

```
[In] Int[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^(3/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (15*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^
3) + (5*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a^2 + (15*b^2*ArcTanh[(Sqrt[a]*Sqrt[
x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 682

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] - Dist[e^2*((m + p)/(c*(p + 1))), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{x^4}{(bx+ax^2)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$=-\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}+\frac{10\text{Subst}\left(\int\frac{x^2}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{a}$$

$$=-\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{(15b)\text{Subst}\left(\int\frac{x}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{2a^2}$$

$$=-\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}+\frac{(15b^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{4a^3}$$

$$=-\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}+\frac{(15b^2)\,\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^3}$$

$$=-\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}+\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int\frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{\sqrt{b\sqrt{x}+ax}\left(-15b^2-5ab\sqrt{x}+2a^2x\right)}{2a^3\left(b+a\sqrt{x}\right)}+\frac{15b^2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{2a^{7/2}}$$

```
[In] Integrate[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*Sqrt[x] + a*x]*(-15*b^2 - 5*a*b*Sqrt[x] + 2*a^2*x))/(2*a^3*(b + a*S
qrt[x])) + (15*b^2*ArcTanh[(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])/(b + a*Sqrt[x])]
)/(2*a^(7/2))
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{a\sqrt{b\sqrt{x}+ax}}-\frac{5b\left(\frac{x}{a\sqrt{b\sqrt{x}+ax}}-\frac{3b\left(-\frac{\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\right)}{2a}+\frac{\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)}{2a}$
default	$\frac{\sqrt{b\sqrt{x}+ax}\left(4x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{9}{2}}+10x\sqrt{b\sqrt{x}+ax}\,a^{\frac{7}{2}}b-32x\,a^{\frac{7}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,b+16x\,a^{3}\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\sqrt{a}+b}{2\sqrt{a}}\right)\right.}{}$

```
[In] int(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(3/2)/a/(b*x^(1/2)+a*x)^(1/2)-5/2*b/a*(x/a/(b*x^(1/2)+a*x)^(1/2)-3/2*b/a*
(-x^(1/2)/a/(b*x^(1/2)+a*x)^(1/2)-1/2*b/a*(-1/a/(b*x^(1/2)+a*x)^(1/2)+1/b/a
*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))+1/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1
/2)+(b*x^(1/2)+a*x)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**(3/2)/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{1}{2}\sqrt{ax+b\sqrt{x}}\left(\frac{2\sqrt{x}}{a^2}-\frac{7\,b}{a^3}\right) - \frac{15\,b^2\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{4\,a^{\frac{7}{2}}} - \frac{4\,b^3}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{7}{2}}}$$

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a^2 - 7*b/a^3) - 15/4*b^2*log(abs(-2*s
qrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(7/2) - 4*b^3/((sq
rt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*a^(7/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{x^{3/2}}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

```
[In] int(x^(3/2)/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^(3/2)/(a*x + b*x^(1/2))^(3/2), x)
```

3.126 $\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}+\frac{4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

```
[Out] 4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(3/2)-4*x^(1/2)/a/(b*x^(
1/2)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2043, 666, 634, 212}

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}-\frac{4\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}$$

```
[In] Int[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*Sqrt[x])/(a*Sqrt[b*Sqrt[x] + a*x]) + (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[
b*Sqrt[x] + a*x]])/a^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 666

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[e*(d + e*x)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))), x] - Dist[
e^2*((p + 2)/(c*(p + 1))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^2}{(bx+ax^2)^{3/2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} + \frac{4\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a}\\
&= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} + \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{a\left(b+a\sqrt{x}\right)} + \frac{4\text{arctanh}\left(\frac{\sqrt{a}\sqrt{b\sqrt{x}+ax}}{b+a\sqrt{x}}\right)}{a^{3/2}}$$

```
[In] Integrate[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(a*(b + a*Sqrt[x])) + (4*ArcTanh[(Sqrt[a]*Sqrt[b
*Sqrt[x] + a*x])/(b + a*Sqrt[x])])/a^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.

Time = 2.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

method	result
derivativedivides	$-\frac{2\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}-\frac{b\left(-\frac{1}{a\sqrt{b\sqrt{x}+ax}}+\frac{b+2a\sqrt{x}}{ba\sqrt{b\sqrt{x}+ax}}\right)}{a}+\frac{2\ln\left(\frac{\frac{b}{2}+a\sqrt{x}}{\sqrt{a}}+\sqrt{b\sqrt{x}+ax}\right)}{a^{\frac{3}{2}}}$
default	$-\frac{2\sqrt{b\sqrt{x}+ax}\left(2x\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{5}{2}}-x\ln\left(\frac{2a\sqrt{x}+2\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,\sqrt{a}+b}{2\sqrt{a}}\right)a^{2}b+4\sqrt{x}\,\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}\,a^{\frac{3}{2}}b-2\sqrt{x}\,\ln\left(2a\sqrt{x}\right.\right.}{a^{\frac{3}{2}}\sqrt{\sqrt{x}\,(a\sqrt{x}+b}}$

```
[In] int(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x^(1/2)/a/(b*x^(1/2)+a*x)^(1/2)-b/a*(-1/a/(b*x^(1/2)+a*x)^(1/2)+1/b/a*(b
+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))+2/a^(3/2)*ln((1/2*b+a*x^(1/2))/a^(1/2)
+(b*x^(1/2)+a*x)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(a*x + b*sqrt(x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(a*x + b*sqrt(x))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{2\,\log\left(\left|2\,\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right|\right)}{a^{\frac{3}{2}}}-\frac{4\,b}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)a^{\frac{3}{2}}}$$

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b))/a^(3/2
) - 4*b/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*a^(3/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{\sqrt{x}}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

```
[In] int(x^(1/2)/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^(1/2))^(3/2), x)
```

3.127 $\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\left(b+2a\sqrt{x}\right)}{b^2\sqrt{b\sqrt{x}+ax}}$$

```
[Out] -4*(b+2*a*x^(1/2))/b^2/(b*x^(1/2)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2038, 627}

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=-\frac{4\left(2a\sqrt{x}+b\right)}{b^2\sqrt{ax+b\sqrt{x}}}$$

```
[In] Int[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(b + 2*a*Sqrt[x]))/(b^2*Sqrt[b*Sqrt[x] + a*x])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{(bx+ax^2)^{3/2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{4\left(b+2a\sqrt{x}\right)}{b^2\sqrt{b\sqrt{x}+ax}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\left(b+2a\sqrt{x}\right)\sqrt{b\sqrt{x}+ax}}{b^2\left(b+a\sqrt{x}\right)\sqrt{x}}$$

```
[In] Integrate[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
[Out] (-4*(b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(b^2*(b + a*Sqrt[x])*Sqrt[x])
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{4\left(b+2a\sqrt{x}\right)}{b^2\sqrt{b\sqrt{x}+ax}}$	25
default	$-\frac{4\sqrt{b\sqrt{x}+ax}\left(x\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^2+2\sqrt{x}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}ab-\left(\sqrt{x}\left(a\sqrt{x}+b\right)\right)^{\frac{3}{2}}a^2x+\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}b^2\right)}{\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,b^3x\left(a\sqrt{x}+b\right)^2}$	111

```
[In] int(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -4*(b+2*a*x^(1/2))/b^2/(b*x^(1/2)+a*x)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4\left(abx-\left(2\,a^2x-b^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{a^2b^2x^2-b^4x}$$

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4*(a*b*x - (2*a^2*x - b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^2*x^2 - b^
4*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\sqrt{x}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\left(\frac{2\,a\sqrt{x}}{b^2}+\frac{1}{b}\right)}{\sqrt{ax+b\sqrt{x}}}$$

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -4*(2*a*sqrt(x)/b^2 + 1/b)/sqrt(a*x + b*sqrt(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\sqrt{x}\,\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.128 $\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx\sqrt{b\sqrt{x}+ax}} - \frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{5b^4\sqrt{x}}$$

```
[Out] 4/b/x/(b*x^(1/2)+a*x)^(1/2)-24/5*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(3/2)+32/5*a*(
b*x^(1/2)+a*x)^(1/2)/b^3/x-64/5*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2040, 2041, 2039}

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{64a^2\sqrt{ax+b\sqrt{x}}}{5b^4\sqrt{x}} + \frac{32a\sqrt{ax+b\sqrt{x}}}{5b^3x} - \frac{24\sqrt{ax+b\sqrt{x}}}{5b^2x^{3/2}} + \frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

```
[In] Int[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x*Sqrt[b*Sqrt[x] + a*x]) - (24*Sqrt[b*Sqrt[x] + a*x])/(5*b^2*x^(3/2))
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(5*b^3*x) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(
5*b^4*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4}{bx\sqrt{b\sqrt{x}+ax}}+\frac{6\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{b}\\
&= \frac{4}{bx\sqrt{b\sqrt{x}+ax}}-\frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}}-\frac{(24a)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{5b^2}\\
&= \frac{4}{bx\sqrt{b\sqrt{x}+ax}}-\frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x}+\frac{(16a^2)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{5b^3}\\
&= \frac{4}{bx\sqrt{b\sqrt{x}+ax}}-\frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x}-\frac{64a^2\sqrt{b\sqrt{x}+ax}}{5b^4\sqrt{x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(b^3-2ab^2\sqrt{x}+8a^2bx+16a^3x^{3/2}\right)}{5b^4\left(b+a\sqrt{x}\right)x^{3/2}}$$

```
[In] Integrate[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(b^3 - 2*a*b^2*Sqrt[x] + 8*a^2*b*x + 16*a^3*x^(3/
2)))/(5*b^4*(b + a*Sqrt[x])*x^(3/2))
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.67

method	result
derivativedivides	$-\frac{4}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{12a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}+\frac{8a\left(b+2a\sqrt{x}\right)}{3b^3\sqrt{b\sqrt{x}+ax}}\right)}{5b}$
default	$2\sqrt{b\sqrt{x}+ax}\left(10x^{\frac{9}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{11}{2}}+10x^{\frac{9}{2}}a^{\frac{11}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}-30x^{\frac{7}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{9}{2}}+10x^{\frac{7}{2}}a^{\frac{9}{2}}\left(\sqrt{x}\left(a\sqrt{x}+b\right)\right)^{\frac{3}{2}}+10x^{\frac{7}{2}}\right.$

```
[In] int(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/5/b/x/(b*x^(1/2)+a*x)^(1/2)-12/5*a/b*(-2/3/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/
2)+8/3*a/b^3*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4\left(8\,a^3bx^2-3\,ab^3x-\left(16\,a^4x^2-10\,a^2b^2x-b^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{5\left(a^2b^4x^3-b^6x^2\right)}$$

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/5*(8*a^3*b*x^2 - 3*a*b^3*x - (16*a^4*x^2 - 10*a^2*b^2*x - b^4)*sqrt(x))*s
qrt(a*x + b*sqrt(x))/(a^2*b^4*x^3 - b^6*x^2)
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(x**(3/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^{3/2}\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.129 $\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} - \frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}} + \frac{512a^3\sqrt{b\sqrt{x}+ax}}{63b^5x} - \frac{1024a^4\sqrt{b\sqrt{x}+ax}}{63b^6\sqrt{x}}$$

```
[Out] 4/b/x^2/(b*x^(1/2)+a*x)^(1/2)-40/9*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(5/2)+320/63
*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^2-128/21*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x^(3/
2)+512/63*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x-1024/63*a^4*(b*x^(1/2)+a*x)^(1/2)
/b^6/x^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2040, 2041, 2039}

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{63b^6\sqrt{x}} + \frac{512a^3\sqrt{ax+b\sqrt{x}}}{63b^5x} - \frac{128a^2\sqrt{ax+b\sqrt{x}}}{21b^4x^{3/2}} + \frac{320a\sqrt{ax+b\sqrt{x}}}{63b^3x^2} - \frac{40\sqrt{ax+b\sqrt{x}}}{9b^2x^{5/2}} + \frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

```
[In] Int[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^2*Sqrt[b*Sqrt[x] + a*x]) - (40*Sqrt[b*Sqrt[x] + a*x])/(9*b^2*x^(5/2)
) + (320*a*Sqrt[b*Sqrt[x] + a*x])/(63*b^3*x^2) - (128*a^2*Sqrt[b*Sqrt[x] +
```

```
a*x])/(21*b^4*x^(3/2)) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(63*b^5*x) - (1024
*a^4*Sqrt[b*Sqrt[x] + a*x])/(63*b^6*Sqrt[x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}+\frac{10\int\frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{b}\\
&= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}-\frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}}-\frac{(80a)\int\frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{9b^2}\\
&= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}-\frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}}+\frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2}+\frac{(160a^2)\int\frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{21b^3}\\
&= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}-\frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}}+\frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2}\\
&\quad-\frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}}-\frac{(128a^3)\int\frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{21b^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} \\
&\quad - \frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}} + \frac{512a^3\sqrt{b\sqrt{x}+ax}}{63b^5x} + \frac{(256a^4)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{63b^5} \\
&= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} \\
&\quad - \frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}} + \frac{512a^3\sqrt{b\sqrt{x}+ax}}{63b^5x} - \frac{1024a^4\sqrt{b\sqrt{x}+ax}}{63b^6\sqrt{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.58

$$
\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\left(7b^5-10ab^4\sqrt{x}+16a^2b^3x-32a^3b^2x^{3/2}+128a^4bx^2+256a^5x^{5/2}\right)}{63b^6\left(b+a\sqrt{x}\right)x^{5/2}}
$$

```
[In] Integrate[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(7*b^5 - 10*a*b^4*Sqrt[x] + 16*a^2*b^3*x - 32*a^3
*b^2*x^(3/2) + 128*a^4*b*x^2 + 256*a^5*x^(5/2)))/(63*b^6*(b + a*Sqrt[x])*x^
(5/2))
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-\frac{4}{9b\,x^{2}\sqrt{b\sqrt{x}+ax}}-\frac{20a\left(-\frac{2}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{8a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{6a\left(-\frac{2}{3b\sqrt{x}\,\sqrt{b\sqrt{x}+ax}}+\frac{8a\left(b+2a\sqrt{x}\right)}{3b^{3}\sqrt{b\sqrt{x}+ax}}\right)}{5b}\right)}{7b}\right)}{9b}$
default	$4\sqrt{b\sqrt{x}+ax}\left(126x^{\frac{13}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{15}{2}}+126x^{\frac{13}{2}}\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}\,a^{\frac{15}{2}}-315x^{\frac{11}{2}}\left(b\sqrt{x}+ax\right)^{\frac{3}{2}}a^{\frac{13}{2}}+63x^{\frac{11}{2}}\left(\sqrt{x}\left(a\sqrt{x}+b\right)\right)^{\frac{3}{2}}a\right.$

```
[In] int(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/9/b/x^2/(b*x^(1/2)+a*x)^(1/2)-20/9*a/b*(-2/7/b/x^(3/2)/(b*x^(1/2)+a*x)^(
1/2)-8/7*a/b*(-2/5/b/x/(b*x^(1/2)+a*x)^(1/2)-6/5*a/b*(-2/3/b/x^(1/2)/(b*x^(
1/2)+a*x)^(1/2)+8/3*a/b^3*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4\,(128\,a^5bx^3 - 48\,a^3b^3x^2 - 17\,ab^5x - (256\,a^6x^3 - 160\,a^4b^2x^2 - 26\,a^2b^4x - 7\,b^6)}{63\left(a^2b^6x^4 - b^8x^3\right)}$$

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/63*(128*a^5*b*x^3 - 48*a^3*b^3*x^2 - 17*a*b^5*x - (256*a^6*x^3 - 160*a^4*
b^2*x^2 - 26*a^2*b^4*x - 7*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^6*x^4
 - b^8*x^3)
```

Sympy [F]

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(5/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^{5/2}\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.130 $\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 223

$$\begin{aligned}\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} \\ &+ \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + \frac{2560a^3\sqrt{b\sqrt{x}+ax}}{429b^5x^2} \\ &- \frac{1024a^4\sqrt{b\sqrt{x}+ax}}{143b^6x^{3/2}} + \frac{4096a^5\sqrt{b\sqrt{x}+ax}}{429b^7x} - \frac{8192a^6\sqrt{b\sqrt{x}+ax}}{429b^8\sqrt{x}}\end{aligned}$$

```
[Out] 4/b/x^3/(b*x^(1/2)+a*x)^(1/2)-56/13*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(7/2)+672/1
43*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^3-2240/429*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x
^(5/2)+2560/429*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^2-1024/143*a^4*(b*x^(1/2)+a
*x)^(1/2)/b^6/x^(3/2)+4096/429*a^5*(b*x^(1/2)+a*x)^(1/2)/b^7/x-8192/429*a^6
*(b*x^(1/2)+a*x)^(1/2)/b^8/x^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2040, 2041, 2039}

$$\begin{aligned}\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= -\frac{8192a^6\sqrt{ax+b\sqrt{x}}}{429b^8\sqrt{x}} + \frac{4096a^5\sqrt{ax+b\sqrt{x}}}{429b^7x} \\ &- \frac{1024a^4\sqrt{ax+b\sqrt{x}}}{143b^6x^{3/2}} + \frac{2560a^3\sqrt{ax+b\sqrt{x}}}{429b^5x^2} - \frac{2240a^2\sqrt{ax+b\sqrt{x}}}{429b^4x^{5/2}} \\ &+ \frac{672a\sqrt{ax+b\sqrt{x}}}{143b^3x^3} - \frac{56\sqrt{ax+b\sqrt{x}}}{13b^2x^{7/2}} + \frac{4}{bx^3\sqrt{ax+b\sqrt{x}}}\end{aligned}$$

```
[In] Int[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^3*Sqrt[b*Sqrt[x] + a*x]) - (56*Sqrt[b*Sqrt[x] + a*x])/(13*b^2*x^(7/2
)) + (672*a*Sqrt[b*Sqrt[x] + a*x])/(143*b^3*x^3) - (2240*a^2*Sqrt[b*Sqrt[x]
 + a*x])/(429*b^4*x^(5/2)) + (2560*a^3*Sqrt[b*Sqrt[x] + a*x])/(429*b^5*x^2)
 - (1024*a^4*Sqrt[b*Sqrt[x] + a*x])/(143*b^6*x^(3/2)) + (4096*a^5*Sqrt[b*Sq
rt[x] + a*x])/(429*b^7*x) - (8192*a^6*Sqrt[b*Sqrt[x] + a*x])/(429*b^8*Sqrt[
x])
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}+\frac{14\int\frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx}{b}\\
&= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}-\frac{(168a)\int\frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{13b^2}\\
&= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}+\frac{(1680a^2)\int\frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{143b^3}
\end{aligned}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} - \frac{(4480a^3)\int\frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{429b^4}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + \frac{2560a^3\sqrt{b\sqrt{x}+ax}}{429b^5x^2} + \frac{(1280a^4)\int\frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{143b^5}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + \frac{2560a^3\sqrt{b\sqrt{x}+ax}}{429b^5x^2} - \frac{1024a^4\sqrt{b\sqrt{x}+ax}}{143b^6x^{3/2}} - \frac{(1024a^5)\int\frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{143b^6}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + \frac{2560a^3\sqrt{b\sqrt{x}+ax}}{429b^5x^2} - \frac{1024a^4\sqrt{b\sqrt{x}+ax}}{143b^6x^{3/2}} + \frac{4096a^5\sqrt{b\sqrt{x}+ax}}{429b^7x} + \frac{(2048a^6)\int\frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{429b^7}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + \frac{2560a^3\sqrt{b\sqrt{x}+ax}}{429b^5x^2} - \frac{1024a^4\sqrt{b\sqrt{x}+ax}}{143b^6x^{3/2}} + \frac{4096a^5\sqrt{b\sqrt{x}+ax}}{429b^7x} - \frac{8192a^6\sqrt{b\sqrt{x}+ax}}{429b^8\sqrt{x}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}\big(33b^7 - 42ab^6\sqrt{x} + 56a^2b^5x - 80a^3b^4x^{3/2} + 128a^4b^3x^2 - 256a^5b^2x^{5/2} + 1024a^6bx^3 + 2048a^7}{429b^8\left(b+a\sqrt{x}\right)x^{7/2}}$$

```
[In] Integrate[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(33*b^7 - 42*a*b^6*Sqrt[x] + 56*a^2*b^5*x - 80*a^
3*b^4*x^(3/2) + 128*a^4*b^3*x^2 - 256*a^5*b^2*x^(5/2) + 1024*a^6*b*x^3 + 20
48*a^7*x^(7/2)))/(429*b^8*(b + a*Sqrt[x])*x^(7/2))
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{4}{13b\,x^{3}\sqrt{b\sqrt{x}+ax}}-\frac{28a\left(-\frac{2}{11b\,x^{\frac{5}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{12a\left(-\frac{2}{9b\,x^{2}\sqrt{b\sqrt{x}+ax}}-\frac{10a\left(-\frac{2}{7b\,x^{\frac{3}{2}}\sqrt{b\sqrt{x}+ax}}-\frac{8a\left(-\frac{2}{5bx\sqrt{b\sqrt{x}+ax}}-\frac{6a(}{}\right.}{}\right.}{9b}\right.}{11b}\right.}{13b}$
default	$\frac{2\sqrt{b\sqrt{x}+ax}\left(2574x^{\frac{17}{2}}\sqrt{b\sqrt{x}+ax}\,a^{\frac{19}{2}}+2574x^{\frac{17}{2}}a^{\frac{19}{2}}\sqrt{\sqrt{x}\,(a\sqrt{x}+b)}-6006x^{\frac{15}{2}}(b\sqrt{x}+ax)^{\frac{3}{2}}a^{\frac{17}{2}}+858x^{\frac{15}{2}}a^{\frac{17}{2}}(\sqrt{x}\,(a\sqrt{x}\right.}{}$

```
[In] int(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/13/b/x^3/(b*x^(1/2)+a*x)^(1/2)-28/13*a/b*(-2/11/b/x^(5/2)/(b*x^(1/2)+a*x
)^(1/2)-12/11*a/b*(-2/9/b/x^2/(b*x^(1/2)+a*x)^(1/2)-10/9*a/b*(-2/7/b/x^(3/2
)/(b*x^(1/2)+a*x)^(1/2)-8/7*a/b*(-2/5/b/x/(b*x^(1/2)+a*x)^(1/2)-6/5*a/b*(-2
/3/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)+8/3*a/b^3*(b+2*a*x^(1/2))/(b*x^(1/2)+a*x
)^(1/2))))))
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\left(1024\,a^{7}bx^{4}-384\,a^{5}b^{3}x^{3}-136\,a^{3}b^{5}x^{2}-75\,ab^{7}x-\left(2048\,a^{8}x^{4}-1280\,a^{6}b^{2}x^{3}-\right.\right.}{429\left(a^{2}b^{8}x^{5}-b^{10}x^{4}\right)}$$

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/429*(1024*a^7*b*x^4 - 384*a^5*b^3*x^3 - 136*a^3*b^5*x^2 - 75*a*b^7*x - (2
048*a^8*x^4 - 1280*a^6*b^2*x^3 - 208*a^4*b^4*x^2 - 98*a^2*b^6*x - 33*b^8)*s
qrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^8*x^5 - b^10*x^4)
```

Sympy [F]

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^{\frac{7}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(x**(7/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^{7/2}\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

```
[In] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

3.131 $\int x^3 \sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 301

$$\begin{aligned}\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx &= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}\\ &\quad+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}\\ &\quad+\frac{442b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14421a^{25/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] -884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^6+884/24035*b^5*x^(2/3)*(b*x^(1/3)+a
*x)^(1/2)/a^5-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+476/19665*b
^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^3-28/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/
a^2+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/9*x^4*(b*x^(1/3)+a*x)^(1/2)+
442/14421*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(25/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2046, 2049, 2036, 335, 226}

$$\begin{aligned}&\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx\\ &= \frac{442b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14421a^{25/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &\quad -\frac{884b^6\sqrt{ax+b\sqrt[3]{x}}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{216315a^4}\\ &\quad +\frac{476b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a^2}+\frac{4bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{207a}+\frac{2}{9}x^4\sqrt{ax+b\sqrt[3]{x}}\end{aligned}$$

```
[In] Int[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-884*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^6) + (884*b^5*x^(2/3)*Sqrt[b*x^(1
/3) + a*x])/(24035*a^5) - (6188*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(216315*
a^4) + (476*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^3) - (28*b^2*x^(8/3)*Sq
rt[b*x^(1/3) + a*x])/(1311*a^2) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(207
*a) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/9 + (442*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^
(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Elliptic
F[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(25/4)*Sqrt[b*x^(1/3)
 + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^{11}\sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right) \\
&= \frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax} + \frac{1}{9}(2b)\text{Subst}\left(\int \frac{x^{12}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right) \\
&= \frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a} + \frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax} - \frac{(14b^2)\,\text{Subst}\left(\int \frac{x^{10}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{69a} \\
&= -\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2} + \frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a} \\
&\quad + \frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax} + \frac{(238b^3)\,\text{Subst}\left(\int \frac{x^{8}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1311a^2} \\
&= \frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3} - \frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2} + \frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a} \\
&\quad + \frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax} - \frac{(3094b^4)\,\text{Subst}\left(\int \frac{x^{6}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{19665a^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2} \\
&\quad +\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}+\frac{(3094b^5)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{24035a^4} \\
&= \frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4} \\
&\quad +\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a} \\
&\quad +\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}-\frac{(442b^6)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{4807a^5} \\
&= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4} \\
&\quad +\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a} \\
&\quad +\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}+\frac{(442b^7)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{14421a^6} \\
&= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4} \\
&\quad +\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a} \\
&\quad +\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}+\frac{\left(442b^7\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{14421a^6\sqrt{b\sqrt[3]{x}+ax}} \\
&= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4} \\
&\quad +\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a} \\
&\quad +\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}+\frac{\left(884b^7\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{14421a^6\sqrt{b\sqrt[3]{x}+ax}} \\
&= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5} \\
&\quad -\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3} \\
&\quad -\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax} \\
&\quad +\frac{442b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14421a^{25/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.51

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$$

$$=\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}(-9945b^6+3978ab^5x^{2/3}-3094a^2b^4x^{4/3}+2618a^3b^3x^2-2310a^4b^2x^{8/3}+2090a^5\right.}{216315a^6\sqrt{1+\frac{ax^{2/3}}{b}}}$$

```
[In] Integrate[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-9945*b^6 + 3978*a*b^5*x
^(2/3) - 3094*a^2*b^4*x^(4/3) + 2618*a^3*b^3*x^2 - 2310*a^4*b^2*x^(8/3) + 2
090*a^5*b*x^(10/3) + 24035*a^6*x^4) + 9945*b^6*Hypergeometric2F1[-1/2, 1/4,
 5/4, -((a*x^(2/3))/b)]))/(216315*a^6*Sqrt[1 + (a*x^(2/3))/b])
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{2x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{4b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207a}-\frac{28b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a^2}+\frac{476b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^3}-\frac{6188b^4x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315a^4}+884b$
default	$\frac{2x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{4b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207a}-\frac{28b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a^2}+\frac{476b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^3}-\frac{6188b^4x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315a^4}+884b$

```
[In] int(x^3*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x^4*(b*x^(1/3)+a*x)^(1/2)+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a-28/1
311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+476/19665*b^3*x^2*(b*x^(1/3)+a*x)
^(1/2)/a^3-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+884/24035*b^5*
x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^6+4
42/14421*b^7/a^7*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(
1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b
)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*
a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x^3\,dx$$

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*x + b*x^(1/3))*x^3, x)
```

Sympy [F]

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x^3\sqrt{ax+b\sqrt[3]{x}}\,dx$$

```
[In] integrate(x**3*(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(x**3*sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x^3\,dx$$

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}x^3\,dx$$

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x^3\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

```
[In] int(x^3*(a*x + b*x^(1/3))^(1/2),x)
[Out] int(x^3*(a*x + b*x^(1/3))^(1/2), x)
```

3.132 $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 411

$$\begin{aligned}
&\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx \\
&= \frac{44b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{9/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} \\
&\quad-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} \\
&\quad-\frac{44b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{19/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad+\frac{22b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{221a^{19/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

```
[Out] 44/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(9/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/
3)+a*x)^(1/2)-44/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+220/4641*b^3*x*(
b*x^(1/3)+a*x)^(1/2)/a^3-60/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+4/11
9*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/7*x^3*(b*x^(1/3)+a*x)^(1/2)-44/221*b^
(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^
(1/2))^2)^(1/2)/a^(19/4)/(b*x^(1/3)+a*x)^(1/2)+22/221*b^(21/4)*x^(1/6)*(cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(
1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(
19/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2043, 2046, 2049, 2057, 335, 311, 226, 1210}

$$\begin{aligned}&\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx\\ &=\frac{22b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{221a^{19/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &\quad-\frac{44b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{19/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &\quad+\frac{44b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{9/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{44b^4\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{663a^4}+\frac{220b^3x\sqrt{ax+b\sqrt[3]{x}}}{4641a^3}\\ &\quad-\frac{60b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^2}+\frac{4bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{119a}+\frac{2}{7}x^3\sqrt{ax+b\sqrt[3]{x}}\end{aligned}$$

```
[In] Int[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (44*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(9/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*S
qrt[b*x^(1/3) + a*x]) - (44*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^4) +
(220*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^3) - (60*b^2*x^(5/3)*Sqrt[b*x^(1/
3) + a*x])/(1547*a^2) + (4*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a) + (2*x^
3*Sqrt[b*x^(1/3) + a*x])/7 - (44*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[
(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(
a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(19/4)*Sqrt[b*x^(1/3) + a*x]) + (22
*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[
a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(221*a^(19/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^8\sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right) \\
&= \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} + \frac{1}{7}(2b)\text{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right) \\
&= \frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a} + \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} - \frac{(30b^2)\,\text{Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{119a} \\
&= -\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2} + \frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a} \\
&\quad + \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} + \frac{(330b^3)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1547a^2} \\
&= \frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} - \frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2} + \frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a} \\
&\quad + \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} - \frac{(110b^4)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{663a^3} \\
&= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4} + \frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} - \frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2} \\
&\quad + \frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a} + \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} + \frac{(22b^5)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{221a^4} \\
&= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4} + \frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} \\
&\quad - \frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2} + \frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a} + \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} \\
&\quad + \frac{\left(22b^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{221a^4\sqrt{b\sqrt[3]{x}+ax}} \\
&= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4} + \frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} \\
&\quad - \frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2} + \frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a} + \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} \\
&\quad + \frac{\left(44b^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{221a^4\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} \\
&\quad -\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} \\
&\quad +\frac{\left(44b^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{221a^{9/2}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad -\frac{\left(44b^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{221a^{9/2}\sqrt{b\sqrt[3]{x}+ax}} \\
&= \frac{44b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{9/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3} \\
&\quad -\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax} \\
&\quad -\frac{44b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{19/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad +\frac{22b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{19/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.33

$$
\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx
= \frac{2\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}\left(-385b^4+110ab^3x^{2/3}-90a^2b^2x^{4/3}+78a^3bx^2+663a^4x^{8/3}\right)+385b^4\,\text{Hyper}\right.}{4641a^4\sqrt{1+\frac{ax^{2/3}}{b}}}
$$

```
[In] Integrate[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-385*b^4 + 110*a
*b^3*x^(2/3) - 90*a^2*b^2*x^(4/3) + 78*a^3*b*x^2 + 663*a^4*x^(8/3)) + 385*b
^4*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(4641*a^4*Sqrt[1 +
 (a*x^(2/3))/b])
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{2x^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{7}+\frac{4b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a}-\frac{60b^{2}x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^{2}}+\frac{220b^{3}x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^{3}}-\frac{44b^{4}x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663a^{4}}+\frac{22b^{5}\sqrt{-a}}{\ }$
default	$\frac{2x^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{7}+\frac{4b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a}-\frac{60b^{2}x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^{2}}+\frac{220b^{3}x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^{3}}-\frac{44b^{4}x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663a^{4}}+\frac{22b^{5}\sqrt{-a}}{\ }$

```
[In] int(x^2*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*x^3*(b*x^(1/3)+a*x)^(1/2)+4/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a-60/15
47*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+220/4641*b^3*x*(b*x^(1/3)+a*x)^(1/
2)/a^3-44/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+22/221*b^5/a^5*(-a*b)^(
1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)
+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b
)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int x^{2}\sqrt{b\sqrt[3]{x}+ax}\,dx=\int\sqrt{ax+bx^{\frac{1}{3}}}x^{2}\,dx$$

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))*x^2, x)
```

Sympy [F]

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x^2\sqrt{ax+b\sqrt[3]{x}}\,dx$$

```
[In] integrate(x**2*(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(x**2*sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx=\int\sqrt{ax+bx^{\frac{1}{3}}}x^2\,dx$$

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx=\int\sqrt{ax+bx^{\frac{1}{3}}}x^2\,dx$$

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x^2\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

```
[In] int(x^2*(a*x + b*x^(1/3))^(1/2),x)
[Out] int(x^2*(a*x + b*x^(1/3))^(1/2), x)
```

3.133 $\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 213

$$
\begin{aligned}
\int x\sqrt{b\sqrt[3]{x}+ax}\,dx
&= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3}-\frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2}+\frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a}+\frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} \\
&\quad -\frac{6b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{13/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

```
[Out] 12/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^3-36/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2
)/a^2+4/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/5*x^2*(b*x^(1/3)+a*x)^(1/2)-
6/77*b^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(
1/2)+b^(1/2))^2)^(1/2)/a^(13/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used

$= \{2043, 2046, 2049, 2036, 335, 226\}$

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$$

$$= -\frac{6b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{13/4}\sqrt{ax+b\sqrt[3]{x}}}$$
$$+\frac{12b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^3}-\frac{36b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^2}+\frac{4bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{55a}+\frac{2}{5}x^2\sqrt{ax+b\sqrt[3]{x}}$$

```
[In] Int[x*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (12*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^3) - (36*b^2*x^(2/3)*Sqrt[b*x^(1/3) +
a*x])/(385*a^2) + (4*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(55*a) + (2*x^2*Sqrt[
b*x^(1/3) + a*x])/5 - (6*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x
^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*
x^(1/6))/b^(1/4)], 1/2])/(77*a^(13/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^5\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{5}(2b)\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a}+\frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(18b^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{55a}\\
&= -\frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2}+\frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a}\\
&\quad+\frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax}+\frac{\left(18b^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{77a^2}\\
&= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3}-\frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2}+\frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a}\\
&\quad+\frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(6b^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{77a^3}\\
&= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3}-\frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2}+\frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a}\\
&\quad+\frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(6b^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{77a^3\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3}-\frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2}+\frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a}\\
&\quad+\frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(12b^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{77a^3\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax}$$
$$- \frac{6b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{13/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.55

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$$
$$= \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\sqrt{1+\frac{ax^{2/3}}{b}}\left(45b^3-18ab^2x^{2/3}+14a^2bx^{4/3}+77a^3x^2\right)-45b^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\right.\right.}{385a^3\sqrt{1+\frac{ax^{2/3}}{b}}}$$

```
[In] Integrate[x*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(45*b^3 - 18*a*b^2*x^(2/3
) + 14*a^2*b*x^(4/3) + 77*a^3*x^2) - 45*b^3*Hypergeometric2F1[-1/2, 1/4, 5/
4, -((a*x^(2/3))/b)]))/(385*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5}+\frac{4b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a}-\frac{36b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^2}+\frac{12b^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^3}-\frac{6b^4\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-}}{}$
default	$\frac{2x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5}+\frac{4b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a}-\frac{36b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^2}+\frac{12b^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^3}-\frac{6b^4\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-}}{}$

```
[In] int(x*(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*x^2*(b*x^(1/3)+a*x)^(1/2)+4/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a-36/385
*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+12/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^3-
6/77*b^4/a^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)
```

```
*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1
/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int\sqrt{ax+bx^{\frac{1}{3}}}x\,dx$$

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*x + b*x^(1/3))*x, x)
```

Sympy [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x\sqrt{ax+b\sqrt[3]{x}}\,dx$$

```
[In] integrate(x*(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(x*sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int\sqrt{ax+bx^{\frac{1}{3}}}x\,dx$$

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

Giac [F]

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int\sqrt{ax+bx^{\frac{1}{3}}}x\,dx$$

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx=\int x\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

```
[In] int(x*(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] int(x*(a*x + b*x^(1/3))^(1/2), x)
```

3.134 $\int \sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 323

$$\begin{aligned}\int \sqrt{b\sqrt[3]{x}+ax}\,dx &= -\frac{4b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{3/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}\\ &+\frac{4b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{b\sqrt[3]{x}+ax}}\\ &-\frac{2b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{7/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] -4/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(3/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)
+a*x)^(1/2)+4/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/3*x*(b*x^(1/3)+a*x)^(1
/2)+4/5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/co
s(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)
/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a
^(1/2)+b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^(1/3)+a*x)^(1/2)-2/5*b^(9/4)*x^(1/6)*
(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*
(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)
/a^(7/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2029, 2043, 2049, 2057, 335, 311, 226, 1210}

$$\begin{aligned}&\int \sqrt{b\sqrt[3]{x}+ax}\,dx\\ &= -\frac{2b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &\quad+\frac{4b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &\quad-\frac{4b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{3/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{4b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{15a}+\frac{2}{3}x\sqrt{ax+b\sqrt[3]{x}}\end{aligned}$$

```
[In] Int[Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-4*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(3/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a) + (2*x*Sqr
t[b*x^(1/3) + a*x])/3 + (4*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*
x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)
*x^(1/6))/b^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[b*x^(1/3) + a*x]) - (2*b^(9/4)*(S
qrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^
2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(7/4)*
Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{9}(2b)\int\frac{\sqrt[3]{x}}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$$

$$
\begin{aligned}
&= \frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{3}(2b)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right) \\
&= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{(2b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{5a} \\
&= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(2b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{5a\sqrt{b\sqrt[3]{x}+ax}} \\
&= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(4b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{5a\sqrt{b\sqrt[3]{x}+ax}} \\
&= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax} \\
&\quad -\frac{\left(4b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{5a^{3/2}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad +\frac{\left(4b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{5a^{3/2}\sqrt{b\sqrt[3]{x}+ax}} \\
&= -\frac{4b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{3/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax} \\
&\quad +\frac{4b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad -\frac{2b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.29

$$
\begin{aligned}
&\int \sqrt{b\sqrt[3]{x}+ax}\,dx \\
&= \frac{2\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)\sqrt{1+\frac{ax^{2/3}}{b}}-b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)\right)}{3a\sqrt{1+\frac{ax^{2/3}}{b}}}
\end{aligned}
$$

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))*Sqrt[1 + (a*x^(2/3))/b] -
 b*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(3*a*Sqrt[1 + (a*x
^(2/3))/b])
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{4b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a}-\frac{2b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\left(x^{\frac{1}{3}}\right.}\right.}{a}\right.}{5a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{4b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a}-\frac{2b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\left(x^{\frac{1}{3}}\right.}\right.}{a}\right.}{5a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$

```
[In] int((b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x*(b*x^(1/3)+a*x)^(1/2)+4/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a-2/5/a^2*
b^2*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(
1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2
)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(
1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/
3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3)), x)
```

Sympy [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+b\sqrt[3]{x}}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\int \sqrt{ax+bx^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3)), x)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.12

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx=\frac{6\,x\,\sqrt{a\,x+b\,x^{1/3}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{7\,\sqrt{\frac{a\,x^{2/3}}{b}+1}}$$

```
[In] int((a*x + b*x^(1/3))^(1/2),x)
[Out] (6*x*(a*x + b*x^(1/3))^(1/2)*hypergeom([-1/2, 7/4], 11/4, -(a*x^(2/3))/b))/
(7*((a*x^(2/3))/b + 1)^(1/2))
```

3.135 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\begin{aligned}\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx &= 2\sqrt{b\sqrt[3]{x}+ax}\\ &\quad+\frac{2b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] 2*(b*x^(1/3)+a*x)^(1/2)+2*b^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(
1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x
^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2043, 2046, 2036, 335, 226}

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx$$

$$=\frac{2b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}}$$
$$+2\sqrt{ax+b\sqrt[3]{x}}$$

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x,x]
```

```
[Out] 2*Sqrt[b*x^(1/3) + a*x] + (2*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b +
a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/
4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= 2\sqrt{b\sqrt[3]{x}+ax} + (2b)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= 2\sqrt{b\sqrt[3]{x}+ax} + \frac{\left(2b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}$$

$$= 2\sqrt{b\sqrt[3]{x}+ax} + \frac{\left(4b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}$$

$$= 2\sqrt{b\sqrt[3]{x}+ax} + \frac{2b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx = \frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}}$$

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x,x]
```

```
[Out] (6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((a*x^(2/3))/b)
])/Sqrt[1 + (a*x^(2/3))/b]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$2\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{2b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{b\,x^{\frac{1}{3}}+ax}}$	132
default	$2\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{2b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a\sqrt{b\,x^{\frac{1}{3}}+ax}}$	132

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2*(b*x^(1/3)+a*x)^(1/2)+2*b/a*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^
(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(
-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int\frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=\int\frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(a*x + b*x^(1/3))/x, x)
```

Sympy [F]

$$\int\frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=\int\frac{\sqrt{ax+b\sqrt[3]{x}}}{x}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x,x)
[Out] Integral(sqrt(a*x + b*x**(1/3))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx=\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x}\,dx$$

```
[In] int((a*x + b*x^(1/3))^(1/2)/x,x)
[Out] int((a*x + b*x^(1/3))^(1/2)/x, x)
```

3.136 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 325

$$\begin{aligned}\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx &= \frac{12a^{3/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}\\ &\quad-\frac{12a^{5/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\\ &\quad+\frac{6a^{5/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] 12/5*a^(3/2)*(b+a*x^(2/3))*x^(1/3)/b/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a
*x)^(1/2)-6/5*(b*x^(1/3)+a*x)^(1/2)/x-12/5*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3
)-12/5*a^(5/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)+6/5*a^(5/4)*x^(1/6)*(
cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6
)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(
x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/
b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2043, 2045, 2050, 2057, 335, 311, 226, 1210}

$$
\begin{aligned}
&\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx \\
&= \frac{6a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}} \\
&\quad - \frac{12a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}} \\
&\quad + \frac{12a^{3/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5b\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}} - \frac{12a\sqrt{ax+b\sqrt[3]{x}}}{5b\sqrt[3]{x}} - \frac{6\sqrt{ax+b\sqrt[3]{x}}}{5x}
\end{aligned}
$$

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

```
[Out] (12*a^(3/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[
b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(5*x) - (12*a*Sqrt[b*x^(1/3)
+ a*x])/(5*b*x^(1/3)) - (12*a^(5/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a
*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4
)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (6*a^(5/4)*(
Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))
^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)
*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^4}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x}+\frac{1}{5}(6a)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}+\frac{(6a^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}+\frac{\left(6a^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{5b\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}+\frac{\left(12a^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5b\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}\\
&\quad+\frac{\left(12a^{3/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}}\\
&\quad-\frac{\left(12a^{3/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{12a^{3/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}\\
&\quad-\frac{12a^{5/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\\
&\quad+\frac{6a^{5/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = -\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{ax^{2/3}}{b}\right)}{5\sqrt{1+\frac{ax^{2/3}}{b}}\,x}$$

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((a*x^(2/3))
/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x)
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.66

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x}-\frac{12\left(b+a\,x^{\frac{2}{3}}\right)a}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{6a\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\left(\right.}\right.}{}\right.}{5b\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x}-\frac{12\left(b+a\,x^{\frac{2}{3}}\right)a}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{6a\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\left(\right.}\right.}{}\right.}{5b\sqrt{b\,x^{\frac{1}{3}}+ax}}$

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -6/5*(b*x^(1/3)+a*x)^(1/2)/x-12/5*(b+a*x^(2/3))*a/b/(x^(1/3)*(b+a*x^(2/3)))
^(1/2)+6/5*a/b*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/
2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^
(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1
/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*Ellipt
icF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(a*x + b*x^(1/3))/x^2, x)
```

Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx=\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^2}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**2,x)
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx=\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^2}\,dx$$

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^2,x)
[Out] int((a*x + b*x^(1/3))^(1/2)/x^2, x)
```

3.137 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 188

$$\begin{aligned}\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}}+\frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}\\ &\quad+\frac{10a^{11/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] -6/11*(b*x^(1/3)+a*x)^(1/2)/x^2-12/77*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+20/
77*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)+10/77*a^(11/4)*x^(1/6)*(cos(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
)*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^
(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(9/4)/(b
*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

= {2043, 2045, 2050, 2036, 335, 226}

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx$$

$$= \frac{10a^{11/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{20a^2\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}} - \frac{12a\sqrt{ax+b\sqrt[3]{x}}}{77bx^{4/3}} - \frac{6\sqrt{ax+b\sqrt[3]{x}}}{11x^2}$$

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(11*x^2) - (12*a*Sqrt[b*x^(1/3) + a*x])/(77*b*x^
(4/3)) + (20*a^2*Sqrt[b*x^(1/3) + a*x])/(77*b^2*x^(2/3)) + (10*a^(11/4)*(Sq
rt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2
]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*b^(9/4)*
Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^7}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2}+\frac{1}{11}(6a)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}}-\frac{(30a^2)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{77b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}}+\frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}+\frac{(10a^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{77b^2}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}}+\frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}\\
&\quad+\frac{\left(10a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{77b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}}+\frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}\\
&\quad+\frac{\left(20a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{77b^2\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{10a^{11/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = -\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{2},-\frac{7}{4},-\frac{ax^{2/3}}{b}\right)}{11\sqrt{1+\frac{ax^{2/3}}{b}}x^2}$$

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((a*x^(2/3)
)/b)])/(11*Sqrt[1 + (a*x^(2/3))/b]*x^2)
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{11x^2}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b\,x^{\frac{4}{3}}}+\frac{20a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^2x^{\frac{2}{3}}}+\frac{10a^2\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{77b^2\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{11x^2}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b\,x^{\frac{4}{3}}}+\frac{20a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^2x^{\frac{2}{3}}}+\frac{10a^2\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{77b^2\sqrt{b\,x^{\frac{1}{3}}+ax}}$

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -6/11*(b*x^(1/3)+a*x)^(1/2)/x^2-12/77*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+20/
77*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)+10/77*a^2/b^2*(-a*b)^(1/2)*((x^(1/
3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a
/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*
EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^3}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral(sqrt(a*x + b*x^(1/3))/x^3, x)
```

Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = \int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^3}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**3,x)
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^3}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^3}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx=\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^3}\,dx$$

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^3,x)
[Out] int((a*x + b*x^(1/3))^(1/2)/x^3, x)
```

3.138 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 413

$$\begin{aligned}
\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx
&= -\frac{308a^{9/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}}\\
&\quad+\frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}}-\frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x}+\frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}}\\
&\quad+\frac{308a^{17/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}\\
&\quad-\frac{154a^{17/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

```
[Out] -308/1105*a^(9/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^
(1/3)+a*x)^(1/2)-6/17*(b*x^(1/3)+a*x)^(1/2)/x^3-12/221*a*(b*x^(1/3)+a*x)^(1
/2)/b/x^(7/3)+44/663*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-308/3315*a^3*(b*
x^(1/3)+a*x)^(1/2)/b^3/x+308/1105*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(1/3)+308
/1105*a^(17/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)-154/1105*a^(17/4)*x^
(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)
*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(
```

```
1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)
^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2043, 2045, 2050, 2057, 335, 311, 226, 1210}

$$\begin{aligned}\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx &= -\frac{154a^{17/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &\quad+\frac{308a^{17/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &\quad-\frac{308a^{9/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{308a^4\sqrt{ax+b\sqrt[3]{x}}}{1105b^4\sqrt[3]{x}}\\ &\quad-\frac{308a^3\sqrt{ax+b\sqrt[3]{x}}}{3315b^3x}+\frac{44a^2\sqrt{ax+b\sqrt[3]{x}}}{663b^2x^{5/3}}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{221bx^{7/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{17x^3}\end{aligned}$$

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

```
[Out] (-308*a^(9/2)*(b + a*x^(2/3))*x^(1/3))/(1105*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(17*x^3) - (12*a*Sqrt[
b*x^(1/3) + a*x])/(221*b*x^(7/3)) + (44*a^2*Sqrt[b*x^(1/3) + a*x])/(663*b^2
*x^(5/3)) - (308*a^3*Sqrt[b*x^(1/3) + a*x])/(3315*b^3*x) + (308*a^4*Sqrt[b*
x^(1/3) + a*x])/(1105*b^4*x^(1/3)) + (308*a^(17/4)*(Sqrt[b] + Sqrt[a]*x^(1/
3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2
*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a
*x]) - (154*a^(17/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt
[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1
/4)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
```

```
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} + \frac{1}{17}(6a)\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} - \frac{(66a^2)\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{221b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} + \frac{(154a^3)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{663b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} - \frac{(154a^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1105b^3}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - \frac{(154a^5)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1105b^4}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - \frac{\left(154a^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{1105b^4\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - \frac{\left(308a^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{1105b^4\sqrt{b\sqrt[3]{x}+ax}}$$

$$
\begin{aligned}
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}}+\frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}}-\frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x}\\
&\quad+\frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}}-\frac{\left(308a^{9/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{1105b^{7/2}\sqrt{b\sqrt[3]{x}+ax}}\\
&\quad+\frac{\left(308a^{9/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\operatorname{Subst}\left(\int\frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{1105b^{7/2}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{308a^{9/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}}\\
&\quad+\frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}}-\frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x}+\frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}}\\
&\quad+\frac{308a^{17/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}\\
&\quad-\frac{154a^{17/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.14

$$
\int\frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx=-\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{17}{4},-\frac{1}{2},-\frac{13}{4},-\frac{ax^{2/3}}{b}\right)}{17\sqrt{1+\frac{ax^{2/3}}{b}}x^3}
$$

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-17/4, -1/2, -13/4, -((a*x^(2/3
))/b)])/(17*Sqrt[1 + (a*x^(2/3))/b]*x^3)
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.68

method	result
derivativedivides	$154 a^{4} \sqrt{-}$ $-\frac{6\sqrt{b\,x^{\frac{1}{3}}+a x}}{17 x^{3}}-\frac{12 a \sqrt{b\,x^{\frac{1}{3}}+a x}}{221 b\,x^{\frac{7}{3}}}+\frac{44 a^{2} \sqrt{b\,x^{\frac{1}{3}}+a x}}{663 b^{2} x^{\frac{5}{3}}}-\frac{308 a^{3} \sqrt{b\,x^{\frac{1}{3}}+a x}}{3315 b^{3} x}+\frac{308 \left(b+a\,x^{\frac{2}{3}}\right) a^{4}}{1105 b^{4} \sqrt{x^{\frac{1}{3}} \left(b+a\,x^{\frac{2}{3}}\right)}}-$
default	$154 a^{4} \sqrt{-}$ $-\frac{6\sqrt{b\,x^{\frac{1}{3}}+a x}}{17 x^{3}}-\frac{12 a \sqrt{b\,x^{\frac{1}{3}}+a x}}{221 b\,x^{\frac{7}{3}}}+\frac{44 a^{2} \sqrt{b\,x^{\frac{1}{3}}+a x}}{663 b^{2} x^{\frac{5}{3}}}-\frac{308 a^{3} \sqrt{b\,x^{\frac{1}{3}}+a x}}{3315 b^{3} x}+\frac{308 \left(b+a\,x^{\frac{2}{3}}\right) a^{4}}{1105 b^{4} \sqrt{x^{\frac{1}{3}} \left(b+a\,x^{\frac{2}{3}}\right)}}-$

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -6/17*(b*x^(1/3)+a*x)^(1/2)/x^3-12/221*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+44
/663*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-308/3315*a^3*(b*x^(1/3)+a*x)^(1/
2)/b^3/x+308/1105*(b+a*x^(2/3))*a^4/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-154/1
105*a^4/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*
(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/
2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*
(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF
(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^{4}}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^4, x)
```

Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^4}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**4,x)
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^4}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^4}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = \int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^4}\,dx$$

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^4,x)
[Out] int((a*x + b*x^(1/3))^(1/2)/x^4, x)
```

3.139 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 276

$$\begin{aligned}
\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}}+\frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}}\\
&\quad-\frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2}+\frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}}-\frac{2652a^5\sqrt{b\sqrt[3]{x}+ax}}{33649b^5x^{2/3}}\\
&\quad-\frac{1326a^{23/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{33649b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

```
[Out] -6/23*(b*x^(1/3)+a*x)^(1/2)/x^4-12/437*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+6
8/2185*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)-884/24035*a^3*(b*x^(1/3)+a*x)^
(1/2)/b^3/x^2+7956/168245*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-2652/33649*
a^5*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-1326/33649*a^(23/4)*x^(1/6)*(cos(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4
)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*
a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(21/4)
/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2043, 2045, 2050, 2036, 335, 226}

$$\begin{aligned}
&\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx \\
&= -\frac{1326a^{23/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{33649b^{21/4}\sqrt{ax+b\sqrt[3]{x}}} \\
&\quad -\frac{2652a^5\sqrt{ax+b\sqrt[3]{x}}}{33649b^5x^{2/3}}+\frac{7956a^4\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}-\frac{884a^3\sqrt{ax+b\sqrt[3]{x}}}{24035b^3x^2} \\
&\quad +\frac{68a^2\sqrt{ax+b\sqrt[3]{x}}}{2185b^2x^{8/3}}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{437bx^{10/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{23x^4}
\end{aligned}$$

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(23*x^4) - (12*a*Sqrt[b*x^(1/3) + a*x])/(437*b*x
^(10/3)) + (68*a^2*Sqrt[b*x^(1/3) + a*x])/(2185*b^2*x^(8/3)) - (884*a^3*Sqr
t[b*x^(1/3) + a*x])/(24035*b^3*x^2) + (7956*a^4*Sqrt[b*x^(1/3) + a*x])/(168
245*b^4*x^(4/3)) - (2652*a^5*Sqrt[b*x^(1/3) + a*x])/(33649*b^5*x^(2/3)) - (
1326*a^(23/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + S
qrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1
/2])/(33649*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4}+\frac{1}{23}(6a)\text{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}}-\frac{\left(102a^2\right)\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{437b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}}+\frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}}\\
&\quad+\frac{\left(442a^3\right)\text{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2185b^2}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4}-\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}}+\frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}}\\
&\quad-\frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2}-\frac{\left(3978a^4\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{24035b^3}
\end{aligned}
$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} + \frac{(3978a^5)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{33649b^4}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} - \frac{2652a^5\sqrt{b\sqrt[3]{x}+ax}}{33649b^5x^{2/3}} - \frac{(1326a^6)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{33649b^5}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} - \frac{2652a^5\sqrt{b\sqrt[3]{x}+ax}}{33649b^5x^{2/3}} - \frac{\left(1326a^6\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{33649b^5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} - \frac{2652a^5\sqrt{b\sqrt[3]{x}+ax}}{33649b^5x^{2/3}} - \frac{\left(2652a^6\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{33649b^5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} - \frac{2652a^5\sqrt{b\sqrt[3]{x}+ax}}{33649b^5x^{2/3}} - \frac{1326a^{23/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{33649b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx=-\frac{6\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{23}{4},-\frac{1}{2},-\frac{19}{4},-\frac{ax^{2/3}}{b}\right)}{23\sqrt{1+\frac{ax^{2/3}}{b}}x^4}$$

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-23/4, -1/2, -19/4, -((a*x^(2/3
))/b)])/(23*Sqrt[1 + (a*x^(2/3))/b]*x^4)
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{23x^{4}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{437b\,x^{\frac{10}{3}}}+\frac{68a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{2185b^{2}x^{\frac{8}{3}}}-\frac{884a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{24035b^{3}x^{2}}+\frac{7956a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{168245b^{4}x^{\frac{4}{3}}}-\frac{2652a^{5}\sqrt{b\,x^{\frac{1}{3}}}}{33649b^{5}x}$
default	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{23x^{4}}-\frac{12a\sqrt{b\,x^{\frac{1}{3}}+ax}}{437b\,x^{\frac{10}{3}}}+\frac{68a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{2185b^{2}x^{\frac{8}{3}}}-\frac{884a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{24035b^{3}x^{2}}+\frac{7956a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{168245b^{4}x^{\frac{4}{3}}}-\frac{2652a^{5}\sqrt{b\,x^{\frac{1}{3}}}}{33649b^{5}x}$

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -6/23*(b*x^(1/3)+a*x)^(1/2)/x^4-12/437*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+6
8/2185*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)-884/24035*a^3*(b*x^(1/3)+a*x)^
(1/2)/b^3/x^2+7956/168245*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-2652/33649*
a^5*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-1326/33649*a^5/b^5*(-a*b)^(1/2)*((x^(
1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))
*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2
)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^5}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
[Out] integral(sqrt(a*x + b*x^(1/3))/x^5, x)
```

Sympy [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx=\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^5}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**5,x)
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^5}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx=\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^5}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx=\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^5}\,dx$$

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^5,x)
```

```
[Out] int((a*x + b*x^(1/3))^(1/2)/x^5, x)
```

3.140 $\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 298

$$\begin{aligned}\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx={}&\frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}\\&+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}\\&+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{884b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{100947a^{21/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] 2/9*x^3*(b*x^(1/3)+a*x)^(3/2)+1768/100947*b^6*(b*x^(1/3)+a*x)^(1/2)/a^5-176
8/168245*b^5*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+1768/216315*b^4*x^(4/3)*(b*x
^(1/3)+a*x)^(1/2)/a^3-136/19665*b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^2+8/1311*b^
2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/a+4/69*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)-884
/100947*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6
)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*
a^(1/2)+b^(1/2))^2)^(1/2)/a^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2043, 2046, 2049, 2036, 335, 226}

$$\begin{aligned}\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx = &-\frac{884b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{100947a^{21/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &+\frac{1768b^6\sqrt{ax+b\sqrt[3]{x}}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{168245a^4}\\ &+\frac{1768b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{216315a^3}-\frac{136b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{19665a^2}\\ &+\frac{8b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}+\frac{2}{9}x^3\left(ax+b\sqrt[3]{x}\right)^{3/2}\end{aligned}$$

```
[In] Int[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (1768*b^6*Sqrt[b*x^(1/3) + a*x])/(100947*a^5) - (1768*b^5*x^(2/3)*Sqrt[b*x^
(1/3) + a*x])/(168245*a^4) + (1768*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(2163
15*a^3) - (136*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^2) + (8*b^2*x^(8/3)*
Sqrt[b*x^(1/3) + a*x])/(1311*a) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/69 +
 (2*x^3*(b*x^(1/3) + a*x)^(3/2))/9 - (884*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^(1/
3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2
*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*a^(21/4)*Sqrt[b*x^(1/3) +
 a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^8\left(bx+ax^3\right)^{3/2}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{3}(2b)\text{Subst}\left(\int x^9\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{69}\left(4b^2\right)\text{Subst}\left(\int \frac{x^{10}}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}\\
&\quad +\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(68b^3\right)\text{Subst}\left(\int \frac{x^8}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{1311a}\\
&= -\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}\\
&\quad +\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{\left(884b^4\right)\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{19665a^2}
\end{aligned}$$

$$=\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(884b^5\right)\text{Subst}\left(\int\frac{x^4}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{24035a^3}$$

$$=-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{\left(884b^6\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{33649a^4}$$

$$=\frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(884b^7\right)\text{Subst}\left(\int\frac{1}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{100947a^5}$$

$$=\frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(884b^7\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{100947a^5\sqrt{b\sqrt[3]{x}+ax}}$$

$$=\frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(1768b^7\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{100947a^5\sqrt{b\sqrt[3]{x}+ax}}$$

$$=\frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{884b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{100947a^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.48

$$\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\left(3315b^4-7293ab^3x^{2/3}+12155a^2b^2x^{4/3}-17765a^3bx^2+\right.\right.}{216315a^5\sqrt{1+\frac{ax^{2/3}}{b}}}$$

```
[In] Integrate[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]*(3315*b
^4 - 7293*a*b^3*x^(2/3) + 12155*a^2*b^2*x^(4/3) - 17765*a^3*b*x^2 + 24035*a
^4*x^(8/3)) - 3315*b^6*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/b)])
)/(216315*a^5*Sqrt[1 + (a*x^(2/3))/b])
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.66

method	result
default	$\frac{\frac{1126x^{\frac{11}{3}}a^6b^2}{3933}+\frac{104x^{\frac{13}{3}}a^7b}{207}-\frac{16a^5b^3x^3}{19665}-\frac{3536x^{\frac{5}{3}}a^3b^5}{1514205}+\frac{272x^{\frac{7}{3}}a^4b^4}{216315}+\frac{2x^5a^8}{9}-\frac{884b^7\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{}}{100947}}{a^6\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$\frac{2a\,x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9}+\frac{58b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207}+\frac{8b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a}-\frac{136b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665a^2}+\frac{1768b^4x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315a^3}-176$

```
[In] int(x^2*(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1514205*(216755*x^(11/3)*a^6*b^2+380380*x^(13/3)*a^7*b-616*a^5*b^3*x^3-17
68*x^(5/3)*a^3*b^5+952*x^(7/3)*a^4*b^4+168245*x^5*a^8-6630*b^7*(-a*b)^(1/2)
*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+5304*a^2*b^6*x+13260*x^(1/3)
*a*b^7)/a^6/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Fricas [F]

$$\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} \, dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2 \, dx$$

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] integral((a*x^3 + b*x^(7/3))*sqrt(a*x + b*x^(1/3)), x)
```

Sympy [F]

$$\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} \, dx=\int x^{2} \left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate(x**2*(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(x**2*(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} \, dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2 \, dx$$

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} \, dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2 \, dx$$

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int x^2\,\left(a\,x+b\,x^{1/3}\right)^{3/2}\,dx$$

```
[In] int(x^2*(a*x + b*x^(1/3))^(3/2),x)
[Out] int(x^2*(a*x + b*x^(1/3))^(3/2), x)
```

3.141 $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 408

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=-\frac{88b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}$$
$$-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}$$
$$+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{88b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105a^{15/4}\sqrt{b\sqrt[3]{x}+ax}}-\frac{44b^{21/4}\left(\sqrt{b}+\right.}{}$$

```
[Out] 2/7*x^2*(b*x^(1/3)+a*x)^(3/2)-88/1105*b^5*(b+a*x^(2/3))*x^(1/3)/a^(7/2)/(x^
(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+88/3315*b^4*x^(1/3)*(b*x^(1/3)
+a*x)^(1/2)/a^3-88/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^2+24/1547*b^2*x^(5/3)
*(b*x^(1/3)+a*x)^(1/2)/a+12/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)+88/1105*b^(
21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))
,1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(
1/2))^2)^(1/2)/a^(15/4)/(b*x^(1/3)+a*x)^(1/2)-44/1105*b^(21/4)*x^(1/6)*(cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(
1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(
15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2043, 2046, 2049, 2057, 335, 311, 226, 1210}

$$\begin{aligned}\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}dx = &-\frac{44b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &+\frac{88b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &-\frac{88b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105a^{7/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{88b^4\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{3315a^3}-\frac{88b^3x\sqrt{ax+b\sqrt[3]{x}}}{4641a^2}\\ &+\frac{24b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}+\frac{2}{7}x^2\left(ax+b\sqrt[3]{x}\right)^{3/2}\end{aligned}$$

```
[In] Int[x*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-88*b^5*(b + a*x^(2/3))*x^(1/3))/(1105*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))
*Sqrt[b*x^(1/3) + a*x]) + (88*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(3315*a^3)
 - (88*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^2) + (24*b^2*x^(5/3)*Sqrt[b*x^(
1/3) + a*x])/(1547*a) + (12*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/119 + (2*x^2*(
b*x^(1/3) + a*x)^(3/2))/7 + (88*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(
b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a
^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) - (44
*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[
a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(1105*a^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int x^5\left(bx+ax^3\right)^{3/2}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{7}(6b)\text{Subst}\left(\int x^6\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{119}\left(12b^2\right)\text{Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}\\
&\quad+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(132b^3\right)\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{1547a}\\
&= -\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}\\
&\quad+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{\left(44b^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{663a^2}\\
&= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}\\
&\quad+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(44b^5\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{1105a^3}\\
&= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}\\
&\quad+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(44b^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{1105a^3\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}\\
&\quad+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(88b^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{1105a^3\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3} - \frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2} \\
&\quad + \frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a} + \frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax} \\
&\quad + \frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2} - \frac{\left(88b^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{1105a^{7/2}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad + \frac{\left(88b^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{1105a^{7/2}\sqrt{b\sqrt[3]{x}+ax}} \\
&= -\frac{88b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3} \\
&\quad - \frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2} + \frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a} + \frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax} \\
&\quad + \frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{88b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105a^{15/4}\sqrt{b\sqrt[3]{x}+ax}} - 44b
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.30

$$
\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx = \frac{2\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\left(\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\left(77b^2-143abx^{2/3}+221a^2x^{4/3}\right)-77b^4\,\text{Hypergeom}\right.}{1547a^3\sqrt{1+\frac{ax^{2/3}}{b}}}
$$

```
[In] Integrate[x*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]
*(77*b^2 - 143*a*b*x^(2/3) + 221*a^2*x^(4/3)) - 77*b^4*Hypergeometric2F1[-3
/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(1547*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.64

method	result
default	$\frac{\frac{622x^{\frac{8}{3}}a^4b^2}{1547}+\frac{80x^{\frac{10}{3}}a^5b}{119}-\frac{16a^3b^3x^2}{4641}-\frac{88b^6\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{1105}+\frac{44b^6\sqrt{\frac{a\,x^{\frac{1}{3}}}{}}}{}}{a^4\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$ $44b^5\sqrt{-}$
derivativedivides	$\frac{2a\,x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7}+\frac{46b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119}+\frac{24b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a}-\frac{88b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^2}+\frac{88b^4x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{3315a^3}-$

```
[In] int(x*(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/23205/a^4*(4665*x^(8/3)*a^4*b^2+7800*x^(10/3)*a^5*b-40*a^3*b^3*x^2-924*b^
6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticE(((a*x^(1/3)
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+462*b^6*((a*x^(1/3)+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))+3315*a^6*x^4+308*x^(2/3)*a*b^5+88*x^(4/3)*a^2*b^
4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Fricas [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x^2 + b*x^(4/3))*sqrt(a*x + b*x^(1/3)), x)
```

Sympy [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int x\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(x*(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x, x)
```

Giac [F]

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int x\,\left(a\,x+b\,x^{1/3}\right)^{3/2}\,dx$$

```
[In] int(x*(a*x + b*x^(1/3))^(3/2),x)
[Out] int(x*(a*x + b*x^(1/3))^(3/2), x)
```

3.142 $\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 208

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=-\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{4b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] 2/5*x*(b*x^(1/3)+a*x)^(3/2)-8/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^2+24/385*b^2*x
^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a+12/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)+4/77*b
^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)
)),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b
^(1/2))^2)^(1/2)/a^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2029, 2043, 2046, 2049, 2036, 335, 226}

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{4b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}+\frac{2}{5}x\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

```
[In] Int[(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-8*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^2) + (24*b^2*x^(2/3)*Sqrt[b*x^(1/3) +
a*x])/(385*a) + (12*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/55 + (2*x*(b*x^(1/3) +
 a*x)^(3/2))/5 + (4*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(77*a^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
```

```
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{5}(2b)\int\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\,dx\\
&= \frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{5}(6b)\text{Subst}\left(\int x^3\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{55}\left(12b^2\right)\text{Subst}\left(\int\frac{x^4}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}\\
&\quad+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(12b^3\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{77a}\\
&= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}\\
&\quad+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{\left(4b^4\right)\text{Subst}\left(\int\frac{1}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{77a^2}\\
&= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}\\
&\quad+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{\left(4b^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{77a^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}\\
&\quad+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{\left(8b^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{77a^2\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2} + \frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a} + \frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}$$
$$+ \frac{4b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.51

$$\int \left(b\sqrt[3]{x} +ax\right)^{3/2}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-\left(\left(5b-11ax^{2/3}\right)\left(b+ax^{2/3}\right)^2\sqrt{1+\frac{ax^{2/3}}{b}}\right)+5b^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\right.\right.}{55a^2\sqrt{1+\frac{ax^{2/3}}{b}}}$$

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-((5*b - 11*a*x^(2/3))*(b + a*x^(2/3))^2*Sqrt[1 +
 (a*x^(2/3))/b]) + 5*b^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/b)
]))/(55*a^2*Sqrt[1 + (a*x^(2/3))/b])
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.78

method	result
default	$\frac{\frac{262a^3b^2x^{\frac{5}{3}}}{385}+\frac{56a^4b\,x^{\frac{7}{3}}}{55}+\frac{4b^4\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{77}-\frac{16a^2b^3x}{385}+\frac{2a^5x^3}{5}}{a^3\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$\frac{2a\,x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5}+\frac{34b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55}+\frac{24b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a}-\frac{8b^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^2}+\frac{4b^4\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{}$

```
[In] int((b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/385*(131*a^3*b^2*x^(5/3)+196*a^4*b*x^(7/3)+10*b^4*(-a*b)^(1/2)*((a*x^(1/3
)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2
```

```
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-8*a^2*b^3*x+77*a^5*x^3-20*a*b^4*x^(1/3)
)/a^3/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Fricas [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] integral((a*x + b*x^(1/3))^(3/2), x)
```

Sympy [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral((a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2), x)
```

Giac [F]

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.19

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx=\frac{2\,x\,\left(a\,x+b\,x^{1/3}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{9}{4};\ \frac{13}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{3\,{\left(\frac{a\,x^{2/3}}{b}+1\right)}^{3/2}}$$

```
[In] int((a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] (2*x*(a*x + b*x^(1/3))^(3/2)*hypergeom([-3/2, 9/4], 13/4, -(a*x^(2/3))/b))/
(3*((a*x^(2/3))/b + 1)^(3/2))
```

3.143 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 319

$$\begin{aligned}
\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx &= \frac{8b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5\sqrt{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} \\
&\quad+\frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{8b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad+\frac{4b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{3/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

```
[Out] 2/3*(b*x^(1/3)+a*x)^(3/2)+8/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(1/2)/(x^(1/3)*a^
(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+4/5*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)-8/
5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/
4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)
+b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^(1/3)+a*x)^(1/2)+4/5*b^(9/4)*x^(1/6)*(cos(2
*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(
1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/
3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(3/
4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2046, 2029, 2057, 335, 311, 226, 1210}

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\frac{4b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

$$-\frac{8b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

$$+\frac{8b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5\sqrt{a}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{4}{5}b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}+\frac{2}{3}\left(ax+b\sqrt[3]{x}\right)^{3/2}$$

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

```
[Out] (8*b^2*(b + a*x^(2/3))*x^(1/3))/(5*Sqrt[a]*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt
[b*x^(1/3) + a*x]) + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/5 + (2*(b*x^(1/3)
+ a*x)^(3/2))/3 - (8*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (4*b^(9/4)*(Sqrt[b]
 + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(
1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b
*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + (2b)\text{Subst}\left(\int \sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{1}{5}\left(4b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{\left(4b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{\left(8b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5\sqrt{b\sqrt[3]{x}+ax}}$$

$$\begin{aligned}= {} & \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} \\ & + \frac{\left(8b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5\sqrt{a}\sqrt{b\sqrt[3]{x}+ax}} \\ & - \frac{\left(8b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5\sqrt{a}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

$$\begin{aligned}= {} & \frac{8b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5\sqrt{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} \\ & - \frac{8b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{b\sqrt[3]{x}+ax}} \\ & + \frac{4b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.19

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx = \frac{2b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}}$$

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

```
[Out] (2*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((a*x
^(2/3))/b)])/Sqrt[1 + (a*x^(2/3))/b]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{2ax\sqrt{b\,x^{\frac{1}{3}}+ax}}{3}+\frac{22b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15}+\frac{4b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\left(x\right.}\right.}{}\right.}{5a\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$\frac{\frac{8b^{3}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{5}-\frac{4b^{3}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F}{5}}{a\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a*x*(b*x^(1/3)+a*x)^(1/2)+22/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)+4/5*b^2
/a*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1
/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)
/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1
/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3
)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x, x)
```

Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x,x)
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x}\,dx$$

```
[In] int((a*x + b*x^(1/3))^(3/2)/x,x)
[Out] int((a*x + b*x^(1/3))^(3/2)/x, x)
```

3.144 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 144

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = 4a\sqrt{b\sqrt[3]{x}+ax}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{4a^{3/4}b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] -2*(b*x^(1/3)+a*x)^(3/2)/x+4*a*(b*x^(1/3)+a*x)^(1/2)+4*a^(3/4)*b^(3/4)*x^(1
/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/
2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(
1/2)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2043, 2045, 2046, 2036, 335, 226}

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = \frac{4a^{3/4}b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt{ax+b\sqrt[3]{x}}} - \frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x}+4a\sqrt{ax+b\sqrt[3]{x}}$$

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] 4*a*Sqrt[b*x^(1/3) + a*x] - (2*(b*x^(1/3) + a*x)^(3/2))/x + (4*a^(3/4)*b^(3
/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(
1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/Sqrt[
b*x^(1/3) + a*x]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
```

```
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{(bx+ax^3)^{3/2}}{x^4}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + (6a)\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x}\,dx, x, \sqrt[3]{x}\right)\\
&= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + (4ab)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{\left(4ab\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}\\
&= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{\left(8ab\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}\\
&= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\\
&\quad + \frac{4a^{3/4}b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = -\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{2/3}}$$

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((a*x^(2/3)
)/b)])/(Sqrt[1 + (a*x^(2/3))/b]*x^(2/3))
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90

method	result
default	$\frac{4x^{\frac{1}{3}}\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b+2x^{\frac{4}{3}}a^{2}-2b^{2}}{x^{\frac{1}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{x^{\frac{2}{3}}}+2a\sqrt{b\,x^{\frac{1}{3}}+ax}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\right.}{\sqrt{-a}}}\right.}{\sqrt{b\,x^{\frac{1}{3}}+ax}}$

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/x^(1/3)*(2*x^(1/3)*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1
/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*b+x^(4/3)*a^2-b^2)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^2}\,dx$$

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^2,x)
[Out] int((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

3.145 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 350

$$\begin{aligned}\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx &= \frac{8a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\ &- \frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} \\ &- \frac{8a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}} \\ &+ \frac{4a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] -2/3*(b*x^(1/3)+a*x)^(3/2)/x^2+8/5*a^(5/2)*(b+a*x^(2/3))*x^(1/3)/b/(x^(1/3)
*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-4/5*a*(b*x^(1/3)+a*x)^(1/2)/x-8/5*a
^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3)-8/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Ellipti
cE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(
1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+
a*x)^(1/2)+4/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(
1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*
x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^
(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2043, 2045, 2050, 2057, 335, 311, 226, 1210}

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\frac{4a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$
$$-\frac{8a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$
$$+\frac{8a^{5/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5b\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{8a^2\sqrt{ax+b\sqrt[3]{x}}}{5b\sqrt[3]{x}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{3x^2}-\frac{4a\sqrt{ax+b\sqrt[3]{x}}}{5x}$$

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (8*a^(5/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b
*x^(1/3) + a*x]) - (4*a*Sqrt[b*x^(1/3) + a*x])/(5*x) - (8*a^2*Sqrt[b*x^(1/3
) + a*x])/(5*b*x^(1/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(3*x^2) - (8*a^(9/4)*
(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3)
)^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4
)*Sqrt[b*x^(1/3) + a*x]) + (4*a^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b +
 a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1
/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + (2a)\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{1}{5}\left(4a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(4a^3\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5b}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(4a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{5b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(8a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(8a^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(8a^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{8a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} - \frac{8a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}} + \frac{4a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.18

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx = -\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2},-\frac{5}{4},-\frac{ax^{2/3}}{b}\right)}{3\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{5/3}}$$

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((a*x^(2/3
))/b)])/(3*Sqrt[1 + (a*x^(2/3))/b]*x^(5/3))
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.67

method	result
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{3x^{\frac{5}{3}}}-\frac{22a\sqrt{b\,x^{\frac{1}{3}}+ax}}{15x}-\frac{8\left(b+a\,x^{\frac{2}{3}}\right)a^2}{5b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x}{\sqrt{}}}}{}$
default	$-\frac{2\left(-12a^2b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+6a^2b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{}}{\sqrt{-a}}}\right.}{}$

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*b*(b*x^(1/3)+a*x)^(1/2)/x^(5/3)-22/15*a*(b*x^(1/3)+a*x)^(1/2)/x-8/5*(b
+a*x^(2/3))*a^2/b/(x^(1/3)*(b+a*x^(2/3)))^(1/2)+4/5/b*a^2*(-a*b)^(1/2)*((x^
(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2)
)*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/
2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))
^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*
a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**3,x)
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}{x^3}\,dx$$

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^3,x)
[Out] int((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

3.146 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 213

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx = -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}} + \frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3} + \frac{4a^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] -2/5*(b*x^(1/3)+a*x)^(3/2)/x^3-12/55*a*(b*x^(1/3)+a*x)^(1/2)/x^2-24/385*a^2
*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+8/77*a^3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)
+4/77*a^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used

$= \{2043, 2045, 2050, 2036, 335, 226\}$

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\frac{4a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}$$
$$+\frac{8a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}}-\frac{24a^2\sqrt{ax+b\sqrt[3]{x}}}{385bx^{4/3}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{5x^3}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{55x^2}$$

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-12*a*Sqrt[b*x^(1/3) + a*x])/(55*x^2) - (24*a^2*Sqrt[b*x^(1/3) + a*x])/(38
5*b*x^(4/3)) + (8*a^3*Sqrt[b*x^(1/3) + a*x])/(77*b^2*x^(2/3)) - (2*(b*x^(1/
3) + a*x)^(3/2))/(5*x^3) + (4*a^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
```

```
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{1}{5}(6a)\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^7}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{1}{55}\left(12a^2\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}\\
&\quad-\frac{\left(12a^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{77b}\\
&= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}\\
&\quad-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{\left(4a^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{77b^2}\\
&= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}\\
&\quad-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{\left(4a^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{77b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}\\
&\quad-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{\left(8a^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{77b^2\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}\\
&+\frac{4a^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.29

$$
\int\frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{15}{4},-\frac{3}{2},-\frac{11}{4},-\frac{ax^{2/3}}{b}\right)}{5\sqrt{1+\frac{ax^{2/3}}{b}}x^{8/3}}
$$

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-15/4, -3/2, -11/4, -((a*x^(2
/3))/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x^(8/3))
```

Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.79

method	result
default	$\frac{\frac{4a^{3}\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{14}{3}}}{77}-\frac{262x^{\frac{11}{3}}a^{2}b^{2}}{385}+\frac{16x^{\frac{13}{3}}a^{3}b}{385}-\frac{56a\,b^{3}x^{3}}{55}+}{b^{2}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{14}{3}}}$
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{5x^{\frac{8}{3}}}-\frac{34a\sqrt{b\,x^{\frac{1}{3}}+ax}}{55x^{2}}-\frac{24a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385b\,x^{\frac{4}{3}}}+\frac{8a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^{2}x^{\frac{2}{3}}}+\frac{4a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-}}{77}$

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/385*(10*a^3*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-
2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1
/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^
(14/3)-131*x^(11/3)*a^2*b^2+8*x^(13/3)*a^3*b-196*a*b^3*x^3+20*a^4*x^5-77*x^
(7/3)*b^4)/b^2/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(14/3)
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**4,x)
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}{x^4}\,dx$$

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^4,x)
[Out] int((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

3.147 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 438

$$\begin{aligned}
\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx &= -\frac{88a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}}+\frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} \\
&\quad -\frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x}+\frac{88a^5\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} \\
&\quad +\frac{88a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad -\frac{44a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

```
[Out] -2/7*(b*x^(1/3)+a*x)^(3/2)/x^4-88/1105*a^(11/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(
x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-12/119*a*(b*x^(1/3)+a*x)^(1/
2)/x^3-24/1547*a^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+88/4641*a^3*(b*x^(1/3)+a
*x)^(1/2)/b^2/x^(5/3)-88/3315*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x+88/1105*a^5*(
b*x^(1/3)+a*x)^(1/2)/b^4/x^(1/3)+88/1105*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(
1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Elli
pticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+
b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1
/3)+a*x)^(1/2)-44/1105*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/
```

```
4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(
2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2045, 2050, 2057, 335, 311, 226, 1210}

$$\begin{aligned}\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx = & -\frac{44a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}\\ & +\frac{88a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}\\ & -\frac{88a^{11/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{88a^5\sqrt{ax+b\sqrt[3]{x}}}{1105b^4\sqrt[3]{x}}-\frac{88a^4\sqrt{ax+b\sqrt[3]{x}}}{3315b^3x}\\ & +\frac{88a^3\sqrt{ax+b\sqrt[3]{x}}}{4641b^2x^{5/3}}-\frac{24a^2\sqrt{ax+b\sqrt[3]{x}}}{1547bx^{7/3}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{7x^4}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{119x^3}\end{aligned}$$

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (-88*a^(11/2)*(b + a*x^(2/3))*x^(1/3))/(1105*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) - (12*a*Sqrt[b*x^(1/3) + a*x])/(119*x^3) - (24*a^2
*Sqrt[b*x^(1/3) + a*x])/(1547*b*x^(7/3)) + (88*a^3*Sqrt[b*x^(1/3) + a*x])/(
4641*b^2*x^(5/3)) - (88*a^4*Sqrt[b*x^(1/3) + a*x])/(3315*b^3*x) + (88*a^5*S
qrt[b*x^(1/3) + a*x])/(1105*b^4*x^(1/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(7*x
^4) + (88*a^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b
] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4
)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a*x]) - (44*a^(21/4)*(Sqrt[b] + S
qrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)
*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*b^(15/4)*Sqrt[b
*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} + \frac{1}{7}(6a)\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} + \frac{1}{119}\left(12a^2\right)\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} - \frac{\left(132a^3\right)\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1547b}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} + \frac{\left(44a^4\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{663b^2}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} - \frac{\left(44a^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1105b^3}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} - \frac{\left(44a^6\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1105b^4}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} - \frac{\left(44a^6\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{1105b^4\sqrt{b\sqrt[3]{x}+ax}}$$

$$
\begin{aligned}
&= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} \\
&\quad + \frac{88a^5\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} - \frac{\left(88a^6\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{1105b^4\sqrt{b\sqrt[3]{x}+ax}} \\
&= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} \\
&\quad - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} \\
&\quad - \frac{\left(88a^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{1105b^{7/2}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad + \frac{\left(88a^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{1105b^{7/2}\sqrt{b\sqrt[3]{x}+ax}} \\
&= -\frac{88a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} \\
&\quad + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} \\
&\quad + \frac{88a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{44a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.14

$$
\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx = -\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{21}{4}, -\frac{3}{2}, -\frac{17}{4}, -\frac{ax^{2/3}}{b}\right)}{7\sqrt{1+\frac{ax^{2/3}}{b}}x^{11/3}}
$$

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-21/4, -3/2, -17/4, -((a*x^(2
/3))/b)])/(7*Sqrt[1 + (a*x^(2/3))/b]*x^(11/3))
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.69

method	result
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{7x^{\frac{11}{3}}}-\frac{46a\sqrt{b\,x^{\frac{1}{3}}+ax}}{119x^{3}}-\frac{24a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547b\,x^{\frac{7}{3}}}+\frac{88a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641b^{2}x^{\frac{5}{3}}}-\frac{88a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{3315b^{3}x}+\frac{88\left(b+a\,x^{\frac{2}{3}}\right)}{1105b^{4}\sqrt{x^{\frac{1}{3}}\left(b+\right.}}$
default	$-\frac{88a^{5}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{20}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{1105}+\frac{44a^{5}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-}}{}$

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*b*(b*x^(1/3)+a*x)^(1/2)/x^(11/3)-46/119*a*(b*x^(1/3)+a*x)^(1/2)/x^3-24
/1547*a^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+88/4641*a^3*(b*x^(1/3)+a*x)^(1/2)
/b^2/x^(5/3)-88/3315*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x+88/1105*(b+a*x^(2/3))*
a^5/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-44/1105*a^5/b^4*(-a*b)^(1/2)*((x^(1/3
)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/
(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(
-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^{5}}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

Sympy [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**5,x)
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^5}\,dx$$

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^5,x)
[Out] int((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

3.148 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 301

$$\begin{aligned}\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx = &-\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4}-\frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}}+\frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}}\\ &-\frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2}+\frac{1768a^5\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}}-\frac{1768a^6\sqrt{b\sqrt[3]{x}+ax}}{100947b^5x^{2/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5}\\ &-\frac{884a^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{100947b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] -2/9*(b*x^(1/3)+a*x)^(3/2)/x^5-4/69*a*(b*x^(1/3)+a*x)^(1/2)/x^4-8/1311*a^2*
(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+136/19665*a^3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^
(8/3)-1768/216315*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2+1768/168245*a^5*(b*x^(1
/3)+a*x)^(1/2)/b^4/x^(4/3)-1768/100947*a^6*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3
)-884/100947*a^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(
1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2043, 2045, 2050, 2036, 335, 226}

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx =$$
$$-\frac{884a^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{100947b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}$$
$$-\frac{1768a^6\sqrt{ax+b\sqrt[3]{x}}}{100947b^5x^{2/3}}+\frac{1768a^5\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}-\frac{1768a^4\sqrt{ax+b\sqrt[3]{x}}}{216315b^3x^2}$$
$$+\frac{136a^3\sqrt{ax+b\sqrt[3]{x}}}{19665b^2x^{8/3}}-\frac{8a^2\sqrt{ax+b\sqrt[3]{x}}}{1311bx^{10/3}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{9x^5}-\frac{4a\sqrt{ax+b\sqrt[3]{x}}}{69x^4}$$

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-4*a*Sqrt[b*x^(1/3) + a*x])/(69*x^4) - (8*a^2*Sqrt[b*x^(1/3) + a*x])/(1311
*b*x^(10/3)) + (136*a^3*Sqrt[b*x^(1/3) + a*x])/(19665*b^2*x^(8/3)) - (1768*
a^4*Sqrt[b*x^(1/3) + a*x])/(216315*b^3*x^2) + (1768*a^5*Sqrt[b*x^(1/3) + a*
x])/(168245*b^4*x^(4/3)) - (1768*a^6*Sqrt[b*x^(1/3) + a*x])/(100947*b^5*x^(
2/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(9*x^5) - (884*a^(27/4)*(Sqrt[b] + Sqrt
[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*El
lipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*b^(21/4)*Sqrt[b*
x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{16}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5}+\frac{1}{3}(2a)\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5}+\frac{1}{69}\left(4a^2\right)\text{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4}-\frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5}\\
&\quad -\frac{\left(68a^3\right)\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1311b}\\
&= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4}-\frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}}+\frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}}\\
&\quad -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5}+\frac{\left(884a^4\right)\text{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{19665b^2}
\end{aligned}
$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} - \frac{(884a^5)\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{24035b^3}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} + \frac{(884a^6)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{33649b^4}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} - \frac{1768a^6\sqrt{b\sqrt[3]{x}+ax}}{100947b^5x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} - \frac{(884a^7)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{100947b^5}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} - \frac{1768a^6\sqrt{b\sqrt[3]{x}+ax}}{100947b^5x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} - \frac{\left(884a^7\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{100947b^5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} - \frac{1768a^6\sqrt{b\sqrt[3]{x}+ax}}{100947b^5x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} - \frac{\left(1768a^7\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{100947b^5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}} - \frac{1768a^6\sqrt{b\sqrt[3]{x}+ax}}{100947b^5x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} - \frac{884a^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{100947b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.21

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=-\frac{2b\sqrt{b\sqrt[3]{x}+ax}\,\text{Hypergeometric2F1}\left(-\frac{27}{4},-\frac{3}{2},-\frac{23}{4},-\frac{ax^{2/3}}{b}\right)}{9\sqrt{1+\frac{ax^{2/3}}{b}}\,x^{14/3}}$$

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-27/4, -3/2, -23/4, -((a*x^(2
/3))/b)])/(9*Sqrt[1 + (a*x^(2/3))/b]*x^(14/3))
```

Maple [A] (verified)

Time = 5.86 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.67

method	result
default	$-\frac{2\left(6630a^6\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{26}{3}}-1768x^{\frac{23}{3}}a^5b^2+5304x^{\frac{25}{3}}\right.}{1514205b^5\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$
derivativedivides	$-\frac{2b\sqrt{b\,x^{\frac{1}{3}}+ax}}{9x^{\frac{14}{3}}}-\frac{58a\sqrt{b\,x^{\frac{1}{3}}+ax}}{207x^4}-\frac{8a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311b\,x^{\frac{10}{3}}}+\frac{136a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{19665b^2x^{\frac{8}{3}}}-\frac{1768a^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{216315b^3x^2}+\frac{1768a^5\sqrt{b\,x^{\frac{1}{3}}}}{168245b^4x}$

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -2/1514205*(6630*a^6*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1
/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*x^(26/3)-1768*x^(23/3)*a^5*b^2+5304*x^(25/3)*a^6*b+952*a^4*b^3*x^7+216
755*x^(17/3)*a^2*b^5-616*x^(19/3)*a^3*b^4+380380*a*b^6*x^5+13260*a^7*x^9+16
8245*x^(13/3)*b^7)/b^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(26/3)
```

Fricas [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\text{Timed out}$$

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**6,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx=\int \frac{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}{x^6}\,dx$$

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^6,x)
[Out] int((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

3.149 $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 304

$$\begin{aligned}
\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx
&= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} \\
&\quad + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} \\
&\quad + \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} \\
&\quad - \frac{5525b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14421a^{29/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

```
[Out] 11050/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^7-2210/4807*b^5*x^(2/3)*(b*x^(1/3)+
a*x)^(1/2)/a^6+15470/43263*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-1190/3933*
b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^4+350/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2
)/a^3-50/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/9*x^4*(b*x^(1/3)+a*x)^(
1/2)/a-5525/14421*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1
/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))
/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(29/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2043, 2049, 2036, 335, 226}

$$\begin{aligned}\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx &= -\frac{5525b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14421a^{29/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &\quad+\frac{11050b^6\sqrt{ax+b\sqrt[3]{x}}}{14421a^7}-\frac{2210b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{4807a^6}\\ &\quad+\frac{15470b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{43263a^5}-\frac{1190b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{3933a^4}\\ &\quad+\frac{350b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a^3}-\frac{50bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}{207a^2}+\frac{2x^4\sqrt{ax+b\sqrt[3]{x}}}{9a}\end{aligned}$$

```
[In] Int[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (11050*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^7) - (2210*b^5*x^(2/3)*Sqrt[b*x^
(1/3) + a*x])/(4807*a^6) + (15470*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(43263
*a^5) - (1190*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(3933*a^4) + (350*b^2*x^(8/3)*
Sqrt[b*x^(1/3) + a*x])/(1311*a^3) - (50*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(
207*a^2) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/(9*a) - (5525*b^(27/4)*(Sqrt[b] +
Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6
)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(29/4)*Sqrt
[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^{14}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} - \frac{(25b)\text{Subst}\left(\int \frac{x^{12}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{9a}\\
&= -\frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} + \frac{(175b^2)\,\text{Subst}\left(\int \frac{x^{10}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{69a^2}\\
&= \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2}\\
&\quad + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} - \frac{(2975b^3)\,\text{Subst}\left(\int \frac{x^{8}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1311a^3}\\
&= -\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} + \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2}\\
&\quad + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} + \frac{(7735b^4)\,\text{Subst}\left(\int \frac{x^{6}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{3933a^4}\\
&= \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} + \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3}\\
&\quad - \frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} - \frac{(7735b^5)\,\text{Subst}\left(\int \frac{x^{4}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{4807a^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6}+\frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} \\
&\quad -\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4}+\frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3}-\frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} \\
&\quad +\frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a}+\frac{(5525b^6)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{4807a^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7}-\frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6}+\frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} \\
&\quad -\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4}+\frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3}-\frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} \\
&\quad +\frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a}-\frac{(5525b^7)\,\text{Subst}\left(\int\frac{1}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{14421a^7}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7}-\frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6}+\frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} \\
&\quad -\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4}+\frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3}-\frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} \\
&\quad +\frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a}-\frac{\left(5525b^7\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{14421a^7\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7}-\frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6}+\frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} \\
&\quad -\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4}+\frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3}-\frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} \\
&\quad +\frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a}-\frac{\left(11050b^7\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{14421a^7\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7}-\frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} \\
&\quad +\frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5}-\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} \\
&\quad +\frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3}-\frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2}+\frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} \\
&\quad -\frac{5525b^{27/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14421a^{29/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(16575b^7+6630ab^6x^{2/3}-2210a^2b^5x^{4/3}+1190a^3b^4x^2-770a^4b^3x^{8/3}+550a^5b^2x^{10/3}-418a^6\right.}{43263a^7\left(b+ax^{2/3}\right)}$$

```
[In] Integrate[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(16575*b^7 + 6630*a*b^6*x^(2/3) - 2210*a^2*b^5*x^(
4/3) + 1190*a^3*b^4*x^2 - 770*a^4*b^3*x^(8/3) + 550*a^5*b^2*x^(10/3) - 418*
a^6*b*x^4 + 4807*a^7*x^(14/3) - 16575*b^7*Sqrt[1 + (a*x^(2/3))/b]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(43263*a^7*(b + a*x^(2/3)))
```

Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.64

method	result
default	$-\frac{-1100x^{\frac{11}{3}}a^6b^2+836x^{\frac{13}{3}}a^7b+1540a^5b^3x^3+4420x^{\frac{5}{3}}a^3b^5-2380x^{\frac{7}{3}}a^4b^4-9614x^5a^8+16575b^7\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-}}{43263\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^8}$
derivativedivides	$\frac{2x^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{9a}-\frac{50b\,x^{\frac{10}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{207a^2}+\frac{350b^2x^{\frac{8}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1311a^3}-\frac{1190b^3x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{3933a^4}+\frac{15470b^4x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{43263a^5}-$

```
[In] int(x^4/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/43263*(-1100*x^(11/3)*a^6*b^2+836*x^(13/3)*a^7*b+1540*a^5*b^3*x^3+4420*x
^(5/3)*a^3*b^5-2380*x^(7/3)*a^4*b^4-9614*x^5*a^8+16575*b^7*(-a*b)^(1/2)*((a
*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-13260*a^2*b^6*x-33150*x^(1/3)*a*
b^7)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^8
```

Fricas [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^4}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] integral((a^2*x^5 - a*b*x^(13/3) + b^2*x^(11/3))*sqrt(a*x + b*x^(1/3))/(a^3
*x^2 + b^3), x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^4}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

```
[In] integrate(x**4/(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(x**4/sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^4}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^4}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^4}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

```
[In] int(x^4/(a*x + b*x^(1/3))^(1/2),x)
[Out] int(x^4/(a*x + b*x^(1/3))^(1/2), x)
```

3.150 $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 414

$$\begin{aligned}
&\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx \\
&= -\frac{418b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{11/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5}-\frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} \\
&\quad+\frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3}-\frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2}+\frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} \\
&\quad+\frac{418b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{23/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad-\frac{209b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{221a^{23/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

```
[Out] -418/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(11/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^
(1/3)+a*x)^(1/2)+418/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-2090/4641*b^
3*x*(b*x^(1/3)+a*x)^(1/2)/a^4+570/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^
3-38/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/7*x^3*(b*x^(1/3)+a*x)^(1/2)/
a+418/221*b^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(23/4)/(b*x^(1/3)+a*x)^(1/2)-209/221*b^(21/4)
*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1
/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*
```

```
2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))
^2)^(1/2)/a^(23/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2043, 2049, 2057, 335, 311, 226, 1210}

$$
\begin{aligned}
&\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx \\
&= -\frac{209b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{221a^{23/4}\sqrt{ax+b\sqrt[3]{x}}} \\
&\quad+\frac{418b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{23/4}\sqrt{ax+b\sqrt[3]{x}}} \\
&\quad-\frac{418b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{11/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{418b^4\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{663a^5}-\frac{2090b^3x\sqrt{ax+b\sqrt[3]{x}}}{4641a^4} \\
&\quad+\frac{570b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^3}-\frac{38bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{119a^2}+\frac{2x^3\sqrt{ax+b\sqrt[3]{x}}}{7a}
\end{aligned}
$$

```
[In] Int[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-418*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(11/2)*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) + (418*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^5
) - (2090*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^4) + (570*b^2*x^(5/3)*Sqrt[b
*x^(1/3) + a*x])/(1547*a^3) - (38*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a^2
) + (2*x^3*Sqrt[b*x^(1/3) + a*x])/(7*a) + (418*b^(21/4)*(Sqrt[b] + Sqrt[a]*
x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Ellipt
icE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(23/4)*Sqrt[b*x^(1/3)
 + a*x]) - (209*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(
Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/
b^(1/4)], 1/2])/(221*a^(23/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^{11}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right) \\
&= \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} - \frac{(19b)\text{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7a} \\
&= -\frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} + \frac{\left(285b^2\right)\text{Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{119a^2} \\
&= \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} \\
&\quad + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} - \frac{\left(3135b^3\right)\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1547a^3} \\
&= -\frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} \\
&\quad + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} + \frac{\left(1045b^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{663a^4} \\
&= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} \\
&\quad - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} - \frac{\left(209b^5\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{221a^5} \\
&= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} \\
&\quad + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} \\
&\quad - \frac{\left(209b^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{221a^5\sqrt{b\sqrt[3]{x}+ax}} \\
&= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} \\
&\quad + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} \\
&\quad - \frac{\left(418b^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{221a^5\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} - \frac{\left(418b^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{221a^{11/2}\sqrt{b\sqrt[3]{x}+ax}} + \frac{\left(418b^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{221a^{11/2}\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{418b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{11/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} + \frac{418b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{23/4}\sqrt{b\sqrt[3]{x}+ax}} - \frac{209b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{23/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.35

$$\int\frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(1463b^5\sqrt[3]{x}+418ab^4x-190a^2b^3x^{5/3}+114a^3b^2x^{7/3}-78a^4bx^3+663a^5x^{11/3}-1463b^5\sqrt{1+\right.}{4641a^5\left(b+ax^{2/3}\right)}$$

```
[In] Integrate[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(1463*b^5*x^(1/3) + 418*a*b^4*x - 190*a^2*b^3*x^(5
/3) + 114*a^3*b^2*x^(7/3) - 78*a^4*b*x^3 + 663*a^5*x^(11/3) - 1463*b^5*Sqrt
[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/3))/
b)]))/(4641*a^5*(b + a*x^(2/3)))
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.63

method	result
default	$-\frac{-228x^{\frac{8}{3}}a^4b^2+156x^{\frac{10}{3}}a^5b+380a^3b^3x^2+8778b^6\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-}{4641a^6\sqrt{x^{\frac{1}{3}}}}$ $209b$
derivativedivides	$\frac{2x^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7a}-\frac{38b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a^2}+\frac{570b^2x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^3}-\frac{2090b^3x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^4}+\frac{418b^4x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663a^5}-$

```
[In] int(x^3/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4641/a^6*(-228*x^(8/3)*a^4*b^2+156*x^(10/3)*a^5*b+380*a^3*b^3*x^2+8778*b
^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticE(((a*x^(1/3
)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-4389*b^6*((a*x^(1/3)+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))-1326*a^6*x^4-2926*x^(2/3)*a*b^5-836*x^(4/3)*a^
2*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^3}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^4 - a*b*x^(10/3) + b^2*x^(8/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^2 + b^3), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^3}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

```
[In] integrate(x**3/(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(x**3/sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^3}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^3}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x^3}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

```
[In] int(x^3/(a*x + b*x^(1/3))^(1/2),x)
[Out] int(x^3/(a*x + b*x^(1/3))^(1/2), x)
```

3.151 $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 216

$$\begin{aligned}\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx &= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4}+\frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3}-\frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2}+\frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a}\\&\quad+\frac{39b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{17/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] -78/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^4+234/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1
/2)/a^3-26/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/5*x^2*(b*x^(1/3)+a*x)^(
1/2)/a+39/77*b^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(
1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(17/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used

$= \{2043, 2049, 2036, 335, 226\}$

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$$

$$= \frac{39b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{17/4}\sqrt{ax+b\sqrt[3]{x}}}$$

$$- \frac{78b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^3} - \frac{26bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{55a^2} + \frac{2x^2\sqrt{ax+b\sqrt[3]{x}}}{5a}$$

```
[In] Int[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-78*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^4) + (234*b^2*x^(2/3)*Sqrt[b*x^(1/3)
+ a*x])/(385*a^3) - (26*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(55*a^2) + (2*x^2*
Sqrt[b*x^(1/3) + a*x])/(5*a) + (39*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*a^(17/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{x^8}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} - \frac{(13b)\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5a}$$

$$= -\frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{(117b^2)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{55a^2}$$

$$= \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} - \frac{(117b^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{77a^3}$$

$$= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{(39b^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{77a^4}$$

$$= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{\left(39b^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{77a^4\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{\left(78b^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{77a^4\sqrt{b\sqrt[3]{x}+ax}}$$

$$
\begin{aligned}
&= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4}+\frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3}\\
&\quad-\frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2}+\frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a}\\
&\quad+\frac{39b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{17/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.57

$$
\int\frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx
$$

$$
=\frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-195b^4-78ab^3x^{2/3}+26a^2b^2x^{4/3}-14a^3bx^2+77a^4x^{8/3}+195b^4\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometr}\right.}{385a^4\left(b+ax^{2/3}\right)}
$$

```
[In] Integrate[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-195*b^4 - 78*a*b^3*x^(2/3) + 26*a^2*b^2*x^(4/3)
- 14*a^3*b*x^2 + 77*a^4*x^(8/3) + 195*b^4*Sqrt[1 + (a*x^(2/3))/b]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(385*a^4*(b + a*x^(2/3)))
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.75

method	result
default	$-\frac{-52a^3b^2x^{\frac{5}{3}}+28a^4b\,x^{\frac{7}{3}}-195b^4\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+156a^2b^3}{385\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^5}$
derivativedivides	$\frac{2x^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5a}-\frac{26b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a^2}+\frac{234b^2x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^3}-\frac{78b^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^4}+\frac{39b^4\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{}$

```
[In] int(x^2/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/385*(-52*a^3*b^2*x^(5/3)+28*a^4*b*x^(7/3)-195*b^4*(-a*b)^(1/2)*((a*x^(1/
3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1
```

```
/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+156*a^2*b^3*x-154*a^5*x^3+390*a*b^4*x^
(1/3))/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^5
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^2}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^3 - a*b*x^(7/3) + b^2*x^(5/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^2 + b^3), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^2}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

```
[In] integrate(x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^2}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^2}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x^2}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

```
[In] int(x^2/(a*x + b*x^(1/3))^(1/2),x)
[Out] int(x^2/(a*x + b*x^(1/3))^(1/2), x)
```

3.152 $\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 326

$$
\begin{aligned}
\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx
&= \frac{14b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{5/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} \\
&\quad - \frac{14b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad + \frac{7b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{11/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

```
[Out] 14/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(5/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)
+a*x)^(1/2)-14/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/3*x*(b*x^(1/3)+a*x)
^(1/2)/a-14/5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(
1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(11/4)/(b*x^(1/3)+a*x)^(1/2)+7/5*b^(9/4)*x
^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^
(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2
)^(1/2)/a^(11/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2043, 2049, 2057, 335, 311, 226, 1210}

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{7b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{14b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{14b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{5/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}} - \frac{14b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{15a^2} + \frac{2x\sqrt{ax+b\sqrt[3]{x}}}{3a}$$

```
[In] Int[x/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (14*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(5/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) - (14*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a^2) + (2*x*
Sqrt[b*x^(1/3) + a*x])/(3*a) - (14*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt
[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[
(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[b*x^(1/3) + a*x]) + (7*b
^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5
*a^(11/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} - \frac{(7b)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{3a}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{(7b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5a^2}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(7b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{5a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(14b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(14b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5a^{5/2}\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(14b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5a^{5/2}\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{14b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{5/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} - \frac{14b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{b\sqrt[3]{x}+ax}} + \frac{7b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.33

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(-7b^2\sqrt[3]{x}-2abx+5a^2x^{5/3}+7b^2\sqrt{1+\frac{ax^{2/3}}{b}}\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)\right)}{15a^2\left(b+ax^{2/3}\right)}$$

```
[In] Integrate[x/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-7*b^2*x^(1/3) - 2*a*b*x + 5*a^2*x^(5/3) + 7*b^2*
Sqrt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/
3))/b)]))/(15*a^2*(b + a*x^(2/3)))
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3a}-\frac{14b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a^{2}}+\frac{7b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\left(x^{\frac{1}{3}}\right.}\right.}{}\right.}{5a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{-42b^{3}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+21b^{3}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}}{15a^{3}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}$

```
[In] int(x/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x*(b*x^(1/3)+a*x)^(1/2)/a-14/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+7/5
*b^2/a^3*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2
*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))
^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((
x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^2 + b^3), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

```
[In] integrate(x/(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(x/sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{x}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

```
[In] int(x/(a*x + b*x^(1/3))^(1/2),x)
[Out] int(x/(a*x + b*x^(1/3))^(1/2), x)
```

3.153 $\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a} - \frac{b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] 2*(b*x^(1/3)+a*x)^(1/2)/a-b^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(
1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x
^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {2035, 2038, 2036, 335, 226}

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$$
$$=\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}$$
$$-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[In] Int[1/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x])/a - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(5/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2035

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Dist[a*((2*n - j - 2)/(b*(n -
2))), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{b\int\frac{1}{x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}\,dx}{3a}\\
&= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{b\text{Subst}\left(\int\frac{1}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{a}\\
&= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{\left(b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{a\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{\left(2b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{a\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{5/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$
\int\frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}\left(b+ax^{2/3}-b\sqrt{1+\frac{ax^{2/3}}{b}}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{ax^{2/3}}{b}\right)\right)}{a\left(b+ax^{2/3}\right)}
$$

```
[In] Integrate[1/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(b + a*x^(2/3) - b*Sqrt[1 + (a*x^(2/3))/b]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a*(b + a*x^(2/3)))
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{-b\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+2ab\,x^{\frac{1}{3}}+2a^{2}x}{\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{2}}$	127
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{a}-\frac{b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$	135

```
[In] int(1/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/
3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*Ellipt
icF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+2*a*b*x^(1/3
)+2*a^2*x)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^2
```

Fricas [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^3 + b^3*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

```
[In] integrate(1/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.32

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx=\frac{2\,x\,\sqrt{\frac{b}{a\,x^{2/3}}+1}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ -\frac{b}{a\,x^{2/3}}\right)}{\sqrt{a\,x+b\,x^{1/3}}}$$

```
[In] int(1/(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] (2*x*(b/(a*x^(2/3)) + 1)^(1/2)*hypergeom([-3/4, 1/2], 1/4, -b/(a*x^(2/3))))
/(a*x + b*x^(1/3))^(1/2)
```

3.154 $\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 294

$$
\begin{aligned}
\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx
&= \frac{6\sqrt{a}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} \\
&\quad - \frac{6\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad + \frac{3\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

```
[Out] 6*(b+a*x^(2/3))*x^(1/3)*a^(1/2)/b/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)
^(1/2)-6*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3)-6*a^(1/4)*x^(1/6)*(cos(2*arctan(a^
(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Ell
ipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)
+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1
/3)+a*x)^(1/2)+3*a^(1/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(
x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2043, 2050, 2057, 335, 311, 226, 1210}

$$\begin{aligned}\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx &= \frac{3\sqrt[4]{a}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &\quad-\frac{6\sqrt[4]{a}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}\\ &\quad+\frac{6\sqrt{a}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{b\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{b\sqrt[3]{x}}\end{aligned}$$

```
[In] Int[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (6*Sqrt[a]*(b + a*x^(2/3))*x^(1/3))/(b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x
^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) - (6*a^(1/4)*(Sqrt[b
] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^
(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*
x^(1/3) + a*x]) + (3*a^(1/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{(3a)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(3a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(6a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}} \\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(6\sqrt{a}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{\left(6\sqrt{a}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}} \\
&= \frac{6\sqrt{a}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} \\
&\quad - \frac{6\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad + \frac{3\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.18

$$
\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{ax^{2/3}}{b}\right)}{\sqrt{b\sqrt[3]{x}+ax}}
$$

```
[In] Integrate[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((a*x^(2/3))
/b)])/Sqrt[b*x^(1/3) + a*x]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.66

method	result
derivativedivides	$-\frac{6\left(b+a\,x^{\frac{2}{3}}\right)}{b\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}\,a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a}\right.}{b\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$\frac{6\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}\,a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b-3\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-}}{\sqrt{-ab}}}}{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)b}$

```
[In] int(1/x/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6*(b+a*x^(2/3))/b/(x^(1/3)*(b+a*x^(2/3)))^(1/2)+3/b*(-a*b)^(1/2)*((x^(1/3)
+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(
-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-
2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2
),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a
*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x}\,dx$$

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^4 + b^3*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

```
[In] integrate(1/x/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*x + b*x**(1/3))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x}\,dx$$

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x}\,dx$$

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

```
[In] int(1/(x*(a*x + b*x^(1/3))^(1/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(1/3))^(1/2)), x)
```

3.155 $\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 163

$$\begin{aligned}\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}}+\frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}}\\ &\quad+\frac{5a^{7/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{7b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] -6/7*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+10/7*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/
3)+5/7*a^(7/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used

= {2043, 2050, 2036, 335, 226}

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx$$
$$= \frac{5a^{7/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{7b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}$$
$$+\frac{10a\sqrt{ax+b\sqrt[3]{x}}}{7b^2x^{2/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}$$

```
[In] Int[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) + (10*a*Sqrt[b*x^(1/3) + a*x])/(7*
b^2*x^(2/3)) + (5*a^(7/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/
(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))
/b^(1/4)], 1/2])/(7*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} - \frac{(15a)\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{(5a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7b^2}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{\left(5a^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{7b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{\left(10a^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{7b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}}\\
&\quad + \frac{5a^{7/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},-\frac{ax^{2/3}}{b}\right)}{7x\sqrt{b\sqrt[3]{x}+ax}}$$

```
[In] Integrate[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((a*x^(2/3)
)/b)])/(7*x*Sqrt[b*x^(1/3) + a*x])
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

method	result
default	$\frac{5a\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{4}{3}}+4abx+10x^{\frac{5}{3}}a^{2}-6b^{2}x^{\frac{1}{3}}}{7b^{2}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{4}{3}}}$
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{7b\,x^{\frac{4}{3}}}+\frac{10a\sqrt{b\,x^{\frac{1}{3}}+ax}}{7b^{2}x^{\frac{2}{3}}}+\frac{5a\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\Bigg(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},}{7b^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$

```
[In] int(1/x^2/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*(5*a*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*
x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*E
llipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(4/3)
+4*a*b*x+10*x^(5/3)*a^2-6*b^2*x^(1/3))/b^2/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^
(4/3)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^5 + b^3*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{x^2\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

```
[In] integrate(1/x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*x**(1/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{x^2\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

```
[In] int(1/(x^2*(a*x + b*x^(1/3))^(1/2)),x)
[Out] int(1/(x^2*(a*x + b*x^(1/3))^(1/2)), x)
```

3.156 $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 388

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx
&= -\frac{154a^{7/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{65b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} \\
&\quad + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} \\
&\quad + \frac{154a^{13/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{77a^{13/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{65b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

```
[Out] -154/65*a^(7/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1
/3)+a*x)^(1/2)-6/13*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+22/39*a*(b*x^(1/3)+a*x)
^(1/2)/b^2/x^(5/3)-154/195*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x+154/65*a^3*(b*x^
(1/3)+a*x)^(1/2)/b^4/x^(1/3)+154/65*a^(13/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*
x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE
(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/
2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a
*x)^(1/2)-77/65*a^(13/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4
```

```
)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(
x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2043, 2050, 2057, 335, 311, 226, 1210}

$$\begin{aligned}
&\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx \\
&= -\frac{77a^{13/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{65b^{15/4}\sqrt{ax+b\sqrt[3]{x}}} \\
&\quad+\frac{154a^{13/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{15/4}\sqrt{ax+b\sqrt[3]{x}}} \\
&\quad-\frac{154a^{7/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{65b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{154a^3\sqrt{ax+b\sqrt[3]{x}}}{65b^4\sqrt[3]{x}} \\
&\quad-\frac{154a^2\sqrt{ax+b\sqrt[3]{x}}}{195b^3x}+\frac{22a\sqrt{ax+b\sqrt[3]{x}}}{39b^2x^{5/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{13bx^{7/3}}
\end{aligned}$$

```
[In] Int[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-154*a^(7/2)*(b + a*x^(2/3))*x^(1/3))/(65*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3))*
Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(13*b*x^(7/3)) + (22*a*S
qrt[b*x^(1/3) + a*x])/(39*b^2*x^(5/3)) - (154*a^2*Sqrt[b*x^(1/3) + a*x])/(1
95*b^3*x) + (154*a^3*Sqrt[b*x^(1/3) + a*x])/(65*b^4*x^(1/3)) + (154*a^(13/4
)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/
3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(65*b^(
15/4)*Sqrt[b*x^(1/3) + a*x]) - (77*a^(13/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(65*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} - \frac{(33a)\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{13b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} + \frac{\left(77a^2\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{39b^2}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x}\\
&\quad - \frac{\left(77a^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{65b^3}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x}\\
&\quad + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{\left(77a^4\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{65b^4}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x}\\
&\quad + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{\left(77a^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{65b^4\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x}\\
&\quad + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{\left(154a^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{65b^4\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x}\\
&\quad + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{\left(154a^{7/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{65b^{7/2}\sqrt{b\sqrt[3]{x}+ax}}\\
&\quad + \frac{\left(154a^{7/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{65b^{7/2}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{154a^{7/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{65b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} \\
&\quad + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} \\
&\quad + \frac{154a^{13/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{77a^{13/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.15

$$
\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},\frac{1}{2},-\frac{9}{4},-\frac{ax^{2/3}}{b}\right)}{13x^2\sqrt{b\sqrt[3]{x}+ax}}
$$

```
[In] Integrate[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-13/4, 1/2, -9/4, -((a*x^(2/3
))/b)])/(13*x^2*Sqrt[b*x^(1/3) + a*x])
```

Maple [A] (verified)

Time = 3.26 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{13b\,x^{\frac{7}{3}}}+\frac{22a\sqrt{b\,x^{\frac{1}{3}}+ax}}{39b^{2}x^{\frac{5}{3}}}-\frac{154a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{195b^{3}x}+\frac{154\left(b+a\,x^{\frac{2}{3}}\right)a^{3}}{65b^{4}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-\frac{77a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-}}{}$
default	$\frac{-462a^{3}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{10}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+231a^{3}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-}}{\sqrt{-ab}}}}{}$

```
[In] int(1/x^3/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6/13*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+22/39*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(
5/3)-154/195*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x+154/65*(b+a*x^(2/3))*a^3/b^4/(
x^(1/3)*(b+a*x^(2/3)))^(1/2)-77/65*a^3/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b
)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2
))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)
^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))
^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{1}{x^{3}\sqrt{b\sqrt[3]{x}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x^{3}}\,dx$$

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^6 + b^3*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^3\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

```
[In] integrate(1/x**3/(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a*x + b*x**(1/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^3\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

```
[In] int(1/(x^3*(a*x + b*x^(1/3))^(1/2)),x)
[Out] int(1/(x^3*(a*x + b*x^(1/3))^(1/2)), x)
```

3.157 $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 251

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}}+\frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}}-\frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} \\
&\quad+\frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}}-\frac{1326a^4\sqrt{b\sqrt[3]{x}+ax}}{1463b^5x^{2/3}} \\
&\quad-\frac{663a^{19/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1463b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

```
[Out] -6/19*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+34/95*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x^
(8/3)-442/1045*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2+3978/7315*a^3*(b*x^(1/3)+a
*x)^(1/2)/b^4/x^(4/3)-1326/1463*a^4*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-663/1
463*a^(19/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2
*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1
/2)+b^(1/2))^2)^(1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2043, 2050, 2036, 335, 226}

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$$
$$= -\frac{663a^{19/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1463b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}$$
$$-\frac{1326a^4\sqrt{ax+b\sqrt[3]{x}}}{1463b^5x^{2/3}}+\frac{3978a^3\sqrt{ax+b\sqrt[3]{x}}}{7315b^4x^{4/3}}$$
$$-\frac{442a^2\sqrt{ax+b\sqrt[3]{x}}}{1045b^3x^2}+\frac{34a\sqrt{ax+b\sqrt[3]{x}}}{95b^2x^{8/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{19bx^{10/3}}$$

```
[In] Int[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(19*b*x^(10/3)) + (34*a*Sqrt[b*x^(1/3) + a*x])/(
95*b^2*x^(8/3)) - (442*a^2*Sqrt[b*x^(1/3) + a*x])/(1045*b^3*x^2) + (3978*a^
3*Sqrt[b*x^(1/3) + a*x])/(7315*b^4*x^(4/3)) - (1326*a^4*Sqrt[b*x^(1/3) + a*
x])/(1463*b^5*x^(2/3)) - (663*a^(19/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(1463*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} - \frac{(51a)\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{19b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} + \frac{(221a^2)\,\text{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{95b^2}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2}\\
&\quad - \frac{(1989a^3)\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1045b^3}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2}\\
&\quad + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} + \frac{(1989a^4)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1463b^4}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}}\\
&\quad - \frac{1326a^4\sqrt{b\sqrt[3]{x}+ax}}{1463b^5x^{2/3}} - \frac{(663a^5)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1463b^5}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}}\\
&\quad - \frac{1326a^4\sqrt{b\sqrt[3]{x}+ax}}{1463b^5x^{2/3}} - \frac{\left(663a^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{1463b^5\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} - \frac{1326a^4\sqrt{b\sqrt[3]{x}+ax}}{1463b^5x^{2/3}} - \frac{\left(1326a^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{1463b^5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} - \frac{1326a^4\sqrt{b\sqrt[3]{x}+ax}}{1463b^5x^{2/3}} - \frac{663a^{19/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1463b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = -\frac{6\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{19}{4},\frac{1}{2},-\frac{15}{4},-\frac{ax^{2/3}}{b}\right)}{19x^3\sqrt{b\sqrt[3]{x}+ax}}$$

```
[In] Integrate[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-19/4, 1/2, -15/4, -((a*x^(2/
3))/b)])/(19*x^3*Sqrt[b*x^(1/3) + a*x])
```

Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.71

method	result
default	$-\frac{3315a^4\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)x^{\frac{16}{3}}+2652a^4b\,x^5+6630x^{\frac{17}{3}}a^5+476}{7315b^5\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{16}{3}}}$
derivativedivides	$-\frac{6\sqrt{b\,x^{\frac{1}{3}}+ax}}{19b\,x^{\frac{10}{3}}}+\frac{34a\sqrt{b\,x^{\frac{1}{3}}+ax}}{95b^2x^{\frac{8}{3}}}-\frac{442a^2\sqrt{b\,x^{\frac{1}{3}}+ax}}{1045b^3x^2}+\frac{3978a^3\sqrt{b\,x^{\frac{1}{3}}+ax}}{7315b^4x^{\frac{4}{3}}}-\frac{1326a^4\sqrt{b\,x^{\frac{1}{3}}+ax}}{1463b^5x^{\frac{2}{3}}}-\frac{663a^4\sqrt{-ab}}{}$

```
[In] int(1/x^4/(b*x^(1/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7315*(3315*a^4*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2)
)^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)
)*x^(16/3)+2652*a^4*b*x^5+6630*x^(17/3)*a^5+476*x^(11/3)*a^2*b^3-884*x^(13/
3)*a^3*b^2-308*a*b^4*x^3+2310*x^(7/3)*b^5)/b^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2
)/x^(16/3)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^7 + b^3*x^5), x)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^4\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

```
[In] integrate(1/x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a*x + b*x**(1/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = \int \frac{1}{x^4\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

```
[In] int(1/(x^4*(a*x + b*x^(1/3))^(1/2)),x)
[Out] int(1/(x^4*(a*x + b*x^(1/3))^(1/2)), x)
```

3.158 $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 437

$$\begin{aligned}\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= -\frac{4807b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{13/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\ &\quad -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6}-\frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} \\ &\quad +\frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4}-\frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3}+\frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} \\ &\quad +\frac{4807b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{27/4}\sqrt{b\sqrt[3]{x}+ax}} \\ &\quad -\frac{4807b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{442a^{27/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] -3*x^4/a/(b*x^(1/3)+a*x)^(1/2)-4807/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(13/2)/
(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+4807/663*b^4*x^(1/3)*(b*x^(
1/3)+a*x)^(1/2)/a^6-24035/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^5+6555/1547*b^
2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^4-437/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)
/a^3+23/7*x^3*(b*x^(1/3)+a*x)^(1/2)/a^2+4807/221*b^(21/4)*x^(1/6)*(cos(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4
)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*
a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(27/4)
/(b*x^(1/3)+a*x)^(1/2)-4807/442*b^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1
```

```
/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*
((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(27/4)/(b*x^(1/3)+a*x)^
(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2047, 2049, 2057, 335, 311, 226, 1210}

$$\begin{aligned}\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = & -\frac{4807b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{442a^{27/4}\sqrt{ax+b\sqrt[3]{x}}} \\ & +\frac{4807b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{27/4}\sqrt{ax+b\sqrt[3]{x}}} \\ & -\frac{4807b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{13/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{4807b^4\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{663a^6} \\ & -\frac{24035b^3x\sqrt{ax+b\sqrt[3]{x}}}{4641a^5}+\frac{6555b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^4} \\ & -\frac{437bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}{119a^3}+\frac{23x^3\sqrt{ax+b\sqrt[3]{x}}}{7a^2}-\frac{3x^4}{a\sqrt{ax+b\sqrt[3]{x}}}\end{aligned}$$

```
[In] Int[x^4/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-4807*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(13/2)*(Sqrt[b] + Sqrt[a]*x^(1/3
))*Sqrt[b*x^(1/3) + a*x]) - (3*x^4)/(a*Sqrt[b*x^(1/3) + a*x]) + (4807*b^4*x
^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^6) - (24035*b^3*x*Sqrt[b*x^(1/3) + a*x
])/(4641*a^5) + (6555*b^2*x^(5/3)*Sqrt[b*x^(1/3) + a*x])/(1547*a^4) - (437*
b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a^3) + (23*x^3*Sqrt[b*x^(1/3) + a*x])
/(7*a^2) + (4807*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/
(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))
/b^(1/4)], 1/2])/(221*a^(27/4)*Sqrt[b*x^(1/3) + a*x]) - (4807*b^(21/4)*(Sqr
t[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]
*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(442*a^(27/4)
*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
```

```
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^{14}}{(bx+ax^3)^{3/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{69\text{Subst}\left(\int \frac{x^{11}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a}\\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} - \frac{(437b)\text{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{14a^2}\\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2}\\
&\quad + \frac{\left(6555b^2\right)\text{Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{238a^3}\\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3}\\
&\quad + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} - \frac{\left(72105b^3\right)\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{3094a^4}\\
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5}\\
&\quad + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3}\\
&\quad + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} + \frac{\left(24035b^4\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1326a^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6}-\frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5}+\frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4}\\
&\quad -\frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3}+\frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2}-\frac{\left(4807b^5\right)\text{Subst}\left(\int\frac{x}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{442a^6}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6}-\frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5}\\
&\quad +\frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4}-\frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3}+\frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2}\\
&\quad -\frac{\left(4807b^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{442a^6\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6}-\frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5}\\
&\quad +\frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4}-\frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3}+\frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2}\\
&\quad -\frac{\left(4807b^5\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{221a^6\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6}-\frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5}\\
&\quad +\frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4}-\frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3}+\frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2}\\
&\quad -\frac{\left(4807b^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{221a^{13/2}\sqrt{b\sqrt[3]{x}+ax}}\\
&\quad +\frac{\left(4807b^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int\frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{221a^{13/2}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4807b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{13/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} \\
&\quad + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} \\
&\quad + \frac{4807b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{27/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{4807b^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{442a^{27/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.30

$$
\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{2x^{2/3}\Bigl(-33649b^5+4807ab^4x^{2/3}-2185a^2b^3x^{4/3}+1311a^3b^2x^2-897a^4bx^{8/3}+663a^5x}{4641a^6\sqrt{b\sqrt[3]{x}+ax}}
$$

```
[In] Integrate[x^4/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(2/3)*(-33649*b^5 + 4807*a*b^4*x^(2/3) - 2185*a^2*b^3*x^(4/3) + 1311*a
^3*b^2*x^2 - 897*a^4*b*x^(8/3) + 663*a^5*x^(10/3) + 33649*b^5*Sqrt[1 + (a*x
^(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(4641*a^6*S
qrt[b*x^(1/3) + a*x])
```

Maple [A] (verified)

Time = 4.53 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{3x^{\frac{2}{3}}b^{5}}{a^{6}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{7a^{2}}-\frac{80b\,x^{\frac{7}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{119a^{3}}+\frac{1914b^{2}x^{\frac{5}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547a^{4}}-\frac{10112b^{3}x\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641a^{5}}+$
default	$5244x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{4}b^{2}-3588x^{\frac{10}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{5}b-8740x^{2}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{3}b^{3}-201894b^{6}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}$

```
[In] int(x^4/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(2/3)/a^6*b^5/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2/7*x^3*(b*x^(1/3)+a*x)^(
1/2)/a^2-80/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a^3+1914/1547*b^2*x^(5/3)*(
b*x^(1/3)+a*x)^(1/2)/a^4-10112/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^5+2818/66
3*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^6-4807/442*b^5/a^7*(-a*b)^(1/2)*((x^(
1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))
*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2
)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^6 + 3*a^2*b^2*x^(14/3) - 2*a*b^3*x^4 - (2*a^3*b*x^5 - b^4*x
^3)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^4}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4/(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(x**4/(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^4}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

```
[In] int(x^4/(a*x + b*x^(1/3))^(3/2),x)
[Out] int(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

3.159 $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 239

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} + \frac{663b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\,\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154a^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] -3*x^3/a/(b*x^(1/3)+a*x)^(1/2)-663/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^5+1989/38
5*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^4-221/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1
/2)/a^3+17/5*x^2*(b*x^(1/3)+a*x)^(1/2)/a^2+663/154*b^(15/4)*x^(1/6)*(cos(2*
arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1
/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3
)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(21/
4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2043, 2047, 2049, 2036, 335, 226}

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{663b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154a^{21/4}\sqrt{ax+b\sqrt[3]{x}}}$$
$$-\frac{663b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^5}+\frac{1989b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^4}$$
$$-\frac{221bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{55a^3}+\frac{17x^2\sqrt{ax+b\sqrt[3]{x}}}{5a^2}-\frac{3x^3}{a\sqrt{ax+b\sqrt[3]{x}}}$$

```
[In] Int[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-3*x^3)/(a*Sqrt[b*x^(1/3) + a*x]) - (663*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^
5) + (1989*b^2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(385*a^4) - (221*b*x^(4/3)*Sq
rt[b*x^(1/3) + a*x])/(55*a^3) + (17*x^2*Sqrt[b*x^(1/3) + a*x])/(5*a^2) + (6
63*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqr
t[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2
])/(154*a^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^{11}}{\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right) \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{51\text{Subst}\left(\int \frac{x^8}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a} \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} - \frac{(221b)\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{10a^2} \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} \\
&\quad + \frac{\left(1989b^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{110a^3} \\
&= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} \\
&\quad + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} - \frac{\left(1989b^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{154a^4}
\end{aligned}
$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} + \frac{(663b^4)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{154a^5}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} + \frac{\left(663b^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{154a^5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} + \frac{\left(663b^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{77a^5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} + \frac{663b^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154a^{21/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.50

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{-3315b^4\sqrt[3]{x} - 1326ab^3x + 442a^2b^2x^{5/3} - 238a^3bx^{7/3} + 154a^4x^3 + 3315b^4\sqrt{1+\frac{ax^{2/3}}{b}}}{385a^5\sqrt{b\sqrt[3]{x}+ax}}$$

```
[In] Integrate[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-3315*b^4*x^(1/3) - 1326*a*b^3*x + 442*a^2*b^2*x^(5/3) - 238*a^3*b*x^(7/3)
 + 154*a^4*x^3 + 3315*b^4*Sqrt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1
[1/4, 1/2, 5/4, -((a*x^(2/3))/b)])/(385*a^5*Sqrt[b*x^(1/3) + a*x])
```

Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{3x^{\frac{1}{3}}b^{4}}{a^{5}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{5a^{2}}-\frac{56b\,x^{\frac{4}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{55a^{3}}+\frac{834b^{2}x^{\frac{2}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385a^{4}}-\frac{432b^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77a^{5}}+$
default	$-\frac{-884\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{5}{3}}a^{3}b^{2}+476\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,x^{\frac{7}{3}}a^{4}b-3315\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}}{}$

```
[In] int(x^3/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3*x^(1/3)/a^5*b^4/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2/5*x^2*(b*x^(1/3)+a*x)^
(1/2)/a^2-56/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^3+834/385*b^2*x^(2/3)*(b*
x^(1/3)+a*x)^(1/2)/a^4-432/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^5+663/154*b^4/a^6
*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3
)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(
b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^{3}}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^5 + 3*a^2*b^2*x^(11/3) - 2*a*b^3*x^3 - (2*a^3*b*x^4 - b^4*x
^2)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^{3}}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

Giac [F]

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{x^3}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

```
[In] int(x^3/(a*x + b*x^(1/3))^(3/2),x)
[Out] int(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

3.160 $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 349

$$
\begin{aligned}
\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= \frac{77b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} \\
&\quad - \frac{77b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad + \frac{77b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10a^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

```
[Out] -3*x^2/a/(b*x^(1/3)+a*x)^(1/2)+77/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(7/2)/(x^(1
/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-77/15*b*x^(1/3)*(b*x^(1/3)+a*x)^
(1/2)/a^3+11/3*x*(b*x^(1/3)+a*x)^(1/2)/a^2-77/5*b^(9/4)*x^(1/6)*(cos(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^
(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(15/4)/(
b*x^(1/3)+a*x)^(1/2)+77/10*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arc
tan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*
x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2043, 2047, 2049, 2057, 335, 311, 226, 1210}

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{77b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10a^{15/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{77b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{15/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{77b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{7/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}} - \frac{77b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}{15a^3} + \frac{11x\sqrt{ax+b\sqrt[3]{x}}}{3a^2} - \frac{3x^2}{a\sqrt{ax+b\sqrt[3]{x}}}$$

```
[In] Int[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (77*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) - (3*x^2)/(a*Sqrt[b*x^(1/3) + a*x]) - (77*b*x^(1/3)*Sqr
t[b*x^(1/3) + a*x])/(15*a^3) + (11*x*Sqrt[b*x^(1/3) + a*x])/(3*a^2) - (77*b
^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5
*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) + (77*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*
Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*Arc
Tan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(10*a^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{x^8}{\left(bx+ax^3\right)^{3/2}}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{33\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{2a}\\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2}-\frac{(77b)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{6a^2}\\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3}\\
&\quad+\frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2}+\frac{\left(77b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{10a^3}\\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3}+\frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2}\\
&\quad+\frac{\left(77b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{10a^3\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3}+\frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2}\\
&\quad+\frac{\left(77b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{5a^3\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3}+\frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2}\\
&\quad+\frac{\left(77b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{5a^{7/2}\sqrt{b\sqrt[3]{x}+ax}}\\
&\quad-\frac{\left(77b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{5a^{7/2}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{77b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} \\
&\quad - \frac{77b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad + \frac{77b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10a^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.27

$$
\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{2x^{2/3}\Bigg(77b^2-11abx^{2/3}+5a^2x^{4/3}-77b^2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\Big(\frac{3}{4},\frac{3}{2},\frac{7}{4},-}{15a^3\sqrt{b\sqrt[3]{x}+ax}}
$$

```
[In] Integrate[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(2/3)*(77*b^2 - 11*a*b*x^(2/3) + 5*a^2*x^(4/3) - 77*b^2*Sqrt[1 + (a*x^
(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(15*a^3*Sqrt
[b*x^(1/3) + a*x])
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-\frac{3x^{\frac{2}{3}}b^{2}}{a^{3}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2x\sqrt{b\,x^{\frac{1}{3}}+ax}}{3a^{2}}-\frac{32b\,x^{\frac{1}{3}}\sqrt{b\,x^{\frac{1}{3}}+ax}}{15a^{3}}+\frac{77b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}}{}$
default	$-\frac{-462b^{3}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+231b^{3}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}}{}$

```
[In] int(x^2/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3*x^(2/3)/a^3*b^2/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2/3*x*(b*x^(1/3)+a*x)^(1
/2)/a^2-32/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^3+77/10*b^2/a^4*(-a*b)^(1/2
)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)
^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*
x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^
(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(
1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^{2}}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 + 3*a^2*b^2*x^(8/3) - 2*a*b^3*x^2 - (2*a^3*b*x^3 - b^4*x)
*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(x**2/(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

```
[In] int(x^2/(a*x + b*x^(1/3))^(3/2),x)
[Out] int(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

3.161 $\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 149

$$\begin{aligned}\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = {} & -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2} \\ & -\frac{5b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{9/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] -3*x/a/(b*x^(1/3)+a*x)^(1/2)+5*(b*x^(1/3)+a*x)^(1/2)/a^2-5/2*b^(3/4)*x^(1/6
)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(
1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2)
)*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/
2)/a^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2043, 2047, 2049, 2036, 335, 226}

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{5b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{9/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{5\sqrt{ax+b\sqrt[3]{x}}}{a^2} - \frac{3x}{a\sqrt{ax+b\sqrt[3]{x}}}$$

```
[In] Int[x/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-3*x)/(a*Sqrt[b*x^(1/3) + a*x]) + (5*Sqrt[b*x^(1/3) + a*x])/a^2 - (5*b^(3/
4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1
/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(
9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2047

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Dist[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1))), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{x^5}{(bx+ax^3)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{15\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a}$$

$$= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2} - \frac{(5b)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a^2}$$

$$= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2} - \frac{\left(5b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{2a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2} - \frac{\left(5b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2}$$
$$- \frac{5b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{\sqrt{b\sqrt[3]{x}+ax}\left(5b+2ax^{2/3}-5b\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{ax^{2/3}}{b}\right)\right)}{a^2\left(b+ax^{2/3}\right)}$$

```
[In] Integrate[x/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*x^(1/3) + a*x]*(5*b + 2*a*x^(2/3) - 5*b*Sqrt[1 + (a*x^(2/3))/b]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a^2*(b + a*x^(2/3)))
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{3x^{\frac{1}{3}}b}{a^{2}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{a^{2}}-\frac{5b\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\Bigg(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)}{\sqrt{-ab}}}}{2a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{5\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,b-6\sqrt{b\,x^{\frac{1}{3}}+ax}\,x^{\frac{1}{3}}ab-}{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)a^{3}}$

```
[In] int(x/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(1/3)/a^2*b/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+2*(b*x^(1/3)+a*x)^(1/2)/a^2
-5/2*b/a^3*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(
-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2
))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

Sympy [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x/(a*x + b*x**(1/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

Giac [F]

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{x}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}} \,d x$$

```
[In] int(x/(a*x + b*x^(1/3))^(3/2),x)
[Out] int(x/(a*x + b*x^(1/3))^(3/2), x)
```

3.162 $\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 296

$$\begin{aligned}\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = & -\frac{3\left(b+ax^{2/3}\right)\sqrt[3]{x}}{\sqrt{a}b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} \\ & +\frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{b\sqrt[3]{x}+ax}} \\ & -\frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] 3*x^(2/3)/b/(b*x^(1/3)+a*x)^(1/2)-3*(b+a*x^(2/3))*x^(1/3)/b/a^(1/2)/(x^(1/3
)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+3*x^(1/6)*(cos(2*arctan(a^(1/4)*x^
(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(s
in(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2)
)*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^(1
/3)+a*x)^(1/2)-3/2*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2031, 2043, 2057, 335, 311, 226, 1210}

$$\begin{aligned}\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx =& -\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}} \\ &+\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}} \\ &-\frac{3\sqrt[3]{x}\left(ax^{2/3}+b\right)}{\sqrt{a}b\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{3x^{2/3}}{b\sqrt{ax+b\sqrt[3]{x}}}\end{aligned}$$

```
[In] Int[(b*x^(1/3) + a*x)^(-3/2),x]
```

```
[Out] (-3*(b + a*x^(2/3))*x^(1/3))/(Sqrt[a]*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*
x^(1/3) + a*x]) + (3*x^(2/3))/(b*Sqrt[b*x^(1/3) + a*x]) + (3*(Sqrt[b] + Sqr
t[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*E
llipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(3/4)*b^(3/4)*Sqrt[b
*x^(1/3) + a*x]) - (3*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqr
t[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(
1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\int \frac{1}{\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}\,dx}{2b} \\
&= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{3\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b} \\
&= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{2b\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

$$= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}}$$
$$+ \frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{3\left(b+ax^{2/3}\right)\sqrt[3]{x}}{\sqrt{a}b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}}$$
$$+ \frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}$$
$$- \frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.21

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{2\sqrt{1+\frac{ax^{2/3}}{b}}x^{2/3}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{ax^{2/3}}{b}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}$$

```
[In] Integrate[(b*x^(1/3) + a*x)^(-3/2),x]
[Out] (2*Sqrt[1 + (a*x^(2/3))/b]*x^(2/3)*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^
(2/3))/b)])/(b*Sqrt[b*x^(1/3) + a*x])
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{3x^{\frac{2}{3}}}{b\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}-\frac{3\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a}+\right.}{2ba\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$\frac{-3\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b+\frac{3\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-}}{\sqrt{-ab}}}}{}}{a\,x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)b}$

```
[In] int(1/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(2/3)/b/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)-3/2/b/a*(-a*b)^(1/2)*((x^(1/3)+
1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2
/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^5 + 2*a^3*b^3*x^3 + b^6*x), x)
```

Sympy [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.14

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\frac{2\,x\,{\left(\frac{a\,x^{2/3}}{b}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{3}{2};\ \frac{7}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}$$

```
[In] int(1/(a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] (2*x*((a*x^(2/3))/b + 1)^(3/2)*hypergeom([3/4, 3/2], 7/4, -(a*x^(2/3))/b))/
(a*x + b*x^(1/3))^(3/2)
```

3.163 $\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 158

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} - \frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}} - \frac{5a^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[Out] 3/b/x^(1/3)/(b*x^(1/3)+a*x)^(1/2)-5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)-5/2*a
^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^
(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2043, 2048, 2050, 2036, 335, 226}

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{5a^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{9/4}\sqrt{ax+b\sqrt[3]{x}}} - \frac{5\sqrt{ax+b\sqrt[3]{x}}}{b^2x^{2/3}} + \frac{3}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[In] Int[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]) - (5*Sqrt[b*x^(1/3) + a*x])/(b^2*x^(2/3
)) - (5*a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] +
 Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)],
 1/2])/(2*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{1}{x\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}+\frac{15\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}\\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}-\frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}}-\frac{(5a)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b^2}\\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}-\frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}}-\frac{\left(5a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{2b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}-\frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}}-\frac{\left(5a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}-\frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}}\\
&\quad-\frac{5a^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{ax^{2/3}}{b}\right)}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}$$

```
[In] Integrate[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((a*x^(2/3))
/b)])/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03

method	result
derivativedivides	$-\frac{3x^{\frac{1}{3}}a}{b^{2}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{b^{2}x^{\frac{2}{3}}}-\frac{5\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\Bigg(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right.}{\sqrt{-ab}}}}{2b^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}$
default	$-\frac{5x^{\frac{2}{3}}\sqrt{-ab}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+6\sqrt{b\,x^{\frac{1}{3}}+ax}\,xa+}{2b^{2}x\left(b+a\,x^{\frac{2}{3}}\right)}$

```
[In] int(1/x/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3*x^(1/3)*a/b^2/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)-2*(b*x^(1/3)+a*x)^(1/2)/b^
2/x^(2/3)-5/2/b^2*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^
(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*
b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))
*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^6 + 2*a^3*b^3*x^4 + b^6*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*x**(1/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}} \,dx$$

```
[In] int(1/(x*(a*x + b*x^(1/3))^(3/2)),x)
[Out] int(1/(x*(a*x + b*x^(1/3))^(3/2)), x)
```

3.164 $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 383

$$\begin{aligned}
\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}+\frac{77a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad -\frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}}+\frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x}-\frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} \\
&\quad -\frac{77a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad +\frac{77a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

```
[Out] 3/b/x^(4/3)/(b*x^(1/3)+a*x)^(1/2)+77/5*a^(5/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(x
^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-11/3*(b*x^(1/3)+a*x)^(1/2)/b^
2/x^(5/3)+77/15*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x-77/5*a^2*(b*x^(1/3)+a*x)^(1/2
)/b^4/x^(1/3)-77/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1
/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))
/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)+77/10*a^
(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))
,1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(
1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2048, 2050, 2057, 335, 311, 226, 1210}

$$\int \frac{1}{x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$
$$-\frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$
$$+\frac{77a^{5/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{77a^2\sqrt{ax+b\sqrt[3]{x}}}{5b^4\sqrt[3]{x}}$$
$$+\frac{77a\sqrt{ax+b\sqrt[3]{x}}}{15b^3x}-\frac{11\sqrt{ax+b\sqrt[3]{x}}}{3b^2x^{5/3}}+\frac{3}{bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[In] Int[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(4/3)*Sqrt[b*x^(1/3) + a*x]) + (77*a^(5/2)*(b + a*x^(2/3))*x^(1/3))/
(5*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (11*Sqrt[b*x^(1
/3) + a*x])/(3*b^2*x^(5/3)) + (77*a*Sqrt[b*x^(1/3) + a*x])/(15*b^3*x) - (77
*a^2*Sqrt[b*x^(1/3) + a*x])/(5*b^4*x^(1/3)) - (77*a^(9/4)*(Sqrt[b] + Sqrt[a
]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Elli
pticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(15/4)*Sqrt[b*x^(1/3)
 + a*x]) + (77*a^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sq
rt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^
(1/4)], 1/2])/(10*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = 3\text{Subst}\left(\int \frac{1}{x^4 \left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{33\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} - \frac{(77a)\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{6b^2}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} + \frac{\left(77a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{10b^3}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^3\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{10b^4}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{10b^4\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5b^4\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5b^{7/2}\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(77a^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5b^{7/2}\sqrt{b\sqrt[3]{x}+ax}}$$

$$
\begin{aligned}
&= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{77a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} \\
&\quad - \frac{77a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{15/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad + \frac{77a^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{15/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.17

$$
\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{3}{2},-\frac{5}{4},-\frac{ax^{2/3}}{b}\right)}{3bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}
$$

```
[In] Integrate[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-9/4, 3/2, -5/4, -((a*x^(2/3)
)/b)])/(3*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])
```

Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{3b^{2}x^{\frac{5}{3}}}+\frac{32a\sqrt{b\,x^{\frac{1}{3}}+ax}}{15b^{3}x}-\frac{62\left(b+a\,x^{\frac{2}{3}}\right)a^{2}}{5b^{4}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}}-\frac{3x^{\frac{2}{3}}a^{3}}{b^{4}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{77a^{2}\sqrt{-ab}\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\sqrt{}}{}$
default	$-\frac{-462a^{2}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{8}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+231a^{2}b\sqrt{\frac{a\,x^{\frac{1}{3}}+}{\sqrt{-}}}}{}$

```
[In] int(1/x^2/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)+32/15*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x-
62/5*(b+a*x^(2/3))*a^2/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-3*x^(2/3)*a^3/b^4/
((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+77/10*a^2/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-
a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(
1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a
*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^{2}}\,dx$$

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^7 + 2*a^3*b^3*x^5 + b^6*x^3), x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^2\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(1/(x**2*(a*x + b*x**(1/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^2\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

```
[In] int(1/(x^2*(a*x + b*x^(1/3))^(3/2)),x)
[Out] int(1/(x^2*(a*x + b*x^(1/3))^(3/2)), x)
```

3.165 $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 246

$$\begin{aligned}\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}}\\ &+\frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2}-\frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}}+\frac{663a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^5x^{2/3}}\\ &+\frac{663a^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

```
[Out] 3/b/x^(7/3)/(b*x^(1/3)+a*x)^(1/2)-17/5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)+22
1/55*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2-1989/385*a^2*(b*x^(1/3)+a*x)^(1/2)/b^4
/x^(4/3)+663/77*a^3*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)+663/154*a^(15/4)*x^(1
/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/
2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(
1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2043, 2048, 2050, 2036, 335, 226}

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{663a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154b^{21/4}\sqrt{ax+b\sqrt[3]{x}}} + \frac{663a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^5x^{2/3}} - \frac{1989a^2\sqrt{ax+b\sqrt[3]{x}}}{385b^4x^{4/3}} + \frac{221a\sqrt{ax+b\sqrt[3]{x}}}{55b^3x^2} - \frac{17\sqrt{ax+b\sqrt[3]{x}}}{5b^2x^{8/3}} + \frac{3}{bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[In] Int[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(7/3)*Sqrt[b*x^(1/3) + a*x]) - (17*Sqrt[b*x^(1/3) + a*x])/(5*b^2*x^(
8/3)) + (221*a*Sqrt[b*x^(1/3) + a*x])/(55*b^3*x^2) - (1989*a^2*Sqrt[b*x^(1/
3) + a*x])/(385*b^4*x^(4/3)) + (663*a^3*Sqrt[b*x^(1/3) + a*x])/(77*b^5*x^(2
/3)) + (663*a^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt
[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1
/4)], 1/2])/(154*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{1}{x^7\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right) \\
&= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{51\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b} \\
&= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} - \frac{(221a)\text{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{10b^2} \\
&= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} \\
&\quad + \frac{\left(1989a^2\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{110b^3} \\
&= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} \\
&\quad - \frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}} - \frac{\left(1989a^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{154b^4}
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}}+\frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2}-\frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}}\\
&\quad+\frac{663a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^5x^{2/3}}+\frac{(663a^4)\,\mathrm{Subst}\left(\int\frac{1}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{154b^5}\\
&=\frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}}+\frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2}-\frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}}\\
&\quad+\frac{663a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^5x^{2/3}}+\frac{\left(663a^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{154b^5\sqrt{b\sqrt[3]{x}+ax}}\\
&=\frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}}+\frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2}-\frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}}\\
&\quad+\frac{663a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^5x^{2/3}}+\frac{\left(663a^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{77b^5\sqrt{b\sqrt[3]{x}+ax}}\\
&=\frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}}+\frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2}\\
&\quad-\frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}}+\frac{663a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^5x^{2/3}}\\
&\quad+\frac{663a^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{154b^{21/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.26

$$
\int\frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=-\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,\mathrm{Hypergeometric2F1}\left(-\frac{15}{4},\frac{3}{2},-\frac{11}{4},-\frac{ax^{2/3}}{b}\right)}{5bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}
$$

```
[In] Integrate[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-15/4, 3/2, -11/4, -((a*x^(2/
3))/b)])/(5*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])
```

Maple [A] (verified)

Time = 5.33 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{5b^{2}x^{\frac{8}{3}}}+\frac{56a\sqrt{b\,x^{\frac{1}{3}}+ax}}{55b^{3}x^{2}}-\frac{834a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{385b^{4}x^{\frac{4}{3}}}+\frac{432a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{77b^{5}x^{\frac{2}{3}}}+\frac{3x^{\frac{1}{3}}a^{4}}{b^{5}\sqrt{\left(x^{\frac{2}{3}}+\frac{b}{a}\right)x^{\frac{1}{3}}a}}+\frac{663a^{3}\sqrt{-ab}}{}$
default	$\frac{3315x^{\frac{14}{3}}\sqrt{-ab}\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,a^{3}-884x^{\frac{11}{3}}\sqrt{x^{\frac{1}{3}}}}{}$

```
[In] int(1/x^3/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)+56/55*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^
2-834/385*a^2*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)+432/77*a^3*(b*x^(1/3)+a*x)^
(1/2)/b^5/x^(2/3)+3*x^(1/3)*a^4/b^5/((x^(2/3)+b/a)*x^(1/3)*a)^(1/2)+663/154
*a^3/b^5*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2
*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))
^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [F]

$$\int \frac{1}{x^{3}\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^{3}}\,dx$$

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^8 + 2*a^3*b^3*x^6 + b^6*x^4), x)
```

Sympy [F]

$$\int \frac{1}{x^{3}\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^{3}\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**3/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a*x + b*x**(1/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^3\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

```
[In] int(1/(x^3*(a*x + b*x^(1/3))^(3/2)),x)
[Out] int(1/(x^3*(a*x + b*x^(1/3))^(3/2)), x)
```

3.166 $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 471

$$\begin{aligned}
\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}-\frac{4807a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221b^7\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad -\frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}}+\frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3}-\frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} \\
&\quad +\frac{24035a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^5x^{5/3}}-\frac{4807a^4\sqrt{b\sqrt[3]{x}+ax}}{663b^6x}+\frac{4807a^5\sqrt{b\sqrt[3]{x}+ax}}{221b^7\sqrt[3]{x}} \\
&\quad +\frac{4807a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{221b^{27/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad -\frac{4807a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{442b^{27/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

```
[Out] 3/b/x^(10/3)/(b*x^(1/3)+a*x)^(1/2)-4807/221*a^(11/2)*(b+a*x^(2/3))*x^(1/3)/
b^7/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-23/7*(b*x^(1/3)+a*x)^(1
/2)/b^2/x^(11/3)+437/119*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^3-6555/1547*a^2*(b*x
^(1/3)+a*x)^(1/2)/b^4/x^(7/3)+24035/4641*a^3*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(5
/3)-4807/663*a^4*(b*x^(1/3)+a*x)^(1/2)/b^6/x+4807/221*a^5*(b*x^(1/3)+a*x)^(
1/2)/b^7/x^(1/3)+4807/221*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arc
tan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*
x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(27/4)/(b*x^(1/3)+a*x)^(1/2)-
4807/442*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/
```

```
cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)
*a^(1/2)+b^(1/2))^2)^(1/2)/b^(27/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2043, 2048, 2050, 2057, 335, 311, 226, 1210}

$$\begin{aligned}
&\int \frac{1}{x^4 \left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \\
&-\frac{4807a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{442b^{27/4}\sqrt{ax+b\sqrt[3]{x}}} \\
&+\frac{4807a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221b^{27/4}\sqrt{ax+b\sqrt[3]{x}}} \\
&-\frac{4807a^{11/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221b^7\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{4807a^5\sqrt{ax+b\sqrt[3]{x}}}{221b^7\sqrt[3]{x}} \\
&-\frac{4807a^4\sqrt{ax+b\sqrt[3]{x}}}{663b^6x}+\frac{24035a^3\sqrt{ax+b\sqrt[3]{x}}}{4641b^5x^{5/3}}-\frac{6555a^2\sqrt{ax+b\sqrt[3]{x}}}{1547b^4x^{7/3}} \\
&+\frac{437a\sqrt{ax+b\sqrt[3]{x}}}{119b^3x^3}-\frac{23\sqrt{ax+b\sqrt[3]{x}}}{7b^2x^{11/3}}+\frac{3}{bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}
\end{aligned}$$

```
[In] Int[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(10/3)*Sqrt[b*x^(1/3) + a*x]) - (4807*a^(11/2)*(b + a*x^(2/3))*x^(1/
3))/(221*b^7*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (23*Sqrt[
b*x^(1/3) + a*x])/(7*b^2*x^(11/3)) + (437*a*Sqrt[b*x^(1/3) + a*x])/(119*b^3
*x^3) - (6555*a^2*Sqrt[b*x^(1/3) + a*x])/(1547*b^4*x^(7/3)) + (24035*a^3*Sq
rt[b*x^(1/3) + a*x])/(4641*b^5*x^(5/3)) - (4807*a^4*Sqrt[b*x^(1/3) + a*x])/
(663*b^6*x) + (4807*a^5*Sqrt[b*x^(1/3) + a*x])/(221*b^7*x^(1/3)) + (4807*a^
(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2
21*b^(27/4)*Sqrt[b*x^(1/3) + a*x]) - (4807*a^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1
/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[
2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(442*b^(27/4)*Sqrt[b*x^(1/3) + a
*x])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 3\text{Subst}\left(\int \frac{1}{x^{10}\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{69\text{Subst}\left(\int \frac{1}{x^{11}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}\\
&= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} - \frac{(437a)\text{Subst}\left(\int \frac{1}{x^{9}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{14b^2}\\
&= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3}\\
&\quad + \frac{\left(6555a^2\right)\text{Subst}\left(\int \frac{1}{x^{7}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{238b^3}\\
&= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3}\\
&\quad - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} - \frac{\left(72105a^3\right)\text{Subst}\left(\int \frac{1}{x^{5}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{3094b^4}\\
&= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}}\\
&\quad + \frac{24035a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^5x^{5/3}} + \frac{\left(24035a^4\right)\text{Subst}\left(\int \frac{1}{x^{3}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1326b^5}\\
&= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}}\\
&\quad + \frac{24035a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^5x^{5/3}} - \frac{4807a^4\sqrt{b\sqrt[3]{x}+ax}}{663b^6x} - \frac{\left(4807a^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{442b^6}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} \\
&\quad - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24035a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^5x^{5/3}} - \frac{4807a^4\sqrt{b\sqrt[3]{x}+ax}}{663b^6x} \\
&\quad + \frac{4807a^5\sqrt{b\sqrt[3]{x}+ax}}{221b^7\sqrt[3]{x}} - \frac{(4807a^6)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{442b^7}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} \\
&\quad - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24035a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^5x^{5/3}} - \frac{4807a^4\sqrt{b\sqrt[3]{x}+ax}}{663b^6x} \\
&\quad + \frac{4807a^5\sqrt{b\sqrt[3]{x}+ax}}{221b^7\sqrt[3]{x}} - \frac{\left(4807a^6\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{442b^7\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} \\
&\quad - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24035a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^5x^{5/3}} - \frac{4807a^4\sqrt{b\sqrt[3]{x}+ax}}{663b^6x} \\
&\quad + \frac{4807a^5\sqrt{b\sqrt[3]{x}+ax}}{221b^7\sqrt[3]{x}} - \frac{\left(4807a^6\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{221b^7\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} \\
&\quad + \frac{24035a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^5x^{5/3}} - \frac{4807a^4\sqrt{b\sqrt[3]{x}+ax}}{663b^6x} + \frac{4807a^5\sqrt{b\sqrt[3]{x}+ax}}{221b^7\sqrt[3]{x}} \\
&\quad - \frac{\left(4807a^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{221b^{13/2}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad + \frac{\left(4807a^{11/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{a}x^2}{\sqrt{b}}}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{221b^{13/2}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{4807a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221b^7\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} \\
&\quad + \frac{24035a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^5x^{5/3}} - \frac{4807a^4\sqrt{b\sqrt[3]{x}+ax}}{663b^6x} + \frac{4807a^5\sqrt{b\sqrt[3]{x}+ax}}{221b^7\sqrt[3]{x}} \\
&\quad + \frac{4807a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221b^{27/4}\sqrt{b\sqrt[3]{x}+ax}} \\
&\quad - \frac{4807a^{21/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{442b^{27/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.14

$$
\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = -\frac{2\sqrt{1+\frac{ax^{2/3}}{b}}\,\text{Hypergeometric2F1}\left(-\frac{21}{4},\frac{3}{2},-\frac{17}{4},-\frac{ax^{2/3}}{b}\right)}{7bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}
$$

```
[In] Integrate[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-21/4, 3/2, -17/4, -((a*x^(2/
3))/b)])/(7*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])
```

Maple [A] (verified)

Time = 6.07 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{2\sqrt{b\,x^{\frac{1}{3}}+ax}}{7b^{2}x^{\frac{11}{3}}}+\frac{80a\sqrt{b\,x^{\frac{1}{3}}+ax}}{119b^{3}x^{3}}-\frac{1914a^{2}\sqrt{b\,x^{\frac{1}{3}}+ax}}{1547b^{4}x^{\frac{7}{3}}}+\frac{10112a^{3}\sqrt{b\,x^{\frac{1}{3}}+ax}}{4641b^{5}x^{\frac{5}{3}}}-\frac{2818a^{4}\sqrt{b\,x^{\frac{1}{3}}+ax}}{663b^{6}x}+\frac{4144(b}{221b^{7}\sqrt{x}}$
default	$-\frac{201894a^{5}b\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{x^{\frac{1}{3}}a}{\sqrt{-ab}}}\,x^{\frac{20}{3}}\sqrt{x^{\frac{1}{3}}\left(b+a\,x^{\frac{2}{3}}\right)}\,E\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-100947a^{5}b\sqrt{}}{}$

```
[In] int(1/x^4/(b*x^(1/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(11/3)+80/119*a*(b*x^(1/3)+a*x)^(1/2)/b^3/
x^3-1914/1547*a^2*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(7/3)+10112/4641*a^3*(b*x^(1/
3)+a*x)^(1/2)/b^5/x^(5/3)-2818/663*a^4*(b*x^(1/3)+a*x)^(1/2)/b^6/x+4144/221
*(b+a*x^(2/3))/b^7*a^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2)+3*x^(2/3)*a^6/b^7/((x^
(2/3)+b/a)*x^(1/3)*a)^(1/2)-4807/442*a^5/b^7*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1
/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*
b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2)))
```

Fricas [F]

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^{4}}\,dx$$

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^9 + 2*a^3*b^3*x^7 + b^6*x^5), x)
```

Sympy [F]

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^4\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**4/(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(1/(x**4*(a*x + b*x**(1/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^4\,{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

```
[In] int(1/(x^4*(a*x + b*x^(1/3))^(3/2)),x)
[Out] int(1/(x^4*(a*x + b*x^(1/3))^(3/2)), x)
```

3.167 $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 371

$$\begin{aligned}\int x^3\sqrt{bx^{2/3}+ax}\,dx={}&-\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}}+\frac{8388608b^{12}\left(bx^{2/3}+ax\right)^{3/2}}{152108775a^{13}x}\\&-\frac{4194304b^{11}\left(bx^{2/3}+ax\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(bx^{2/3}+ax\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}\\&+\frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9}-\frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8}\\&+\frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7}-\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6}\\&+\frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5}-\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}\\&+\frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}-\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}\end{aligned}$$

```
[Out] -524288/4345965*b^9*(b*x^(2/3)+a*x)^(3/2)/a^10+8388608/152108775*b^12*(b*x^
(2/3)+a*x)^(3/2)/a^13/x-4194304/50702925*b^11*(b*x^(2/3)+a*x)^(3/2)/a^12/x^
(2/3)+1048576/10140585*b^10*(b*x^(2/3)+a*x)^(3/2)/a^11/x^(1/3)+65536/482885
*b^8*x^(1/3)*(b*x^(2/3)+a*x)^(3/2)/a^9-360448/2414425*b^7*x^(2/3)*(b*x^(2/3
)+a*x)^(3/2)/a^8+90112/557175*b^6*x*(b*x^(2/3)+a*x)^(3/2)/a^7-45056/260015*
b^5*x^(4/3)*(b*x^(2/3)+a*x)^(3/2)/a^6+2816/15295*b^4*x^(5/3)*(b*x^(2/3)+a*x
)^(3/2)/a^5-1408/7245*b^3*x^2*(b*x^(2/3)+a*x)^(3/2)/a^4+352/1725*b^2*x^(7/3
)*(b*x^(2/3)+a*x)^(3/2)/a^3-16/75*b*x^(8/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/9*x
^3*(b*x^(2/3)+a*x)^(3/2)/a
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2041, 2027, 2039}

$$\begin{aligned}\int x^3\sqrt{bx^{2/3}+ax}\,dx &= \frac{8388608b^{12}\left(ax+bx^{2/3}\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(ax+bx^{2/3}\right)^{3/2}}{50702925a^{12}x^{2/3}}\\ &+\frac{1048576b^{10}\left(ax+bx^{2/3}\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}-\frac{524288b^{9}\left(ax+bx^{2/3}\right)^{3/2}}{4345965a^{10}}\\ &+\frac{65536b^{8}\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{482885a^{9}}-\frac{360448b^{7}x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{2414425a^{8}}\\ &+\frac{90112b^{6}x\left(ax+bx^{2/3}\right)^{3/2}}{557175a^{7}}-\frac{45056b^{5}x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{260015a^{6}}\\ &+\frac{2816b^{4}x^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{15295a^{5}}-\frac{1408b^{3}x^{2}\left(ax+bx^{2/3}\right)^{3/2}}{7245a^{4}}\\ &+\frac{352b^{2}x^{7/3}\left(ax+bx^{2/3}\right)^{3/2}}{1725a^{3}}-\frac{16bx^{8/3}\left(ax+bx^{2/3}\right)^{3/2}}{75a^{2}}+\frac{2x^{3}\left(ax+bx^{2/3}\right)^{3/2}}{9a}\end{aligned}$$

```
[In] Int[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-524288*b^9*(b*x^(2/3) + a*x)^(3/2))/(4345965*a^10) + (8388608*b^12*(b*x^(
2/3) + a*x)^(3/2))/(152108775*a^13*x) - (4194304*b^11*(b*x^(2/3) + a*x)^(3/
2))/(50702925*a^12*x^(2/3)) + (1048576*b^10*(b*x^(2/3) + a*x)^(3/2))/(10140
585*a^11*x^(1/3)) + (65536*b^8*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(482885*a^9
) - (360448*b^7*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))/(2414425*a^8) + (90112*b^6
*x*(b*x^(2/3) + a*x)^(3/2))/(557175*a^7) - (45056*b^5*x^(4/3)*(b*x^(2/3) +
a*x)^(3/2))/(260015*a^6) + (2816*b^4*x^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(1529
5*a^5) - (1408*b^3*x^2*(b*x^(2/3) + a*x)^(3/2))/(7245*a^4) + (352*b^2*x^(7/
3)*(b*x^(2/3) + a*x)^(3/2))/(1725*a^3) - (16*b*x^(8/3)*(b*x^(2/3) + a*x)^(3
/2))/(75*a^2) + (2*x^3*(b*x^(2/3) + a*x)^(3/2))/(9*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
```

```
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}-\frac{(8b)\int x^{8/3}\sqrt{bx^{2/3}+ax}\,dx}{9a}\\
&= -\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}+\frac{(176b^2)\int x^{7/3}\sqrt{bx^{2/3}+ax}\,dx}{225a^2}\\
&= \frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}-\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}\\
&\quad+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}-\frac{(704b^3)\int x^2\sqrt{bx^{2/3}+ax}\,dx}{1035a^3}\\
&= -\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}+\frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}-\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}\\
&\quad+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}+\frac{(1408b^4)\int x^{5/3}\sqrt{bx^{2/3}+ax}\,dx}{2415a^4}\\
&= \frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5}-\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}+\frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}\\
&\quad-\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}-\frac{(22528b^5)\int x^{4/3}\sqrt{bx^{2/3}+ax}\,dx}{45885a^5}\\
&= -\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6}+\frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5}\\
&\quad-\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}+\frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}\\
&\quad-\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}+\frac{(45056b^6)\int x\sqrt{bx^{2/3}+ax}\,dx}{111435a^6}
\end{aligned}$$

$$= \frac{90112 b^6 x \left(b x^{2/3}+a x\right)^{3/2}}{557175 a^7}-\frac{45056 b^5 x^{4/3} \left(b x^{2/3}+a x\right)^{3/2}}{260015 a^6}+\frac{2816 b^4 x^{5/3} \left(b x^{2/3}+a x\right)^{3/2}}{15295 a^5} - \frac{1408 b^3 x^2 \left(b x^{2/3}+a x\right)^{3/2}}{7245 a^4}+\frac{352 b^2 x^{7/3} \left(b x^{2/3}+a x\right)^{3/2}}{1725 a^3}-\frac{16 b x^{8/3} \left(b x^{2/3}+a x\right)^{3/2}}{75 a^2} + \frac{2 x^3 \left(b x^{2/3}+a x\right)^{3/2}}{9 a}-\frac{\left(180224 b^7\right) \int x^{2/3} \sqrt{b x^{2/3}+a x} \, dx}{557175 a^7}$$

$$= -\frac{360448 b^7 x^{2/3} \left(b x^{2/3}+a x\right)^{3/2}}{2414425 a^8}+\frac{90112 b^6 x \left(b x^{2/3}+a x\right)^{3/2}}{557175 a^7}-\frac{45056 b^5 x^{4/3} \left(b x^{2/3}+a x\right)^{3/2}}{260015 a^6} + \frac{2816 b^4 x^{5/3} \left(b x^{2/3}+a x\right)^{3/2}}{15295 a^5}-\frac{1408 b^3 x^2 \left(b x^{2/3}+a x\right)^{3/2}}{7245 a^4}+\frac{352 b^2 x^{7/3} \left(b x^{2/3}+a x\right)^{3/2}}{1725 a^3} - \frac{16 b x^{8/3} \left(b x^{2/3}+a x\right)^{3/2}}{75 a^2}+\frac{2 x^3 \left(b x^{2/3}+a x\right)^{3/2}}{9 a}+\frac{\left(360448 b^8\right) \int \sqrt[3]{x} \sqrt{b x^{2/3}+a x} \, dx}{1448655 a^8}$$

$$= \frac{65536 b^8 \sqrt[3]{x} \left(b x^{2/3}+a x\right)^{3/2}}{482885 a^9}-\frac{360448 b^7 x^{2/3} \left(b x^{2/3}+a x\right)^{3/2}}{2414425 a^8} + \frac{90112 b^6 x \left(b x^{2/3}+a x\right)^{3/2}}{557175 a^7}-\frac{45056 b^5 x^{4/3} \left(b x^{2/3}+a x\right)^{3/2}}{260015 a^6} + \frac{2816 b^4 x^{5/3} \left(b x^{2/3}+a x\right)^{3/2}}{15295 a^5}-\frac{1408 b^3 x^2 \left(b x^{2/3}+a x\right)^{3/2}}{7245 a^4} + \frac{352 b^2 x^{7/3} \left(b x^{2/3}+a x\right)^{3/2}}{1725 a^3}-\frac{16 b x^{8/3} \left(b x^{2/3}+a x\right)^{3/2}}{75 a^2} + \frac{2 x^3 \left(b x^{2/3}+a x\right)^{3/2}}{9 a}-\frac{\left(262144 b^9\right) \int \sqrt{b x^{2/3}+a x} \, dx}{1448655 a^9}$$

$$= -\frac{524288 b^9 \left(b x^{2/3}+a x\right)^{3/2}}{4345965 a^{10}}+\frac{65536 b^8 \sqrt[3]{x} \left(b x^{2/3}+a x\right)^{3/2}}{482885 a^9} - \frac{360448 b^7 x^{2/3} \left(b x^{2/3}+a x\right)^{3/2}}{2414425 a^8}+\frac{90112 b^6 x \left(b x^{2/3}+a x\right)^{3/2}}{557175 a^7} - \frac{45056 b^5 x^{4/3} \left(b x^{2/3}+a x\right)^{3/2}}{260015 a^6}+\frac{2816 b^4 x^{5/3} \left(b x^{2/3}+a x\right)^{3/2}}{15295 a^5} - \frac{1408 b^3 x^2 \left(b x^{2/3}+a x\right)^{3/2}}{7245 a^4}+\frac{352 b^2 x^{7/3} \left(b x^{2/3}+a x\right)^{3/2}}{1725 a^3} - \frac{16 b x^{8/3} \left(b x^{2/3}+a x\right)^{3/2}}{75 a^2}+\frac{2 x^3 \left(b x^{2/3}+a x\right)^{3/2}}{9 a}+\frac{\left(524288 b^{10}\right) \int \frac{\sqrt{b x^{2/3}+a x}}{\sqrt[3]{x}} \, dx}{4345965 a^{10}}$$

$$
\begin{aligned}
&= -\frac{524288 b^9 \left(b x^{2/3}+a x\right)^{3/2}}{4345965 a^{10}}+\frac{1048576 b^{10} \left(b x^{2/3}+a x\right)^{3/2}}{10140585 a^{11} \sqrt[3]{x}}+\frac{65536 b^8 \sqrt[3]{x} \left(b x^{2/3}+a x\right)^{3/2}}{482885 a^9} \\
&\quad -\frac{360448 b^7 x^{2/3} \left(b x^{2/3}+a x\right)^{3/2}}{2414425 a^8}+\frac{90112 b^6 x \left(b x^{2/3}+a x\right)^{3/2}}{557175 a^7}-\frac{45056 b^5 x^{4/3} \left(b x^{2/3}+a x\right)^{3/2}}{260015 a^6} \\
&\quad +\frac{2816 b^4 x^{5/3} \left(b x^{2/3}+a x\right)^{3/2}}{15295 a^5}-\frac{1408 b^3 x^2 \left(b x^{2/3}+a x\right)^{3/2}}{7245 a^4}+\frac{352 b^2 x^{7/3} \left(b x^{2/3}+a x\right)^{3/2}}{1725 a^3} \\
&\quad -\frac{16 b x^{8/3} \left(b x^{2/3}+a x\right)^{3/2}}{75 a^2}+\frac{2 x^3 \left(b x^{2/3}+a x\right)^{3/2}}{9 a}-\frac{\left(2097152 b^{11}\right) \int \frac{\sqrt{b x^{2/3}+a x}}{x^{2/3}} \, dx}{30421755 a^{11}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{524288 b^9 \left(b x^{2/3}+a x\right)^{3/2}}{4345965 a^{10}}-\frac{4194304 b^{11} \left(b x^{2/3}+a x\right)^{3/2}}{50702925 a^{12} x^{2/3}} \\
&\quad +\frac{1048576 b^{10} \left(b x^{2/3}+a x\right)^{3/2}}{10140585 a^{11} \sqrt[3]{x}}+\frac{65536 b^8 \sqrt[3]{x} \left(b x^{2/3}+a x\right)^{3/2}}{482885 a^9} \\
&\quad -\frac{360448 b^7 x^{2/3} \left(b x^{2/3}+a x\right)^{3/2}}{2414425 a^8}+\frac{90112 b^6 x \left(b x^{2/3}+a x\right)^{3/2}}{557175 a^7} \\
&\quad -\frac{45056 b^5 x^{4/3} \left(b x^{2/3}+a x\right)^{3/2}}{260015 a^6}+\frac{2816 b^4 x^{5/3} \left(b x^{2/3}+a x\right)^{3/2}}{15295 a^5} \\
&\quad -\frac{1408 b^3 x^2 \left(b x^{2/3}+a x\right)^{3/2}}{7245 a^4}+\frac{352 b^2 x^{7/3} \left(b x^{2/3}+a x\right)^{3/2}}{1725 a^3} \\
&\quad -\frac{16 b x^{8/3} \left(b x^{2/3}+a x\right)^{3/2}}{75 a^2}+\frac{2 x^3 \left(b x^{2/3}+a x\right)^{3/2}}{9 a}+\frac{\left(4194304 b^{12}\right) \int \frac{\sqrt{b x^{2/3}+a x}}{x} \, dx}{152108775 a^{12}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{524288 b^9 \left(b x^{2/3}+a x\right)^{3/2}}{4345965 a^{10}}+\frac{8388608 b^{12} \left(b x^{2/3}+a x\right)^{3/2}}{152108775 a^{13} x} \\
&\quad -\frac{4194304 b^{11} \left(b x^{2/3}+a x\right)^{3/2}}{50702925 a^{12} x^{2/3}}+\frac{1048576 b^{10} \left(b x^{2/3}+a x\right)^{3/2}}{10140585 a^{11} \sqrt[3]{x}} \\
&\quad +\frac{65536 b^8 \sqrt[3]{x} \left(b x^{2/3}+a x\right)^{3/2}}{482885 a^9}-\frac{360448 b^7 x^{2/3} \left(b x^{2/3}+a x\right)^{3/2}}{2414425 a^8} \\
&\quad +\frac{90112 b^6 x \left(b x^{2/3}+a x\right)^{3/2}}{557175 a^7}-\frac{45056 b^5 x^{4/3} \left(b x^{2/3}+a x\right)^{3/2}}{260015 a^6} \\
&\quad +\frac{2816 b^4 x^{5/3} \left(b x^{2/3}+a x\right)^{3/2}}{15295 a^5}-\frac{1408 b^3 x^2 \left(b x^{2/3}+a x\right)^{3/2}}{7245 a^4} \\
&\quad +\frac{352 b^2 x^{7/3} \left(b x^{2/3}+a x\right)^{3/2}}{1725 a^3}-\frac{16 b x^{8/3} \left(b x^{2/3}+a x\right)^{3/2}}{75 a^2}+\frac{2 x^3 \left(b x^{2/3}+a x\right)^{3/2}}{9 a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.50

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\big(4194304b^{13}-2097152ab^{12}\sqrt[3]{x}+1572864a^2b^{11}x^{2/3}-1310720a^3b^{10}x+}{}$$

```
[In] Integrate[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(4194304*b^13 - 2097152*a*b^12*x^(1/3) + 1572864*a
^2*b^11*x^(2/3) - 1310720*a^3*b^10*x + 1146880*a^4*b^9*x^(4/3) - 1032192*a^
5*b^8*x^(5/3) + 946176*a^6*b^7*x^2 - 878592*a^7*b^6*x^(7/3) + 823680*a^8*b^
5*x^(8/3) - 777920*a^9*b^4*x^3 + 739024*a^10*b^3*x^(10/3) - 705432*a^11*b^2
*x^(11/3) + 676039*a^12*b*x^4 + 16900975*a^13*x^(13/3)))/(152108775*a^13*x^
(1/3))
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.42

method	result
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(16900975a^{12}x^{4}-16224936a^{11}b\,x^{\frac{11}{3}}+15519504a^{10}b^{2}x^{\frac{10}{3}}-14780480a^{9}x^{3}b^{3}+14002560a^{8}b^{4}x^{\frac{8}{3}}-\right.}{}$
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(16224936a^{11}b\,x^{\frac{11}{3}}-15519504a^{10}b^{2}x^{\frac{10}{3}}-14002560a^{8}b^{4}x^{\frac{8}{3}}+13178880a^{7}b^{5}x^{\frac{7}{3}}+11354112a^{5}b^{7}\right.}{}$

```
[In] int(x^3*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/152108775*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(16900975*a^12*x^4-16224936
*a^11*b*x^(11/3)+15519504*a^10*b^2*x^(10/3)-14780480*a^9*x^3*b^3+14002560*a
^8*b^4*x^(8/3)-13178880*a^7*b^5*x^(7/3)+12300288*a^6*b^6*x^2-11354112*a^5*b
^7*x^(5/3)+10321920*a^4*b^8*x^(4/3)-9175040*a^3*b^9*x+7864320*a^2*b^10*x^(2
/3)-6291456*a*b^11*x^(1/3)+4194304*b^12)/x^(1/3)/a^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. $2(277) = 554$.

Time = 179.80 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.49

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/304217550*((211106232532992*b^19 + 43980465111040*b^18 + 206158430208*(6
4*a^3 - 3)*b^16 - 4123168604160*b^17 - 1073741824*(11264*a^3 - 53)*b^15 - 3
93725113600*a^15 - 402653184*(5504*a^3 + 1)*b^14 + 12582912*(3194880*a^6 -
114688*a^3 - 3)*b^13 + 469762048*(18816*a^6 + 103*a^3)*b^12 - 50331648*(488
16*a^6 + 23*a^3)*b^11 - 786432*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^10 -
7340032*(1349120*a^9 + 3439*a^6)*b^9 + 250478592*(5600*a^9 + 3*a^6)*b^8 + 1
2288*(2616979456*a^12 - 21542400*a^9 - 693*a^6)*b^7 + 212992*(43743616*a^12
 + 89111*a^9)*b^6 - 638976*(1652476*a^12 + 935*a^9)*b^5 + 42432*(7217086464
*a^15 + 4969216*a^12 + 165*a^9)*b^4 + 7524608*(20570112*a^15 - 2101*a^12)*b
^3 + 2821728*(7815168*a^15 + 181*a^12)*b^2 + 2028117*(2072576*a^15 - 3*a^12
)*b)*x - 4*(16900975*(16777216*a^13*b^6 + 6291456*a^13*b^5 + 196608*a^13*b^
4 - 262144*a^16 - 114688*a^13*b^3 - 2304*a^13*b^2 + 864*a^13*b - 27*a^13)*x
^5 + 739024*(16777216*a^10*b^9 + 6291456*a^10*b^8 + 196608*a^10*b^7 - 11468
8*a^10*b^6 - 2304*a^10*b^5 + 864*a^10*b^4 - (262144*a^13 + 27*a^10)*b^3)*x^
4 - 878592*(16777216*a^7*b^12 + 6291456*a^7*b^11 + 196608*a^7*b^10 - 114688
*a^7*b^9 - 2304*a^7*b^8 + 864*a^7*b^7 - (262144*a^10 + 27*a^7)*b^6)*x^3 + 1
146880*(16777216*a^4*b^15 + 6291456*a^4*b^14 + 196608*a^4*b^13 - 114688*a^4
*b^12 - 2304*a^4*b^11 + 864*a^4*b^10 - (262144*a^7 + 27*a^4)*b^9)*x^2 - 209
7152*(16777216*a*b^18 + 6291456*a*b^17 + 196608*a*b^16 - 114688*a*b^15 - 23
04*a*b^14 + 864*a*b^13 - (262144*a^4 + 27*a)*b^12)*x + (70368744177664*b^19
 + 26388279066624*b^18 + 824633720832*b^17 - 481036337152*b^16 - 9663676416
*b^15 - 4194304*(262144*a^3 + 27)*b^13 + 3623878656*b^14 + 676039*(16777216
*a^12*b^7 + 6291456*a^12*b^6 + 196608*a^12*b^5 - 114688*a^12*b^4 - 2304*a^1
2*b^3 + 864*a^12*b^2 - (262144*a^15 + 27*a^12)*b)*x^4 - 777920*(16777216*a^
9*b^10 + 6291456*a^9*b^9 + 196608*a^9*b^8 - 114688*a^9*b^7 - 2304*a^9*b^6 +
 864*a^9*b^5 - (262144*a^12 + 27*a^9)*b^4)*x^3 + 946176*(16777216*a^6*b^13
+ 6291456*a^6*b^12 + 196608*a^6*b^11 - 114688*a^6*b^10 - 2304*a^6*b^9 + 864
*a^6*b^8 - (262144*a^9 + 27*a^6)*b^7)*x^2 - 1310720*(16777216*a^3*b^16 + 62
91456*a^3*b^15 + 196608*a^3*b^14 - 114688*a^3*b^13 - 2304*a^3*b^12 + 864*a^
3*b^11 - (262144*a^6 + 27*a^3)*b^10)*x)*x^(2/3) - 24*(29393*(16777216*a^11*
b^8 + 6291456*a^11*b^7 + 196608*a^11*b^6 - 114688*a^11*b^5 - 2304*a^11*b^4
+ 864*a^11*b^3 - (262144*a^14 + 27*a^11)*b^2)*x^4 - 34320*(16777216*a^8*b^1
1 + 6291456*a^8*b^10 + 196608*a^8*b^9 - 114688*a^8*b^8 - 2304*a^8*b^7 + 864
*a^8*b^6 - (262144*a^11 + 27*a^8)*b^5)*x^3 + 43008*(16777216*a^5*b^14 + 629
1456*a^5*b^13 + 196608*a^5*b^12 - 114688*a^5*b^11 - 2304*a^5*b^10 + 864*a^5
*b^9 - (262144*a^8 + 27*a^5)*b^8)*x^2 - 65536*(16777216*a^2*b^17 + 6291456*
a^2*b^16 + 196608*a^2*b^15 - 114688*a^2*b^14 - 2304*a^2*b^13 + 864*a^2*b^12
 - (262144*a^5 + 27*a^2)*b^11)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((1677721
6*a^13*b^6 + 6291456*a^13*b^5 + 196608*a^13*b^4 - 262144*a^16 - 114688*a^13
*b^3 - 2304*a^13*b^2 + 864*a^13*b - 27*a^13)*x)
```

Sympy [F]

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=\int x^3\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x**3*(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(x**3*sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=\int \sqrt{ax+bx^{\frac{2}{3}}}x^3\,dx$$

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(2/3))*x^3, x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.07

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=-\frac{8388608\,b^{\frac{27}{2}}}{152108775\,a^{13}}+\frac{2\left(\frac{27\left(676039\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}-8817900\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b+53117350\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}b^2-195695500\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b^3+492116625\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^4-89237148\right.}{}\right.}{}$$

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
[Out] -8388608/152108775*b^(27/2)/a^13 + 2/152108775*(27*(676039*(a*x^(1/3) + b)^
(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/2)
*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^(17
/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3) + b)
^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3)
+ b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x^(1/3)
 + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sqrt(a*x^
(1/3) + b)*b^12)*b/a^12 + 13*(1300075*(a*x^(1/3) + b)^(27/2) - 18253053*(a*
x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^2 - 478056150*(a
*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2)*b^4 - 26574297
```

```
75*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^(15/2)*b^6 - 463
3467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3) + b)^(11/2)*b^8
- 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1/3) + b)^(7/2)*b
^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3) + b)^(3/2
)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^12)/a
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{bx^{2/3}+ax}\,dx=\int x^3\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

```
[In] int(x^3*(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^3*(a*x + b*x^(2/3))^(1/2), x)
```

3.168 $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 283

$$\begin{aligned}\int x^2\sqrt{bx^{2/3}+ax}\,dx &= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7}-\frac{131072b^9\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^{10}x}\\&\quad+\frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}}-\frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6}\\&\quad+\frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5}-\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4}\\&\quad+\frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3}-\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}\end{aligned}$$

```
[Out] 8192/46189*b^6*(b*x^(2/3)+a*x)^(3/2)/a^7-131072/1616615*b^9*(b*x^(2/3)+a*x)
^(3/2)/a^10/x+196608/1616615*b^8*(b*x^(2/3)+a*x)^(3/2)/a^9/x^(2/3)-49152/32
3323*b^7*(b*x^(2/3)+a*x)^(3/2)/a^8/x^(1/3)-9216/46189*b^5*x^(1/3)*(b*x^(2/3
)+a*x)^(3/2)/a^6+4608/20995*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(3/2)/a^5-384/1615*
b^3*x*(b*x^(2/3)+a*x)^(3/2)/a^4+576/2261*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(3/2)/
a^3-36/133*b*x^(5/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/7*x^2*(b*x^(2/3)+a*x)^(3/2
)/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2041, 2027, 2039}

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx=-\frac{131072b^9\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(ax+bx^{2/3}\right)^{3/2}}{323323a^8\sqrt[3]{x}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{3/2}}{46189a^7}-\frac{9216b^5\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{46189a^6}+\frac{4608b^4x^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{20995a^5}-\frac{384b^3x\left(ax+bx^{2/3}\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}\left(ax+bx^{2/3}\right)^{3/2}}{2261a^3}-\frac{36bx^{5/3}\left(ax+bx^{2/3}\right)^{3/2}}{133a^2}+\frac{2x^2\left(ax+bx^{2/3}\right)^{3/2}}{7a}$$

```
[In] Int[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (8192*b^6*(b*x^(2/3) + a*x)^(3/2))/(46189*a^7) - (131072*b^9*(b*x^(2/3) + a
*x)^(3/2))/(1616615*a^10*x) + (196608*b^8*(b*x^(2/3) + a*x)^(3/2))/(1616615
*a^9*x^(2/3)) - (49152*b^7*(b*x^(2/3) + a*x)^(3/2))/(323323*a^8*x^(1/3)) -
(9216*b^5*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(46189*a^6) + (4608*b^4*x^(2/3)*
(b*x^(2/3) + a*x)^(3/2))/(20995*a^5) - (384*b^3*x*(b*x^(2/3) + a*x)^(3/2))/
(1615*a^4) + (576*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(3/2))/(2261*a^3) - (36*b*x
^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(133*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(3/2))
/(7*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}-\frac{(6b)\int x^{5/3}\sqrt{bx^{2/3}+ax}\,dx}{7a}\\
&= -\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}+\frac{\left(96b^2\right)\int x^{4/3}\sqrt{bx^{2/3}+ax}\,dx}{133a^2}\\
&= \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3}-\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}\\
&\quad+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}-\frac{\left(192b^3\right)\int x\sqrt{bx^{2/3}+ax}\,dx}{323a^3}\\
&= -\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3}-\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}\\
&\quad+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}+\frac{\left(768b^4\right)\int x^{2/3}\sqrt{bx^{2/3}+ax}\,dx}{1615a^4}\\
&= \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5}-\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3}\\
&\quad-\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}-\frac{\left(1536b^5\right)\int \sqrt[3]{x}\sqrt{bx^{2/3}+ax}\,dx}{4199a^5}\\
&= -\frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6}+\frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5}\\
&\quad-\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3}\\
&\quad-\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}+\frac{\left(12288b^6\right)\int \sqrt{bx^{2/3}+ax}\,dx}{46189a^6}\\
&= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7}-\frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6}+\frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5}\\
&\quad-\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3}\\
&\quad-\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}-\frac{\left(8192b^7\right)\int \frac{\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}\,dx}{46189a^7}
\end{aligned}$$

$$= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} - \frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}} - \frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} - \frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4} + \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} - \frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a} + \frac{(32768b^8)\int\frac{\sqrt{bx^{2/3}+ax}}{x^{2/3}}\,dx}{323323a^8}$$

$$= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} + \frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}} - \frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}} - \frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} - \frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4} + \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} - \frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a} - \frac{(65536b^9)\int\frac{\sqrt{bx^{2/3}+ax}}{x}\,dx}{1616615a^9}$$

$$= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} - \frac{131072b^9\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^{10}x} + \frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}} - \frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}} - \frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} - \frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4} + \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} - \frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.47

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}\left(-65536b^9+98304ab^8\sqrt[3]{x}-122880a^2b^7x^{2/3}+143360a^3b^6x-161\right.}{}$$

```
[In] Integrate[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2)*(-65536*b^9 + 98304*a*b^8*x^(1/3) - 122880*a^2*b
^7*x^(2/3) + 143360*a^3*b^6*x - 161280*a^4*b^5*x^(4/3) + 177408*a^5*b^4*x^(
5/3) - 192192*a^6*b^3*x^2 + 205920*a^7*b^2*x^(7/3) - 218790*a^8*b*x^(8/3) +
 230945*a^9*x^3))/(1616615*a^10*x)
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.43

method	result
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(230945a^{9}x^{3}-218790a^{8}b\,x^{\frac{8}{3}}+205920a^{7}b^{2}x^{\frac{7}{3}}-192192a^{6}b^{3}x^{2}+177408a^{5}b^{4}x^{\frac{5}{3}}-161280a^{4}b^{5}x^{\frac{4}{3}}+1\right.}{1616615x^{\frac{1}{3}}\,a^{10}}$
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(218790a^{8}b\,x^{\frac{8}{3}}-205920a^{7}b^{2}x^{\frac{7}{3}}-177408a^{5}b^{4}x^{\frac{5}{3}}+161280a^{4}b^{5}x^{\frac{4}{3}}-230945a^{9}x^{3}+122880a^{2}b^{7}x^{\frac{2}{3}}\right.}{1616615x^{\frac{1}{3}}\,a^{10}}$

```
[In] int(x^2*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1616615*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(230945*a^9*x^3-218790*a^8*b*
x^(8/3)+205920*a^7*b^2*x^(7/3)-192192*a^6*b^3*x^2+177408*a^5*b^4*x^(5/3)-16
1280*a^4*b^5*x^(4/3)+143360*a^3*b^6*x-122880*a^2*b^7*x^(2/3)+98304*a*b^8*x^
(1/3)-65536*b^9)/x^(1/3)/a^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. 2(211) = 422.

Time = 150.03 (sec) , antiderivative size = 1031, normalized size of antiderivative = 3.64

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3233230*((3298534883328*b^16 + 687194767360*b^15 + 3221225472*(64*a^3 - 3
)*b^13 - 64424509440*b^14 - 16777216*(11264*a^3 - 53)*b^12 + 5380094720*a^1
2 - 6291456*(5504*a^3 + 1)*b^11 + 196608*(3194880*a^6 - 114688*a^3 - 3)*b^1
0 + 7340032*(18816*a^6 + 103*a^3)*b^9 - 786432*(48816*a^6 + 23*a^3)*b^8 - 1
2288*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^7 - 114688*(1349120*a^9 + 3439*
a^6)*b^6 + 3913728*(5600*a^9 + 3*a^6)*b^5 - 2112*(2027683840*a^12 + 1958400
*a^9 + 63*a^6)*b^4 - 36608*(59351040*a^12 - 8101*a^9)*b^3 - 549120*(566272*
a^12 + 17*a^9)*b^2 - 109395*(516096*a^12 - a^9)*b)*x + 4*(230945*(16777216*
a^10*b^6 + 6291456*a^10*b^5 + 196608*a^10*b^4 - 262144*a^13 - 114688*a^10*b
^3 - 2304*a^10*b^2 + 864*a^10*b - 27*a^10)*x^4 + 13728*(16777216*a^7*b^9 +
6291456*a^7*b^8 + 196608*a^7*b^7 - 114688*a^7*b^6 - 2304*a^7*b^5 + 864*a^7*
b^4 - (262144*a^10 + 27*a^7)*b^3)*x^3 - 17920*(16777216*a^4*b^12 + 6291456*
a^4*b^11 + 196608*a^4*b^10 - 114688*a^4*b^9 - 2304*a^4*b^8 + 864*a^4*b^7 -
(262144*a^7 + 27*a^4)*b^6)*x^2 + 32768*(16777216*a*b^15 + 6291456*a*b^14 +
196608*a*b^13 - 114688*a*b^12 - 2304*a*b^11 + 864*a*b^10 - (262144*a^4 + 27
*a)*b^9)*x - (1099511627776*b^16 + 412316860416*b^15 + 12884901888*b^14 - 7
516192768*b^13 - 150994944*b^12 - 65536*(262144*a^3 + 27)*b^10 + 56623104*b
^11 - 12155*(16777216*a^9*b^7 + 6291456*a^9*b^6 + 196608*a^9*b^5 - 114688*a
```

```
^9*b^4 - 2304*a^9*b^3 + 864*a^9*b^2 - (262144*a^12 + 27*a^9)*b)*x^3 + 14784
*(16777216*a^6*b^10 + 6291456*a^6*b^9 + 196608*a^6*b^8 - 114688*a^6*b^7 - 2
304*a^6*b^6 + 864*a^6*b^5 - (262144*a^9 + 27*a^6)*b^4)*x^2 - 20480*(1677721
6*a^3*b^13 + 6291456*a^3*b^12 + 196608*a^3*b^11 - 114688*a^3*b^10 - 2304*a^
3*b^9 + 864*a^3*b^8 - (262144*a^6 + 27*a^3)*b^7)*x)*x^(2/3) - 6*(2145*(1677
7216*a^8*b^8 + 6291456*a^8*b^7 + 196608*a^8*b^6 - 114688*a^8*b^5 - 2304*a^8
*b^4 + 864*a^8*b^3 - (262144*a^11 + 27*a^8)*b^2)*x^3 - 2688*(16777216*a^5*b
^11 + 6291456*a^5*b^10 + 196608*a^5*b^9 - 114688*a^5*b^8 - 2304*a^5*b^7 + 8
64*a^5*b^6 - (262144*a^8 + 27*a^5)*b^5)*x^2 + 4096*(16777216*a^2*b^14 + 629
1456*a^2*b^13 + 196608*a^2*b^12 - 114688*a^2*b^11 - 2304*a^2*b^10 + 864*a^2
*b^9 - (262144*a^5 + 27*a^2)*b^8)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((1677
7216*a^10*b^6 + 6291456*a^10*b^5 + 196608*a^10*b^4 - 262144*a^13 - 114688*a
^10*b^3 - 2304*a^10*b^2 + 864*a^10*b - 27*a^10)*x)
```

Sympy [F]

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \int x^2\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x**2*(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(x**2*sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \int \sqrt{ax+bx^{\frac{2}{3}}}x^2\,dx$$

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(2/3))*x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.10

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \frac{131072\,b^{\frac{21}{2}}}{1616615\,a^{10}} + \frac{2\Bigg(\frac{21\Big(12155\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}-122265\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b+554268\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^2-1492260\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^3+2645370\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^4-3233230\Big(ax^{\frac{1}{3}}+}{a^9}}{}$$

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 131072/1616615*b^(21/2)/a^10 + 2/1616615*(21*(12155*(a*x^(1/3) + b)^(19/2)
- 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)^(15/2)*b^2 - 149
2260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)*b^4 - 3233
230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^6 - 1662804
*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 - 230945*sqrt
(a*x^(1/3) + b)*b^9)*b/a^9 + 5*(46189*(a*x^(1/3) + b)^(21/2) - 510510*(a*x^
(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752*(a*x^(1/
3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 22221108*(a*x^(1
/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 16628040*(a*x^(1
/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(a*x^(1/3)
 + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^9)/a
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx=\int x^2\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

```
[In] int(x^2*(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^2*(a*x + b*x^(2/3))^(1/2), x)
```

3.169 $\int x\sqrt{bx^{2/3}+ax}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 195

$$\int x\sqrt{bx^{2/3}+ax}\,dx = -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{2048b^6\left(bx^{2/3}+ax\right)^{3/2}}{15015a^7x} -\frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}} +\frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}$$

```
[Out] -128/429*b^3*(b*x^(2/3)+a*x)^(3/2)/a^4+2048/15015*b^6*(b*x^(2/3)+a*x)^(3/2)
/a^7/x-1024/5005*b^5*(b*x^(2/3)+a*x)^(3/2)/a^6/x^(2/3)+256/1001*b^4*(b*x^(2
/3)+a*x)^(3/2)/a^5/x^(1/3)+48/143*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(3/2)/a^3-24/
65*b*x^(2/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/5*x*(b*x^(2/3)+a*x)^(3/2)/a
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2041, 2027, 2039}

$$\int x\sqrt{bx^{2/3}+ax}\,dx = \frac{2048b^6\left(ax+bx^{2/3}\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(ax+bx^{2/3}\right)^{3/2}}{5005a^6x^{2/3}} +\frac{256b^4\left(ax+bx^{2/3}\right)^{3/2}}{1001a^5\sqrt[3]{x}}-\frac{128b^3\left(ax+bx^{2/3}\right)^{3/2}}{429a^4}+\frac{48b^2\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{3/2}}{143a^3} -\frac{24bx^{2/3}\left(ax+bx^{2/3}\right)^{3/2}}{65a^2}+\frac{2x\left(ax+bx^{2/3}\right)^{3/2}}{5a}$$

```
[In] Int[x*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-128*b^3*(b*x^(2/3) + a*x)^(3/2))/(429*a^4) + (2048*b^6*(b*x^(2/3) + a*x)^
(3/2))/(15015*a^7*x) - (1024*b^5*(b*x^(2/3) + a*x)^(3/2))/(5005*a^6*x^(2/3)
) + (256*b^4*(b*x^(2/3) + a*x)^(3/2))/(1001*a^5*x^(1/3)) + (48*b^2*x^(1/3)*
(b*x^(2/3) + a*x)^(3/2))/(143*a^3) - (24*b*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))
/(65*a^2) + (2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}-\frac{(4b)\int x^{2/3}\sqrt{bx^{2/3}+ax}\,dx}{5a}\\
&= -\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}+\frac{\left(8b^2\right)\int \sqrt[3]{x}\sqrt{bx^{2/3}+ax}\,dx}{13a^2}\\
&= \frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}\\
&\quad+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}-\frac{\left(64b^3\right)\int \sqrt{bx^{2/3}+ax}\,dx}{143a^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}\\
&\quad -\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}+\frac{(128b^4)\int\frac{\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}\,dx}{429a^4}\\
&= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}}+\frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}\\
&\quad -\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}-\frac{(512b^5)\int\frac{\sqrt{bx^{2/3}+ax}}{x^{2/3}}\,dx}{3003a^5}\\
&= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}-\frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}}\\
&\quad +\frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}\\
&\quad +\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}+\frac{(1024b^6)\int\frac{\sqrt{bx^{2/3}+ax}}{x}\,dx}{15015a^6}\\
&= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{2048b^6\left(bx^{2/3}+ax\right)^{3/2}}{15015a^7x}\\
&\quad -\frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}}\\
&\quad +\frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.49

$$
\int x\sqrt{bx^{2/3}+ax}\,dx=\frac{2\left(bx^{2/3}+ax\right)^{3/2}\left(1024b^6-1536ab^5\sqrt[3]{x}+1920a^2b^4x^{2/3}-2240a^3b^3x+2520a^4b^2x^{4/3}\right.}{15015a^7x}
$$

```
[In] Integrate[x*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2)*(1024*b^6 - 1536*a*b^5*x^(1/3) + 1920*a^2*b^4*x^
(2/3) - 2240*a^3*b^3*x + 2520*a^4*b^2*x^(4/3) - 2772*a^5*b*x^(5/3) + 3003*a
^6*x^2))/(15015*a^7*x)
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.46

method	result	si
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(3003a^{6}x^{2}-2772a^{5}b\,x^{\frac{5}{3}}+2520a^{4}b^{2}x^{\frac{4}{3}}-2240a^{3}b^{3}x+1920a^{2}x^{\frac{2}{3}}b^{4}-1536a\,b^{5}x^{\frac{1}{3}}+1024b^{6}\right)}{15015x^{\frac{1}{3}}a^{7}}$	90
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(2772a^{5}b\,x^{\frac{5}{3}}-2520a^{4}b^{2}x^{\frac{4}{3}}-1920a^{2}x^{\frac{2}{3}}b^{4}-3003a^{6}x^{2}+1536a\,b^{5}x^{\frac{1}{3}}+2240a^{3}b^{3}x-1024b^{6}\right)}{15015x^{\frac{1}{3}}a^{7}}$	90

```
[In] int(x*(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15015*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(3003*a^6*x^2-2772*a^5*b*x^(5/3
)+2520*a^4*b^2*x^(4/3)-2240*a^3*b^3*x+1920*a^2*x^(2/3)*b^4-1536*a*b^5*x^(1/
3)+1024*b^6)/x^(1/3)/a^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(145) = 290$.

Time = 147.95 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.93

$$\int x\sqrt{bx^{2/3}+ax}\,dx =$$

$$-\,(51539607552\,b^{13} + 10737418240\,b^{12} + 50331648\,(64\,a^{3} - 3)b^{10} - 1006632960\,b^{11} - 262144\,(11264\,a^{3} - 53)$$

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30030*((51539607552*b^13 + 10737418240*b^12 + 50331648*(64*a^3 - 3)*b^10
 - 1006632960*b^11 - 262144*(11264*a^3 - 53)*b^9 - 69957888*a^9 - 98304*(55
04*a^3 + 1)*b^8 + 3072*(3194880*a^6 - 114688*a^3 - 3)*b^7 + 114688*(18816*a
^6 + 103*a^3)*b^6 - 12288*(48816*a^6 + 23*a^3)*b^5 + 192*(302776320*a^9 + 4
95872*a^6 + 15*a^3)*b^4 + 1792*(16588800*a^9 - 3439*a^6)*b^3 + 26208*(16384
0*a^9 + 7*a^6)*b^2 + 693*(1024000*a^9 - 3*a^6)*b)*x - 4*(3003*(16777216*a^7
*b^6 + 6291456*a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b^3 - 23
04*a^7*b^2 + 864*a^7*b - 27*a^7)*x^3 + 280*(16777216*a^4*b^9 + 6291456*a^4*
b^8 + 196608*a^4*b^7 - 114688*a^4*b^6 - 2304*a^4*b^5 + 864*a^4*b^4 - (26214
4*a^7 + 27*a^4)*b^3)*x^2 - 512*(16777216*a*b^12 + 6291456*a*b^11 + 196608*a
*b^10 - 114688*a*b^9 - 2304*a*b^8 + 864*a*b^7 - (262144*a^4 + 27*a)*b^6)*x
+ (17179869184*b^13 + 6442450944*b^12 + 201326592*b^11 - 117440512*b^10 - 2
359296*b^9 - 1024*(262144*a^3 + 27)*b^7 + 884736*b^8 + 231*(16777216*a^6*b^
7 + 6291456*a^6*b^6 + 196608*a^6*b^5 - 114688*a^6*b^4 - 2304*a^6*b^3 + 864*
a^6*b^2 - (262144*a^9 + 27*a^6)*b)*x^2 - 320*(16777216*a^3*b^10 + 6291456*a
^3*b^9 + 196608*a^3*b^8 - 114688*a^3*b^7 - 2304*a^3*b^6 + 864*a^3*b^5 - (26
```

```
2144*a^6 + 27*a^3)*b^4)*x)*x^(2/3) - 12*(21*(16777216*a^5*b^8 + 6291456*a^5
*b^7 + 196608*a^5*b^6 - 114688*a^5*b^5 - 2304*a^5*b^4 + 864*a^5*b^3 - (2621
44*a^8 + 27*a^5)*b^2)*x^2 - 32*(16777216*a^2*b^11 + 6291456*a^2*b^10 + 1966
08*a^2*b^9 - 114688*a^2*b^8 - 2304*a^2*b^7 + 864*a^2*b^6 - (262144*a^5 + 27
*a^2)*b^5)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^7*b^6 + 6291456*
a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b^3 - 2304*a^7*b^2 + 86
4*a^7*b - 27*a^7)*x)
```

Sympy [F]

$$\int x\sqrt{bx^{2/3}+ax}\,dx = \int x\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

```
[In] integrate(x*(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(x*sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int x\sqrt{bx^{2/3}+ax}\,dx = \int \sqrt{ax+bx^{\frac{2}{3}}}x\,dx$$

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(2/3))*x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.17

$$\int x\sqrt{bx^{2/3}+ax}\,dx = -\frac{2048\,b^{\frac{15}{2}}}{15015\,a^{7}} + \frac{2\left(\frac{15\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}-1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b+5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{2}-8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{3}+9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{4}-6006\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{5}+3003\sqrt{ax^{\frac{1}{3}}}\right)}{a^{6}}\right)}{}$$

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
[Out] -2048/15015*b^(15/2)/a^7 + 2/15015*(15*(231*(a*x^(1/3) + b)^(13/2) - 1638*(
a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3)
```

```
+ b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(3/2
)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*x^(1/3) + b)^(15/2)
 - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025
*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(
1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3)
 + b)*b^7)/a^6)/a
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{bx^{2/3}+ax}\,dx=\int x\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

```
[In] int(x*(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x*(a*x + b*x^(2/3))^(1/2), x)
```

3.170 $\int \sqrt{bx^{2/3}+ax}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \sqrt{bx^{2/3}+ax}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a} - \frac{32b^3\left(bx^{2/3}+ax\right)^{3/2}}{105a^4x} + \frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}} - \frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}$$

```
[Out] 2/3*(b*x^(2/3)+a*x)^(3/2)/a-32/105*b^3*(b*x^(2/3)+a*x)^(3/2)/a^4/x+16/35*b^
2*(b*x^(2/3)+a*x)^(3/2)/a^3/x^(2/3)-4/7*b*(b*x^(2/3)+a*x)^(3/2)/a^2/x^(1/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2027, 2041, 2039}

$$\int \sqrt{bx^{2/3}+ax}\,dx = -\frac{32b^3\left(ax+bx^{2/3}\right)^{3/2}}{105a^4x} + \frac{16b^2\left(ax+bx^{2/3}\right)^{3/2}}{35a^3x^{2/3}} - \frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{7a^2\sqrt[3]{x}} + \frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}$$

```
[In] Int[Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (32*b^3*(b*x^(2/3) + a*x)^(3/2))/(105*a
^4*x) + (16*b^2*(b*x^(2/3) + a*x)^(3/2))/(35*a^3*x^(2/3)) - (4*b*(b*x^(2/3)
 + a*x)^(3/2))/(7*a^2*x^(1/3))
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{(2b)\int \frac{\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}\,dx}{3a}\\
&= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}+\frac{(8b^2)\int \frac{\sqrt{bx^{2/3}+ax}}{x^{2/3}}\,dx}{21a^2}\\
&= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}+\frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}-\frac{(16b^3)\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx}{105a^3}\\
&= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a}-\frac{32b^3\left(bx^{2/3}+ax\right)^{3/2}}{105a^4x}+\frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int \sqrt{bx^{2/3}+ax}\,dx=\frac{2\sqrt{bx^{2/3}+ax}\left(-16b^4+8ab^3\sqrt[3]{x}-6a^2b^2x^{2/3}+5a^3bx+35a^4x^{4/3}\right)}{105a^4\sqrt[3]{x}}$$

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-16*b^4 + 8*a*b^3*x^(1/3) - 6*a^2*b^2*x^(2/3) + 5
*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^4*x^(1/3))
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^3x-30a^2b\,x^{\frac{2}{3}}+24a\,b^2x^{\frac{1}{3}}-16b^3\right)}{105x^{\frac{1}{3}}a^4}$	57
default	$-\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)\left(30a^2b\,x^{\frac{2}{3}}-24a\,b^2x^{\frac{1}{3}}-35a^3x+16b^3\right)}{105x^{\frac{1}{3}}a^4}$	57

```
[In] int((b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(35*a^3*x-30*a^2*b*x^(2/3)+24*a*b
^2*x^(1/3)-16*b^3)/x^(1/3)/a^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(81) = 162$.

Time = 162.93 (sec) , antiderivative size = 501, normalized size of antiderivative = 4.60

$$\int \sqrt{bx^{2/3}+ax}\,dx=\frac{(805306368\,b^{10}+167772160\,b^{9}+786432\,(64\,a^{3}-3)b^{7}-15728640\,b^{8}-4096\,(11264\,a^{3}}{}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/210*((805306368*b^10 + 167772160*b^9 + 786432*(64*a^3 - 3)*b^7 - 15728640
*b^8 - 4096*(11264*a^3 - 53)*b^6 + 815360*a^6 - 1536*(5504*a^3 + 1)*b^5 - 4
8*(15728640*a^6 + 114688*a^3 + 3)*b^4 - 1792*(221184*a^6 - 103*a^3)*b^3 - 1
92*(307200*a^6 + 23*a^3)*b^2 - 15*(499712*a^6 - 3*a^3)*b)*x + 4*(35*(167772
16*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^4*b^3
 - 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x^2 - 6*(16777216*a^2*b^8 + 6291456*a
```

```
^2*b^7 + 196608*a^2*b^6 - 114688*a^2*b^5 - 2304*a^2*b^4 + 864*a^2*b^3 - (26
2144*a^5 + 27*a^2)*b^2)*x^(4/3) + 8*(16777216*a*b^9 + 6291456*a*b^8 + 19660
8*a*b^7 - 114688*a*b^6 - 2304*a*b^5 + 864*a*b^4 - (262144*a^4 + 27*a)*b^3)*
x - (268435456*b^10 + 100663296*b^9 + 3145728*b^8 - 1835008*b^7 - 36864*b^6
 - 16*(262144*a^3 + 27)*b^4 + 13824*b^5 - 5*(16777216*a^3*b^7 + 6291456*a^3
*b^6 + 196608*a^3*b^5 - 114688*a^3*b^4 - 2304*a^3*b^3 + 864*a^3*b^2 - (2621
44*a^6 + 27*a^3)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^4*b^6 +
 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^4*b^3 - 2304*a^4*
b^2 + 864*a^4*b - 27*a^4)*x)
```

Sympy [F]

$$\int \sqrt{bx^{2/3}+ax}\,dx = \int \sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \sqrt{bx^{2/3}+ax}\,dx = \int \sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31

$$\int \sqrt{bx^{2/3}+ax}\,dx = \frac{32\,b^{\frac{9}{2}}}{105\,a^{4}} + \frac{2\left(\frac{9\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{2}-35\sqrt{ax^{\frac{1}{3}}+b}b^{3}\right)b}{a^{3}} + \frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{2}-420\left(ax^{\frac{1}{3}}+\right.}{a^{3}}\right.}{105\,a}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 32/105*b^(9/2)/a^4 + 2/105*(9*(5*(a*x^(1/3) + b)^(7/2) - 21*(a*x^(1/3) + b)
^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(1/3) + b)*b^3)*b/a^3
 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b)^(7/2)*b + 378*(a*x^(1/3)
 + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 + 315*sqrt(a*x^(1/3) + b)*b
^4)/a^3)/a
```

Mupad [B] (verification not implemented)

Time = 11.74 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.37

$$\int \sqrt{bx^{2/3}+ax}\,dx=\frac{3\,x\,\sqrt{a\,x+b\,x^{2/3}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},4;\ 5;\ -\frac{a\,x^{1/3}}{b}\right)}{4\,\sqrt{\frac{a\,x^{1/3}}{b}+1}}$$

```
[In] int((a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] (3*x*(a*x + b*x^(2/3))^(1/2)*hypergeom([-1/2, 4], 5, -(a*x^(1/3))/b))/(4*((
a*x^(1/3))/b + 1)^(1/2))
```

3.171 $\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 23

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

```
[Out] 2*(b*x^(2/3)+a*x)^(3/2)/a/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2039}

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x,x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x,x]
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{3}}\,a}$	27
default	$\frac{2\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(b+a\,x^{\frac{1}{3}}\right)}{x^{\frac{1}{3}}\,a}$	27

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2*(b*x^(2/3)+a*x)^(1/2)/x^(1/3)*(b+a*x^(1/3))/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(19) = 38.

Time = 170.84 (sec) , antiderivative size = 224, normalized size of antiderivative = 9.74

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx =$$

$$-\frac{(50331648\,b^7 + 10485760\,b^6 + 49152\,(1024\,a^3 - 3)b^4 - 983040\,b^5 + 256\,(73728\,a^3 + 53)b^3 - 23296\,a^3 + 96}{}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="fricas")
[Out] -1/2*((50331648*b^7 + 10485760*b^6 + 49152*(1024*a^3 - 3)*b^4 - 983040*b^5
+ 256*(73728*a^3 + 53)*b^3 - 23296*a^3 + 96*(16384*a^3 - 1)*b^2 - 9*(8192*a
^3 + 1)*b)*x - 4*((16777216*a*b^6 + 6291456*a*b^5 + 196608*a*b^4 - 262144*a
^4 - 114688*a*b^3 - 2304*a*b^2 + 864*a*b - 27*a)*x + (16777216*b^7 + 629145
6*b^6 + 196608*b^5 - 114688*b^4 - 2304*b^3 - (262144*a^3 + 27)*b + 864*b^2)
*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a*b^6 + 6291456*a*b^5 + 196608*
a*b^4 - 262144*a^4 - 114688*a*b^3 - 2304*a*b^2 + 864*a*b - 27*a)*x)
```

Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x,x)
[Out] Integral(sqrt(a*x + b*x**(2/3))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(2/3))/x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \frac{2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}}{a} - \frac{2\,b^{\frac{3}{2}}}{a}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="giac")
[Out] 2*(a*x^(1/3) + b)^(3/2)/a - 2*b^(3/2)/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx = \int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x}\,dx$$

```
[In] int((a*x + b*x^(2/3))^(1/2)/x,x)
[Out] int((a*x + b*x^(2/3))^(1/2)/x, x)
```

3.172 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx=-\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}$$

```
[Out] 3/4*a^2*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)-3/2*(b*x^(2/
3)+a*x)^(1/2)/x-3/4*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(2/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 2050, 2054, 212}

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx=\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{4b^{3/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4bx^{2/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(2*x) - (3*a*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(2/3)
) + (3*a^2*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(4*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}+\frac{1}{4}a\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}-\frac{a^2\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{8b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = -\frac{3\left(2b+a\sqrt[3]{x}\right)\sqrt{bx^{2/3}+ax}}{4bx}+\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}$$

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

```
[Out] (-3*(2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(4*b*x) + (3*a^2*ArcTanh[(Sqrt
[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(4*b^(3/2))
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{3\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}} b^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right) b\,a^{2} x^{\frac{2}{3}}+\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}\right)}{4x\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}}$	79
default	$\frac{3\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right) b\,a^{2} x^{\frac{2}{3}}-\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}} b^{\frac{3}{2}}-\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}\right)}{4x\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}}$	80

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*(b*x^(2/3)+a*x)^(1/2)*((b+a*x^(1/3))^(3/2)*b^(3/2)-arctanh((b+a*x^(1/3
))^(1/2)/b^(1/2))*b*a^2*x^(2/3)+(b+a*x^(1/3))^(1/2)*b^(5/2))/x/(b+a*x^(1/3)
)^(1/2)/b^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx=\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^2}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx=\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^2}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx=-\frac{3\left(\frac{a^3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^3+\sqrt{ax^{\frac{1}{3}}+b}\,ba^3b}{a^2bx^{\frac{2}{3}}}\right)}{4\,a}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -3/4*(a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + ((a*x^(1/3) +
 b)^(3/2)*a^3 + sqrt(a*x^(1/3) + b)*a^3*b)/(a^2*b*x^(2/3)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx=\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^2}\,dx$$

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^2,x)
[Out] int((a*x + b*x^(2/3))^(1/2)/x^2, x)
```

3.173 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 178

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx=-\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}+\frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}}-\frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x}+\frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}}-\frac{21a^5\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{128b^{9/2}}$$

```
[Out] -21/128*a^5*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)-3/5*(b*x
^(2/3)+a*x)^(1/2)/x^2-3/40*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(5/3)+7/80*a^2*(b*x^
(2/3)+a*x)^(1/2)/b^2/x^(4/3)-7/64*a^3*(b*x^(2/3)+a*x)^(1/2)/b^3/x+21/128*a^
4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 2050, 2054, 212}

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx=-\frac{21a^5\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{128b^{9/2}}+\frac{21a^4\sqrt{ax+bx^{2/3}}}{128b^4x^{2/3}}-\frac{7a^3\sqrt{ax+bx^{2/3}}}{64b^3x}+\frac{7a^2\sqrt{ax+bx^{2/3}}}{80b^2x^{4/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{40bx^{5/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(5*x^2) - (3*a*Sqrt[b*x^(2/3) + a*x])/(40*b*x^(5
/3)) + (7*a^2*Sqrt[b*x^(2/3) + a*x])/(80*b^2*x^(4/3)) - (7*a^3*Sqrt[b*x^(2/
```

```
3) + a*x])/(64*b^3*x) + (21*a^4*Sqrt[b*x^(2/3) + a*x])/(128*b^4*x^(2/3)) -
(21*a^5*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(128*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}+\frac{1}{10}a\int\frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}-\frac{(7a^2)\int\frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{80b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}+\frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}}+\frac{(7a^3)\int\frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{96b^2}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2}-\frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}}+\frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}}\\
&\quad-\frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x}-\frac{(7a^4)\int\frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{128b^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} \\
&\quad - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}} + \frac{(7a^5)\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{256b^4} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} \\
&\quad + \frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}} - \frac{(21a^5)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{128b^4} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} \\
&\quad - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}} - \frac{21a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{128b^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$
\begin{aligned}
\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx &= \frac{\sqrt{bx^{2/3}+ax}\left(-384b^4 - 48ab^3\sqrt[3]{x} + 56a^2b^2x^{2/3} - 70a^3bx + 105a^4x^{4/3}\right)}{640b^4x^2} \\
&\quad - \frac{21a^5\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{128b^{9/2}}
\end{aligned}
$$

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-384*b^4 - 48*a*b^3*x^(1/3) + 56*a^2*b^2*x^(2/3) -
70*a^3*b*x + 105*a^4*x^(4/3)))/(640*b^4*x^2) - (21*a^5*ArcTanh[(Sqrt[b]*x^(
1/3))/Sqrt[b*x^(2/3) + a*x]])/(128*b^(9/2))
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(-105b^{\frac{9}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}+490b^{\frac{11}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}-896b^{\frac{13}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}+105\,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b^{4}a^{5}x^{\frac{5}{3}}+79\right.}{640x^{2}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{17}{2}}}$
default	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(-105b^{\frac{9}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}+490b^{\frac{11}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}-896b^{\frac{13}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}+105\,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b^{4}a^{5}x^{\frac{5}{3}}+79\right.}{640x^{2}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{17}{2}}}$

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/640*(b*x^(2/3)+a*x)^(1/2)*(-105*b^(9/2)*(b+a*x^(1/3))^(9/2)+490*b^(11/2)
*(b+a*x^(1/3))^(7/2)-896*b^(13/2)*(b+a*x^(1/3))^(5/2)+105*arctanh((b+a*x^(1
/3))^(1/2)/b^(1/2))*b^4*a^5*x^(5/3)+790*b^(15/2)*(b+a*x^(1/3))^(3/2)+105*b^
(17/2)*(b+a*x^(1/3))^(1/2))/x^2/(b+a*x^(1/3))^(1/2)/b^(17/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^3}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx=\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^3}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^3, x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx=\frac{\frac{105\,a^6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4}+\frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^6-490\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^6b+896\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^6b^2-790\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^6b^3-105}{a^5b^4x^{\frac{5}{3}}}}{640\,a}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/640*(105*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(
a*x^(1/3) + b)^(9/2)*a^6 - 490*(a*x^(1/3) + b)^(7/2)*a^6*b + 896*(a*x^(1/3)
 + b)^(5/2)*a^6*b^2 - 790*(a*x^(1/3) + b)^(3/2)*a^6*b^3 - 105*sqrt(a*x^(1/3
) + b)*a^6*b^4)/(a^5*b^4*x^(5/3)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx=\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^3}\,dx$$

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^3,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^3, x)
```

3.174 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 266

$$\begin{aligned}\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = & -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} \\ & -\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}-\frac{429a^5\sqrt{bx^{2/3}+ax}}{10240b^5x^{4/3}} \\ & +\frac{429a^6\sqrt{bx^{2/3}+ax}}{8192b^6x}-\frac{1287a^7\sqrt{bx^{2/3}+ax}}{16384b^7x^{2/3}}+\frac{1287a^8\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{16384b^{15/2}}\end{aligned}$$

```
[Out] 1287/16384*a^8*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)-3/8*
(b*x^(2/3)+a*x)^(1/2)/x^3-3/112*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(8/3)+13/448*a^
2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-143/4480*a^3*(b*x^(2/3)+a*x)^(1/2)/b^3/
x^2+1287/35840*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(5/3)-429/10240*a^5*(b*x^(2/
3)+a*x)^(1/2)/b^5/x^(4/3)+429/8192*a^6*(b*x^(2/3)+a*x)^(1/2)/b^6/x-1287/163
84*a^7*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used

$= \{2045, 2050, 2054, 212\}$

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \frac{1287a^8\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{16384b^{15/2}} - \frac{1287a^7\sqrt{ax+bx^{2/3}}}{16384b^7x^{2/3}} + \frac{429a^6\sqrt{ax+bx^{2/3}}}{8192b^6x} - \frac{429a^5\sqrt{ax+bx^{2/3}}}{10240b^5x^{4/3}} + \frac{1287a^4\sqrt{ax+bx^{2/3}}}{35840b^4x^{5/3}} - \frac{143a^3\sqrt{ax+bx^{2/3}}}{4480b^3x^2} + \frac{13a^2\sqrt{ax+bx^{2/3}}}{448b^2x^{7/3}} - \frac{3a\sqrt{ax+bx^{2/3}}}{112bx^{8/3}} - \frac{3\sqrt{ax+bx^{2/3}}}{8x^3}$$

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(8*x^3) - (3*a*Sqrt[b*x^(2/3) + a*x])/(112*b*x^(
8/3)) + (13*a^2*Sqrt[b*x^(2/3) + a*x])/(448*b^2*x^(7/3)) - (143*a^3*Sqrt[b*
x^(2/3) + a*x])/(4480*b^3*x^2) + (1287*a^4*Sqrt[b*x^(2/3) + a*x])/(35840*b^
4*x^(5/3)) - (429*a^5*Sqrt[b*x^(2/3) + a*x])/(10240*b^5*x^(4/3)) + (429*a^6
*Sqrt[b*x^(2/3) + a*x])/(8192*b^6*x) - (1287*a^7*Sqrt[b*x^(2/3) + a*x])/(16
384*b^7*x^(2/3)) + (1287*a^8*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x
]])/(16384*b^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}+\frac{1}{16}a\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}-\frac{(13a^2)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{224b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}+\frac{(143a^3)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{2688b^2}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}\\
&\quad-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}-\frac{(429a^4)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{8960b^3}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}\\
&\quad-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}+\frac{(429a^5)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{10240b^4}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}\\
&\quad+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}-\frac{429a^5\sqrt{bx^{2/3}+ax}}{10240b^5x^{4/3}}-\frac{(143a^6)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{4096b^5}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}\\
&\quad+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}-\frac{429a^5\sqrt{bx^{2/3}+ax}}{10240b^5x^{4/3}}+\frac{429a^6\sqrt{bx^{2/3}+ax}}{8192b^6x}\\
&\quad+\frac{(429a^7)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{16384b^6}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}\\
&\quad-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}-\frac{429a^5\sqrt{bx^{2/3}+ax}}{10240b^5x^{4/3}}\\
&\quad+\frac{429a^6\sqrt{bx^{2/3}+ax}}{8192b^6x}-\frac{1287a^7\sqrt{bx^{2/3}+ax}}{16384b^7x^{2/3}}-\frac{(429a^8)\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{32768b^7}
\end{aligned}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}} - \frac{429a^5\sqrt{bx^{2/3}+ax}}{10240b^5x^{4/3}} + \frac{429a^6\sqrt{bx^{2/3}+ax}}{8192b^6x} - \frac{1287a^7\sqrt{bx^{2/3}+ax}}{16384b^7x^{2/3}} + \frac{(1287a^8)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{16384b^7}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}} - \frac{429a^5\sqrt{bx^{2/3}+ax}}{10240b^5x^{4/3}} + \frac{429a^6\sqrt{bx^{2/3}+ax}}{8192b^6x} - \frac{1287a^7\sqrt{bx^{2/3}+ax}}{16384b^7x^{2/3}} + \frac{1287a^8\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{16384b^{15/2}}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-215040b^7 - 15360ab^6\sqrt[3]{x} + 16640a^2b^5x^{2/3} - 18304a^3b^4x + 20592a^4b^3x^{4/}\right.}{573440b^7x^3} + \frac{1287a^8\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{16384b^{15/2}}$$

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-215040*b^7 - 15360*a*b^6*x^(1/3) + 16640*a^2*b^5*x
^(2/3) - 18304*a^3*b^4*x + 20592*a^4*b^3*x^(4/3) - 24024*a^5*b^2*x^(5/3) +
30030*a^6*b*x^2 - 45045*a^7*x^(7/3)))/(573440*b^7*x^3) + (1287*a^8*ArcTanh[
(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(16384*b^(15/2))
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.63

method	result
derivativedivides	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(45045b^{\frac{15}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{15}{2}}-345345b^{\frac{17}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{13}{2}}+1150149b^{\frac{19}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}-2167737b^{\frac{21}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}\right.}{5}$
default	$\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,\left(-45045b^{\frac{15}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{15}{2}}+345345b^{\frac{17}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{13}{2}}-1150149b^{\frac{19}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}+2167737b^{\frac{21}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}\right.}{57}$

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/573440*(b*x^(2/3)+a*x)^(1/2)*(45045*b^(15/2)*(b+a*x^(1/3))^(15/2)-345345
*b^(17/2)*(b+a*x^(1/3))^(13/2)+1150149*b^(19/2)*(b+a*x^(1/3))^(11/2)-216773
7*b^(21/2)*(b+a*x^(1/3))^(9/2)+2518087*b^(23/2)*(b+a*x^(1/3))^(7/2)-1831739
*b^(25/2)*(b+a*x^(1/3))^(5/2)+801535*b^(27/2)*(b+a*x^(1/3))^(3/2)-45045*arc
tanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^7*a^8*x^(8/3)+45045*b^(29/2)*(b+a*x^(1/
3))^(1/2))/x^3/(b+a*x^(1/3))^(1/2)/b^(29/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^4}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^4}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^4, x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = -\frac{\frac{45045\,a^9\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^9-345345\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^9b+1150149\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^9b^2-2167737\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^9b^3+2518087\,(}{a^8b^7x^{\frac{8}{3}}}}{573440\,a}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/573440*(45045*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) +
(45045*(a*x^(1/3) + b)^(15/2)*a^9 - 345345*(a*x^(1/3) + b)^(13/2)*a^9*b + 1
150149*(a*x^(1/3) + b)^(11/2)*a^9*b^2 - 2167737*(a*x^(1/3) + b)^(9/2)*a^9*b
^3 + 2518087*(a*x^(1/3) + b)^(7/2)*a^9*b^4 - 1831739*(a*x^(1/3) + b)^(5/2)*
a^9*b^5 + 801535*(a*x^(1/3) + b)^(3/2)*a^9*b^6 + 45045*sqrt(a*x^(1/3) + b)*
a^9*b^7)/(a^8*b^7*x^(8/3)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = \int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^4}\,dx$$

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^4,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^4, x)
```

3.175 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 354

$$\begin{aligned}\int \frac{\sqrt{bx^{2/3}+ax}}{x^5} \, dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} \\ &\quad - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3} + \frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}} - \frac{4199a^5\sqrt{bx^{2/3}+ax}}{236544b^5x^{7/3}} \\ &\quad + \frac{4199a^6\sqrt{bx^{2/3}+ax}}{215040b^6x^2} - \frac{12597a^7\sqrt{bx^{2/3}+ax}}{573440b^7x^{5/3}} + \frac{4199a^8\sqrt{bx^{2/3}+ax}}{163840b^8x^{4/3}} \\ &\quad - \frac{4199a^9\sqrt{bx^{2/3}+ax}}{131072b^9x} + \frac{12597a^{10}\sqrt{bx^{2/3}+ax}}{262144b^{10}x^{2/3}} - \frac{12597a^{11}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{262144b^{21/2}}\end{aligned}$$

```
[Out] -12597/262144*a^11*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)-
3/11*(b*x^(2/3)+a*x)^(1/2)/x^4-3/220*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/
1320*a^2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(10/3)-323/21120*a^3*(b*x^(2/3)+a*x)^(
1/2)/b^3/x^3+323/19712*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(8/3)-4199/236544*a^
5*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)+4199/215040*a^6*(b*x^(2/3)+a*x)^(1/2)/b
^6/x^2-12597/573440*a^7*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(5/3)+4199/163840*a^8*(
b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)-4199/131072*a^9*(b*x^(2/3)+a*x)^(1/2)/b^9/
x+12597/262144*a^10*(b*x^(2/3)+a*x)^(1/2)/b^10/x^(2/3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 2050, 2054, 212}

$$\begin{aligned}\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = & -\frac{12597a^{11}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{262144b^{21/2}}+\frac{12597a^{10}\sqrt{ax+bx^{2/3}}}{262144b^{10}x^{2/3}}\\ & -\frac{4199a^9\sqrt{ax+bx^{2/3}}}{131072b^9x}+\frac{4199a^8\sqrt{ax+bx^{2/3}}}{163840b^8x^{4/3}}-\frac{12597a^7\sqrt{ax+bx^{2/3}}}{573440b^7x^{5/3}}\\ & +\frac{4199a^6\sqrt{ax+bx^{2/3}}}{215040b^6x^2}-\frac{4199a^5\sqrt{ax+bx^{2/3}}}{236544b^5x^{7/3}}+\frac{323a^4\sqrt{ax+bx^{2/3}}}{19712b^4x^{8/3}}\\ & -\frac{323a^3\sqrt{ax+bx^{2/3}}}{21120b^3x^3}+\frac{19a^2\sqrt{ax+bx^{2/3}}}{1320b^2x^{10/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{220bx^{11/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{11x^4}\end{aligned}$$

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4) - (3*a*Sqrt[b*x^(2/3) + a*x])/(220*b*x^
(11/3)) + (19*a^2*Sqrt[b*x^(2/3) + a*x])/(1320*b^2*x^(10/3)) - (323*a^3*Sqr
t[b*x^(2/3) + a*x])/(21120*b^3*x^3) + (323*a^4*Sqrt[b*x^(2/3) + a*x])/(1971
2*b^4*x^(8/3)) - (4199*a^5*Sqrt[b*x^(2/3) + a*x])/(236544*b^5*x^(7/3)) + (4
199*a^6*Sqrt[b*x^(2/3) + a*x])/(215040*b^6*x^2) - (12597*a^7*Sqrt[b*x^(2/3)
 + a*x])/(573440*b^7*x^(5/3)) + (4199*a^8*Sqrt[b*x^(2/3) + a*x])/(163840*b^
8*x^(4/3)) - (4199*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^9*x) + (12597*a^10*
Sqrt[b*x^(2/3) + a*x])/(262144*b^10*x^(2/3)) - (12597*a^11*ArcTanh[(Sqrt[b]
*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(262144*b^(21/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}+\frac{1}{22}a\int\frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}-\frac{\left(19a^2\right)\int\frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{440b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}+\frac{\left(323a^3\right)\int\frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{7920b^2}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}\\
&\quad-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}-\frac{\left(323a^4\right)\int\frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx}{8448b^3}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}\\
&\quad-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}+\frac{\left(4199a^5\right)\int\frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{118272b^4}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}\\
&\quad+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-\frac{4199a^5\sqrt{bx^{2/3}+ax}}{236544b^5x^{7/3}}-\frac{\left(4199a^6\right)\int\frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{129024b^5}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}\\
&\quad-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-\frac{4199a^5\sqrt{bx^{2/3}+ax}}{236544b^5x^{7/3}}\\
&\quad+\frac{4199a^6\sqrt{bx^{2/3}+ax}}{215040b^6x^2}+\frac{\left(4199a^7\right)\int\frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{143360b^6}
\end{aligned}
$$

$$\begin{aligned}&=-\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}\\&\quad-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-\frac{4199a^5\sqrt{bx^{2/3}+ax}}{236544b^5x^{7/3}}\\&\quad+\frac{4199a^6\sqrt{bx^{2/3}+ax}}{215040b^6x^2}-\frac{12597a^7\sqrt{bx^{2/3}+ax}}{573440b^7x^{5/3}}-\frac{(4199a^8)\int\frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{163840b^7}\end{aligned}$$

$$\begin{aligned}&=-\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}\\&\quad+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-\frac{4199a^5\sqrt{bx^{2/3}+ax}}{236544b^5x^{7/3}}+\frac{4199a^6\sqrt{bx^{2/3}+ax}}{215040b^6x^2}\\&\quad-\frac{12597a^7\sqrt{bx^{2/3}+ax}}{573440b^7x^{5/3}}+\frac{4199a^8\sqrt{bx^{2/3}+ax}}{163840b^8x^{4/3}}+\frac{(4199a^9)\int\frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{196608b^8}\end{aligned}$$

$$\begin{aligned}&=-\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}\\&\quad-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-\frac{4199a^5\sqrt{bx^{2/3}+ax}}{236544b^5x^{7/3}}\\&\quad+\frac{4199a^6\sqrt{bx^{2/3}+ax}}{215040b^6x^2}-\frac{12597a^7\sqrt{bx^{2/3}+ax}}{573440b^7x^{5/3}}+\frac{4199a^8\sqrt{bx^{2/3}+ax}}{163840b^8x^{4/3}}\\&\quad-\frac{4199a^9\sqrt{bx^{2/3}+ax}}{131072b^9x}-\frac{(4199a^{10})\int\frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{262144b^9}\end{aligned}$$

$$\begin{aligned}&=-\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}\\&\quad+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-\frac{4199a^5\sqrt{bx^{2/3}+ax}}{236544b^5x^{7/3}}+\frac{4199a^6\sqrt{bx^{2/3}+ax}}{215040b^6x^2}\\&\quad-\frac{12597a^7\sqrt{bx^{2/3}+ax}}{573440b^7x^{5/3}}+\frac{4199a^8\sqrt{bx^{2/3}+ax}}{163840b^8x^{4/3}}-\frac{4199a^9\sqrt{bx^{2/3}+ax}}{131072b^9x}\\&\quad+\frac{12597a^{10}\sqrt{bx^{2/3}+ax}}{262144b^{10}x^{2/3}}+\frac{(4199a^{11})\int\frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{524288b^{10}}\end{aligned}$$

$$\begin{aligned}&=-\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}\\&\quad+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-\frac{4199a^5\sqrt{bx^{2/3}+ax}}{236544b^5x^{7/3}}+\frac{4199a^6\sqrt{bx^{2/3}+ax}}{215040b^6x^2}\\&\quad-\frac{12597a^7\sqrt{bx^{2/3}+ax}}{573440b^7x^{5/3}}+\frac{4199a^8\sqrt{bx^{2/3}+ax}}{163840b^8x^{4/3}}-\frac{4199a^9\sqrt{bx^{2/3}+ax}}{131072b^9x}\\&\quad+\frac{12597a^{10}\sqrt{bx^{2/3}+ax}}{262144b^{10}x^{2/3}}-\frac{(12597a^{11})\,\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{262144b^{10}}\end{aligned}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4} - \frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}} + \frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}} - \frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}$$
$$+ \frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}} - \frac{4199a^5\sqrt{bx^{2/3}+ax}}{236544b^5x^{7/3}} + \frac{4199a^6\sqrt{bx^{2/3}+ax}}{215040b^6x^2}$$
$$- \frac{12597a^7\sqrt{bx^{2/3}+ax}}{573440b^7x^{5/3}} + \frac{4199a^8\sqrt{bx^{2/3}+ax}}{163840b^8x^{4/3}} - \frac{4199a^9\sqrt{bx^{2/3}+ax}}{131072b^9x}$$
$$+ \frac{12597a^{10}\sqrt{bx^{2/3}+ax}}{262144b^{10}x^{2/3}} - \frac{12597a^{11}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{262144b^{21/2}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \frac{\sqrt{bx^{2/3}+ax}\,(-82575360b^{10} - 4128768ab^9\sqrt[3]{x} + 4358144a^2b^8x^{2/3} - 4630528a^3b^7x + 4}{}$$
$$- \frac{12597a^{11}\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{262144b^{21/2}}$$

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-82575360*b^10 - 4128768*a*b^9*x^(1/3) + 4358144*a^
2*b^8*x^(2/3) - 4630528*a^3*b^7*x + 4961280*a^4*b^6*x^(4/3) - 5374720*a^5*b
^5*x^(5/3) + 5912192*a^6*b^4*x^2 - 6651216*a^7*b^3*x^(7/3) + 7759752*a^8*b^
2*x^(8/3) - 9699690*a^9*b*x^3 + 14549535*a^10*x^(10/3)))/(302776320*b^10*x^
4) - (12597*a^11*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(262144*
b^(21/2))
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.59

method	result
derivativedivides	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,(-14549535\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{21}{2}} b^{\frac{21}{2}}+155195040\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{19}{2}} b^{\frac{23}{2}}-749786037\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{17}{2}} b^{\frac{25}{2}}+2163862272(}{}$
default	$-\frac{\sqrt{b\,x^{\frac{2}{3}}+ax}\,(-14549535\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{21}{2}} b^{\frac{21}{2}}+155195040\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{19}{2}} b^{\frac{23}{2}}-749786037\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{17}{2}} b^{\frac{25}{2}}+2163862272(}{}$

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/302776320*(b*x^(2/3)+a*x)^(1/2)*(-14549535*(b+a*x^(1/3))^(21/2)*b^(21/2)
+155195040*(b+a*x^(1/3))^(19/2)*b^(23/2)-749786037*(b+a*x^(1/3))^(17/2)*b^(
25/2)+2163862272*(b+a*x^(1/3))^(15/2)*b^(27/2)-4139920070*(b+a*x^(1/3))^(13
/2)*b^(29/2)+5503713280*(b+a*x^(1/3))^(11/2)*b^(31/2)-5174056250*(b+a*x^(1/
3))^(9/2)*b^(33/2)+3424523520*(b+a*x^(1/3))^(7/2)*b^(35/2)-1551313995*(b+a*
x^(1/3))^(5/2)*b^(37/2)+14549535*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^10*
a^11*x^(11/3)+450357600*(b+a*x^(1/3))^(3/2)*b^(39/2)+14549535*(b+a*x^(1/3))
^(1/2)*b^(41/2))/x^4/(b+a*x^(1/3))^(1/2)/b^(41/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^5}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^5}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^5, x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \frac{\frac{14549535\,a^{12}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}} + \frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{12}-155195040\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{12}b+749786037\left(ax^{\frac{1}{3}}+b\right)}{}}{}$$

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/302776320*(14549535*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b
^10) + (14549535*(a*x^(1/3) + b)^(21/2)*a^12 - 155195040*(a*x^(1/3) + b)^(1
9/2)*a^12*b + 749786037*(a*x^(1/3) + b)^(17/2)*a^12*b^2 - 2163862272*(a*x^(
1/3) + b)^(15/2)*a^12*b^3 + 4139920070*(a*x^(1/3) + b)^(13/2)*a^12*b^4 - 55
03713280*(a*x^(1/3) + b)^(11/2)*a^12*b^5 + 5174056250*(a*x^(1/3) + b)^(9/2)
*a^12*b^6 - 3424523520*(a*x^(1/3) + b)^(7/2)*a^12*b^7 + 1551313995*(a*x^(1/
3) + b)^(5/2)*a^12*b^8 - 450357600*(a*x^(1/3) + b)^(3/2)*a^12*b^9 - 1454953
5*sqrt(a*x^(1/3) + b)*a^12*b^10)/(a^11*b^10*x^(11/3)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = \int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^5}\,dx$$

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^5,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^5, x)
```

3.176 $\int x^2 \left(bx^{2/3}+ax\right)^{3/2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 343

$$\begin{aligned}\int x^2 \left(bx^{2/3}+ax\right)^{3/2} \, dx &= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} - \frac{1048576b^{11}\left(bx^{2/3}+ax\right)^{5/2}}{152108775a^{12}x^{5/3}} \\ &+ \frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}} - \frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x} + \frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}} \\ &- \frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}} - \frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6} \\ &+ \frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5} - \frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4} \\ &+ \frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3} - \frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}\end{aligned}$$

```
[Out] 45056/557175*b^6*(b*x^(2/3)+a*x)^(5/2)/a^7-1048576/152108775*b^11*(b*x^(2/3
)+a*x)^(5/2)/a^12/x^(5/3)+524288/30421755*b^10*(b*x^(2/3)+a*x)^(5/2)/a^11/x
^(4/3)-131072/4345965*b^9*(b*x^(2/3)+a*x)^(5/2)/a^10/x+65536/1448655*b^8*(b
*x^(2/3)+a*x)^(5/2)/a^9/x^(2/3)-90112/1448655*b^7*(b*x^(2/3)+a*x)^(5/2)/a^8
/x^(1/3)-11264/111435*b^5*x^(1/3)*(b*x^(2/3)+a*x)^(5/2)/a^6+5632/45885*b^4*
x^(2/3)*(b*x^(2/3)+a*x)^(5/2)/a^5-352/2415*b^3*x*(b*x^(2/3)+a*x)^(5/2)/a^4+
176/1035*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(5/2)/a^3-44/225*b*x^(5/3)*(b*x^(2/3)+
a*x)^(5/2)/a^2+2/9*x^2*(b*x^(2/3)+a*x)^(5/2)/a
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2041, 2027, 2039}

$$\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx=-\frac{1048576b^{11}\left(ax+bx^{2/3}\right)^{5/2}}{152108775a^{12}x^{5/3}}+\frac{524288b^{10}\left(ax+bx^{2/3}\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^{9}\left(ax+bx^{2/3}\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^{8}\left(ax+bx^{2/3}\right)^{5/2}}{1448655a^{9}x^{2/3}}-\frac{90112b^{7}\left(ax+bx^{2/3}\right)^{5/2}}{1448655a^{8}\sqrt[3]{x}}+\frac{45056b^{6}\left(ax+bx^{2/3}\right)^{5/2}}{557175a^{7}}-\frac{11264b^{5}\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{111435a^{6}}+\frac{5632b^{4}x^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{45885a^{5}}-\frac{352b^{3}x\left(ax+bx^{2/3}\right)^{5/2}}{2415a^{4}}+\frac{176b^{2}x^{4/3}\left(ax+bx^{2/3}\right)^{5/2}}{1035a^{3}}-\frac{44bx^{5/3}\left(ax+bx^{2/3}\right)^{5/2}}{225a^{2}}+\frac{2x^{2}\left(ax+bx^{2/3}\right)^{5/2}}{9a}$$

```
[In] Int[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (45056*b^6*(b*x^(2/3) + a*x)^(5/2))/(557175*a^7) - (1048576*b^11*(b*x^(2/3)
 + a*x)^(5/2))/(152108775*a^12*x^(5/3)) + (524288*b^10*(b*x^(2/3) + a*x)^(5
/2))/(30421755*a^11*x^(4/3)) - (131072*b^9*(b*x^(2/3) + a*x)^(5/2))/(434596
5*a^10*x) + (65536*b^8*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^9*x^(2/3)) - (90
112*b^7*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^8*x^(1/3)) - (11264*b^5*x^(1/3)
*(b*x^(2/3) + a*x)^(5/2))/(111435*a^6) + (5632*b^4*x^(2/3)*(b*x^(2/3) + a*x
)^(5/2))/(45885*a^5) - (352*b^3*x*(b*x^(2/3) + a*x)^(5/2))/(2415*a^4) + (17
6*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(5/2))/(1035*a^3) - (44*b*x^(5/3)*(b*x^(2/3
) + a*x)^(5/2))/(225*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(5/2))/(9*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}-\frac{(22b)\int x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}\,dx}{27a}\\
&= -\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}+\frac{\left(88b^2\right)\int x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}\,dx}{135a^2}\\
&= \frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}-\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}\\
&\quad+\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}-\frac{\left(176b^3\right)\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx}{345a^3}\\
&= -\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4}+\frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}-\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}\\
&\quad+\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}+\frac{\left(2816b^4\right)\int x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}\,dx}{7245a^4}\\
&= \frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5}-\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4}+\frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}\\
&\quad-\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}-\frac{\left(5632b^5\right)\int \sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}\,dx}{19665a^5}\\
&= -\frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6}+\frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5}\\
&\quad-\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4}+\frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}-\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}\\
&\quad+\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}+\frac{\left(22528b^6\right)\int \left(bx^{2/3}+ax\right)^{3/2}\,dx}{111435a^6}\\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}-\frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6}+\frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5}\\
&\quad-\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4}+\frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}\\
&\quad-\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}-\frac{\left(45056b^7\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{\sqrt[3]{x}}\,dx}{334305a^7}
\end{aligned}$$

$$= \frac{45056 b^6 \left(b x^{2/3}+a x\right)^{5/2}}{557175 a^7}-\frac{90112 b^7 \left(b x^{2/3}+a x\right)^{5/2}}{1448655 a^8 \sqrt[3]{x}}-\frac{11264 b^5 \sqrt[3]{x} \left(b x^{2/3}+a x\right)^{5/2}}{111435 a^6}$$

$$+\frac{5632 b^4 x^{2/3} \left(b x^{2/3}+a x\right)^{5/2}}{45885 a^5}-\frac{352 b^3 x \left(b x^{2/3}+a x\right)^{5/2}}{2415 a^4}+\frac{176 b^2 x^{4/3} \left(b x^{2/3}+a x\right)^{5/2}}{1035 a^3}$$

$$-\frac{44 b x^{5/3} \left(b x^{2/3}+a x\right)^{5/2}}{225 a^2}+\frac{2 x^2 \left(b x^{2/3}+a x\right)^{5/2}}{9 a}+\frac{\left(360448 b^8\right) \int \frac{\left(b x^{2/3}+a x\right)^{3/2}}{x^{2/3}} \, dx}{4345965 a^8}$$

$$= \frac{45056 b^6 \left(b x^{2/3}+a x\right)^{5/2}}{557175 a^7}+\frac{65536 b^8 \left(b x^{2/3}+a x\right)^{5/2}}{1448655 a^9 x^{2/3}}-\frac{90112 b^7 \left(b x^{2/3}+a x\right)^{5/2}}{1448655 a^8 \sqrt[3]{x}}$$

$$-\frac{11264 b^5 \sqrt[3]{x} \left(b x^{2/3}+a x\right)^{5/2}}{111435 a^6}+\frac{5632 b^4 x^{2/3} \left(b x^{2/3}+a x\right)^{5/2}}{45885 a^5}$$

$$-\frac{352 b^3 x \left(b x^{2/3}+a x\right)^{5/2}}{2415 a^4}+\frac{176 b^2 x^{4/3} \left(b x^{2/3}+a x\right)^{5/2}}{1035 a^3}$$

$$-\frac{44 b x^{5/3} \left(b x^{2/3}+a x\right)^{5/2}}{225 a^2}+\frac{2 x^2 \left(b x^{2/3}+a x\right)^{5/2}}{9 a}-\frac{\left(65536 b^9\right) \int \frac{\left(b x^{2/3}+a x\right)^{3/2}}{x} \, dx}{1448655 a^9}$$

$$= \frac{45056 b^6 \left(b x^{2/3}+a x\right)^{5/2}}{557175 a^7}-\frac{131072 b^9 \left(b x^{2/3}+a x\right)^{5/2}}{4345965 a^{10} x}+\frac{65536 b^8 \left(b x^{2/3}+a x\right)^{5/2}}{1448655 a^9 x^{2/3}}$$

$$-\frac{90112 b^7 \left(b x^{2/3}+a x\right)^{5/2}}{1448655 a^8 \sqrt[3]{x}}-\frac{11264 b^5 \sqrt[3]{x} \left(b x^{2/3}+a x\right)^{5/2}}{111435 a^6}$$

$$+\frac{5632 b^4 x^{2/3} \left(b x^{2/3}+a x\right)^{5/2}}{45885 a^5}-\frac{352 b^3 x \left(b x^{2/3}+a x\right)^{5/2}}{2415 a^4}+\frac{176 b^2 x^{4/3} \left(b x^{2/3}+a x\right)^{5/2}}{1035 a^3}$$

$$-\frac{44 b x^{5/3} \left(b x^{2/3}+a x\right)^{5/2}}{225 a^2}+\frac{2 x^2 \left(b x^{2/3}+a x\right)^{5/2}}{9 a}+\frac{\left(262144 b^{10}\right) \int \frac{\left(b x^{2/3}+a x\right)^{3/2}}{x^{4/3}} \, dx}{13037895 a^{10}}$$

$$= \frac{45056 b^6 \left(b x^{2/3}+a x\right)^{5/2}}{557175 a^7}+\frac{524288 b^{10} \left(b x^{2/3}+a x\right)^{5/2}}{30421755 a^{11} x^{4/3}}-\frac{131072 b^9 \left(b x^{2/3}+a x\right)^{5/2}}{4345965 a^{10} x}$$

$$+\frac{65536 b^8 \left(b x^{2/3}+a x\right)^{5/2}}{1448655 a^9 x^{2/3}}-\frac{90112 b^7 \left(b x^{2/3}+a x\right)^{5/2}}{1448655 a^8 \sqrt[3]{x}}-\frac{11264 b^5 \sqrt[3]{x} \left(b x^{2/3}+a x\right)^{5/2}}{111435 a^6}$$

$$+\frac{5632 b^4 x^{2/3} \left(b x^{2/3}+a x\right)^{5/2}}{45885 a^5}-\frac{352 b^3 x \left(b x^{2/3}+a x\right)^{5/2}}{2415 a^4}+\frac{176 b^2 x^{4/3} \left(b x^{2/3}+a x\right)^{5/2}}{1035 a^3}$$

$$-\frac{44 b x^{5/3} \left(b x^{2/3}+a x\right)^{5/2}}{225 a^2}+\frac{2 x^2 \left(b x^{2/3}+a x\right)^{5/2}}{9 a}-\frac{\left(524288 b^{11}\right) \int \frac{\left(b x^{2/3}+a x\right)^{3/2}}{x^{5/3}} \, dx}{91265265 a^{11}}$$

$$= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7} - \frac{1048576b^{11}\left(bx^{2/3}+ax\right)^{5/2}}{152108775a^{12}x^{5/3}} + \frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}} - \frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x} + \frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}} - \frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}} - \frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6} + \frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5} - \frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4} + \frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3} - \frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}$$

Mathematica [A] (verified)

Time = 6.33 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.49

$$\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx = 2\left(b+a\sqrt[3]{x}\right)\left(bx^{2/3}+ax\right)^{3/2}\left(-524288b^{11}+1310720ab^{10}\sqrt[3]{x}-2293760a^2b^9x^{2/3}+3440640a^3b^8\right.$$

```
[In] Integrate[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(-524288*b^11 + 1310720*a*b^10*x
^(1/3) - 2293760*a^2*b^9*x^(2/3) + 3440640*a^3*b^8*x - 4730880*a^4*b^7*x^(4
/3) + 6150144*a^5*b^6*x^(5/3) - 7687680*a^6*b^5*x^2 + 9335040*a^7*b^4*x^(7/
3) - 11085360*a^8*b^3*x^(8/3) + 12932920*a^9*b^2*x^3 - 14872858*a^10*b*x^(1
0/3) + 16900975*a^11*x^(11/3)))/(152108775*a^12*x)
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.42

method	result
derivativedivides	$\frac{2\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}}\left(b+a x^{\frac{1}{3}}\right)\left(16900975 a^{11} x^{\frac{11}{3}}-14872858 a^{10} b x^{\frac{10}{3}}+12932920 b^{2} a^{9} x^{3}-11085360 a^{8} b^{3} x^{\frac{8}{3}}+9335040 a^{7} b^{4} x^{\frac{7}{3}}-\right.}{152108775}$
default	$\frac{2\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}}\left(b+a x^{\frac{1}{3}}\right)\left(16900975 a^{11} x^{\frac{11}{3}}-14872858 a^{10} b x^{\frac{10}{3}}+12932920 b^{2} a^{9} x^{3}-11085360 a^{8} b^{3} x^{\frac{8}{3}}+9335040 a^{7} b^{4} x^{\frac{7}{3}}-\right.}{152108775}$

```
[In] int(x^2*(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/152108775*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(16900975*a^11*x^(11/3)-148
72858*a^10*b*x^(10/3)+12932920*b^2*a^9*x^3-11085360*a^8*b^3*x^(8/3)+9335040
*a^7*b^4*x^(7/3)-7687680*a^6*b^5*x^2+6150144*a^5*b^6*x^(5/3)-4730880*a^4*b^
7*x^(4/3)+3440640*a^3*b^8*x-2293760*a^2*b^9*x^(2/3)+1310720*a*b^10*x^(1/3)-
524288*b^11)/x/a^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. 2(255) = 510.

Time = 199.50 (sec) , antiderivative size = 1293, normalized size of antiderivative = 3.77

$$\int x^2 \left(bx^{2/3}+ax\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/152108775*((6597069766656*b^19 + 1374389534720*b^18 + 6442450944*(64*a^3
- 3)*b^16 - 128849018880*b^17 - 33554432*(11264*a^3 - 53)*b^15 + 9843127840
0*a^15 - 12582912*(5504*a^3 + 1)*b^14 + 393216*(3194880*a^6 - 114688*a^3 -
3)*b^13 + 14680064*(18816*a^6 + 103*a^3)*b^12 - 1572864*(48816*a^6 + 23*a^3
)*b^11 - 24576*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^10 - 229376*(1349120*
a^9 + 3439*a^6)*b^9 + 7827456*(5600*a^9 + 3*a^6)*b^8 - 384*(620420562944*a^
12 + 21542400*a^9 + 693*a^6)*b^7 - 6656*(7444688384*a^12 - 89111*a^9)*b^6 +
 19968*(232361024*a^12 - 935*a^9)*b^5 - 1326*(173210075136*a^15 - 533564416
*a^12 - 165*a^9)*b^4 - 1881152*(45121536*a^15 + 34547*a^12)*b^3 - 352716*(1
9243008*a^15 - 1339*a^12)*b^2 + 2028117*(237568*a^15 + 21*a^12)*b)*x + (169
00975*(16777216*a^13*b^6 + 6291456*a^13*b^5 + 196608*a^13*b^4 - 262144*a^16
 - 114688*a^13*b^3 - 2304*a^13*b^2 + 864*a^13*b - 27*a^13)*x^5 - 92378*(167
77216*a^10*b^9 + 6291456*a^10*b^8 + 196608*a^10*b^7 - 114688*a^10*b^6 - 230
4*a^10*b^5 + 864*a^10*b^4 - (262144*a^13 + 27*a^10)*b^3)*x^4 + 109824*(1677
7216*a^7*b^12 + 6291456*a^7*b^11 + 196608*a^7*b^10 - 114688*a^7*b^9 - 2304*
a^7*b^8 + 864*a^7*b^7 - (262144*a^10 + 27*a^7)*b^6)*x^3 - 143360*(16777216*
a^4*b^15 + 6291456*a^4*b^14 + 196608*a^4*b^13 - 114688*a^4*b^12 - 2304*a^4*
b^11 + 864*a^4*b^10 - (262144*a^7 + 27*a^4)*b^9)*x^2 + 262144*(16777216*a*b
^18 + 6291456*a*b^17 + 196608*a*b^16 - 114688*a*b^15 - 2304*a*b^14 + 864*a*
b^13 - (262144*a^4 + 27*a)*b^12)*x - 4*(2199023255552*b^19 + 824633720832*b
^18 + 25769803776*b^17 - 15032385536*b^16 - 301989888*b^15 - 131072*(262144
*a^3 + 27)*b^13 + 113246208*b^14 - 4732273*(16777216*a^12*b^7 + 6291456*a^1
2*b^6 + 196608*a^12*b^5 - 114688*a^12*b^4 - 2304*a^12*b^3 + 864*a^12*b^2 -
(262144*a^15 + 27*a^12)*b)*x^4 - 24310*(16777216*a^9*b^10 + 6291456*a^9*b^9
 + 196608*a^9*b^8 - 114688*a^9*b^7 - 2304*a^9*b^6 + 864*a^9*b^5 - (262144*a
^12 + 27*a^9)*b^4)*x^3 + 29568*(16777216*a^6*b^13 + 6291456*a^6*b^12 + 1966
08*a^6*b^11 - 114688*a^6*b^10 - 2304*a^6*b^9 + 864*a^6*b^8 - (262144*a^9 +
27*a^6)*b^7)*x^2 - 40960*(16777216*a^3*b^16 + 6291456*a^3*b^15 + 196608*a^3
*b^14 - 114688*a^3*b^13 - 2304*a^3*b^12 + 864*a^3*b^11 - (262144*a^6 + 27*a
^3)*b^10)*x)*x^(2/3) + 3*(29393*(16777216*a^11*b^8 + 6291456*a^11*b^7 + 196
608*a^11*b^6 - 114688*a^11*b^5 - 2304*a^11*b^4 + 864*a^11*b^3 - (262144*a^1
4 + 27*a^11)*b^2)*x^4 - 34320*(16777216*a^8*b^11 + 6291456*a^8*b^10 + 19660
8*a^8*b^9 - 114688*a^8*b^8 - 2304*a^8*b^7 + 864*a^8*b^6 - (262144*a^11 + 27
*a^8)*b^5)*x^3 + 43008*(16777216*a^5*b^14 + 6291456*a^5*b^13 + 196608*a^5*b
^12 - 114688*a^5*b^11 - 2304*a^5*b^10 + 864*a^5*b^9 - (262144*a^8 + 27*a^5)
```

```
*b^8)*x^2 - 65536*(16777216*a^2*b^17 + 6291456*a^2*b^16 + 196608*a^2*b^15 -
 114688*a^2*b^14 - 2304*a^2*b^13 + 864*a^2*b^12 - (262144*a^5 + 27*a^2)*b^1
1)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^12*b^6 + 6291456*a^12*b^
5 + 196608*a^12*b^4 - 262144*a^15 - 114688*a^12*b^3 - 2304*a^12*b^2 + 864*a
^12*b - 27*a^12)*x)
```

Sympy [F]

$$\int x^2 \left(bx^{2/3}+ax\right)^{3/2}\,dx=\int x^{2} \left(a x+b x^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2*(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int x^2 \left(bx^{2/3}+ax\right)^{3/2}\,dx=\int \left(a x+b x^{\frac{2}{3}}\right)^{\frac{3}{2}} x^{2}\,dx$$

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)*x^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(255) = 510$.

Time = 0.31 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.24

$$\int x^2 \left(bx^{2/3}+ax\right)^{3/2}\,dx=\text{Too large to display}$$

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/16900975*b*(524288*b^(25/2)/a^12 + (25*(88179*(a*x^(1/3) + b)^(23/2) - 10
62347*(a*x^(1/3) + b)^(21/2)*b + 5870865*(a*x^(1/3) + b)^(19/2)*b^2 - 19684
665*(a*x^(1/3) + b)^(17/2)*b^3 + 44618574*(a*x^(1/3) + b)^(15/2)*b^4 - 7207
6158*(a*x^(1/3) + b)^(13/2)*b^5 + 85180914*(a*x^(1/3) + b)^(11/2)*b^6 - 743
64290*(a*x^(1/3) + b)^(9/2)*b^7 + 47805615*(a*x^(1/3) + b)^(7/2)*b^8 - 2230
9287*(a*x^(1/3) + b)^(5/2)*b^9 + 7436429*(a*x^(1/3) + b)^(3/2)*b^10 - 20281
17*sqrt(a*x^(1/3) + b)*b^11)*b/a^11 + 3*(676039*(a*x^(1/3) + b)^(25/2) - 88
17900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/2)*b^2 - 1956
```

```
95500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^(17/2)*b^4 - 8
92371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3) + b)^(13/2)*b^6
 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3) + b)^(9/2)*
b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x^(1/3) + b)^(5/2)
*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sqrt(a*x^(1/3) + b)*
b^12)/a^11)/a) - 2/152108775*a*(4194304*b^(27/2)/a^13 - (27*(676039*(a*x^(1
/3) + b)^(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) +
b)^(21/2)*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3)
 + b)^(17/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(
1/3) + b)^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a
*x^(1/3) + b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(
a*x^(1/3) + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*
sqrt(a*x^(1/3) + b)*b^12)*b/a^12 + 13*(1300075*(a*x^(1/3) + b)^(27/2) - 182
53053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^2 - 478
056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2)*b^4 -
 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^(15/2)*
b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3) + b)^(1
1/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1/3) + b
)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3)
+ b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^12)/a)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(bx^{2/3}+ax\right)^{3/2}\,dx=\int x^2\,\left(a\,x+b\,x^{2/3}\right)^{3/2}\,dx$$

```
[In] int(x^2*(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^2*(a*x + b*x^(2/3))^(3/2), x)
```

3.177 $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 255

$$\begin{aligned}\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=&-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{4849845a^9x^{5/3}}\\&-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}\\&-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}\\&+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}\end{aligned}$$

```
[Out] -256/1615*b^3*(b*x^(2/3)+a*x)^(5/2)/a^4+65536/4849845*b^8*(b*x^(2/3)+a*x)^(
5/2)/a^9/x^(5/3)-32768/969969*b^7*(b*x^(2/3)+a*x)^(5/2)/a^8/x^(4/3)+8192/13
8567*b^6*(b*x^(2/3)+a*x)^(5/2)/a^7/x-4096/46189*b^5*(b*x^(2/3)+a*x)^(5/2)/a
^6/x^(2/3)+512/4199*b^4*(b*x^(2/3)+a*x)^(5/2)/a^5/x^(1/3)+64/323*b^2*x^(1/3
)*(b*x^(2/3)+a*x)^(5/2)/a^3-32/133*b*x^(2/3)*(b*x^(2/3)+a*x)^(5/2)/a^2+2/7*
x*(b*x^(2/3)+a*x)^(5/2)/a
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used

$= \{2041, 2027, 2039\}$

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(ax+bx^{2/3}\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(ax+bx^{2/3}\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(ax+bx^{2/3}\right)^{5/2}}{4199a^5\sqrt[3]{x}}-\frac{256b^3\left(ax+bx^{2/3}\right)^{5/2}}{1615a^4}+\frac{64b^2\sqrt[3]{x}\left(ax+bx^{2/3}\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(ax+bx^{2/3}\right)^{5/2}}{133a^2}+\frac{2x\left(ax+bx^{2/3}\right)^{5/2}}{7a}$$

```
[In] Int[x*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-256*b^3*(b*x^(2/3) + a*x)^(5/2))/(1615*a^4) + (65536*b^8*(b*x^(2/3) + a*x
)^(5/2))/(4849845*a^9*x^(5/3)) - (32768*b^7*(b*x^(2/3) + a*x)^(5/2))/(96996
9*a^8*x^(4/3)) + (8192*b^6*(b*x^(2/3) + a*x)^(5/2))/(138567*a^7*x) - (4096*
b^5*(b*x^(2/3) + a*x)^(5/2))/(46189*a^6*x^(2/3)) + (512*b^4*(b*x^(2/3) + a*
x)^(5/2))/(4199*a^5*x^(1/3)) + (64*b^2*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(32
3*a^3) - (32*b*x^(2/3)*(b*x^(2/3) + a*x)^(5/2))/(133*a^2) + (2*x*(b*x^(2/3)
 + a*x)^(5/2))/(7*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{(16b)\int x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}\,dx}{21a}\\
&= -\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}+\frac{\left(32b^2\right)\int \sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}\,dx}{57a^2}\\
&= \frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}\\
&\quad+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{\left(128b^3\right)\int \left(bx^{2/3}+ax\right)^{3/2}\,dx}{323a^3}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}\\
&\quad-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}+\frac{\left(256b^4\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{\sqrt[3]{x}}\,dx}{969a^4}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}\\
&\quad-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{\left(2048b^5\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{2/3}}\,dx}{12597a^5}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}\\
&\quad+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}\\
&\quad+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}+\frac{\left(4096b^6\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx}{46189a^6}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}\\
&\quad+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}\\
&\quad+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{\left(16384b^7\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{4/3}}\,dx}{415701a^7}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}\\
&\quad-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}\\
&\quad-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}+\frac{\left(32768b^8\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{5/3}}\,dx}{2909907a^8}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4} + \frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{4849845a^9x^{5/3}} - \frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}} \\
&\quad + \frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x} - \frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}} + \frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}} \\
&\quad + \frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3} - \frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2} + \frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.51

$$
\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx = \frac{2\left(b+a\sqrt[3]{x}\right)\left(bx^{2/3}+ax\right)^{3/2}\left(32768b^8-81920ab^7\sqrt[3]{x}+143360a^2b^6x^{2/3}-215040a^3b^5x+2956\right.}{4849845a^9x}
$$

```
[In] Integrate[x*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(32768*b^8 - 81920*a*b^7*x^(1/3)
 + 143360*a^2*b^6*x^(2/3) - 215040*a^3*b^5*x + 295680*a^4*b^4*x^(4/3) - 384
384*a^5*b^3*x^(5/3) + 480480*a^6*b^2*x^2 - 583440*a^7*b*x^(7/3) + 692835*a^
8*x^(8/3)))/(4849845*a^9*x)
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.44

method	result
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(692835a^8x^{\frac{8}{3}}-583440a^7b\,x^{\frac{7}{3}}+480480a^6x^2b^2-384384a^5b^3x^{\frac{5}{3}}+295680x^{\frac{4}{3}}a^4b^4-215040a^3b^5x\right.}{4849845x\,a^9}$
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(692835a^8x^{\frac{8}{3}}-583440a^7b\,x^{\frac{7}{3}}+480480a^6x^2b^2-384384a^5b^3x^{\frac{5}{3}}+295680x^{\frac{4}{3}}a^4b^4-215040a^3b^5x\right.}{4849845x\,a^9}$

```
[In] int(x*(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/4849845*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(692835*a^8*x^(8/3)-583440*a^
7*b*x^(7/3)+480480*a^6*x^2*b^2-384384*a^5*b^3*x^(5/3)+295680*x^(4/3)*a^4*b^
4-215040*a^3*b^5*x+143360*a^2*b^6*x^(2/3)-81920*x^(1/3)*a*b^7+32768*b^8)/x/
a^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(189) = 378$.

Time = 144.93 (sec) , antiderivative size = 1031, normalized size of antiderivative = 4.04

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4849845*((824633720832*b^16 + 171798691840*b^15 + 805306368*(64*a^3 - 3)
*b^13 - 16106127360*b^14 - 4194304*(11264*a^3 - 53)*b^12 - 8070142080*a^12
- 1572864*(5504*a^3 + 1)*b^11 + 49152*(3194880*a^6 - 114688*a^3 - 3)*b^10 +
 1835008*(18816*a^6 + 103*a^3)*b^9 - 196608*(48816*a^6 + 23*a^3)*b^8 + 3072
*(6575923200*a^9 + 495872*a^6 + 15*a^3)*b^7 + 28672*(146455680*a^9 - 3439*a
^6)*b^6 - 419328*(934400*a^9 - 7*a^6)*b^5 + 1584*(12166103040*a^12 - 382758
40*a^9 - 21*a^6)*b^4 + 164736*(43008000*a^12 + 33737*a^9)*b^3 + 51480*(1083
8016*a^12 - 799*a^9)*b^2 - 109395*(401408*a^12 + 33*a^9)*b)*x - 2*(692835*(
16777216*a^10*b^6 + 6291456*a^10*b^5 + 196608*a^10*b^4 - 262144*a^13 - 1146
88*a^10*b^3 - 2304*a^10*b^2 + 864*a^10*b - 27*a^10)*x^4 - 6864*(16777216*a^
7*b^9 + 6291456*a^7*b^8 + 196608*a^7*b^7 - 114688*a^7*b^6 - 2304*a^7*b^5 +
864*a^7*b^4 - (262144*a^10 + 27*a^7)*b^3)*x^3 + 8960*(16777216*a^4*b^12 + 6
291456*a^4*b^11 + 196608*a^4*b^10 - 114688*a^4*b^9 - 2304*a^4*b^8 + 864*a^4
*b^7 - (262144*a^7 + 27*a^4)*b^6)*x^2 - 16384*(16777216*a*b^15 + 6291456*a*
b^14 + 196608*a*b^13 - 114688*a*b^12 - 2304*a*b^11 + 864*a*b^10 - (262144*a
^4 + 27*a)*b^9)*x + 2*(274877906944*b^16 + 103079215104*b^15 + 3221225472*b
^14 - 1879048192*b^13 - 37748736*b^12 - 16384*(262144*a^3 + 27)*b^10 + 1415
5776*b^11 + 401115*(16777216*a^9*b^7 + 6291456*a^9*b^6 + 196608*a^9*b^5 - 1
14688*a^9*b^4 - 2304*a^9*b^3 + 864*a^9*b^2 - (262144*a^12 + 27*a^9)*b)*x^3
+ 3696*(16777216*a^6*b^10 + 6291456*a^6*b^9 + 196608*a^6*b^8 - 114688*a^6*b
^7 - 2304*a^6*b^6 + 864*a^6*b^5 - (262144*a^9 + 27*a^6)*b^4)*x^2 - 5120*(16
777216*a^3*b^13 + 6291456*a^3*b^12 + 196608*a^3*b^11 - 114688*a^3*b^10 - 23
04*a^3*b^9 + 864*a^3*b^8 - (262144*a^6 + 27*a^3)*b^7)*x)*x^(2/3) + 3*(2145*
(16777216*a^8*b^8 + 6291456*a^8*b^7 + 196608*a^8*b^6 - 114688*a^8*b^5 - 230
4*a^8*b^4 + 864*a^8*b^3 - (262144*a^11 + 27*a^8)*b^2)*x^3 - 2688*(16777216*
a^5*b^11 + 6291456*a^5*b^10 + 196608*a^5*b^9 - 114688*a^5*b^8 - 2304*a^5*b^
7 + 864*a^5*b^6 - (262144*a^8 + 27*a^5)*b^5)*x^2 + 4096*(16777216*a^2*b^14
+ 6291456*a^2*b^13 + 196608*a^2*b^12 - 114688*a^2*b^11 - 2304*a^2*b^10 + 86
4*a^2*b^9 - (262144*a^5 + 27*a^2)*b^8)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/(
(16777216*a^9*b^6 + 6291456*a^9*b^5 + 196608*a^9*b^4 - 262144*a^12 - 114688
*a^9*b^3 - 2304*a^9*b^2 + 864*a^9*b - 27*a^9)*x)
```

Sympy [F]

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\int x\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x*(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\int\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^(2/3))^(3/2)*x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(189)=378$.

Time = 0.32 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.36

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=$$

$$-\frac{2}{692835}b\left(\frac{32768\,b^{\frac{19}{2}}}{a^9}-\frac{\frac{19\left(6435\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}-58344\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b+235620\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^2-556920\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^3+850850\left(ax^{\frac{1}{3}}+b\right)\right.}{a^8}}{}\right.$$

$$+\frac{2}{1616615}a\left(\frac{65536\,b^{\frac{21}{2}}}{a^{10}}+\frac{\frac{21\left(12155\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}-122265\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b+554268\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^2-1492260\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^3+2645370\left(ax\right.\right.}{}}{}\right.$$

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
[Out] -2/692835*b*(32768*b^(19/2)/a^9 - (19*(6435*(a*x^(1/3) + b)^(17/2) - 58344*
(a*x^(1/3) + b)^(15/2)*b + 235620*(a*x^(1/3) + b)^(13/2)*b^2 - 556920*(a*x^
(1/3) + b)^(11/2)*b^3 + 850850*(a*x^(1/3) + b)^(9/2)*b^4 - 875160*(a*x^(1/3
) + b)^(7/2)*b^5 + 612612*(a*x^(1/3) + b)^(5/2)*b^6 - 291720*(a*x^(1/3) + b
)^(3/2)*b^7 + 109395*sqrt(a*x^(1/3) + b)*b^8)*b/a^8 + 9*(12155*(a*x^(1/3) +
```

```
 b)^(19/2) - 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)^(15/2
)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)
*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^
6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 -
230945*sqrt(a*x^(1/3) + b)*b^9)/a^8)/a) + 2/1616615*a*(65536*b^(21/2)/a^10
+ (21*(12155*(a*x^(1/3) + b)^(19/2) - 122265*(a*x^(1/3) + b)^(17/2)*b + 554
268*(a*x^(1/3) + b)^(15/2)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)*b^3 + 26453
70*(a*x^(1/3) + b)^(11/2)*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340
*(a*x^(1/3) + b)^(7/2)*b^6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*
x^(1/3) + b)^(3/2)*b^8 - 230945*sqrt(a*x^(1/3) + b)*b^9)*b/a^9 + 5*(46189*(
a*x^(1/3) + b)^(21/2) - 510510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3
) + b)^(17/2)*b^2 - 7759752*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3
) + b)^(13/2)*b^4 - 22221108*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/
3) + b)^(9/2)*b^6 - 16628040*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3)
 + b)^(5/2)*b^8 - 3233230*(a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3)
 + b)*b^10)/a^9)/a)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx=\int x\,\left(a\,x+b\,x^{2/3}\right)^{3/2}\,dx$$

```
[In] int(x*(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x*(a*x + b*x^(2/3))^(3/2), x)
```

3.178 $\int \left(bx^{2/3}+ax\right)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 169

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}-\frac{512b^5\left(bx^{2/3}+ax\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x}+\frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}}$$

```
[Out] 2/5*(b*x^(2/3)+a*x)^(5/2)/a-512/15015*b^5*(b*x^(2/3)+a*x)^(5/2)/a^6/x^(5/3)
+256/3003*b^4*(b*x^(2/3)+a*x)^(5/2)/a^5/x^(4/3)-64/429*b^3*(b*x^(2/3)+a*x)^
(5/2)/a^4/x+32/143*b^2*(b*x^(2/3)+a*x)^(5/2)/a^3/x^(2/3)-4/13*b*(b*x^(2/3)+
a*x)^(5/2)/a^2/x^(1/3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2027, 2041, 2039}

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=-\frac{512b^5\left(ax+bx^{2/3}\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(ax+bx^{2/3}\right)^{5/2}}{429a^4x}+\frac{32b^2\left(ax+bx^{2/3}\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{5/2}}{13a^2\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{5a}$$

```
[In] Int[(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (512*b^5*(b*x^(2/3) + a*x)^(5/2))/(1501
5*a^6*x^(5/3)) + (256*b^4*(b*x^(2/3) + a*x)^(5/2))/(3003*a^5*x^(4/3)) - (64
```

```
*b^3*(b*x^(2/3) + a*x)^(5/2))/(429*a^4*x) + (32*b^2*(b*x^(2/3) + a*x)^(5/2)
)/(143*a^3*x^(2/3)) - (4*b*(b*x^(2/3) + a*x)^(5/2))/(13*a^2*x^(1/3))
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}-\frac{(2b)\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{\sqrt[3]{x}}\,dx}{3a}\\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}-\frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}}+\frac{(16b^2)\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{2/3}}\,dx}{39a^2}\\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}+\frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}}-\frac{(32b^3)\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx}{143a^3}\\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}-\frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x}+\frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}}\\
&\quad-\frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}}+\frac{(128b^4)\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{4/3}}\,dx}{1287a^4}
\end{aligned}$$

$$= \frac{2(bx^{2/3}+ax)^{5/2}}{5a} + \frac{256b^4(bx^{2/3}+ax)^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3(bx^{2/3}+ax)^{5/2}}{429a^4x} + \frac{32b^2(bx^{2/3}+ax)^{5/2}}{143a^3x^{2/3}} - \frac{4b(bx^{2/3}+ax)^{5/2}}{13a^2\sqrt[3]{x}} - \frac{(256b^5)\int \frac{(bx^{2/3}+ax)^{3/2}}{x^{5/3}}\,dx}{9009a^5}$$

$$= \frac{2(bx^{2/3}+ax)^{5/2}}{5a} - \frac{512b^5(bx^{2/3}+ax)^{5/2}}{15015a^6x^{5/3}} + \frac{256b^4(bx^{2/3}+ax)^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3(bx^{2/3}+ax)^{5/2}}{429a^4x} + \frac{32b^2(bx^{2/3}+ax)^{5/2}}{143a^3x^{2/3}} - \frac{4b(bx^{2/3}+ax)^{5/2}}{13a^2\sqrt[3]{x}}$$

Mathematica [A] (verified)

Time = 6.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

$$\int (bx^{2/3} + ax)^{3/2}\,dx = \frac{2(b+a\sqrt[3]{x})\,(bx^{2/3}+ax)^{3/2}\,(-256b^5+640ab^4\sqrt[3]{x}-1120a^2b^3x^{2/3}+1680a^3b^2x-2310a^4bx^{4/3}}{15015a^6x}$$

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))*(b*x^(2/3) + a*x)^(3/2)*(-256*b^5 + 640*a*b^4*x^(1/3) -
1120*a^2*b^3*x^(2/3) + 1680*a^3*b^2*x - 2310*a^4*b*x^(4/3) + 3003*a^5*x^(5/
3)))/(15015*a^6*x)
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.47

method	result	size
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(3003a^{5}x^{\frac{5}{3}}-2310a^{4}b\,x^{\frac{4}{3}}+1680a^{3}b^{2}x-1120a^{2}b^{3}x^{\frac{2}{3}}+640a\,b^{4}x^{\frac{1}{3}}-256b^{5}\right)}{15015x\,a^{6}}$	79
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(3003a^{5}x^{\frac{5}{3}}-2310a^{4}b\,x^{\frac{4}{3}}+1680a^{3}b^{2}x-1120a^{2}b^{3}x^{\frac{2}{3}}+640a\,b^{4}x^{\frac{1}{3}}-256b^{5}\right)}{15015x\,a^{6}}$	79

```
[In] int((b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15015*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(3003*a^5*x^(5/3)-2310*a^4*b*x^
(4/3)+1680*a^3*b^2*x-1120*a^2*b^3*x^(2/3)+640*a*b^4*x^(1/3)-256*b^5)/x/a^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. 2(125) = 250.

Time = 153.11 (sec) , antiderivative size = 768, normalized size of antiderivative = 4.54

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx = \frac{2\left(4\left(805306368\,b^{13}+167772160\,b^{12}+786432\left(64\,a^3-3\right)b^{10}-15728640\,b^{11}-4096\left(11264\,a^3-\right.\right.\right.}{}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(4*(805306368*b^13 + 167772160*b^12 + 786432*(64*a^3 - 3)*b^10 - 15
728640*b^11 - 4096*(11264*a^3 - 53)*b^9 + 4372368*a^9 - 1536*(5504*a^3 + 1)
*b^8 - 48*(242810880*a^6 + 114688*a^3 + 3)*b^7 - 1792*(1353984*a^6 - 103*a^
3)*b^6 + 192*(1152384*a^6 - 23*a^3)*b^5 - 3*(3633315840*a^9 - 12027392*a^6
- 15*a^3)*b^4 - 112*(35389440*a^9 + 29281*a^6)*b^3 - 819*(368640*a^9 - 31*a
^6)*b^2 + 693*(40960*a^9 + 3*a^6)*b)*x + (3003*(16777216*a^7*b^6 + 6291456*
a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b^3 - 2304*a^7*b^2 + 86
4*a^7*b - 27*a^7)*x^3 - 70*(16777216*a^4*b^9 + 6291456*a^4*b^8 + 196608*a^4
*b^7 - 114688*a^4*b^6 - 2304*a^4*b^5 + 864*a^4*b^4 - (262144*a^7 + 27*a^4)*
b^3)*x^2 + 128*(16777216*a*b^12 + 6291456*a*b^11 + 196608*a*b^10 - 114688*a
*b^9 - 2304*a*b^8 + 864*a*b^7 - (262144*a^4 + 27*a)*b^6)*x - 16*(268435456*
b^13 + 100663296*b^12 + 3145728*b^11 - 1835008*b^10 - 36864*b^9 - 16*(26214
4*a^3 + 27)*b^7 + 13824*b^8 - 231*(16777216*a^6*b^7 + 6291456*a^6*b^6 + 196
608*a^6*b^5 - 114688*a^6*b^4 - 2304*a^6*b^3 + 864*a^6*b^2 - (262144*a^9 + 2
7*a^6)*b)*x^2 - 5*(16777216*a^3*b^10 + 6291456*a^3*b^9 + 196608*a^3*b^8 - 1
14688*a^3*b^7 - 2304*a^3*b^6 + 864*a^3*b^5 - (262144*a^6 + 27*a^3)*b^4)*x)*
x^(2/3) + 3*(21*(16777216*a^5*b^8 + 6291456*a^5*b^7 + 196608*a^5*b^6 - 1146
88*a^5*b^5 - 2304*a^5*b^4 + 864*a^5*b^3 - (262144*a^8 + 27*a^5)*b^2)*x^2 -
32*(16777216*a^2*b^11 + 6291456*a^2*b^10 + 196608*a^2*b^9 - 114688*a^2*b^8
- 2304*a^2*b^7 + 864*a^2*b^6 - (262144*a^5 + 27*a^2)*b^5)*x)*x^(1/3))*sqrt(
a*x + b*x^(2/3)))/((16777216*a^6*b^6 + 6291456*a^6*b^5 + 196608*a^6*b^4 - 2
62144*a^9 - 114688*a^6*b^3 - 2304*a^6*b^2 + 864*a^6*b - 27*a^6)*x)
```

Sympy [F]

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx = \int \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\int \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(125) = 250$.

Time = 0.31 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.57

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{2}{3003}\,b\left(\frac{256\,b^{\frac{13}{2}}}{a^6}+\frac{13\left(63\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}-385\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b+990\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^2-1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^3+1155\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^4-\right.}{a^5}\right.$$
$$-\frac{2}{15015}\,a\left(\frac{1024\,b^{\frac{15}{2}}}{a^7}-\frac{15\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}-1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b+5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^2-8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^3+9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^4-6006\left(a\right.\right.}{a^6}\right.$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3003*b*(256*b^(13/2)/a^6 + (13*(63*(a*x^(1/3) + b)^(11/2) - 385*(a*x^(1/3
) + b)^(9/2)*b + 990*(a*x^(1/3) + b)^(7/2)*b^2 - 1386*(a*x^(1/3) + b)^(5/2)
*b^3 + 1155*(a*x^(1/3) + b)^(3/2)*b^4 - 693*sqrt(a*x^(1/3) + b)*b^5)*b/a^5
+ 3*(231*(a*x^(1/3) + b)^(13/2) - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x
^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) +
b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^
6)/a^5)/a) - 2/15015*a*(1024*b^(15/2)/a^7 - (15*(231*(a*x^(1/3) + b)^(13/2)
 - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a
*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3)
+ b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*x^(1/3) +
b)^(15/2) - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^
2 - 25025*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 270
27*(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(
a*x^(1/3) + b)*b^7)/a^6)/a)
```

Mupad [B] (verification not implemented)

Time = 10.89 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.24

$$\int \left(bx^{2/3}+ax\right)^{3/2}\,dx=\frac{x\,\left(a\,x+b\,x^{2/3}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},6;\ 7;\ -\frac{a\,x^{1/3}}{b}\right)}{2\,{\left(\frac{a\,x^{1/3}}{b}+1\right)}^{3/2}}$$

```
[In] int((a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] (x*(a*x + b*x^(2/3))^(3/2)*hypergeom([-3/2, 6], 7, -(a*x^(1/3))/b))/(2*((a*
x^(1/3))/b + 1)^(3/2))
```

3.179 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx=\frac{16b^2\left(bx^{2/3}+ax\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}$$

```
[Out] 16/105*b^2*(b*x^(2/3)+a*x)^(5/2)/a^3/x^(5/3)-8/21*b*(b*x^(2/3)+a*x)^(5/2)/a
^2/x^(4/3)+2/3*(b*x^(2/3)+a*x)^(5/2)/a/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2039}

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx=\frac{16b^2\left(ax+bx^{2/3}\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(ax+bx^{2/3}\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}$$

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

```
[Out] (16*b^2*(b*x^(2/3) + a*x)^(5/2))/(105*a^3*x^(5/3)) - (8*b*(b*x^(2/3) + a*x)
^(5/2))/(21*a^2*x^(4/3)) + (2*(b*x^(2/3) + a*x)^(5/2))/(3*a*x)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}-\frac{(4b)\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{4/3}}\,dx}{9a}\\ &= -\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}+\frac{(8b^2)\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{5/3}}\,dx}{63a^2}\\ &= \frac{16b^2\left(bx^{2/3}+ax\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}\end{aligned}$$

Mathematica [A] (verified)

Time = 6.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx=\frac{2\left(b+a\sqrt[3]{x}\right)\left(8b^2-20ab\sqrt[3]{x}+35a^2x^{2/3}\right)\left(bx^{2/3}+ax\right)^{3/2}}{105a^3x}$$

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x,x]
[Out] (2*(b + a*x^(1/3))*(8*b^2 - 20*a*b*x^(1/3) + 35*a^2*x^(2/3))*(b*x^(2/3) + a
*x)^(3/2))/(105*a^3*x)
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.57

method	result	size
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^2x^{\frac{2}{3}}-20ab\,x^{\frac{1}{3}}+8b^2\right)}{105x\,a^3}$	48
default	$\frac{2\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^2x^{\frac{2}{3}}-20ab\,x^{\frac{1}{3}}+8b^2\right)}{105x\,a^3}$	48

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(35*a^2*x^(2/3)-20*a*b*x^(1/3)+8*
b^2)/x/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(62) = 124$.

Time = 175.32 (sec) , antiderivative size = 501, normalized size of antiderivative = 5.96

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx = -\frac{(201326592\,b^{10} + 41943040\,b^9 + 196608\,(6784\,a^3-3)b^7 - 3932160\,b^8 + 1024\,(257536\,a^3+53)b^6 - 407680}{}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/105*((201326592*b^10 + 41943040*b^9 + 196608*(6784*a^3 - 3)*b^7 - 393216
0*b^8 + 1024*(257536*a^3 + 53)*b^6 - 407680*a^6 - 384*(72704*a^3 + 1)*b^5 +
 12*(94371840*a^6 - 437248*a^3 - 3)*b^4 + 896*(442368*a^6 + 449*a^3)*b^3 +
24*(1105920*a^6 - 151*a^3)*b^2 - 15*(253952*a^6 + 15*a^3)*b)*x - 2*(35*(167
77216*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^4*
b^3 - 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x^2 + 3*(16777216*a^2*b^8 + 629145
6*a^2*b^7 + 196608*a^2*b^6 - 114688*a^2*b^5 - 2304*a^2*b^4 + 864*a^2*b^3 -
(262144*a^5 + 27*a^2)*b^2)*x^(4/3) - 4*(16777216*a*b^9 + 6291456*a*b^8 + 19
6608*a*b^7 - 114688*a*b^6 - 2304*a*b^5 + 864*a*b^4 - (262144*a^4 + 27*a)*b^
3)*x + 2*(67108864*b^10 + 25165824*b^9 + 786432*b^8 - 458752*b^7 - 9216*b^6
 - 4*(262144*a^3 + 27)*b^4 + 3456*b^5 + 25*(16777216*a^3*b^7 + 6291456*a^3*
b^6 + 196608*a^3*b^5 - 114688*a^3*b^4 - 2304*a^3*b^3 + 864*a^3*b^2 - (26214
4*a^6 + 27*a^3)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^3*b^6 +
6291456*a^3*b^5 + 196608*a^3*b^4 - 262144*a^6 - 114688*a^3*b^3 - 2304*a^3*b
^2 + 864*a^3*b - 27*a^3)*x)
```

Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(62) = 124$.

Time = 0.30 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.15

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx =$$

$$-\frac{2}{35}\,b\left(\frac{8\,b^{\frac{7}{2}}}{a^3}-\frac{\frac{7\left(3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}-10\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b+15\sqrt{ax^{\frac{1}{3}}+b}b^2\right)b}{a^2}+\frac{3\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^2-35\sqrt{ax^{\frac{1}{3}}+b}b^3\right.}{a^2}}{a}\right.$$

$$\left.+\frac{2}{105}\,a\left(\frac{16\,b^{\frac{9}{2}}}{a^4}+\frac{\frac{9\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^2-35\sqrt{ax^{\frac{1}{3}}+b}b^3\right)b}{a^3}+\frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}+b\right.}{a^3}}{a}\right.\right.$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -2/35*b*(8*b^(7/2)/a^3 - (7*(3*(a*x^(1/3) + b)^(5/2) - 10*(a*x^(1/3) + b)^(
3/2)*b + 15*sqrt(a*x^(1/3) + b)*b^2)*b/a^2 + 3*(5*(a*x^(1/3) + b)^(7/2) - 2
1*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(1/3
) + b)*b^3)/a^2)/a) + 2/105*a*(16*b^(9/2)/a^4 + (9*(5*(a*x^(1/3) + b)^(7/2)
 - 21*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^
(1/3) + b)*b^3)*b/a^3 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b)^(7/
2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 + 315*
sqrt(a*x^(1/3) + b)*b^4)/a^3)/a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx=\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x}\,dx$$

```
[In] int((a*x + b*x^(2/3))^(3/2)/x,x)
[Out] int((a*x + b*x^(2/3))^(3/2)/x, x)
```

3.180 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=\frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-6b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)$$

```
[Out] 2*(b*x^(2/3)+a*x)^(3/2)/x-6*b^(3/2)*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)
^(1/2))+6*b*(b*x^(2/3)+a*x)^(1/2)/x^(1/3)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2046, 2054, 212}

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=-6b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)+\frac{6b\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (6*b*Sqrt[b*x^(2/3) + a*x])/x^(1/3) + (2*(b*x^(2/3) + a*x)^(3/2))/x - 6*b^(
3/2)*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}+b\int \frac{\sqrt{bx^{2/3}+ax}}{x^{4/3}}\,dx\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}+b^2\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-\left(6b^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx=\frac{2\sqrt{bx^{2/3}+ax}\left(\sqrt{b+a\sqrt[3]{x}}\left(4b+a\sqrt[3]{x}\right)-3b^{3/2}\text{arctanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)\right)}{\sqrt{b+a\sqrt[3]{x}}\sqrt[3]{x}}
$$

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(Sqrt[b + a*x^(1/3)]*(4*b + a*x^(1/3)) - 3*b^(3/2)
*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]]))/(Sqrt[b + a*x^(1/3)]*x^(1/3))
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(-3 b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)+\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}+3 b \sqrt{b+a\,x^{\frac{1}{3}}}\right)}{x \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}}$	67
default	$-\frac{2\left(b\,x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \left(3 b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)-\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}-3 b \sqrt{b+a\,x^{\frac{1}{3}}}\right)}{x \left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}}$	69

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*x^(2/3)+a*x)^(3/2)*(-3*b^(3/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))+(b
+a*x^(1/3))^(3/2)+3*b*(b+a*x^(1/3))^(1/2))/x/(b+a*x^(1/3))^(3/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx = \frac{6\,b^2\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}} + 6\sqrt{ax^{\frac{1}{3}}+b}b - \frac{2\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\sqrt{-b}b^{\frac{3}{2}}\right)}{\sqrt{-b}}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 6*b^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/sqrt(-b) + 2*(a*x^(1/3) + b)^(3/
2) + 6*sqrt(a*x^(1/3) + b)*b - 2*(3*b^2*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt(-
b)*b^(3/2))/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx = \int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^2}\,dx$$

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^2,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

3.181 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=-\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{3a^3\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{3/2}}$$

```
[Out] -(b*x^(2/3)+a*x)^(3/2)/x^2+3/8*a^3*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^
(1/2))/b^(3/2)-3/4*a*(b*x^(2/3)+a*x)^(1/2)/x-3/8*a^2*(b*x^(2/3)+a*x)^(1/2)/
b/x^(2/3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 2050, 2054, 212}

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=\frac{3a^3\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{3/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{8bx^{2/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4x}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(4*x) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(8*b*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/x^2 + (3*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt
[b*x^(2/3) + a*x]])/(8*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{1}{2}a\int\frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{1}{8}a^2\int\frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}-\frac{a^3\int\frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{16b}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\\
&\quad+\frac{\left(3a^3\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b}
\end{aligned}
$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2} + \frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx = \frac{6a^3\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},4,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^4\sqrt[3]{x}}$$

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (6*a^3*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 4, 7/
2, 1 + (a*x^(1/3))/b])/(5*b^4*x^(1/3))
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(3\,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b\,a^{3}x-3b^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}-8b^{\frac{5}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}+3b^{\frac{7}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\right)}{8x^{2}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	93
default	$\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(3\,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b\,a^{3}x-3b^{\frac{3}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}-8b^{\frac{5}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}+3b^{\frac{7}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\right)}{8x^{2}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	93

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(b*x^(2/3)+a*x)^(3/2)*(3*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b*a^3*x-3
*b^(3/2)*(b+a*x^(1/3))^(5/2)-8*b^(5/2)*(b+a*x^(1/3))^(3/2)+3*b^(7/2)*(b+a*x
^(1/3))^(1/2))/x^2/(b+a*x^(1/3))^(3/2)/b^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**3,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx = -\frac{\frac{3\,a^4\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^4+8\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^4b-3\sqrt{ax^{\frac{1}{3}}+ba^4b^2}}{a^3bx}}{8\,a}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^4*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(a*x^(1/
3) + b)^(5/2)*a^4 + 8*(a*x^(1/3) + b)^(3/2)*a^4*b - 3*sqrt(a*x^(1/3) + b)*a
^4*b^2)/(a^3*b*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx=\int \frac{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}{x^3}\,dx$$

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^3,x)
[Out] int((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

3.182 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x} + \frac{21a^5\sqrt{bx^{2/3}+ax}}{512b^4x^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3} - \frac{21a^6\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{512b^{9/2}}$$

```
[Out] -1/2*(b*x^(2/3)+a*x)^(3/2)/x^3-21/512*a^6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3
)+a*x)^(1/2))/b^(9/2)-3/20*a*(b*x^(2/3)+a*x)^(1/2)/x^2-3/160*a^2*(b*x^(2/3)
+a*x)^(1/2)/b/x^(5/3)+7/320*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)-7/256*a^4
*(b*x^(2/3)+a*x)^(1/2)/b^3/x+21/512*a^5*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 2050, 2054, 212}

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = -\frac{21a^6\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{9/2}} + \frac{21a^5\sqrt{ax+bx^{2/3}}}{512b^4x^{2/3}} - \frac{7a^4\sqrt{ax+bx^{2/3}}}{256b^3x} + \frac{7a^3\sqrt{ax+bx^{2/3}}}{320b^2x^{4/3}} - \frac{3a^2\sqrt{ax+bx^{2/3}}}{160bx^{5/3}} - \frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3} - \frac{3a\sqrt{ax+bx^{2/3}}}{20x^2}$$

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(20*x^2) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(160*
b*x^(5/3)) + (7*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^2*x^(4/3)) - (7*a^4*Sqrt[
b*x^(2/3) + a*x])/(256*b^3*x) + (21*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^4*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/(2*x^3) - (21*a^6*ArcTanh[(Sqrt[b]*x^(1/3))
/Sqrt[b*x^(2/3) + a*x]])/(512*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}+\frac{1}{4}a\int\frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}+\frac{1}{40}a^2\int\frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}-\frac{\left(7a^3\right)\int\frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{320b}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} \\
&\quad - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3} + \frac{(7a^4)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{384b^2} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} \\
&\quad - \frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3} - \frac{(7a^5)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{512b^3} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x} \\
&\quad + \frac{21a^5\sqrt{bx^{2/3}+ax}}{512b^4x^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3} + \frac{(7a^6)\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{1024b^4} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x} \\
&\quad + \frac{21a^5\sqrt{bx^{2/3}+ax}}{512b^4x^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3} - \frac{(21a^6)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{512b^4} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}} + \frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}} - \frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x} \\
&\quad + \frac{21a^5\sqrt{bx^{2/3}+ax}}{512b^4x^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3} - \frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{512b^{9/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.30

$$
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = -\frac{6a^6\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},7,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^7\sqrt[3]{x}}
$$

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-6*a^6*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 7, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^7*x^(1/3))
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(105\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}b^{\frac{9}{2}}-595\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}b^{\frac{11}{2}}+1386\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}b^{\frac{13}{2}}-1686\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}b^{\frac{15}{2}}-105\operatorname{arctanh}\left(\right.\right.}{2560x^{3}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}b^{\frac{17}{2}}}$
default	$\frac{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(105\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{11}{2}}b^{\frac{9}{2}}-595\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{9}{2}}b^{\frac{11}{2}}+1386\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}b^{\frac{13}{2}}-1686\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}b^{\frac{15}{2}}-105\operatorname{arctanh}\left(\right.\right.}{2560x^{3}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}b^{\frac{17}{2}}}$

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2560*(b*x^(2/3)+a*x)^(3/2)*(105*(b+a*x^(1/3))^(11/2)*b^(9/2)-595*(b+a*x^(
1/3))^(9/2)*b^(11/2)+1386*(b+a*x^(1/3))^(7/2)*b^(13/2)-1686*(b+a*x^(1/3))^(
5/2)*b^(15/2)-105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^4*a^6*x^2-595*(b+a
*x^(1/3))^(3/2)*b^(17/2)+105*(b+a*x^(1/3))^(1/2)*b^(19/2))/x^3/(b+a*x^(1/3)
)^(3/2)/b^(17/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx=\text{Timed out}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**4,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.70

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = \frac{\frac{105\,a^7\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^7-595\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^7b+1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^7b^2-1686\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}}{a^6b^4x^2}}{2560\,a}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
[Out] 1/2560*(105*a^7*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*
(a*x^(1/3) + b)^(11/2)*a^7 - 595*(a*x^(1/3) + b)^(9/2)*a^7*b + 1386*(a*x^(1
/3) + b)^(7/2)*a^7*b^2 - 1686*(a*x^(1/3) + b)^(5/2)*a^7*b^3 - 595*(a*x^(1/3
) + b)^(3/2)*a^7*b^4 + 105*sqrt(a*x^(1/3) + b)*a^7*b^5)/(a^6*b^4*x^2))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = \int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^4}\,dx$$

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^4,x)
[Out] int((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

3.183 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 291

$$\begin{aligned}\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx &= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}\\ &\quad -\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}}-\frac{143a^6\sqrt{bx^{2/3}+ax}}{20480b^5x^{4/3}}+\frac{143a^7\sqrt{bx^{2/3}+ax}}{16384b^6x}\\ &\quad -\frac{429a^8\sqrt{bx^{2/3}+ax}}{32768b^7x^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{429a^9\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{32768b^{15/2}}\end{aligned}$$

```
[Out] -1/3*(b*x^(2/3)+a*x)^(3/2)/x^4+429/32768*a^9*arctanh(x^(1/3)*b^(1/2)/(b*x^(
2/3)+a*x)^(1/2))/b^(15/2)-1/16*a*(b*x^(2/3)+a*x)^(1/2)/x^3-1/224*a^2*(b*x^(
2/3)+a*x)^(1/2)/b/x^(8/3)+13/2688*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-143
/26880*a^4*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2+429/71680*a^5*(b*x^(2/3)+a*x)^(1/2
)/b^4/x^(5/3)-143/20480*a^6*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+143/16384*a^7
*(b*x^(2/3)+a*x)^(1/2)/b^6/x-429/32768*a^8*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used

$= \{2045,\ 2050,\ 2054,\ 212\}$

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx = \frac{429a^9\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{32768b^{15/2}} - \frac{429a^8\sqrt{ax+bx^{2/3}}}{32768b^7x^{2/3}} + \frac{143a^7\sqrt{ax+bx^{2/3}}}{16384b^6x} - \frac{143a^6\sqrt{ax+bx^{2/3}}}{20480b^5x^{4/3}} + \frac{429a^5\sqrt{ax+bx^{2/3}}}{71680b^4x^{5/3}} - \frac{143a^4\sqrt{ax+bx^{2/3}}}{26880b^3x^2} + \frac{13a^3\sqrt{ax+bx^{2/3}}}{2688b^2x^{7/3}} - \frac{a^2\sqrt{ax+bx^{2/3}}}{224bx^{8/3}} - \frac{\left(ax+bx^{2/3}\right)^{3/2}}{3x^4} - \frac{a\sqrt{ax+bx^{2/3}}}{16x^3}$$

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] -1/16*(a*Sqrt[b*x^(2/3) + a*x])/x^3 - (a^2*Sqrt[b*x^(2/3) + a*x])/(224*b*x^
(8/3)) + (13*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^2*x^(7/3)) - (143*a^4*Sqrt[
b*x^(2/3) + a*x])/(26880*b^3*x^2) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(71680*
b^4*x^(5/3)) - (143*a^6*Sqrt[b*x^(2/3) + a*x])/(20480*b^5*x^(4/3)) + (143*a
^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^6*x) - (429*a^8*Sqrt[b*x^(2/3) + a*x])/(
32768*b^7*x^(2/3)) - (b*x^(2/3) + a*x)^(3/2)/(3*x^4) + (429*a^9*ArcTanh[(Sq
rt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{1}{6}a\int\frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx\\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{1}{96}a^2\int\frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}-\frac{\left(13a^3\right)\int\frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{1344b}\\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}\\
&\quad-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{\left(143a^4\right)\int\frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{16128b^2}\\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}\\
&\quad-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}-\frac{\left(143a^5\right)\int\frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{17920b^3}\\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}\\
&\quad+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{\left(143a^6\right)\int\frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{20480b^4}\\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}\\
&\quad+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}}-\frac{143a^6\sqrt{bx^{2/3}+ax}}{20480b^5x^{4/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}\\
&\quad-\frac{\left(143a^7\right)\int\frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{24576b^5}\\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}\\
&\quad-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}}-\frac{143a^6\sqrt{bx^{2/3}+ax}}{20480b^5x^{4/3}}\\
&\quad+\frac{143a^7\sqrt{bx^{2/3}+ax}}{16384b^6x}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{\left(143a^8\right)\int\frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{32768b^6}
\end{aligned}
$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2} + \frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}} - \frac{143a^6\sqrt{bx^{2/3}+ax}}{20480b^5x^{4/3}} + \frac{143a^7\sqrt{bx^{2/3}+ax}}{16384b^6x} - \frac{429a^8\sqrt{bx^{2/3}+ax}}{32768b^7x^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4} - \frac{(143a^9)\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{65536b^7}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2} + \frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}} - \frac{143a^6\sqrt{bx^{2/3}+ax}}{20480b^5x^{4/3}} + \frac{143a^7\sqrt{bx^{2/3}+ax}}{16384b^6x} - \frac{429a^8\sqrt{bx^{2/3}+ax}}{32768b^7x^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4} + \frac{(429a^9)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{32768b^7}$$

$$= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3} - \frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}} + \frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}} - \frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2} + \frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}} - \frac{143a^6\sqrt{bx^{2/3}+ax}}{20480b^5x^{4/3}} + \frac{143a^7\sqrt{bx^{2/3}+ax}}{16384b^6x} - \frac{429a^8\sqrt{bx^{2/3}+ax}}{32768b^7x^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4} + \frac{429a^9\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{32768b^{15/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.21

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx = \frac{6a^9\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},10,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^{10}\sqrt[3]{x}}$$

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (6*a^9*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 10, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^10*x^(1/3))
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}} \left(45045b^{\frac{31}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}-390390b^{\frac{29}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}-2633274b^{\frac{27}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}+4349826b^{\frac{25}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}-46858\right.$
default	$\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}} \left(45045b^{\frac{31}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}-390390b^{\frac{29}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{3}{2}}-2633274b^{\frac{27}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{5}{2}}+4349826b^{\frac{25}{2}}\left(b+a\,x^{\frac{1}{3}}\right)^{\frac{7}{2}}-46858\right.$

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3440640*(b*x^(2/3)+a*x)^(3/2)*(45045*b^(31/2)*(b+a*x^(1/3))^(1/2)-390390*
b^(29/2)*(b+a*x^(1/3))^(3/2)-2633274*b^(27/2)*(b+a*x^(1/3))^(5/2)+4349826*b
^(25/2)*(b+a*x^(1/3))^(7/2)-4685824*b^(23/2)*(b+a*x^(1/3))^(9/2)+3317886*b^
(21/2)*(b+a*x^(1/3))^(11/2)-1495494*b^(19/2)*(b+a*x^(1/3))^(13/2)+390390*b^
(17/2)*(b+a*x^(1/3))^(15/2)-45045*b^(15/2)*(b+a*x^(1/3))^(17/2)+45045*arcta
nh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^7*a^9*x^3)/x^4/(b+a*x^(1/3))^(3/2)/b^(29/
2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=\text{Timed out}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**5,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.67

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=$$

$$-\frac{\frac{45045\,a^{10}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{7}}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{10}-390390\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{10}b+1495494\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{10}b^{2}-3317886\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{10}b^{3}+}{}}{3440640\,a}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/3440640*(45045*a^10*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7)
+ (45045*(a*x^(1/3) + b)^(17/2)*a^10 - 390390*(a*x^(1/3) + b)^(15/2)*a^10*b
 + 1495494*(a*x^(1/3) + b)^(13/2)*a^10*b^2 - 3317886*(a*x^(1/3) + b)^(11/2)
*a^10*b^3 + 4685824*(a*x^(1/3) + b)^(9/2)*a^10*b^4 - 4349826*(a*x^(1/3) + b
)^(7/2)*a^10*b^5 + 2633274*(a*x^(1/3) + b)^(5/2)*a^10*b^6 + 390390*(a*x^(1/
3) + b)^(3/2)*a^10*b^7 - 45045*sqrt(a*x^(1/3) + b)*a^10*b^8)/(a^9*b^7*x^3))
/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx=\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^5}\,dx$$

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^5,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

3.184 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 379

$$\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx &= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} \\
&\quad+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}} \\
&\quad-\frac{4199a^6\sqrt{bx^{2/3}+ax}}{1892352b^5x^{7/3}}+\frac{4199a^7\sqrt{bx^{2/3}+ax}}{1720320b^6x^2}-\frac{12597a^8\sqrt{bx^{2/3}+ax}}{4587520b^7x^{5/3}} \\
&\quad+\frac{4199a^9\sqrt{bx^{2/3}+ax}}{1310720b^8x^{4/3}}-\frac{4199a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^9x}+\frac{12597a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{10}x^{2/3}} \\
&\quad-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}-\frac{12597a^{12}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{2097152b^{21/2}}
\end{aligned}$$

```
[Out] -1/4*(b*x^(2/3)+a*x)^(3/2)/x^5-12597/2097152*a^12*arctanh(x^(1/3)*b^(1/2)/(
b*x^(2/3)+a*x)^(1/2))/b^(21/2)-3/88*a*(b*x^(2/3)+a*x)^(1/2)/x^4-3/1760*a^2*
(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/10560*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(
10/3)-323/168960*a^4*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3+323/157696*a^5*(b*x^(2/3
)+a*x)^(1/2)/b^4/x^(8/3)-4199/1892352*a^6*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)
+4199/1720320*a^7*(b*x^(2/3)+a*x)^(1/2)/b^6/x^2-12597/4587520*a^8*(b*x^(2/3
)+a*x)^(1/2)/b^7/x^(5/3)+4199/1310720*a^9*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)
-4199/1048576*a^10*(b*x^(2/3)+a*x)^(1/2)/b^9/x+12597/2097152*a^11*(b*x^(2/3
)+a*x)^(1/2)/b^10/x^(2/3)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2045, 2050, 2054, 212}

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = -\frac{12597a^{12}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{2097152b^{21/2}} + \frac{12597a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{10}x^{2/3}} - \frac{4199a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^9x} + \frac{4199a^9\sqrt{ax+bx^{2/3}}}{1310720b^8x^{4/3}} - \frac{12597a^8\sqrt{ax+bx^{2/3}}}{4587520b^7x^{5/3}} + \frac{4199a^7\sqrt{ax+bx^{2/3}}}{1720320b^6x^2} - \frac{4199a^6\sqrt{ax+bx^{2/3}}}{1892352b^5x^{7/3}} + \frac{323a^5\sqrt{ax+bx^{2/3}}}{157696b^4x^{8/3}} - \frac{323a^4\sqrt{ax+bx^{2/3}}}{168960b^3x^3} + \frac{19a^3\sqrt{ax+bx^{2/3}}}{10560b^2x^{10/3}} - \frac{3a^2\sqrt{ax+bx^{2/3}}}{1760bx^{11/3}} - \frac{\left(ax+bx^{2/3}\right)^{3/2}}{4x^5} - \frac{3a\sqrt{ax+bx^{2/3}}}{88x^4}$$

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(88*x^4) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(1760
*b*x^(11/3)) + (19*a^3*Sqrt[b*x^(2/3) + a*x])/(10560*b^2*x^(10/3)) - (323*a
^4*Sqrt[b*x^(2/3) + a*x])/(168960*b^3*x^3) + (323*a^5*Sqrt[b*x^(2/3) + a*x]
)/(157696*b^4*x^(8/3)) - (4199*a^6*Sqrt[b*x^(2/3) + a*x])/(1892352*b^5*x^(7
/3)) + (4199*a^7*Sqrt[b*x^(2/3) + a*x])/(1720320*b^6*x^2) - (12597*a^8*Sqrt
[b*x^(2/3) + a*x])/(4587520*b^7*x^(5/3)) + (4199*a^9*Sqrt[b*x^(2/3) + a*x])
/(1310720*b^8*x^(4/3)) - (4199*a^10*Sqrt[b*x^(2/3) + a*x])/(1048576*b^9*x)
+ (12597*a^11*Sqrt[b*x^(2/3) + a*x])/(2097152*b^10*x^(2/3)) - (b*x^(2/3) +
a*x)^(3/2)/(4*x^5) - (12597*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) +
 a*x]])/(2097152*b^(21/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{1}{8}a\int\frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{1}{176}a^2\int\frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}-\frac{\left(19a^3\right)\int\frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{3520b}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}\\
&\quad-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{\left(323a^4\right)\int\frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{63360b^2}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}\\
&\quad-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}-\frac{\left(323a^5\right)\int\frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx}{67584b^3}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}\\
&\quad+\frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{\left(4199a^6\right)\int\frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{946176b^4}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}\\
&\quad+\frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}}-\frac{4199a^6\sqrt{bx^{2/3}+ax}}{1892352b^5x^{7/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}-\frac{\left(4199a^7\right)\int\frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{1032192b^5}
\end{aligned}$$

$$\begin{aligned}&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} \\ &\quad - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}} - \frac{4199a^6\sqrt{bx^{2/3}+ax}}{1892352b^5x^{7/3}} \\ &\quad + \frac{4199a^7\sqrt{bx^{2/3}+ax}}{1720320b^6x^2} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5} + \frac{(4199a^8)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{1146880b^6}\end{aligned}$$

$$\begin{aligned}&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3} \\ &\quad + \frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}} - \frac{4199a^6\sqrt{bx^{2/3}+ax}}{1892352b^5x^{7/3}} + \frac{4199a^7\sqrt{bx^{2/3}+ax}}{1720320b^6x^2} \\ &\quad - \frac{12597a^8\sqrt{bx^{2/3}+ax}}{4587520b^7x^{5/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5} - \frac{(4199a^9)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{1310720b^7}\end{aligned}$$

$$\begin{aligned}&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} \\ &\quad - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}} - \frac{4199a^6\sqrt{bx^{2/3}+ax}}{1892352b^5x^{7/3}} \\ &\quad + \frac{4199a^7\sqrt{bx^{2/3}+ax}}{1720320b^6x^2} - \frac{12597a^8\sqrt{bx^{2/3}+ax}}{4587520b^7x^{5/3}} + \frac{4199a^9\sqrt{bx^{2/3}+ax}}{1310720b^8x^{4/3}} \\ &\quad - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5} + \frac{(4199a^{10})\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{1572864b^8}\end{aligned}$$

$$\begin{aligned}&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} \\ &\quad - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}} - \frac{4199a^6\sqrt{bx^{2/3}+ax}}{1892352b^5x^{7/3}} \\ &\quad + \frac{4199a^7\sqrt{bx^{2/3}+ax}}{1720320b^6x^2} - \frac{12597a^8\sqrt{bx^{2/3}+ax}}{4587520b^7x^{5/3}} + \frac{4199a^9\sqrt{bx^{2/3}+ax}}{1310720b^8x^{4/3}} \\ &\quad - \frac{4199a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^9x} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5} - \frac{(4199a^{11})\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{2097152b^9}\end{aligned}$$

$$\begin{aligned}&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3} \\ &\quad + \frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}} - \frac{4199a^6\sqrt{bx^{2/3}+ax}}{1892352b^5x^{7/3}} + \frac{4199a^7\sqrt{bx^{2/3}+ax}}{1720320b^6x^2} \\ &\quad - \frac{12597a^8\sqrt{bx^{2/3}+ax}}{4587520b^7x^{5/3}} + \frac{4199a^9\sqrt{bx^{2/3}+ax}}{1310720b^8x^{4/3}} - \frac{4199a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^9x} \\ &\quad + \frac{12597a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{10}x^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5} + \frac{(4199a^{12})\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{4194304b^{10}}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} \\
&\quad - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3} + \frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}} - \frac{4199a^6\sqrt{bx^{2/3}+ax}}{1892352b^5x^{7/3}} \\
&\quad + \frac{4199a^7\sqrt{bx^{2/3}+ax}}{1720320b^6x^2} - \frac{12597a^8\sqrt{bx^{2/3}+ax}}{4587520b^7x^{5/3}} + \frac{4199a^9\sqrt{bx^{2/3}+ax}}{1310720b^8x^{4/3}} \\
&\quad - \frac{4199a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^9x} + \frac{12597a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{10}x^{2/3}} \\
&\quad - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5} - \frac{(12597a^{12})\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{2097152b^{10}} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4} - \frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}} + \frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}} - \frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3} \\
&\quad + \frac{323a^5\sqrt{bx^{2/3}+ax}}{157696b^4x^{8/3}} - \frac{4199a^6\sqrt{bx^{2/3}+ax}}{1892352b^5x^{7/3}} + \frac{4199a^7\sqrt{bx^{2/3}+ax}}{1720320b^6x^2} \\
&\quad - \frac{12597a^8\sqrt{bx^{2/3}+ax}}{4587520b^7x^{5/3}} + \frac{4199a^9\sqrt{bx^{2/3}+ax}}{1310720b^8x^{4/3}} - \frac{4199a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^9x} \\
&\quad + \frac{12597a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{10}x^{2/3}} - \frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5} - \frac{12597a^{12}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{2097152b^{21/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

$$
\begin{aligned}
&\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \\
&-\frac{6a^{12}\left(b+a\sqrt[3]{x}\right)^2\sqrt{bx^{2/3}+ax}\,\text{Hypergeometric2F1}\left(\frac{5}{2},13,\frac{7}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{5b^{13}\sqrt[3]{x}}
\end{aligned}
$$

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-6*a^12*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 13,
 7/2, 1 + (a*x^(1/3))/b])/(5*b^13*x^(1/3))
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \Bigg(145495535 b^{\frac{21}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{23}{2}}-169744575 b^{\frac{23}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{21}{2}}+904981077 b^{\frac{25}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{19}{2}}-29136483309 b^{\frac{27}{2}}}{}$
default	$-\frac{\left(b x^{\frac{2}{3}}+a x\right)^{\frac{3}{2}} \Bigg(-145495535 b^{\frac{21}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{23}{2}}+169744575 b^{\frac{23}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{21}{2}}-904981077 b^{\frac{25}{2}} \left(b+a x^{\frac{1}{3}}\right)^{\frac{19}{2}}+2913648309}{}$

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2422210560*(b*x^(2/3)+a*x)^(3/2)*(14549535*b^(21/2)*(b+a*x^(1/3))^(23/2)-
169744575*b^(23/2)*(b+a*x^(1/3))^(21/2)+904981077*b^(25/2)*(b+a*x^(1/3))^(1
9/2)-2913648309*b^(27/2)*(b+a*x^(1/3))^(17/2)+6303782342*b^(29/2)*(b+a*x^(1
/3))^(15/2)-9643633350*b^(31/2)*(b+a*x^(1/3))^(13/2)+10677769530*b^(33/2)*(
b+a*x^(1/3))^(11/2)-8598579770*b^(35/2)*(b+a*x^(1/3))^(9/2)+4975837515*b^(3
7/2)*(b+a*x^(1/3))^(7/2)-2001671595*b^(39/2)*(b+a*x^(1/3))^(5/2)-14549535*a
rctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^10*a^12*x^4-169744575*b^(41/2)*(b+a*x
^(1/3))^(3/2)+14549535*b^(43/2)*(b+a*x^(1/3))^(1/2))/x^5/(b+a*x^(1/3))^(3/2
)/b^(41/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**6,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.65

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \frac{\frac{14549535\,a^{13}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}} + \frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{13}-169744575\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{13}b+904981077\left(ax^{\frac{1}{3}}+b\right.}{}}{}$$

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2422210560*(14549535*a^13*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*
b^10) + (14549535*(a*x^(1/3) + b)^(23/2)*a^13 - 169744575*(a*x^(1/3) + b)^(
21/2)*a^13*b + 904981077*(a*x^(1/3) + b)^(19/2)*a^13*b^2 - 2913648309*(a*x^
(1/3) + b)^(17/2)*a^13*b^3 + 6303782342*(a*x^(1/3) + b)^(15/2)*a^13*b^4 - 9
643633350*(a*x^(1/3) + b)^(13/2)*a^13*b^5 + 10677769530*(a*x^(1/3) + b)^(11
/2)*a^13*b^6 - 8598579770*(a*x^(1/3) + b)^(9/2)*a^13*b^7 + 4975837515*(a*x^
(1/3) + b)^(7/2)*a^13*b^8 - 2001671595*(a*x^(1/3) + b)^(5/2)*a^13*b^9 - 169
744575*(a*x^(1/3) + b)^(3/2)*a^13*b^10 + 14549535*sqrt(a*x^(1/3) + b)*a^13*
b^11)/(a^12*b^10*x^4))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx = \int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^6}\,dx$$

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^6,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

3.185 $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 401

$$\begin{aligned}\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx &= \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}} - \frac{16777216b^{13}\sqrt{bx^{2/3}+ax}}{11700675a^{14}\sqrt[3]{x}} \\ &- \frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} \\ &- \frac{131072b^{9}x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^{8}x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^{9}} \\ &- \frac{180224b^{7}x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^{8}} + \frac{1171456b^{6}x^{2}\sqrt{bx^{2/3}+ax}}{3900225a^{7}} \\ &- \frac{73216b^{5}x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^{6}} + \frac{36608b^{4}x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^{5}} - \frac{9152b^{3}x^{3}\sqrt{bx^{2/3}+ax}}{36225a^{4}} \\ &+ \frac{416b^{2}x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^{3}} - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^{2}} + \frac{2x^{4}\sqrt{bx^{2/3}+ax}}{9a}\end{aligned}$$

```
[Out] 8388608/11700675*b^12*(b*x^(2/3)+a*x)^(1/2)/a^13-16777216/11700675*b^13*(b*
x^(2/3)+a*x)^(1/2)/a^14/x^(1/3)-2097152/3900225*b^11*x^(1/3)*(b*x^(2/3)+a*x
)^(1/2)/a^12+1048576/2340135*b^10*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^11-131072
/334305*b^9*x*(b*x^(2/3)+a*x)^(1/2)/a^10+65536/185725*b^8*x^(4/3)*(b*x^(2/3
)+a*x)^(1/2)/a^9-180224/557175*b^7*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^8+117145
6/3900225*b^6*x^2*(b*x^(2/3)+a*x)^(1/2)/a^7-73216/260015*b^5*x^(7/3)*(b*x^(
2/3)+a*x)^(1/2)/a^6+36608/137655*b^4*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-9152
/36225*b^3*x^3*(b*x^(2/3)+a*x)^(1/2)/a^4+416/1725*b^2*x^(10/3)*(b*x^(2/3)+a
*x)^(1/2)/a^3-52/225*b*x^(11/3)*(b*x^(2/3)+a*x)^(1/2)/a^2+2/9*x^4*(b*x^(2/3
)+a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2041, 2027, 2039}

$$\begin{aligned}\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = &-\frac{16777216b^{13}\sqrt{ax+bx^{2/3}}}{11700675a^{14}\sqrt[3]{x}}+\frac{8388608b^{12}\sqrt{ax+bx^{2/3}}}{11700675a^{13}}\\ &-\frac{2097152b^{11}\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{3900225a^{12}}+\frac{1048576b^{10}x^{2/3}\sqrt{ax+bx^{2/3}}}{2340135a^{11}}\\ &-\frac{131072b^9x\sqrt{ax+bx^{2/3}}}{334305a^{10}}+\frac{65536b^8x^{4/3}\sqrt{ax+bx^{2/3}}}{185725a^9}\\ &-\frac{180224b^7x^{5/3}\sqrt{ax+bx^{2/3}}}{557175a^8}+\frac{1171456b^6x^2\sqrt{ax+bx^{2/3}}}{3900225a^7}\\ &-\frac{73216b^5x^{7/3}\sqrt{ax+bx^{2/3}}}{260015a^6}+\frac{36608b^4x^{8/3}\sqrt{ax+bx^{2/3}}}{137655a^5}-\frac{9152b^3x^3\sqrt{ax+bx^{2/3}}}{36225a^4}\\ &+\frac{416b^2x^{10/3}\sqrt{ax+bx^{2/3}}}{1725a^3}-\frac{52bx^{11/3}\sqrt{ax+bx^{2/3}}}{225a^2}+\frac{2x^4\sqrt{ax+bx^{2/3}}}{9a}\end{aligned}$$

```
[In] Int[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (8388608*b^12*Sqrt[b*x^(2/3) + a*x])/(11700675*a^13) - (16777216*b^13*Sqrt[
b*x^(2/3) + a*x])/(11700675*a^14*x^(1/3)) - (2097152*b^11*x^(1/3)*Sqrt[b*x^
(2/3) + a*x])/(3900225*a^12) + (1048576*b^10*x^(2/3)*Sqrt[b*x^(2/3) + a*x])
/(2340135*a^11) - (131072*b^9*x*Sqrt[b*x^(2/3) + a*x])/(334305*a^10) + (655
36*b^8*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(185725*a^9) - (180224*b^7*x^(5/3)*Sq
rt[b*x^(2/3) + a*x])/(557175*a^8) + (1171456*b^6*x^2*Sqrt[b*x^(2/3) + a*x])
/(3900225*a^7) - (73216*b^5*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(260015*a^6) + (
36608*b^4*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(137655*a^5) - (9152*b^3*x^3*Sqrt[
b*x^(2/3) + a*x])/(36225*a^4) + (416*b^2*x^(10/3)*Sqrt[b*x^(2/3) + a*x])/(1
725*a^3) - (52*b*x^(11/3)*Sqrt[b*x^(2/3) + a*x])/(225*a^2) + (2*x^4*Sqrt[b*
x^(2/3) + a*x])/(9*a)
```

<u>Rule 2027</u>

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

<u>Rule 2039</u>

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
```

```
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{(26b)\int \frac{x^{11/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{27a} \\
&= -\frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} + \frac{(208b^2)\int \frac{x^{10/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{225a^2} \\
&= \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{(4576b^3)\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx}{5175a^3} \\
&= -\frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} \\
&\quad - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} + \frac{(18304b^4)\int \frac{x^{8/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{21735a^4} \\
&= \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} \\
&\quad - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{(36608b^5)\int \frac{x^{7/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{45885a^5} \\
&= -\frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} \\
&\quad - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} \\
&\quad - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} + \frac{(585728b^6)\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx}{780045a^6} \\
&= \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} \\
&\quad + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} \\
&\quad - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{(1171456b^7)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{1671525a^7}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} \\
&\quad + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} \\
&\quad - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} + \frac{(360448b^8)\int \frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{557175a^8}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} \\
&\quad + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} \\
&\quad + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} \\
&\quad - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{(65536b^9)\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{111435a^9}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} \\
&\quad + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} \\
&\quad + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} \\
&\quad - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} + \frac{(524288b^{10})\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{1002915a^{10}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} \\
&\quad - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} \\
&\quad + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} \\
&\quad - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{(1048576b^{11})\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{2340135a^{11}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} \\
&\quad - \frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} \\
&\quad + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} \\
&\quad + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} \\
&\quad - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} + \frac{(4194304b^{12})\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{11700675a^{12}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}} - \frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} \\
&\quad - \frac{131072b^{9}x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^{8}x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^{9}} - \frac{180224b^{7}x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^{8}} \\
&\quad + \frac{1171456b^{6}x^{2}\sqrt{bx^{2/3}+ax}}{3900225a^{7}} - \frac{73216b^{5}x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^{6}} \\
&\quad + \frac{36608b^{4}x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^{5}} - \frac{9152b^{3}x^{3}\sqrt{bx^{2/3}+ax}}{36225a^{4}} + \frac{416b^{2}x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^{3}} \\
&\quad - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^{2}} + \frac{2x^{4}\sqrt{bx^{2/3}+ax}}{9a} - \frac{(8388608b^{13})\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{35102025a^{13}} \\
&= \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}} - \frac{16777216b^{13}\sqrt{bx^{2/3}+ax}}{11700675a^{14}\sqrt[3]{x}} - \frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} \\
&\quad + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} - \frac{131072b^{9}x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^{8}x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^{9}} \\
&\quad - \frac{180224b^{7}x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^{8}} + \frac{1171456b^{6}x^{2}\sqrt{bx^{2/3}+ax}}{3900225a^{7}} \\
&\quad - \frac{73216b^{5}x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^{6}} + \frac{36608b^{4}x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^{5}} - \frac{9152b^{3}x^{3}\sqrt{bx^{2/3}+ax}}{36225a^{4}} \\
&\quad + \frac{416b^{2}x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^{3}} - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^{2}} + \frac{2x^{4}\sqrt{bx^{2/3}+ax}}{9a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.46

$$
\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\big(-8388608b^{13}+4194304ab^{12}\sqrt[3]{x}-3145728a^2b^{11}x^{2/3}+2621440a^3b^{10}x-}{}
$$

```
[In] Integrate[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-8388608*b^13 + 4194304*a*b^12*x^(1/3) - 3145728*
a^2*b^11*x^(2/3) + 2621440*a^3*b^10*x - 2293760*a^4*b^9*x^(4/3) + 2064384*a
^5*b^8*x^(5/3) - 1892352*a^6*b^7*x^2 + 1757184*a^7*b^6*x^(7/3) - 1647360*a^
8*b^5*x^(8/3) + 1555840*a^9*b^4*x^3 - 1478048*a^10*b^3*x^(10/3) + 1410864*a
^11*b^2*x^(11/3) - 1352078*a^12*b*x^4 + 1300075*a^13*x^(13/3)))/(11700675*a
^14*x^(1/3))
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.42

method	result
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(1300075a^{13}x^{\frac{13}{3}}-1352078a^{12}b\,x^{4}+1410864a^{11}b^{2}x^{\frac{11}{3}}-1478048a^{10}b^{3}x^{\frac{10}{3}}+1555840a^{9}b^{4}x^{3}-1647360a^{8}b^{5}\right.}{11}$
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(1300075a^{13}x^{\frac{13}{3}}-1352078a^{12}b\,x^{4}+1410864a^{11}b^{2}x^{\frac{11}{3}}-1478048a^{10}b^{3}x^{\frac{10}{3}}+1555840a^{9}b^{4}x^{3}-1647360a^{8}b^{5}\right.}{11}$

```
[In] int(x^4/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/11700675*x^(1/3)*(b+a*x^(1/3))*(1300075*a^13*x^(13/3)-1352078*a^12*b*x^4+
1410864*a^11*b^2*x^(11/3)-1478048*a^10*b^3*x^(10/3)+1555840*a^9*b^4*x^3-164
7360*a^8*b^5*x^(8/3)+1757184*a^7*b^6*x^(7/3)-1892352*a^6*b^7*x^2+2064384*a^
5*b^8*x^(5/3)-2293760*a^4*b^9*x^(4/3)+2621440*a^3*b^10*x-3145728*a^2*b^11*x
^(2/3)+4194304*a*b^12*x^(1/3)-8388608*b^13)/(b*x^(2/3)+a*x)^(1/2)/a^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. 2(299) = 598.

Time = 148.43 (sec) , antiderivative size = 1294, normalized size of antiderivative = 3.23

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/11700675*((211106232532992*b^19 + 43980465111040*b^18 + 206158430208*(64*
a^3 - 3)*b^16 - 4123168604160*b^17 - 1073741824*(11264*a^3 - 53)*b^15 + 151
43273600*a^15 - 402653184*(5504*a^3 + 1)*b^14 + 12582912*(3194880*a^6 - 114
688*a^3 - 3)*b^13 + 469762048*(18816*a^6 + 103*a^3)*b^12 - 50331648*(48816*
a^6 + 23*a^3)*b^11 - 786432*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^10 - 734
0032*(1349120*a^9 + 3439*a^6)*b^9 + 250478592*(5600*a^9 + 3*a^6)*b^8 + 1228
8*(2616979456*a^12 - 21542400*a^9 - 693*a^6)*b^7 + 212992*(43743616*a^12 +
89111*a^9)*b^6 - 638976*(1652476*a^12 + 935*a^9)*b^5 + 3264*(3608543232*a^1
5 + 64599808*a^12 + 2145*a^9)*b^4 + 578816*(13049856*a^15 - 27313*a^12)*b^3
 + 217056*(6211584*a^15 + 2353*a^12)*b^2 - 156009*(2547712*a^15 + 39*a^12)*
b)*x + 2*(1300075*(16777216*a^13*b^6 + 6291456*a^13*b^5 + 196608*a^13*b^4 -
 262144*a^16 - 114688*a^13*b^3 - 2304*a^13*b^2 + 864*a^13*b - 27*a^13)*x^5
- 1478048*(16777216*a^10*b^9 + 6291456*a^10*b^8 + 196608*a^10*b^7 - 114688*
a^10*b^6 - 2304*a^10*b^5 + 864*a^10*b^4 - (262144*a^13 + 27*a^10)*b^3)*x^4
+ 1757184*(16777216*a^7*b^12 + 6291456*a^7*b^11 + 196608*a^7*b^10 - 114688*
a^7*b^9 - 2304*a^7*b^8 + 864*a^7*b^7 - (262144*a^10 + 27*a^7)*b^6)*x^3 - 22
```

```
93760*(16777216*a^4*b^15 + 6291456*a^4*b^14 + 196608*a^4*b^13 - 114688*a^4*
b^12 - 2304*a^4*b^11 + 864*a^4*b^10 - (262144*a^7 + 27*a^4)*b^9)*x^2 + 4194
304*(16777216*a*b^18 + 6291456*a*b^17 + 196608*a*b^16 - 114688*a*b^15 - 230
4*a*b^14 + 864*a*b^13 - (262144*a^4 + 27*a)*b^12)*x - 2*(70368744177664*b^1
9 + 26388279066624*b^18 + 824633720832*b^17 - 481036337152*b^16 - 966367641
6*b^15 - 4194304*(262144*a^3 + 27)*b^13 + 3623878656*b^14 + 676039*(1677721
6*a^12*b^7 + 6291456*a^12*b^6 + 196608*a^12*b^5 - 114688*a^12*b^4 - 2304*a^
12*b^3 + 864*a^12*b^2 - (262144*a^15 + 27*a^12)*b)*x^4 - 777920*(16777216*a
^9*b^10 + 6291456*a^9*b^9 + 196608*a^9*b^8 - 114688*a^9*b^7 - 2304*a^9*b^6
+ 864*a^9*b^5 - (262144*a^12 + 27*a^9)*b^4)*x^3 + 946176*(16777216*a^6*b^13
 + 6291456*a^6*b^12 + 196608*a^6*b^11 - 114688*a^6*b^10 - 2304*a^6*b^9 + 86
4*a^6*b^8 - (262144*a^9 + 27*a^6)*b^7)*x^2 - 1310720*(16777216*a^3*b^16 + 6
291456*a^3*b^15 + 196608*a^3*b^14 - 114688*a^3*b^13 - 2304*a^3*b^12 + 864*a
^3*b^11 - (262144*a^6 + 27*a^3)*b^10)*x)*x^(2/3) + 48*(29393*(16777216*a^11
*b^8 + 6291456*a^11*b^7 + 196608*a^11*b^6 - 114688*a^11*b^5 - 2304*a^11*b^4
 + 864*a^11*b^3 - (262144*a^14 + 27*a^11)*b^2)*x^4 - 34320*(16777216*a^8*b^
11 + 6291456*a^8*b^10 + 196608*a^8*b^9 - 114688*a^8*b^8 - 2304*a^8*b^7 + 86
4*a^8*b^6 - (262144*a^11 + 27*a^8)*b^5)*x^3 + 43008*(16777216*a^5*b^14 + 62
91456*a^5*b^13 + 196608*a^5*b^12 - 114688*a^5*b^11 - 2304*a^5*b^10 + 864*a^
5*b^9 - (262144*a^8 + 27*a^5)*b^8)*x^2 - 65536*(16777216*a^2*b^17 + 6291456
*a^2*b^16 + 196608*a^2*b^15 - 114688*a^2*b^14 - 2304*a^2*b^13 + 864*a^2*b^1
2 - (262144*a^5 + 27*a^2)*b^11)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((167772
16*a^14*b^6 + 6291456*a^14*b^5 + 196608*a^14*b^4 - 262144*a^17 - 114688*a^1
4*b^3 - 2304*a^14*b^2 + 864*a^14*b - 27*a^14)*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(x**4/(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(x**4/sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^4}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{16777216\,b^{\frac{27}{2}}}{11700675\,a^{14}}$$
$$+ \frac{2\left(1300075\left(ax^{\frac{1}{3}}+b\right)^{\frac{27}{2}} - 18253053\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}b + 119041650\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b^2 - 478056150\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}b\right.}{}$$

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 16777216/11700675*b^(27/2)/a^14 + 2/11700675*(1300075*(a*x^(1/3) + b)^(27/2
) - 18253053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^
2 - 478056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2
)*b^4 - 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^
(15/2)*b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3)
+ b)^(11/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1
/3) + b)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x
^(1/3) + b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^14
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^4}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

```
[In] int(x^4/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^4/(a*x + b*x^(2/3))^(1/2), x)
```

3.186 $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 313

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}}+\frac{524288b^{10}\sqrt{bx^{2/3}+ax}}{323323a^{11}\sqrt[3]{x}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9}-\frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8}+\frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6}+\frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5}-\frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3}-\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2}+\frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}$$

```
[Out] -262144/323323*b^9*(b*x^(2/3)+a*x)^(1/2)/a^10+524288/323323*b^10*(b*x^(2/3)
+a*x)^(1/2)/a^11/x^(1/3)+196608/323323*b^8*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^
9-163840/323323*b^7*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^8+20480/46189*b^6*x*(b*
x^(2/3)+a*x)^(1/2)/a^7-18432/46189*b^5*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a^6+15
36/4199*b^4*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-768/2261*b^3*x^2*(b*x^(2/3)+a
*x)^(1/2)/a^4+720/2261*b^2*x^(7/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-40/133*b*x^(8/
3)*(b*x^(2/3)+a*x)^(1/2)/a^2+2/7*x^3*(b*x^(2/3)+a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

$= \{2041, 2027, 2039\}$

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{524288b^{10}\sqrt{ax+bx^{2/3}}}{323323a^{11}\sqrt[3]{x}} - \frac{262144b^9\sqrt{ax+bx^{2/3}}}{323323a^{10}} + \frac{196608b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{323323a^8} + \frac{20480b^6x\sqrt{ax+bx^{2/3}}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{ax+bx^{2/3}}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{ax+bx^{2/3}}}{4199a^5} - \frac{768b^3x^2\sqrt{ax+bx^{2/3}}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{ax+bx^{2/3}}}{2261a^3} - \frac{40bx^{8/3}\sqrt{ax+bx^{2/3}}}{133a^2} + \frac{2x^3\sqrt{ax+bx^{2/3}}}{7a}$$

```
[In] Int[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-262144*b^9*Sqrt[b*x^(2/3) + a*x])/(323323*a^10) + (524288*b^10*Sqrt[b*x^(
2/3) + a*x])/(323323*a^11*x^(1/3)) + (196608*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a
*x])/(323323*a^9) - (163840*b^7*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(323323*a^8)
 + (20480*b^6*x*Sqrt[b*x^(2/3) + a*x])/(46189*a^7) - (18432*b^5*x^(4/3)*Sqr
t[b*x^(2/3) + a*x])/(46189*a^6) + (1536*b^4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/
(4199*a^5) - (768*b^3*x^2*Sqrt[b*x^(2/3) + a*x])/(2261*a^4) + (720*b^2*x^(7
/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^3) - (40*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]
)/(133*a^2) + (2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}-\frac{(20b)\int\frac{x^{8/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a}\\
&= -\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2}+\frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}+\frac{(120b^2)\int\frac{x^{7/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{133a^2}\\
&= \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3}-\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2}+\frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}-\frac{(1920b^3)\int\frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx}{2261a^3}\\
&= -\frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4}+\frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3}\\
&\quad -\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2}+\frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}+\frac{(256b^4)\int\frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{323a^4}\\
&= \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5}-\frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4}+\frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3}\\
&\quad -\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2}+\frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}-\frac{(3072b^5)\int\frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{4199a^5}\\
&= -\frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6}+\frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5}\\
&\quad -\frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4}+\frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3}\\
&\quad -\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2}+\frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}+\frac{(30720b^6)\int\frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{46189a^6}\\
&= \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7}-\frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6}\\
&\quad +\frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5}-\frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4}+\frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3}\\
&\quad -\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2}+\frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}-\frac{(81920b^7)\int\frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{138567a^7}\\
&= -\frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8}+\frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7}-\frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6}\\
&\quad +\frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5}-\frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4}+\frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3}\\
&\quad -\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2}+\frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}+\frac{(163840b^8)\int\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{323323a^8}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} \\
&\quad + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} \\
&\quad + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} \\
&\quad - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} - \frac{(131072b^9)\int\frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{323323a^9} \\
&= -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} \\
&\quad + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} \\
&\quad + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} \\
&\quad - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} + \frac{(262144b^{10})\int\frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{969969a^{10}} \\
&= -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}} + \frac{524288b^{10}\sqrt{bx^{2/3}+ax}}{323323a^{11}\sqrt[3]{x}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} \\
&\quad - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} \\
&\quad - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} \\
&\quad + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.47

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\left(262144b^{10} - 131072ab^9\sqrt[3]{x} + 98304a^2b^8x^{2/3} - 81920a^3b^7x + 71680a^4b^6x\right.}{}$$

```
[In] Integrate[x^3/Sqrt[b*x^(2/3) + a*x],x]

[Out] (2*Sqrt[b*x^(2/3) + a*x]*(262144*b^10 - 131072*a*b^9*x^(1/3) + 98304*a^2*b^
8*x^(2/3) - 81920*a^3*b^7*x + 71680*a^4*b^6*x^(4/3) - 64512*a^5*b^5*x^(5/3)
 + 59136*a^6*b^4*x^2 - 54912*a^7*b^3*x^(7/3) + 51480*a^8*b^2*x^(8/3) - 4862
0*a^9*b*x^3 + 46189*a^10*x^(10/3)))/(323323*a^11*x^(1/3))
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.43

method	result
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(46189a^{10}x^{\frac{10}{3}}-48620a^{9}b\,x^{3}+51480a^{8}b^{2}x^{\frac{8}{3}}-54912a^{7}b^{3}x^{\frac{7}{3}}+59136x^{2}a^{6}b^{4}-64512a^{5}b^{5}x^{\frac{5}{3}}+71680a^{4}x^{\frac{4}{3}}b\right.}{323323\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{11}}$
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(46189a^{10}x^{\frac{10}{3}}-48620a^{9}b\,x^{3}+51480a^{8}b^{2}x^{\frac{8}{3}}-54912a^{7}b^{3}x^{\frac{7}{3}}+59136x^{2}a^{6}b^{4}-64512a^{5}b^{5}x^{\frac{5}{3}}+71680a^{4}x^{\frac{4}{3}}b\right.}{323323\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{11}}$

```
[In] int(x^3/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/323323*x^(1/3)*(b+a*x^(1/3))*(46189*a^10*x^(10/3)-48620*a^9*b*x^3+51480*a
^8*b^2*x^(8/3)-54912*a^7*b^3*x^(7/3)+59136*x^2*a^6*b^4-64512*a^5*b^5*x^(5/3
)+71680*a^4*x^(4/3)*b^6-81920*a^3*b^7*x+98304*a^2*b^8*x^(2/3)-131072*a*b^9*
x^(1/3)+262144*b^10)/(b*x^(2/3)+a*x)^(1/2)/a^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. 2(233) = 466.

Time = 176.92 (sec) , antiderivative size = 1031, normalized size of antiderivative = 3.29

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/323323*((3298534883328*b^16 + 687194767360*b^15 + 3221225472*(64*a^3 - 3
)*b^13 - 64424509440*b^14 - 16777216*(11264*a^3 - 53)*b^12 - 269004736*a^12
 - 6291456*(5504*a^3 + 1)*b^11 + 196608*(3194880*a^6 - 114688*a^3 - 3)*b^10
 + 7340032*(18816*a^6 + 103*a^3)*b^9 - 786432*(48816*a^6 + 23*a^3)*b^8 - 12
288*(45731840*a^9 - 495872*a^6 - 15*a^3)*b^7 - 114688*(1349120*a^9 + 3439*a
^6)*b^6 + 3913728*(5600*a^9 + 3*a^6)*b^5 - 2112*(101384192*a^12 + 1958400*a
^9 + 63*a^6)*b^4 - 36608*(3784704*a^12 - 8101*a^9)*b^3 - 109824*(226688*a^1
2 + 85*a^9)*b^2 + 7293*(974848*a^12 + 15*a^9)*b)*x - (46189*(16777216*a^10*
b^6 + 6291456*a^10*b^5 + 196608*a^10*b^4 - 262144*a^13 - 114688*a^10*b^3 -
2304*a^10*b^2 + 864*a^10*b - 27*a^10)*x^4 - 54912*(16777216*a^7*b^9 + 62914
56*a^7*b^8 + 196608*a^7*b^7 - 114688*a^7*b^6 - 2304*a^7*b^5 + 864*a^7*b^4 -
 (262144*a^10 + 27*a^7)*b^3)*x^3 + 71680*(16777216*a^4*b^12 + 6291456*a^4*b
^11 + 196608*a^4*b^10 - 114688*a^4*b^9 - 2304*a^4*b^8 + 864*a^4*b^7 - (2621
44*a^7 + 27*a^4)*b^6)*x^2 - 131072*(16777216*a*b^15 + 6291456*a*b^14 + 1966
08*a*b^13 - 114688*a*b^12 - 2304*a*b^11 + 864*a*b^10 - (262144*a^4 + 27*a)*
b^9)*x + 4*(1099511627776*b^16 + 412316860416*b^15 + 12884901888*b^14 - 751
6192768*b^13 - 150994944*b^12 - 65536*(262144*a^3 + 27)*b^10 + 56623104*b^1
```

```
1 - 12155*(16777216*a^9*b^7 + 6291456*a^9*b^6 + 196608*a^9*b^5 - 114688*a^9
*b^4 - 2304*a^9*b^3 + 864*a^9*b^2 - (262144*a^12 + 27*a^9)*b)*x^3 + 14784*(
16777216*a^6*b^10 + 6291456*a^6*b^9 + 196608*a^6*b^8 - 114688*a^6*b^7 - 230
4*a^6*b^6 + 864*a^6*b^5 - (262144*a^9 + 27*a^6)*b^4)*x^2 - 20480*(16777216*
a^3*b^13 + 6291456*a^3*b^12 + 196608*a^3*b^11 - 114688*a^3*b^10 - 2304*a^3*
b^9 + 864*a^3*b^8 - (262144*a^6 + 27*a^3)*b^7)*x)*x^(2/3) + 24*(2145*(16777
216*a^8*b^8 + 6291456*a^8*b^7 + 196608*a^8*b^6 - 114688*a^8*b^5 - 2304*a^8*
b^4 + 864*a^8*b^3 - (262144*a^11 + 27*a^8)*b^2)*x^3 - 2688*(16777216*a^5*b^
11 + 6291456*a^5*b^10 + 196608*a^5*b^9 - 114688*a^5*b^8 - 2304*a^5*b^7 + 86
4*a^5*b^6 - (262144*a^8 + 27*a^5)*b^5)*x^2 + 4096*(16777216*a^2*b^14 + 6291
456*a^2*b^13 + 196608*a^2*b^12 - 114688*a^2*b^11 - 2304*a^2*b^10 + 864*a^2*
b^9 - (262144*a^5 + 27*a^2)*b^8)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((16777
216*a^11*b^6 + 6291456*a^11*b^5 + 196608*a^11*b^4 - 262144*a^14 - 114688*a^
11*b^3 - 2304*a^11*b^2 + 864*a^11*b - 27*a^11)*x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^3}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(x**3/(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(x**3/sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^3}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.52

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = -\frac{524288\,b^{\frac{21}{2}}}{323323\,a^{11}} + \frac{2\left(46189\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}} - 510510\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b + 2567565\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^2 - 7759752\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^3 + 1566\right.}{}$$

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -524288/323323*b^(21/2)/a^11 + 2/323323*(46189*(a*x^(1/3) + b)^(21/2) - 510
510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752
*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 2222110
8*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 1662804
0*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(
a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^11
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x^3}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

```
[In] int(x^3/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(2/3))^(1/2), x)
```

3.187 $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 225

$$\begin{aligned}\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx &= \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7}-\frac{4096b^7\sqrt{bx^{2/3}+ax}}{2145a^8\sqrt[3]{x}}\\&-\frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6}+\frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5}-\frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4}\\&+\frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3}-\frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2}+\frac{2x^2\sqrt{bx^{2/3}+ax}}{5a}\end{aligned}$$

```
[Out] 2048/2145*b^6*(b*x^(2/3)+a*x)^(1/2)/a^7-4096/2145*b^7*(b*x^(2/3)+a*x)^(1/2)
/a^8/x^(1/3)-512/715*b^5*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^6+256/429*b^4*x^(2
/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-224/429*b^3*x*(b*x^(2/3)+a*x)^(1/2)/a^4+336/7
15*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-28/65*b*x^(5/3)*(b*x^(2/3)+a*x)^(1
/2)/a^2+2/5*x^2*(b*x^(2/3)+a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2041, 2027, 2039}

$$\begin{aligned}\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx &= -\frac{4096b^7\sqrt{ax+bx^{2/3}}}{2145a^8\sqrt[3]{x}}+\frac{2048b^6\sqrt{ax+bx^{2/3}}}{2145a^7}\\&-\frac{512b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^6}+\frac{256b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^5}-\frac{224b^3x\sqrt{ax+bx^{2/3}}}{429a^4}\\&+\frac{336b^2x^{4/3}\sqrt{ax+bx^{2/3}}}{715a^3}-\frac{28bx^{5/3}\sqrt{ax+bx^{2/3}}}{65a^2}+\frac{2x^2\sqrt{ax+bx^{2/3}}}{5a}\end{aligned}$$

```
[In] Int[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2048*b^6*Sqrt[b*x^(2/3) + a*x])/(2145*a^7) - (4096*b^7*Sqrt[b*x^(2/3) + a*
x])/(2145*a^8*x^(1/3)) - (512*b^5*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^6)
+ (256*b^4*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(429*a^5) - (224*b^3*x*Sqrt[b*x^(
2/3) + a*x])/(429*a^4) + (336*b^2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^3)
- (28*b*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(65*a^2) + (2*x^2*Sqrt[b*x^(2/3) + a
*x])/(5*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{(14b)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{15a}\\
&= -\frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} + \frac{(56b^2)\int \frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{65a^2}\\
&= \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{(112b^3)\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{143a^3}\\
&= -\frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3}\\
&\quad - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} + \frac{(896b^4)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{1287a^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} \\
&\quad - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{(256b^5)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{429a^5} \\
&= -\frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} \\
&\quad - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} \\
&\quad - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} + \frac{(1024b^6)\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{2145a^6} \\
&= \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} \\
&\quad - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} \\
&\quad + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{(2048b^7)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{6435a^7} \\
&= \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{4096b^7\sqrt{bx^{2/3}+ax}}{2145a^8\sqrt[3]{x}} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} \\
&\quad - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.49

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\big(-2048b^7 + 1024ab^6\sqrt[3]{x} - 768a^2b^5x^{2/3} + 640a^3b^4x - 560a^4b^3x^{4/3} + 504a^5}{2145a^8\sqrt[3]{x}}$$

```
[In] Integrate[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-2048*b^7 + 1024*a*b^6*x^(1/3) - 768*a^2*b^5*x^(2
/3) + 640*a^3*b^4*x - 560*a^4*b^3*x^(4/3) + 504*a^5*b^2*x^(5/3) - 462*a^6*b
*x^2 + 429*a^7*x^(7/3)))/(2145*a^8*x^(1/3))
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.45

method	result	size
derivativedivides	$\frac{2x^{\frac{1}{3}} \left(b+a\,x^{\frac{1}{3}}\right) \left(429a^{7} x^{\frac{7}{3}}-462a^{6} b\,x^{2}+504a^{5} b^{2} x^{\frac{5}{3}}-560a^{4} b^{3} x^{\frac{4}{3}}+640a^{3} b^{4} x-768a^{2} b^{5} x^{\frac{2}{3}}+1024a\,b^{6} x^{\frac{1}{3}}-2048b^{7}\right)}{2145\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{8}}$	101
default	$\frac{2x^{\frac{1}{3}} \left(b+a\,x^{\frac{1}{3}}\right) \left(429a^{7} x^{\frac{7}{3}}-462a^{6} b\,x^{2}+504a^{5} b^{2} x^{\frac{5}{3}}-560a^{4} b^{3} x^{\frac{4}{3}}+640a^{3} b^{4} x-768a^{2} b^{5} x^{\frac{2}{3}}+1024a\,b^{6} x^{\frac{1}{3}}-2048b^{7}\right)}{2145\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{8}}$	101

```
[In] int(x^2/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/2145*x^(1/3)*(b+a*x^(1/3))*(429*a^7*x^(7/3)-462*a^6*b*x^2+504*a^5*b^2*x^(
5/3)-560*a^4*b^3*x^(4/3)+640*a^3*b^4*x-768*a^2*b^5*x^(2/3)+1024*a*b^6*x^(1/
3)-2048*b^7)/(b*x^(2/3)+a*x)^(1/2)/a^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. 2(167) = 334.

Time = 180.27 (sec) , antiderivative size = 768, normalized size of antiderivative = 3.41

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{(51539607552\,b^{13}+10737418240\,b^{12}+50331648\,(64\,a^3-3)b^{10}-1006632960\,b^{11}-26}{}$$

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2145*((51539607552*b^13 + 10737418240*b^12 + 50331648*(64*a^3 - 3)*b^10 -
 1006632960*b^11 - 262144*(11264*a^3 - 53)*b^9 + 4996992*a^9 - 98304*(5504*
a^3 + 1)*b^8 + 3072*(3194880*a^6 - 114688*a^3 - 3)*b^7 + 114688*(18816*a^6
+ 103*a^3)*b^6 - 12288*(48816*a^6 + 23*a^3)*b^5 + 192*(21626880*a^9 + 49587
2*a^6 + 15*a^3)*b^4 + 256*(10690560*a^9 - 24073*a^6)*b^3 + 3744*(133120*a^9
 + 49*a^6)*b^2 - 297*(450560*a^9 + 7*a^6)*b)*x + 2*(429*(16777216*a^7*b^6 +
 6291456*a^7*b^5 + 196608*a^7*b^4 - 262144*a^10 - 114688*a^7*b^3 - 2304*a^7
*b^2 + 864*a^7*b - 27*a^7)*x^3 - 560*(16777216*a^4*b^9 + 6291456*a^4*b^8 +
196608*a^4*b^7 - 114688*a^4*b^6 - 2304*a^4*b^5 + 864*a^4*b^4 - (262144*a^7
+ 27*a^4)*b^3)*x^2 + 1024*(16777216*a*b^12 + 6291456*a*b^11 + 196608*a*b^10
 - 114688*a*b^9 - 2304*a*b^8 + 864*a*b^7 - (262144*a^4 + 27*a)*b^6)*x - 2*(
17179869184*b^13 + 6442450944*b^12 + 201326592*b^11 - 117440512*b^10 - 2359
296*b^9 - 1024*(262144*a^3 + 27)*b^7 + 884736*b^8 + 231*(16777216*a^6*b^7 +
 6291456*a^6*b^6 + 196608*a^6*b^5 - 114688*a^6*b^4 - 2304*a^6*b^3 + 864*a^6
*b^2 - (262144*a^9 + 27*a^6)*b)*x^2 - 320*(16777216*a^3*b^10 + 6291456*a^3*
b^9 + 196608*a^3*b^8 - 114688*a^3*b^7 - 2304*a^3*b^6 + 864*a^3*b^5 - (26214
4*a^6 + 27*a^3)*b^4)*x)*x^(2/3) + 24*(21*(16777216*a^5*b^8 + 6291456*a^5*b^
7 + 196608*a^5*b^6 - 114688*a^5*b^5 - 2304*a^5*b^4 + 864*a^5*b^3 - (262144*
```

```
a^8 + 27*a^5)*b^2)*x^2 - 32*(16777216*a^2*b^11 + 6291456*a^2*b^10 + 196608*
a^2*b^9 - 114688*a^2*b^8 - 2304*a^2*b^7 + 864*a^2*b^6 - (262144*a^5 + 27*a^
2)*b^5)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^8*b^6 + 6291456*a^8
*b^5 + 196608*a^8*b^4 - 262144*a^11 - 114688*a^8*b^3 - 2304*a^8*b^2 + 864*a
^8*b - 27*a^8)*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x^2}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{4096\,b^{\frac{15}{2}}}{2145\,a^8} + \frac{2\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} - 3465\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b + 12285\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^2 - 25025\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^3 + 32175\left(ax^{\frac{1}{3}}+b\right.\right.}{2145\,a^8}$$

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4096/2145*b^(15/2)/a^8 + 2/2145*(429*(a*x^(1/3) + b)^(15/2) - 3465*(a*x^(1/
3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025*(a*x^(1/3) + b)
^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(1/3) + b)^(5/2)*
b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a^8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x^2}{\sqrt{a\,x+b\,x^{2/3}}} \,dx$$

```
[In] int(x^2/(a*x + b*x^(2/3))^(1/2),x)
[Out] int(x^2/(a*x + b*x^(2/3))^(1/2), x)
```

3.188 $\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 137

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4}+\frac{256b^4\sqrt{bx^{2/3}+ax}}{105a^5\sqrt[3]{x}} +\frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3}-\frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2}+\frac{2x\sqrt{bx^{2/3}+ax}}{3a}$$

```
[Out] -128/105*b^3*(b*x^(2/3)+a*x)^(1/2)/a^4+256/105*b^4*(b*x^(2/3)+a*x)^(1/2)/a^
5/x^(1/3)+32/35*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-16/21*b*x^(2/3)*(b*x^
(2/3)+a*x)^(1/2)/a^2+2/3*x*(b*x^(2/3)+a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2041, 2027, 2039}

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{256b^4\sqrt{ax+bx^{2/3}}}{105a^5\sqrt[3]{x}}-\frac{128b^3\sqrt{ax+bx^{2/3}}}{105a^4} +\frac{32b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{35a^3}-\frac{16bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^2}+\frac{2x\sqrt{ax+bx^{2/3}}}{3a}$$

```
[In] Int[x/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-128*b^3*Sqrt[b*x^(2/3) + a*x])/(105*a^4) + (256*b^4*Sqrt[b*x^(2/3) + a*x]
)/(105*a^5*x^(1/3)) + (32*b^2*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(35*a^3) - (16
*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^2) + (2*x*Sqrt[b*x^(2/3) + a*x])/(3
*a)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x\sqrt{bx^{2/3}+ax}}{3a} - \frac{(8b)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{9a} \\
&= -\frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} + \frac{(16b^2)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^2} \\
&= \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} - \frac{(64b^3)\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{105a^3} \\
&= -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4} + \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} \\
&\quad + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} + \frac{(128b^4)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{315a^4} \\
&= -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4} + \frac{256b^4\sqrt{bx^{2/3}+ax}}{105a^5\sqrt[3]{x}} + \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3} \\
&\quad - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\sqrt{bx^{2/3}+ax}\left(128b^4-64ab^3\sqrt[3]{x}+48a^2b^2x^{2/3}-40a^3bx+35a^4x^{4/3}\right)}{105a^5\sqrt[3]{x}}$$

```
[In] Integrate[x/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(128*b^4 - 64*a*b^3*x^(1/3) + 48*a^2*b^2*x^(2/3) -
 40*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^5*x^(1/3))
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.50

method	result	size
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^4x^{\frac{4}{3}}-40a^3bx+48a^2b^2x^{\frac{2}{3}}-64a\,x^{\frac{1}{3}}b^3+128b^4\right)}{105\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^5}$	68
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(35a^4x^{\frac{4}{3}}-40a^3bx+48a^2b^2x^{\frac{2}{3}}-64a\,x^{\frac{1}{3}}b^3+128b^4\right)}{105\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^5}$	68

```
[In] int(x/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*x^(1/3)*(b+a*x^(1/3))*(35*a^4*x^(4/3)-40*a^3*b*x+48*a^2*b^2*x^(2/3)-6
4*a*x^(1/3)*b^3+128*b^4)/(b*x^(2/3)+a*x)^(1/2)/a^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(101) = 202.

Time = 170.47 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.66

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx =$$

$$-\frac{2\left(2\,(805306368\,b^{10}+167772160\,b^9+786432\,(64\,a^3-3)b^7-15728640\,b^8-4096\,(11264\,a^3-53)b^6-1019\right.}{}$$

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(2*(805306368*b^10 + 167772160*b^9 + 786432*(64*a^3 - 3)*b^7 - 15728
640*b^8 - 4096*(11264*a^3 - 53)*b^6 - 101920*a^6 - 1536*(5504*a^3 + 1)*b^5
- 48*(1966080*a^6 + 114688*a^3 + 3)*b^4 - 1792*(36864*a^6 - 103*a^3)*b^3 -
192*(65280*a^6 + 23*a^3)*b^2 + 15*(188416*a^6 + 3*a^3)*b)*x - (35*(16777216
```

```
*a^4*b^6 + 6291456*a^4*b^5 + 196608*a^4*b^4 - 262144*a^7 - 114688*a^4*b^3 -
 2304*a^4*b^2 + 864*a^4*b - 27*a^4)*x^2 + 48*(16777216*a^2*b^8 + 6291456*a^
2*b^7 + 196608*a^2*b^6 - 114688*a^2*b^5 - 2304*a^2*b^4 + 864*a^2*b^3 - (262
144*a^5 + 27*a^2)*b^2)*x^(4/3) - 64*(16777216*a*b^9 + 6291456*a*b^8 + 19660
8*a*b^7 - 114688*a*b^6 - 2304*a*b^5 + 864*a*b^4 - (262144*a^4 + 27*a)*b^3)*
x + 8*(268435456*b^10 + 100663296*b^9 + 3145728*b^8 - 1835008*b^7 - 36864*b
^6 - 16*(262144*a^3 + 27)*b^4 + 13824*b^5 - 5*(16777216*a^3*b^7 + 6291456*a
^3*b^6 + 196608*a^3*b^5 - 114688*a^3*b^4 - 2304*a^3*b^3 + 864*a^3*b^2 - (26
2144*a^6 + 27*a^3)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^5*b^6
 + 6291456*a^5*b^5 + 196608*a^5*b^4 - 262144*a^8 - 114688*a^5*b^3 - 2304*a^
5*b^2 + 864*a^5*b - 27*a^5)*x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(x/(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(x/sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{x}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = -\frac{256\,b^{\frac{9}{2}}}{105\,a^5} + \frac{2\left(35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}} - 180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b + 378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^2 - 420\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^3 + 315\sqrt{ax^{\frac{1}{3}}+b}\,b^4\right)}{105\,a^5}$$

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -256/105*b^(9/2)/a^5 + 2/105*(35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b
)^(7/2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 +
 315*sqrt(a*x^(1/3) + b)*b^4)/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{x}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

```
[In] int(x/(a*x + b*x^(2/3))^(1/2),x)
[Out] int(x/(a*x + b*x^(2/3))^(1/2), x)
```

3.189 $\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx=\frac{2\sqrt{bx^{2/3}+ax}}{a}-\frac{4b\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}$$

```
[Out] 2*(b*x^(2/3)+a*x)^(1/2)/a-4*b*(b*x^(2/3)+a*x)^(1/2)/a^2/x^(1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2027, 2039}

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx=\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

```
[In] Int[1/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
```

```
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sqrt{bx^{2/3}+ax}}{a} - \frac{(2b)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{3a}\\ &= \frac{2\sqrt{bx^{2/3}+ax}}{a} - \frac{4b\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2\left(-2b+a\sqrt[3]{x}\right)\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}$$

```
[In] Integrate[1/Sqrt[b*x^(2/3) + a*x],x]
[Out] (2*(-2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(a\,x^{\frac{1}{3}}-2b\right)}{\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{2}}$	36
default	$\frac{2x^{\frac{1}{3}}\left(b+a\,x^{\frac{1}{3}}\right)\left(a\,x^{\frac{1}{3}}-2b\right)}{\sqrt{b\,x^{\frac{2}{3}}+ax}\,a^{2}}$	36

```
[In] int(1/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x^(1/3)*(b+a*x^(1/3))*(a*x^(1/3)-2*b)/(b*x^(2/3)+a*x)^(1/2)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(37) = 74$.

Time = 138.46 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.06

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{(50331648\,b^7 + 10485760\,b^6 + 49152\,(512\,a^3-3)b^4 - 983040\,b^5 + 256\,(24576\,a^3+53)}{}$$

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] ((50331648*b^7 + 10485760*b^6 + 49152*(512*a^3 - 3)*b^4 - 983040*b^5 + 256*
(24576*a^3 + 53)*b^3 + 11648*a^3 - 96*(2048*a^3 + 1)*b^2 - 3*(155648*a^3 +
3)*b)*x + 2*((16777216*a*b^6 + 6291456*a*b^5 + 196608*a*b^4 - 262144*a^4 -
114688*a*b^3 - 2304*a*b^2 + 864*a*b - 27*a)*x - 2*(16777216*b^7 + 6291456*b
^6 + 196608*b^5 - 114688*b^4 - 2304*b^3 - (262144*a^3 + 27)*b + 864*b^2)*x^
(2/3))*sqrt(a*x + b*x^(2/3)))/((16777216*a^2*b^6 + 6291456*a^2*b^5 + 196608
*a^2*b^4 - 262144*a^5 - 114688*a^2*b^3 - 2304*a^2*b^2 + 864*a^2*b - 27*a^2)
*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(1/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(2/3)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{bx^{2/3} + ax}}\,dx = \frac{4\,b^{\frac{3}{2}}}{a^2} + \frac{2\left(\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} - 3\,\sqrt{ax^{\frac{1}{3}} + b}b\right)}{a^2}$$

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/a^2 + 2*((a*x^(1/3) + b)^(3/2) - 3*sqrt(a*x^(1/3) + b)*b)/a^2
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{bx^{2/3} + ax}}\,dx = \frac{3\,x\,\sqrt{\frac{a\,x^{1/3}}{b} + 1}\,{}_2\mathrm{F}_1\left(\frac{1}{2}, 2;\ 3;\ -\frac{a\,x^{1/3}}{b}\right)}{2\,\sqrt{a\,x + b\,x^{2/3}}}$$

```
[In] int(1/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] (3*x*((a*x^(1/3))/b + 1)^(1/2)*hypergeom([1/2, 2], 3, -(a*x^(1/3))/b))/(2*(
a*x + b*x^(2/3))^(1/2))
```

3.190 $\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}} + \frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

```
[Out] 3*a*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)-3*(b*x^(2/3)+a*x
)^(1/2)/b/x^(2/3)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2050, 2054, 212}

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = \frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}} - \frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

```
[In] Int[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqr
t[b*x^(2/3) + a*x]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}} - \frac{a\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{2b} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}} + \frac{(3a)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}} + \frac{3a\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

```
[In] Integrate[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqr
t[b*x^(2/3) + a*x]])/b^(3/2)
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{3\sqrt{b+a\,x^{\frac{1}{3}}}\left(\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b a\,x^{\frac{1}{3}}\right)}{\sqrt{b\,x^{\frac{2}{3}}+a x}\,b^{\frac{5}{2}}}$	61
default	$\frac{3\sqrt{b+a\,x^{\frac{1}{3}}}\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b a\,x^{\frac{1}{3}}-\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{3}{2}}\right)}{\sqrt{b\,x^{\frac{2}{3}}+a x}\,b^{\frac{5}{2}}}$	61

```
[In] int(1/x/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -3*(b+a*x^(1/3))^(1/2)*((b+a*x^(1/3))^(1/2)*b^(3/2)-arctanh((b+a*x^(1/3))^(
1/2)/b^(1/2))*b*a*x^(1/3))/(b*x^(2/3)+a*x)^(1/2)/b^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx=\text{Timed out}$$

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{1}{x\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(1/x/(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(1/(x*sqrt(a*x + b*x**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{bx^{2/3} + ax}}\,dx = \int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}\,x}\,dx$$

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\sqrt{bx^{2/3} + ax}}\,dx = -\frac{3\left(\frac{a^2 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} + \frac{\sqrt{ax^{\frac{1}{3}}+b}a}{bx^{\frac{1}{3}}}\right)}{a}$$

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -3*(a^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(a*x^(1/3)
+ b)*a/(b*x^(1/3)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{bx^{2/3} + ax}}\,dx = \int \frac{1}{x\,\sqrt{a\,x + b\,x^{2/3}}}\,dx$$

```
[In] int(1/(x*(a*x + b*x^(2/3))^(1/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(2/3))^(1/2)), x)
```

3.191 $\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} - \frac{105a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{64b^{9/2}}$$

```
[Out] -105/64*a^4*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)-3/4*(b*x
^(2/3)+a*x)^(1/2)/b/x^(5/3)+7/8*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)-35/32*a
^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x+105/64*a^3*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2050, 2054, 212}

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = -\frac{105a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{64b^{9/2}} + \frac{105a^3\sqrt{ax+bx^{2/3}}}{64b^4x^{2/3}} - \frac{35a^2\sqrt{ax+bx^{2/3}}}{32b^3x} + \frac{7a\sqrt{ax+bx^{2/3}}}{8b^2x^{4/3}} - \frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

```
[In] Int[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) + (7*a*Sqrt[b*x^(2/3) + a*x])/(8*b
^2*x^(4/3)) - (35*a^2*Sqrt[b*x^(2/3) + a*x])/(32*b^3*x) + (105*a^3*Sqrt[b*x
```

```
^(2/3) + a*x])/(64*b^4*x^(2/3)) - (105*a^4*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/(64*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} - \frac{(7a)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{8b} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} + \frac{(35a^2)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{48b^2} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} - \frac{(35a^3)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{64b^3} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} \\
&\quad + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} + \frac{(35a^4)\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{128b^4} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} \\
&\quad + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} - \frac{(105a^4)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{64b^4}
\end{aligned}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} - \frac{105a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{64b^{9/2}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-48b^3+56ab^2\sqrt[3]{x}-70a^2bx^{2/3}+105a^3x\right)}{64b^4x^{5/3}} - \frac{105a^4\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{64b^{9/2}}$$

```
[In] Integrate[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-48*b^3 + 56*a*b^2*x^(1/3) - 70*a^2*b*x^(2/3) + 105
*a^3*x))/(64*b^4*x^(5/3)) - (105*a^4*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/
3) + a*x]])/(64*b^(9/2))
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(48\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{9}{2}}-56b^{\frac{7}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a\,x^{\frac{1}{3}}+70b^{\frac{5}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{2}x^{\frac{2}{3}}-105b^{\frac{3}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{3}x+105\operatorname{arctanh}\left(\right.\right.}{64x\sqrt{b\,x^{\frac{2}{3}}+ax}\,b^{\frac{11}{2}}}$
default	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(105x^{\frac{7}{3}}\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)a^{4}b+70x^{\frac{5}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}a^{2}-56x^{\frac{4}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{7}{2}}a+48\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{9}{2}}x-105x^{2}\right.}{64x^{2}\sqrt{b\,x^{\frac{2}{3}}+ax}\,b^{\frac{11}{2}}}$

```
[In] int(1/x^2/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(b+a*x^(1/3))^(1/2)*(48*(b+a*x^(1/3))^(1/2)*b^(9/2)-56*b^(7/2)*(b+a*x
^(1/3))^(1/2)*a*x^(1/3)+70*b^(5/2)*(b+a*x^(1/3))^(1/2)*a^2*x^(2/3)-105*b^(3
/2)*(b+a*x^(1/3))^(1/2)*a^3*x+105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*a^4*
b*x^(4/3))/x/(b*x^(2/3)+a*x)^(1/2)/b^(11/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^2\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*x**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = \frac{\frac{105\,a^5\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^5 - 385\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^5b + 511\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^5b^2 - 279\sqrt{ax^{\frac{1}{3}}+b}\,a^5b^3}{a^4b^4x^{\frac{4}{3}}}}{64\,a}$$

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(105*a^5*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(a
*x^(1/3) + b)^(7/2)*a^5 - 385*(a*x^(1/3) + b)^(5/2)*a^5*b + 511*(a*x^(1/3)
+ b)^(3/2)*a^5*b^2 - 279*sqrt(a*x^(1/3) + b)*a^5*b^3)/(a^4*b^4*x^(4/3)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{1}{x^2\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

```
[In] int(1/(x^2*(a*x + b*x^(2/3))^(1/2)),x)
[Out] int(1/(x^2*(a*x + b*x^(2/3))^(1/2)), x)
```

3.192 $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 241

$$\begin{aligned}\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx =& -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}}+\frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}}\\&-\frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2}+\frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}}-\frac{429a^4\sqrt{bx^{2/3}+ax}}{640b^5x^{4/3}}\\&+\frac{429a^5\sqrt{bx^{2/3}+ax}}{512b^6x}-\frac{1287a^6\sqrt{bx^{2/3}+ax}}{1024b^7x^{2/3}}+\frac{1287a^7\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{1024b^{15/2}}\end{aligned}$$

```
[Out] 1287/1024*a^7*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)-3/7*(
b*x^(2/3)+a*x)^(1/2)/b/x^(8/3)+13/28*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-14
3/280*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2+1287/2240*a^3*(b*x^(2/3)+a*x)^(1/2)
/b^4/x^(5/3)-429/640*a^4*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+429/512*a^5*(b*x
^(2/3)+a*x)^(1/2)/b^6/x-1287/1024*a^6*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2050, 2054, 212}

$$\begin{aligned}\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx =& \frac{1287a^7\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{1024b^{15/2}}-\frac{1287a^6\sqrt{ax+bx^{2/3}}}{1024b^7x^{2/3}}\\&+\frac{429a^5\sqrt{ax+bx^{2/3}}}{512b^6x}-\frac{429a^4\sqrt{ax+bx^{2/3}}}{640b^5x^{4/3}}+\frac{1287a^3\sqrt{ax+bx^{2/3}}}{2240b^4x^{5/3}}\\&-\frac{143a^2\sqrt{ax+bx^{2/3}}}{280b^3x^2}+\frac{13a\sqrt{ax+bx^{2/3}}}{28b^2x^{7/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{7bx^{8/3}}\end{aligned}$$

```
[In] Int[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) + (13*a*Sqrt[b*x^(2/3) + a*x])/(28
*b^2*x^(7/3)) - (143*a^2*Sqrt[b*x^(2/3) + a*x])/(280*b^3*x^2) + (1287*a^3*S
qrt[b*x^(2/3) + a*x])/(2240*b^4*x^(5/3)) - (429*a^4*Sqrt[b*x^(2/3) + a*x])/
(640*b^5*x^(4/3)) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^6*x) - (1287*a^6
*Sqrt[b*x^(2/3) + a*x])/(1024*b^7*x^(2/3)) + (1287*a^7*ArcTanh[(Sqrt[b]*x^(
1/3))/Sqrt[b*x^(2/3) + a*x]])/(1024*b^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}}-\frac{(13a)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{14b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}}+\frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}}+\frac{(143a^2)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{168b^2}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}}+\frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}}-\frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2}-\frac{(429a^3)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{560b^3}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}}+\frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}}-\frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2}\\
&\quad+\frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}}+\frac{(429a^4)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{640b^4}
\end{aligned}
$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3}+ax}}{640b^5x^{4/3}} - \frac{(143a^5)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{256b^5}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3}+ax}}{640b^5x^{4/3}} + \frac{429a^5\sqrt{bx^{2/3}+ax}}{512b^6x} + \frac{(429a^6)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{1024b^6}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3}+ax}}{640b^5x^{4/3}} + \frac{429a^5\sqrt{bx^{2/3}+ax}}{512b^6x} - \frac{1287a^6\sqrt{bx^{2/3}+ax}}{1024b^7x^{2/3}} - \frac{(429a^7)\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{2048b^7}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3}+ax}}{640b^5x^{4/3}} + \frac{429a^5\sqrt{bx^{2/3}+ax}}{512b^6x} - \frac{1287a^6\sqrt{bx^{2/3}+ax}}{1024b^7x^{2/3}} + \frac{(1287a^7)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{1024b^7}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3}+ax}}{640b^5x^{4/3}} + \frac{429a^5\sqrt{bx^{2/3}+ax}}{512b^6x} - \frac{1287a^6\sqrt{bx^{2/3}+ax}}{1024b^7x^{2/3}} + \frac{1287a^7\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{1024b^{15/2}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \frac{\sqrt{bx^{2/3}+ax}\,(-15360b^6 + 16640ab^5\sqrt[3]{x} - 18304a^2b^4x^{2/3} + 20592a^3b^3x - 24024a^4b^2x^{4}}{35840b^7x^{8/3}} + \frac{1287a^7\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{1024b^{15/2}}$$

```
[In] Integrate[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-15360*b^6 + 16640*a*b^5*x^(1/3) - 18304*a^2*b^4*x^
(2/3) + 20592*a^3*b^3*x - 24024*a^4*b^2*x^(4/3) + 30030*a^5*b*x^(5/3) - 450
45*a^6*x^2))/(35840*b^7*x^(8/3)) + (1287*a^7*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt
[b*x^(2/3) + a*x]])/(1024*b^(15/2))
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(45045b^{\frac{3}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{6}x^{2}-45045\,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)a^{7}b\,x^{\frac{7}{3}}-30030b^{\frac{5}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{5}x^{\frac{5}{3}}+24024b^{\frac{7}{2}}\sqrt{b+a}\right.}{35840x^{2}\sqrt{b\,x}}$
default	$\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\,\left(45045x^{\frac{13}{3}}\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)a^{7}b+30030x^{\frac{11}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}a^{5}-24024x^{\frac{10}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{7}{2}}a^{4}-18304x^{\frac{8}{3}}\sqrt{b+}\right.}{35840x^{4}\sqrt{b\,x}}$

```
[In] int(1/x^3/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35840/x^2*(b+a*x^(1/3))^(1/2)*(45045*b^(3/2)*(b+a*x^(1/3))^(1/2)*a^6*x^2
-45045*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*a^7*b*x^(7/3)-30030*b^(5/2)*(b+
a*x^(1/3))^(1/2)*a^5*x^(5/3)+24024*b^(7/2)*(b+a*x^(1/3))^(1/2)*a^4*x^(4/3)-
20592*b^(9/2)*(b+a*x^(1/3))^(1/2)*a^3*x+18304*b^(11/2)*(b+a*x^(1/3))^(1/2)*
a^2*x^(2/3)-16640*b^(13/2)*(b+a*x^(1/3))^(1/2)*a*x^(1/3)+15360*(b+a*x^(1/3)
)^(1/2)*b^(15/2))/(b*x^(2/3)+a*x)^(1/2)/b^(17/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{1}{x^3\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a*x + b*x**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx=\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx= -\frac{\frac{45045\,a^8\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^8-300300\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^8b+849849\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^8b^2-1317888\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^8b^3+1200199\,(a}{a^7b^7x^{\frac{7}{3}}}}{35840\,a}$$

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
[Out] -1/35840*(45045*a^8*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + (
45045*(a*x^(1/3) + b)^(13/2)*a^8 - 300300*(a*x^(1/3) + b)^(11/2)*a^8*b + 84
9849*(a*x^(1/3) + b)^(9/2)*a^8*b^2 - 1317888*(a*x^(1/3) + b)^(7/2)*a^8*b^3
+ 1200199*(a*x^(1/3) + b)^(5/2)*a^8*b^4 - 631540*(a*x^(1/3) + b)^(3/2)*a^8*
b^5 + 169995*sqrt(a*x^(1/3) + b)*a^8*b^6)/(a^7*b^7*x^(7/3)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^3\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

```
[In] int(1/(x^3*(a*x + b*x^(2/3))^(1/2)),x)
[Out] int(1/(x^3*(a*x + b*x^(2/3))^(1/2)), x)
```

3.193 $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 329

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} \\
&\quad -\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}} \\
&\quad +\frac{46189a^5\sqrt{bx^{2/3}+ax}}{107520b^6x^2}-\frac{138567a^6\sqrt{bx^{2/3}+ax}}{286720b^7x^{5/3}}+\frac{46189a^7\sqrt{bx^{2/3}+ax}}{81920b^8x^{4/3}} \\
&\quad -\frac{46189a^8\sqrt{bx^{2/3}+ax}}{65536b^9x}+\frac{138567a^9\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{2/3}}-\frac{138567a^{10}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{131072b^{21/2}}
\end{aligned}$$

```
[Out] -138567/131072*a^10*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)
-3/10*(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/60*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^
(10/3)-323/960*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3+323/896*a^3*(b*x^(2/3)+a*x
)^(1/2)/b^4/x^(8/3)-4199/10752*a^4*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)+46189/
107520*a^5*(b*x^(2/3)+a*x)^(1/2)/b^6/x^2-138567/286720*a^6*(b*x^(2/3)+a*x)^
(1/2)/b^7/x^(5/3)+46189/81920*a^7*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)-46189/6
5536*a^8*(b*x^(2/3)+a*x)^(1/2)/b^9/x+138567/131072*a^9*(b*x^(2/3)+a*x)^(1/2
)/b^10/x^(2/3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2050, 2054, 212}

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = -\frac{138567a^{10}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{131072b^{21/2}} + \frac{138567a^9\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{2/3}} - \frac{46189a^8\sqrt{ax+bx^{2/3}}}{65536b^9x} + \frac{46189a^7\sqrt{ax+bx^{2/3}}}{81920b^8x^{4/3}} - \frac{138567a^6\sqrt{ax+bx^{2/3}}}{286720b^7x^{5/3}} + \frac{46189a^5\sqrt{ax+bx^{2/3}}}{107520b^6x^2} - \frac{4199a^4\sqrt{ax+bx^{2/3}}}{10752b^5x^{7/3}} + \frac{323a^3\sqrt{ax+bx^{2/3}}}{896b^4x^{8/3}} - \frac{323a^2\sqrt{ax+bx^{2/3}}}{960b^3x^3} + \frac{19a\sqrt{ax+bx^{2/3}}}{60b^2x^{10/3}} - \frac{3\sqrt{ax+bx^{2/3}}}{10bx^{11/3}}$$

```
[In] Int[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) + (19*a*Sqrt[b*x^(2/3) + a*x])/(
60*b^2*x^(10/3)) - (323*a^2*Sqrt[b*x^(2/3) + a*x])/(960*b^3*x^3) + (323*a^3
*Sqrt[b*x^(2/3) + a*x])/(896*b^4*x^(8/3)) - (4199*a^4*Sqrt[b*x^(2/3) + a*x]
)/(10752*b^5*x^(7/3)) + (46189*a^5*Sqrt[b*x^(2/3) + a*x])/(107520*b^6*x^2)
- (138567*a^6*Sqrt[b*x^(2/3) + a*x])/(286720*b^7*x^(5/3)) + (46189*a^7*Sqrt
[b*x^(2/3) + a*x])/(81920*b^8*x^(4/3)) - (46189*a^8*Sqrt[b*x^(2/3) + a*x])/
(65536*b^9*x) + (138567*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(2/3)) -
(138567*a^10*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(2
1/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&& !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\text{integral} = -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} - \frac{(19a)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{20b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} + \frac{(323a^2)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{360b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} - \frac{(323a^3)\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx}{384b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} + \frac{(4199a^4)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{5376b^4}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}} - \frac{(46189a^5)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{64512b^5}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}} + \frac{46189a^5\sqrt{bx^{2/3}+ax}}{107520b^6x^2} + \frac{(46189a^6)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{71680b^6}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}} + \frac{46189a^5\sqrt{bx^{2/3}+ax}}{107520b^6x^2} - \frac{138567a^6\sqrt{bx^{2/3}+ax}}{286720b^7x^{5/3}} - \frac{(46189a^7)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{81920b^7}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}} + \frac{46189a^5\sqrt{bx^{2/3}+ax}}{107520b^6x^2} - \frac{138567a^6\sqrt{bx^{2/3}+ax}}{286720b^7x^{5/3}} + \frac{46189a^7\sqrt{bx^{2/3}+ax}}{81920b^8x^{4/3}} + \frac{(46189a^8)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{98304b^8}$$

$$\begin{aligned}
&=-\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}\\
&\quad-\frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}}+\frac{46189a^5\sqrt{bx^{2/3}+ax}}{107520b^6x^2}-\frac{138567a^6\sqrt{bx^{2/3}+ax}}{286720b^7x^{5/3}}\\
&\quad+\frac{46189a^7\sqrt{bx^{2/3}+ax}}{81920b^8x^{4/3}}-\frac{46189a^8\sqrt{bx^{2/3}+ax}}{65536b^9x}-\frac{(46189a^9)\int\frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{131072b^9}
\end{aligned}$$

$$\begin{aligned}
&=-\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}\\
&\quad+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}}+\frac{46189a^5\sqrt{bx^{2/3}+ax}}{107520b^6x^2}\\
&\quad-\frac{138567a^6\sqrt{bx^{2/3}+ax}}{286720b^7x^{5/3}}+\frac{46189a^7\sqrt{bx^{2/3}+ax}}{81920b^8x^{4/3}}-\frac{46189a^8\sqrt{bx^{2/3}+ax}}{65536b^9x}\\
&\quad+\frac{138567a^9\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{2/3}}+\frac{(46189a^{10})\int\frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{262144b^{10}}
\end{aligned}$$

$$\begin{aligned}
&=-\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}\\
&\quad+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}}+\frac{46189a^5\sqrt{bx^{2/3}+ax}}{107520b^6x^2}\\
&\quad-\frac{138567a^6\sqrt{bx^{2/3}+ax}}{286720b^7x^{5/3}}+\frac{46189a^7\sqrt{bx^{2/3}+ax}}{81920b^8x^{4/3}}-\frac{46189a^8\sqrt{bx^{2/3}+ax}}{65536b^9x}\\
&\quad+\frac{138567a^9\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{2/3}}-\frac{(138567a^{10})\,\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{131072b^{10}}
\end{aligned}$$

$$\begin{aligned}
&=-\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}\\
&\quad+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}+ax}}{10752b^5x^{7/3}}+\frac{46189a^5\sqrt{bx^{2/3}+ax}}{107520b^6x^2}\\
&\quad-\frac{138567a^6\sqrt{bx^{2/3}+ax}}{286720b^7x^{5/3}}+\frac{46189a^7\sqrt{bx^{2/3}+ax}}{81920b^8x^{4/3}}-\frac{46189a^8\sqrt{bx^{2/3}+ax}}{65536b^9x}\\
&\quad+\frac{138567a^9\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{2/3}}-\frac{138567a^{10}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{131072b^{21/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \frac{\sqrt{bx^{2/3}+ax}\left(-4128768b^9+4358144ab^8\sqrt[3]{x}-4630528a^2b^7x^{2/3}+4961280a^3b^6x-53\right.}{}$$

$$-\frac{138567a^{10}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{131072b^{21/2}}$$

```
[In] Integrate[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (Sqrt[b*x^(2/3) + a*x]*(-4128768*b^9 + 4358144*a*b^8*x^(1/3) - 4630528*a^2*
b^7*x^(2/3) + 4961280*a^3*b^6*x - 5374720*a^4*b^5*x^(4/3) + 5912192*a^5*b^4
*x^(5/3) - 6651216*a^6*b^3*x^2 + 7759752*a^7*b^2*x^(7/3) - 9699690*a^8*b*x^
(8/3) + 14549535*a^9*x^3))/(13762560*b^10*x^(11/3)) - (138567*a^10*ArcTanh[
(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(21/2))
```

Maple [A] (verified)

Time = 10.48 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\left(4128768\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{21}{2}}-4358144b^{\frac{19}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a\,x^{\frac{1}{3}}+4630528b^{\frac{17}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\,a^2x^{\frac{2}{3}}-4961280b^{\frac{15}{2}}\sqrt{b+a\,x^{\frac{1}{3}}}\right.}{}$
default	$-\frac{\sqrt{b+a\,x^{\frac{1}{3}}}\left(14549535x^{\frac{19}{3}}\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)a^{10}b+9699690x^{\frac{17}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{5}{2}}a^8-7759752x^{\frac{16}{3}}\sqrt{b+a\,x^{\frac{1}{3}}}\,b^{\frac{7}{2}}a^7-5912\right.}{}$

```
[In] int(1/x^4/(b*x^(2/3)+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/13762560*(b+a*x^(1/3))^(1/2)*(4128768*(b+a*x^(1/3))^(1/2)*b^(21/2)-43581
44*b^(19/2)*(b+a*x^(1/3))^(1/2)*a*x^(1/3)+4630528*b^(17/2)*(b+a*x^(1/3))^(1
/2)*a^2*x^(2/3)-4961280*b^(15/2)*(b+a*x^(1/3))^(1/2)*a^3*x+5374720*b^(13/2)
*(b+a*x^(1/3))^(1/2)*a^4*x^(4/3)-5912192*b^(11/2)*(b+a*x^(1/3))^(1/2)*a^5*x
^(5/3)+6651216*b^(9/2)*(b+a*x^(1/3))^(1/2)*a^6*x^2-7759752*b^(7/2)*(b+a*x^(
1/3))^(1/2)*a^7*x^(7/3)+9699690*b^(5/2)*(b+a*x^(1/3))^(1/2)*a^8*x^(8/3)-145
49535*b^(3/2)*(b+a*x^(1/3))^(1/2)*a^9*x^3+14549535*arctanh((b+a*x^(1/3))^(1
/2)/b^(1/2))*a^10*b*x^(10/3))/x^3/(b*x^(2/3)+a*x)^(1/2)/b^(23/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^4\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a*x + b*x**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \frac{\frac{14549535\,a^{11}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}} + \frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{11} - 140645505\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{11}b + 609140532\,(ax^{\frac{1}{3}}+}{}}{}$$

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/13762560*(14549535*a^11*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^
10) + (14549535*(a*x^(1/3) + b)^(19/2)*a^11 - 140645505*(a*x^(1/3) + b)^(17
/2)*a^11*b + 609140532*(a*x^(1/3) + b)^(15/2)*a^11*b^2 - 1554721740*(a*x^(1
```

```
/3) + b)^(13/2)*a^11*b^3 + 2585198330*(a*x^(1/3) + b)^(11/2)*a^11*b^4 - 291
8514950*(a*x^(1/3) + b)^(9/2)*a^11*b^5 + 2255541300*(a*x^(1/3) + b)^(7/2)*a
^11*b^6 - 1168982220*(a*x^(1/3) + b)^(5/2)*a^11*b^7 + 382331775*(a*x^(1/3)
+ b)^(3/2)*a^11*b^8 - 68025825*sqrt(a*x^(1/3) + b)*a^11*b^9)/(a^10*b^10*x^(
10/3)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = \int \frac{1}{x^4\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

```
[In] int(1/(x^4*(a*x + b*x^(2/3))^(1/2)),x)
```

```
[Out] int(1/(x^4*(a*x + b*x^(2/3))^(1/2)), x)
```

3.194 $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 336

$$\begin{aligned}\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = &-\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}}\\ &+\frac{1048576b^{10}\sqrt{bx^{2/3}+ax}}{29393a^{12}\sqrt[3]{x}}+\frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}}\\ &-\frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9}+\frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8}\\ &-\frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7}+\frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6}-\frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5}\\ &+\frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4}-\frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3}+\frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2}\end{aligned}$$

```
[Out] -6*x^4/a/(b*x^(2/3)+a*x)^(1/2)-524288/29393*b^9*(b*x^(2/3)+a*x)^(1/2)/a^11+
1048576/29393*b^10*(b*x^(2/3)+a*x)^(1/2)/a^12/x^(1/3)+393216/29393*b^8*x^(1
/3)*(b*x^(2/3)+a*x)^(1/2)/a^10-327680/29393*b^7*x^(2/3)*(b*x^(2/3)+a*x)^(1/
2)/a^9+40960/4199*b^6*x*(b*x^(2/3)+a*x)^(1/2)/a^8-36864/4199*b^5*x^(4/3)*(b
*x^(2/3)+a*x)^(1/2)/a^7+33792/4199*b^4*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^6-16
896/2261*b^3*x^2*(b*x^(2/3)+a*x)^(1/2)/a^5+15840/2261*b^2*x^(7/3)*(b*x^(2/3
)+a*x)^(1/2)/a^4-880/133*b*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+44/7*x^3*(b*x^
(2/3)+a*x)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2040, 2041, 2027, 2039}

$$\begin{aligned}\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{1048576b^{10}\sqrt{ax+bx^{2/3}}}{29393a^{12}\sqrt[3]{x}}-\frac{524288b^9\sqrt{ax+bx^{2/3}}}{29393a^{11}}\\ &\quad+\frac{393216b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{29393a^{10}}-\frac{327680b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{29393a^9}\\ &\quad+\frac{40960b^6x\sqrt{ax+bx^{2/3}}}{4199a^8}-\frac{36864b^5x^{4/3}\sqrt{ax+bx^{2/3}}}{4199a^7}+\frac{33792b^4x^{5/3}\sqrt{ax+bx^{2/3}}}{4199a^6}\\ &\quad-\frac{16896b^3x^2\sqrt{ax+bx^{2/3}}}{2261a^5}+\frac{15840b^2x^{7/3}\sqrt{ax+bx^{2/3}}}{2261a^4}\\ &\quad-\frac{880bx^{8/3}\sqrt{ax+bx^{2/3}}}{133a^3}+\frac{44x^3\sqrt{ax+bx^{2/3}}}{7a^2}-\frac{6x^4}{a\sqrt{ax+bx^{2/3}}}\end{aligned}$$

```
[In] Int[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^4)/(a*Sqrt[b*x^(2/3) + a*x]) - (524288*b^9*Sqrt[b*x^(2/3) + a*x])/(29
393*a^11) + (1048576*b^10*Sqrt[b*x^(2/3) + a*x])/(29393*a^12*x^(1/3)) + (39
3216*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(29393*a^10) - (327680*b^7*x^(2/3)*
Sqrt[b*x^(2/3) + a*x])/(29393*a^9) + (40960*b^6*x*Sqrt[b*x^(2/3) + a*x])/(4
199*a^8) - (36864*b^5*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^7) + (33792*b^
4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^6) - (16896*b^3*x^2*Sqrt[b*x^(2/3)
 + a*x])/(2261*a^5) + (15840*b^2*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^4)
- (880*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(133*a^3) + (44*x^3*Sqrt[b*x^(2/3)
+ a*x])/(7*a^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{22\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} - \frac{(440b)\int \frac{x^{8/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^2}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} + \frac{(2640b^2)\int \frac{x^{7/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{133a^3}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4} - \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} - \frac{(42240b^3)\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx}{2261a^4}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4} - \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} + \frac{(5632b^4)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{323a^5}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4} - \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} - \frac{(67584b^5)\int \frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{4199a^6}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6}$$
$$- \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4}$$
$$- \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} + \frac{(61440b^6)\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{4199a^7}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7}$$
$$+ \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4}$$
$$- \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} - \frac{(163840b^7)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{12597a^8}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9} + \frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8}$$
$$- \frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6}$$
$$- \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4}$$
$$- \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} + \frac{(327680b^8)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{29393a^9}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9}$$
$$+ \frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6}$$
$$- \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4}$$
$$- \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} - \frac{(262144b^9)\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{29393a^{10}}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}}$$
$$- \frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9} + \frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7}$$
$$+ \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4}$$
$$- \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} + \frac{(524288b^{10})\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{88179a^{11}}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}} + \frac{1048576b^{10}\sqrt{bx^{2/3}+ax}}{29393a^{12}\sqrt[3]{x}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9} + \frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4} - \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2}$$

Mathematica [A] (verified)

Time = 5.59 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.48

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{2\sqrt[3]{x}\left(524288b^{11} + 262144ab^{10}\sqrt[3]{x} - 65536a^2b^9x^{2/3} + 32768a^3b^8x - 20480a^4b^7x^{4/3} +\right.}{}$$

```
[In] Integrate[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(1/3)*(524288*b^11 + 262144*a*b^10*x^(1/3) - 65536*a^2*b^9*x^(2/3) + 3
2768*a^3*b^8*x - 20480*a^4*b^7*x^(4/3) + 14336*a^5*b^6*x^(5/3) - 10752*a^6*
b^5*x^2 + 8448*a^7*b^4*x^(7/3) - 6864*a^8*b^3*x^(8/3) + 5720*a^9*b^2*x^3 -
4862*a^10*b*x^(10/3) + 4199*a^11*x^(11/3)))/(29393*a^12*Sqrt[b*x^(2/3) + a*
x])
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.43

method	result
derivativedivides	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(4199a^{11}x^{\frac{11}{3}}-4862a^{10}b\,x^{\frac{10}{3}}+5720b^{2}a^{9}x^{3}-6864a^{8}b^{3}x^{\frac{8}{3}}+8448a^{7}b^{4}x^{\frac{7}{3}}-10752a^{6}b^{5}x^{2}+14336a^{5}b^{6}x^{\frac{5}{3}}-20\right.}{29393\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{12}}$
default	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(4199a^{11}x^{\frac{11}{3}}-4862a^{10}b\,x^{\frac{10}{3}}+5720b^{2}a^{9}x^{3}-6864a^{8}b^{3}x^{\frac{8}{3}}+8448a^{7}b^{4}x^{\frac{7}{3}}-10752a^{6}b^{5}x^{2}+14336a^{5}b^{6}x^{\frac{5}{3}}-20\right.}{29393\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{12}}$

```
[In] int(x^4/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/29393*x*(b+a*x^(1/3))*(4199*a^11*x^(11/3)-4862*a^10*b*x^(10/3)+5720*b^2*a
^9*x^3-6864*a^8*b^3*x^(8/3)+8448*a^7*b^4*x^(7/3)-10752*a^6*b^5*x^2+14336*a^
5*b^6*x^(5/3)-20480*a^4*b^7*x^(4/3)+32768*a^3*b^8*x-65536*a^2*b^9*x^(2/3)+2
62144*a*b^10*x^(1/3)+524288*b^11)/(b*x^(2/3)+a*x)^(3/2)/a^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2566 vs. 2(252) = 504.

Time = 142.89 (sec) , antiderivative size = 2566, normalized size of antiderivative = 7.64

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/29393*((6442450944*a^3*b^19 + 5368709120*a^3*b^18 - 2013265920*a^3*b^17
- 6113744*a^18 + 402653184*(17*a^6 - 3*a^3)*b^16 + 8388608*(464*a^6 + 53*a^
3)*b^15 - 12582912*(246*a^6 + a^3)*b^14 + 1572864*(1036*a^9 - 2560*a^6 - 3*
a^3)*b^13 - 524288*(758*a^9 - 1569*a^6)*b^12 - 393216*(5803*a^9 + 124*a^6)*
b^11 + 98304*(1315*a^12 - 20924*a^9 - 33*a^6)*b^10 - 57344*(2264*a^12 - 315
3*a^9)*b^9 - 6144*(83789*a^12 + 2066*a^9)*b^8 - 1536*(46256*a^15 - 159272*a
^12 - 267*a^9)*b^7 - 128*(264488*a^15 + 382229*a^12)*b^6 + 9984*(15547*a^15
 + 482*a^12)*b^5 - 24*(2376192*a^18 + 4735792*a^15 + 7887*a^12)*b^4 - 1664*
(107856*a^18 - 16759*a^15)*b^3 - 156*(935424*a^18 + 17935*a^15)*b^2 + 663*(
97664*a^18 + 123*a^15)*b)*x^2 + (6442450944*b^22 + 5368709120*b^21 + 402653
184*(17*a^3 - 3)*b^19 - 2013265920*b^20 + 8388608*(464*a^3 + 53)*b^18 - 611
3744*a^15*b^3 - 12582912*(246*a^3 + 1)*b^17 + 1572864*(1036*a^6 - 2560*a^3
- 3)*b^16 - 524288*(758*a^6 - 1569*a^3)*b^15 - 393216*(5803*a^6 + 124*a^3)*
b^14 + 98304*(1315*a^9 - 20924*a^6 - 33*a^3)*b^13 - 57344*(2264*a^9 - 3153*
a^6)*b^12 - 6144*(83789*a^9 + 2066*a^6)*b^11 - 1536*(46256*a^12 - 159272*a^
9 - 267*a^6)*b^10 - 128*(264488*a^12 + 382229*a^9)*b^9 + 9984*(15547*a^12 +
 482*a^9)*b^8 - 24*(2376192*a^15 + 4735792*a^12 + 7887*a^9)*b^7 - 1664*(107
856*a^15 - 16759*a^12)*b^6 - 156*(935424*a^15 + 17935*a^12)*b^5 + 663*(9766
4*a^15 + 123*a^12)*b^4)*x - 2*(4199*(4096*a^13*b^9 + 6144*a^13*b^8 + 768*a^
13*b^7 - 4096*a^19 - 144*a^16*b^2 + 216*a^16*b - 27*a^16 + 256*(16*a^16 - 7
*a^13)*b^6 + 48*(128*a^16 - 3*a^13)*b^5 + 24*(32*a^16 + 9*a^13)*b^4 - (5888
*a^16 + 27*a^13)*b^3)*x^5 - 17446*(4096*a^10*b^12 + 6144*a^10*b^11 + 768*a^
10*b^10 - 144*a^13*b^5 + 216*a^13*b^4 + 256*(16*a^13 - 7*a^10)*b^9 + 48*(12
8*a^13 - 3*a^10)*b^8 + 24*(32*a^13 + 9*a^10)*b^7 - (5888*a^13 + 27*a^10)*b^
6 - (4096*a^16 + 27*a^13)*b^3)*x^4 + 33536*(4096*a^7*b^15 + 6144*a^7*b^14 +
 768*a^7*b^13 - 144*a^10*b^8 + 216*a^10*b^7 + 256*(16*a^10 - 7*a^7)*b^12 +
48*(128*a^10 - 3*a^7)*b^11 + 24*(32*a^10 + 9*a^7)*b^10 - (5888*a^10 + 27*a^
7)*b^9 - (4096*a^13 + 27*a^10)*b^6)*x^3 - 118784*(4096*a^4*b^18 + 6144*a^4*
b^17 + 768*a^4*b^16 - 144*a^7*b^11 + 216*a^7*b^10 + 256*(16*a^7 - 7*a^4)*b^
15 + 48*(128*a^7 - 3*a^4)*b^14 + 24*(32*a^7 + 9*a^4)*b^13 - (5888*a^7 + 27*
a^4)*b^12 - (4096*a^10 + 27*a^7)*b^9)*x^2 - 262144*(4096*a*b^21 + 6144*a*b^
20 + 768*a*b^19 + 256*(16*a^4 - 7*a)*b^18 - 144*a^4*b^14 + 48*(128*a^4 - 3*
a)*b^17 + 216*a^4*b^13 + 24*(32*a^4 + 9*a)*b^16 - (5888*a^4 + 27*a)*b^15 -
(4096*a^7 + 27*a^4)*b^12)*x + (2147483648*b^22 + 3221225472*b^21 + 13421772
8*(16*a^3 - 7)*b^19 + 402653184*b^20 + 25165824*(128*a^3 - 3)*b^18 - 754974
```

```
72*a^3*b^15 + 12582912*(32*a^3 + 9)*b^17 + 113246208*a^3*b^14 - 524288*(588
8*a^3 + 27)*b^16 - 524288*(4096*a^6 + 27*a^3)*b^13 - 9061*(4096*a^12*b^10 +
 6144*a^12*b^9 + 768*a^12*b^8 - 144*a^15*b^3 + 216*a^15*b^2 + 256*(16*a^15
- 7*a^12)*b^7 + 48*(128*a^15 - 3*a^12)*b^6 + 24*(32*a^15 + 9*a^12)*b^5 - (5
888*a^15 + 27*a^12)*b^4 - (4096*a^18 + 27*a^15)*b)*x^4 + 21032*(4096*a^9*b^
13 + 6144*a^9*b^12 + 768*a^9*b^11 - 144*a^12*b^6 + 216*a^12*b^5 + 256*(16*a
^12 - 7*a^9)*b^10 + 48*(128*a^12 - 3*a^9)*b^9 + 24*(32*a^12 + 9*a^9)*b^8 -
(5888*a^12 + 27*a^9)*b^7 - (4096*a^15 + 27*a^12)*b^4)*x^3 - 45568*(4096*a^6
*b^16 + 6144*a^6*b^15 + 768*a^6*b^14 - 144*a^9*b^9 + 216*a^9*b^8 + 256*(16*
a^9 - 7*a^6)*b^13 + 48*(128*a^9 - 3*a^6)*b^12 + 24*(32*a^9 + 9*a^6)*b^11 -
(5888*a^9 + 27*a^6)*b^10 - (4096*a^12 + 27*a^9)*b^7)*x^2 + 360448*(4096*a^3
*b^19 + 6144*a^3*b^18 + 768*a^3*b^17 - 144*a^6*b^12 + 216*a^6*b^11 + 256*(1
6*a^6 - 7*a^3)*b^16 + 48*(128*a^6 - 3*a^3)*b^15 + 24*(32*a^6 + 9*a^3)*b^14
- (5888*a^6 + 27*a^3)*b^13 - (4096*a^9 + 27*a^6)*b^10)*x)*x^(2/3) + 3*(4927
*(4096*a^11*b^11 + 6144*a^11*b^10 + 768*a^11*b^9 - 144*a^14*b^4 + 216*a^14*
b^3 + 256*(16*a^14 - 7*a^11)*b^8 + 48*(128*a^14 - 3*a^11)*b^7 + 24*(32*a^14
 + 9*a^11)*b^6 - (5888*a^14 + 27*a^11)*b^5 - (4096*a^17 + 27*a^14)*b^2)*x^4
 - 8688*(4096*a^8*b^14 + 6144*a^8*b^13 + 768*a^8*b^12 - 144*a^11*b^7 + 216*
a^11*b^6 + 256*(16*a^11 - 7*a^8)*b^11 + 48*(128*a^11 - 3*a^8)*b^10 + 24*(32
*a^11 + 9*a^8)*b^9 - (5888*a^11 + 27*a^8)*b^8 - (4096*a^14 + 27*a^11)*b^5)*
x^3 + 22528*(4096*a^5*b^17 + 6144*a^5*b^16 + 768*a^5*b^15 - 144*a^8*b^10 +
216*a^8*b^9 + 256*(16*a^8 - 7*a^5)*b^14 + 48*(128*a^8 - 3*a^5)*b^13 + 24*(3
2*a^8 + 9*a^5)*b^12 - (5888*a^8 + 27*a^5)*b^11 - (4096*a^11 + 27*a^8)*b^8)*
x^2 + 65536*(4096*a^2*b^20 + 6144*a^2*b^19 + 768*a^2*b^18 - 144*a^5*b^13 +
256*(16*a^5 - 7*a^2)*b^17 + 216*a^5*b^12 + 48*(128*a^5 - 3*a^2)*b^16 + 24*(
32*a^5 + 9*a^2)*b^15 - (5888*a^5 + 27*a^2)*b^14 - (4096*a^8 + 27*a^5)*b^11)
*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((4096*a^15*b^9 + 6144*a^15*b^8 + 768*a
^15*b^7 - 4096*a^21 - 144*a^18*b^2 + 216*a^18*b - 27*a^18 + 256*(16*a^18 -
7*a^15)*b^6 + 48*(128*a^18 - 3*a^15)*b^5 + 24*(32*a^18 + 9*a^15)*b^4 - (588
8*a^18 + 27*a^15)*b^3)*x^2 + (4096*a^12*b^12 + 6144*a^12*b^11 + 768*a^12*b^
10 - 144*a^15*b^5 + 216*a^15*b^4 + 256*(16*a^15 - 7*a^12)*b^9 + 48*(128*a^1
5 - 3*a^12)*b^8 + 24*(32*a^15 + 9*a^12)*b^7 - (5888*a^15 + 27*a^12)*b^6 - (
4096*a^18 + 27*a^15)*b^3)*x)
```

Sympy [F]

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**4/(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=-\frac{1048576\,b^{\frac{21}{2}}}{29393\,a^{12}}+\frac{6\,b^{11}}{\sqrt{ax^{\frac{1}{3}}+b}a^{12}}$$

$$+\ 2\left(4199\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{240}-51051\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{240}b+285285\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{240}b^2-969969\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{24}\right.$$

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1048576/29393*b^(21/2)/a^12 + 6*b^11/(sqrt(a*x^(1/3) + b)*a^12) + 2/29393*
(4199*(a*x^(1/3) + b)^(21/2)*a^240 - 51051*(a*x^(1/3) + b)^(19/2)*a^240*b +
 285285*(a*x^(1/3) + b)^(17/2)*a^240*b^2 - 969969*(a*x^(1/3) + b)^(15/2)*a^
240*b^3 + 2238390*(a*x^(1/3) + b)^(13/2)*a^240*b^4 - 3703518*(a*x^(1/3) + b
)^(11/2)*a^240*b^5 + 4526522*(a*x^(1/3) + b)^(9/2)*a^240*b^6 - 4157010*(a*x
^(1/3) + b)^(7/2)*a^240*b^7 + 2909907*(a*x^(1/3) + b)^(5/2)*a^240*b^8 - 161
6615*(a*x^(1/3) + b)^(3/2)*a^240*b^9 + 969969*sqrt(a*x^(1/3) + b)*a^240*b^1
0)/a^252
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^4}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}\,dx$$

```
[In] int(x^4/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

3.195 $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 248

$$\begin{aligned}\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8}-\frac{65536b^7\sqrt{bx^{2/3}+ax}}{2145a^9\sqrt[3]{x}}\\ &\quad-\frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6}-\frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5}\\ &\quad+\frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4}-\frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3}+\frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2}\end{aligned}$$

```
[Out] -6*x^3/a/(b*x^(2/3)+a*x)^(1/2)+32768/2145*b^6*(b*x^(2/3)+a*x)^(1/2)/a^8-655
36/2145*b^7*(b*x^(2/3)+a*x)^(1/2)/a^9/x^(1/3)-8192/715*b^5*x^(1/3)*(b*x^(2/
3)+a*x)^(1/2)/a^7+4096/429*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^6-3584/429*b
^3*x*(b*x^(2/3)+a*x)^(1/2)/a^5+5376/715*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a
^4-448/65*b*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+32/5*x^2*(b*x^(2/3)+a*x)^(1/2
)/a^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2040, 2041, 2027, 2039}

$$\begin{aligned}\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= -\frac{65536b^7\sqrt{ax+bx^{2/3}}}{2145a^9\sqrt[3]{x}}+\frac{32768b^6\sqrt{ax+bx^{2/3}}}{2145a^8}\\ &\quad-\frac{8192b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^6}-\frac{3584b^3x\sqrt{ax+bx^{2/3}}}{429a^5}\\ &\quad+\frac{5376b^2x^{4/3}\sqrt{ax+bx^{2/3}}}{715a^4}-\frac{448bx^{5/3}\sqrt{ax+bx^{2/3}}}{65a^3}+\frac{32x^2\sqrt{ax+bx^{2/3}}}{5a^2}-\frac{6x^3}{a\sqrt{ax+bx^{2/3}}}\end{aligned}$$

```
[In] Int[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^3)/(a*Sqrt[b*x^(2/3) + a*x]) + (32768*b^6*Sqrt[b*x^(2/3) + a*x])/(214
5*a^8) - (65536*b^7*Sqrt[b*x^(2/3) + a*x])/(2145*a^9*x^(1/3)) - (8192*b^5*x
^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^7) + (4096*b^4*x^(2/3)*Sqrt[b*x^(2/3)
+ a*x])/(429*a^6) - (3584*b^3*x*Sqrt[b*x^(2/3) + a*x])/(429*a^5) + (5376*b^
2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^4) - (448*b*x^(5/3)*Sqrt[b*x^(2/3)
+ a*x])/(65*a^3) + (32*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{16\int\frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2} - \frac{(224b)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{15a^2}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2} + \frac{(896b^2)\int \frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{65a^3}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2} - \frac{(1792b^3)\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{143a^4}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} - \frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2} + \frac{(14336b^4)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{1287a^5}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6} - \frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2} - \frac{(4096b^5)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{429a^6}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} - \frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6} - \frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2} + \frac{(16384b^6)\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{2145a^7}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8} - \frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6} - \frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2} - \frac{(32768b^7)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{6435a^8}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}} + \frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8} - \frac{65536b^7\sqrt{bx^{2/3}+ax}}{2145a^9\sqrt[3]{x}} - \frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7} + \frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6} - \frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5} + \frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4} - \frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3} + \frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2}$$

Mathematica [A] (verified)

Time = 5.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.49

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{2\bigl(-32768b^8\sqrt[3]{x}-16384ab^7x^{2/3}+4096a^2b^6x-2048a^3b^5x^{4/3}+1280a^4b^4x^{5/3}-896a^5}{2145a^9\sqrt{bx^{2/3}+ax}}$$

```
[In] Integrate[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(-32768*b^8*x^(1/3) - 16384*a*b^7*x^(2/3) + 4096*a^2*b^6*x - 2048*a^3*b^
5*x^(4/3) + 1280*a^4*b^4*x^(5/3) - 896*a^5*b^3*x^2 + 672*a^6*b^2*x^(7/3) -
528*a^7*b*x^(8/3) + 429*a^8*x^3))/(2145*a^9*Sqrt[b*x^(2/3) + a*x])
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.44

method	result
derivativedivides	$\frac{2x\bigl(b+a\,x^{\frac{1}{3}}\bigr)\bigl(429a^8x^{\frac{8}{3}}-528a^7b\,x^{\frac{7}{3}}+672a^6x^2b^2-896a^5b^3x^{\frac{5}{3}}+1280x^{\frac{4}{3}}a^4b^4-2048a^3b^5x+4096a^2b^6x^{\frac{2}{3}}-16384x^{\frac{1}{3}}a\,b^7-32}{2145\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^9}$
default	$\frac{2x\bigl(b+a\,x^{\frac{1}{3}}\bigr)\bigl(429a^8x^{\frac{8}{3}}-528a^7b\,x^{\frac{7}{3}}+672a^6x^2b^2-896a^5b^3x^{\frac{5}{3}}+1280x^{\frac{4}{3}}a^4b^4-2048a^3b^5x+4096a^2b^6x^{\frac{2}{3}}-16384x^{\frac{1}{3}}a\,b^7-32}{2145\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^9}$

```
[In] int(x^3/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/2145*x*(b+a*x^(1/3))*(429*a^8*x^(8/3)-528*a^7*b*x^(7/3)+672*a^6*x^2*b^2-8
96*a^5*b^3*x^(5/3)+1280*x^(4/3)*a^4*b^4-2048*a^3*b^5*x+4096*a^2*b^6*x^(2/3)
-16384*x^(1/3)*a*b^7-32768*b^8)/(b*x^(2/3)+a*x)^(3/2)/a^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2083 vs. $2(186) = 372$.

Time = 123.02 (sec) , antiderivative size = 2083, normalized size of antiderivative = 8.40

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2145*((402653184*a^3*b^16 + 335544320*a^3*b^15 - 125829120*a^3*b^14 + 624
624*a^15 + 25165824*(17*a^6 - 3*a^3)*b^13 + 524288*(464*a^6 + 53*a^3)*b^12
- 786432*(246*a^6 + a^3)*b^11 + 98304*(1036*a^9 - 2560*a^6 - 3*a^3)*b^10 -
32768*(758*a^9 - 1569*a^6)*b^9 - 24576*(5803*a^9 + 124*a^6)*b^8 + 6144*(600
```

```
*a^12 - 20924*a^9 - 33*a^6)*b^7 - 1536*(7666*a^12 - 7357*a^9)*b^6 - 768*(40
107*a^12 + 1033*a^9)*b^5 + 96*(63360*a^15 + 167852*a^12 + 267*a^9)*b^4 + 32
*(613440*a^15 - 105031*a^12)*b^3 + 468*(34560*a^15 + 661*a^12)*b^2 - 99*(68
480*a^15 + 87*a^12)*b)*x^2 + (402653184*b^19 + 335544320*b^18 + 25165824*(1
7*a^3 - 3)*b^16 - 125829120*b^17 + 524288*(464*a^3 + 53)*b^15 + 624624*a^12
*b^3 - 786432*(246*a^3 + 1)*b^14 + 98304*(1036*a^6 - 2560*a^3 - 3)*b^13 - 3
2768*(758*a^6 - 1569*a^3)*b^12 - 24576*(5803*a^6 + 124*a^3)*b^11 + 6144*(60
0*a^9 - 20924*a^6 - 33*a^3)*b^10 - 1536*(7666*a^9 - 7357*a^6)*b^9 - 768*(40
107*a^9 + 1033*a^6)*b^8 + 96*(63360*a^12 + 167852*a^9 + 267*a^6)*b^7 + 32*(
613440*a^12 - 105031*a^9)*b^6 + 468*(34560*a^12 + 661*a^9)*b^5 - 99*(68480*
a^12 + 87*a^9)*b^4)*x + 2*(429*(4096*a^10*b^9 + 6144*a^10*b^8 + 768*a^10*b^
7 - 4096*a^16 - 144*a^13*b^2 + 216*a^13*b - 27*a^13 + 256*(16*a^13 - 7*a^10
)*b^6 + 48*(128*a^13 - 3*a^10)*b^5 + 24*(32*a^13 + 9*a^10)*b^4 - (5888*a^13
 + 27*a^10)*b^3)*x^4 - 2096*(4096*a^7*b^12 + 6144*a^7*b^11 + 768*a^7*b^10 -
 144*a^10*b^5 + 216*a^10*b^4 + 256*(16*a^10 - 7*a^7)*b^9 + 48*(128*a^10 - 3
*a^7)*b^8 + 24*(32*a^10 + 9*a^7)*b^7 - (5888*a^10 + 27*a^7)*b^6 - (4096*a^1
3 + 27*a^10)*b^3)*x^3 + 7424*(4096*a^4*b^15 + 6144*a^4*b^14 + 768*a^4*b^13
- 144*a^7*b^8 + 216*a^7*b^7 + 256*(16*a^7 - 7*a^4)*b^12 + 48*(128*a^7 - 3*a
^4)*b^11 + 24*(32*a^7 + 9*a^4)*b^10 - (5888*a^7 + 27*a^4)*b^9 - (4096*a^10
+ 27*a^7)*b^6)*x^2 + 16384*(4096*a*b^18 + 6144*a*b^17 + 768*a*b^16 + 256*(1
6*a^4 - 7*a)*b^15 - 144*a^4*b^11 + 48*(128*a^4 - 3*a)*b^14 + 216*a^4*b^10 +
 24*(32*a^4 + 9*a)*b^13 - (5888*a^4 + 27*a)*b^12 - (4096*a^7 + 27*a^4)*b^9)
*x - (134217728*b^19 + 201326592*b^18 + 8388608*(16*a^3 - 7)*b^16 + 2516582
4*b^17 + 1572864*(128*a^3 - 3)*b^15 - 4718592*a^3*b^12 + 786432*(32*a^3 + 9
)*b^14 + 7077888*a^3*b^11 - 32768*(5888*a^3 + 27)*b^13 - 32768*(4096*a^6 +
27*a^3)*b^10 + 957*(4096*a^9*b^10 + 6144*a^9*b^9 + 768*a^9*b^8 - 144*a^12*b
^3 + 216*a^12*b^2 + 256*(16*a^12 - 7*a^9)*b^7 + 48*(128*a^12 - 3*a^9)*b^6 +
 24*(32*a^12 + 9*a^9)*b^5 - (5888*a^12 + 27*a^9)*b^4 - (4096*a^15 + 27*a^12
)*b)*x^3 - 2848*(4096*a^6*b^13 + 6144*a^6*b^12 + 768*a^6*b^11 - 144*a^9*b^6
 + 216*a^9*b^5 + 256*(16*a^9 - 7*a^6)*b^10 + 48*(128*a^9 - 3*a^6)*b^9 + 24*
(32*a^9 + 9*a^6)*b^8 - (5888*a^9 + 27*a^6)*b^7 - (4096*a^12 + 27*a^9)*b^4)*
x^2 + 22528*(4096*a^3*b^16 + 6144*a^3*b^15 + 768*a^3*b^14 - 144*a^6*b^9 + 2
16*a^6*b^8 + 256*(16*a^6 - 7*a^3)*b^13 + 48*(128*a^6 - 3*a^3)*b^12 + 24*(32
*a^6 + 9*a^3)*b^11 - (5888*a^6 + 27*a^3)*b^10 - (4096*a^9 + 27*a^6)*b^7)*x)
*x^(2/3) + 3*(543*(4096*a^8*b^11 + 6144*a^8*b^10 + 768*a^8*b^9 - 144*a^11*b
^4 + 216*a^11*b^3 + 256*(16*a^11 - 7*a^8)*b^8 + 48*(128*a^11 - 3*a^8)*b^7 +
 24*(32*a^11 + 9*a^8)*b^6 - (5888*a^11 + 27*a^8)*b^5 - (4096*a^14 + 27*a^11
)*b^2)*x^3 - 1408*(4096*a^5*b^14 + 6144*a^5*b^13 + 768*a^5*b^12 - 144*a^8*b
^7 + 216*a^8*b^6 + 256*(16*a^8 - 7*a^5)*b^11 + 48*(128*a^8 - 3*a^5)*b^10 +
24*(32*a^8 + 9*a^5)*b^9 - (5888*a^8 + 27*a^5)*b^8 - (4096*a^11 + 27*a^8)*b^
5)*x^2 - 4096*(4096*a^2*b^17 + 6144*a^2*b^16 + 768*a^2*b^15 - 144*a^5*b^10
+ 256*(16*a^5 - 7*a^2)*b^14 + 216*a^5*b^9 + 48*(128*a^5 - 3*a^2)*b^13 + 24*
(32*a^5 + 9*a^2)*b^12 - (5888*a^5 + 27*a^2)*b^11 - (4096*a^8 + 27*a^5)*b^8)
*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((4096*a^12*b^9 + 6144*a^12*b^8 + 768*a
^12*b^7 - 4096*a^18 - 144*a^15*b^2 + 216*a^15*b - 27*a^15 + 256*(16*a^15 -
```

```
7*a^12)*b^6 + 48*(128*a^15 - 3*a^12)*b^5 + 24*(32*a^15 + 9*a^12)*b^4 - (588
8*a^15 + 27*a^12)*b^3)*x^2 + (4096*a^9*b^12 + 6144*a^9*b^11 + 768*a^9*b^10
- 144*a^12*b^5 + 216*a^12*b^4 + 256*(16*a^12 - 7*a^9)*b^9 + 48*(128*a^12 -
3*a^9)*b^8 + 24*(32*a^12 + 9*a^9)*b^7 - (5888*a^12 + 27*a^9)*b^6 - (4096*a^
15 + 27*a^12)*b^3)*x)
```

Sympy [F]

$$\int \frac{x^3}{(bx^{2/3}+ax)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x**3/(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(bx^{2/3}+ax)^{3/2}}\,dx = \int \frac{x^3}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{(bx^{2/3}+ax)^{3/2}}\,dx = \frac{65536\,b^{\frac{15}{2}}}{2145\,a^9} - \frac{6\,b^8}{\sqrt{ax^{\frac{1}{3}}+b}a^9} + \frac{2\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{126} - 3960\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{126}b + 16380\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{126}b^2 - 40040\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{126}b^3 + \right.}{2145}$$

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
[Out] 65536/2145*b^(15/2)/a^9 - 6*b^8/(sqrt(a*x^(1/3) + b)*a^9) + 2/2145*(429*(a*
x^(1/3) + b)^(15/2)*a^126 - 3960*(a*x^(1/3) + b)^(13/2)*a^126*b + 16380*(a*
x^(1/3) + b)^(11/2)*a^126*b^2 - 40040*(a*x^(1/3) + b)^(9/2)*a^126*b^3 + 643
50*(a*x^(1/3) + b)^(7/2)*a^126*b^4 - 72072*(a*x^(1/3) + b)^(5/2)*a^126*b^5
+ 60060*(a*x^(1/3) + b)^(3/2)*a^126*b^6 - 51480*sqrt(a*x^(1/3) + b)*a^126*b
^7)/a^135
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^3}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}} \,d x$$

```
[In] int(x^3/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

3.196 $\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5} + \frac{512b^4\sqrt{bx^{2/3}+ax}}{21a^6\sqrt[3]{x}} + \frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4} - \frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2}$$

```
[Out] -6*x^2/a/(b*x^(2/3)+a*x)^(1/2)-256/21*b^3*(b*x^(2/3)+a*x)^(1/2)/a^5+512/21*
b^4*(b*x^(2/3)+a*x)^(1/2)/a^6/x^(1/3)+64/7*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(1/2
)/a^4-160/21*b*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+20/3*x*(b*x^(2/3)+a*x)^(1/
2)/a^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2040, 2041, 2027, 2039}

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{512b^4\sqrt{ax+bx^{2/3}}}{21a^6\sqrt[3]{x}} - \frac{256b^3\sqrt{ax+bx^{2/3}}}{21a^5} + \frac{64b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{7a^4} - \frac{160bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^3} + \frac{20x\sqrt{ax+bx^{2/3}}}{3a^2} - \frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

```
[In] Int[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^2)/(a*Sqrt[b*x^(2/3) + a*x]) - (256*b^3*Sqrt[b*x^(2/3) + a*x])/(21*a^
5) + (512*b^4*Sqrt[b*x^(2/3) + a*x])/(21*a^6*x^(1/3)) + (64*b^2*x^(1/3)*Sqr
```

```
t[b*x^(2/3) + a*x])/(7*a^4) - (160*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^3
) + (20*x*Sqrt[b*x^(2/3) + a*x])/(3*a^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} + \frac{10\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2} - \frac{(80b)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{9a^2}$$

$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2} + \frac{(160b^2)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^3}$$

$$
\begin{aligned}
&= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}}+\frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4}-\frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3}\\
&\quad+\frac{20x\sqrt{bx^{2/3}+ax}}{3a^2}-\frac{(128b^3)\int\frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^4}\\
&= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}}-\frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5}+\frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4}\\
&\quad-\frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3}+\frac{20x\sqrt{bx^{2/3}+ax}}{3a^2}+\frac{(256b^4)\int\frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{63a^5}\\
&= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}}-\frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5}+\frac{512b^4\sqrt{bx^{2/3}+ax}}{21a^6\sqrt[3]{x}}\\
&\quad+\frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4}-\frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3}+\frac{20x\sqrt{bx^{2/3}+ax}}{3a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.69 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.53

$$
\int\frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{512b^5\sqrt[3]{x}+256ab^4x^{2/3}-64a^2b^3x+32a^3b^2x^{4/3}-20a^4bx^{5/3}+14a^5x^2}{21a^6\sqrt{bx^{2/3}+ax}}
$$

```
[In] Integrate[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (512*b^5*x^(1/3) + 256*a*b^4*x^(2/3) - 64*a^2*b^3*x + 32*a^3*b^2*x^(4/3) -
20*a^4*b*x^(5/3) + 14*a^5*x^2)/(21*a^6*Sqrt[b*x^(2/3) + a*x])
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.48

method	result	size
derivativedivides	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(7a^5x^{\frac{5}{3}}-10a^4b\,x^{\frac{4}{3}}+16a^3b^2x-32a^2b^3x^{\frac{2}{3}}+128a\,b^4x^{\frac{1}{3}}+256b^5\right)}{21\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^6}$	77
default	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(7a^5x^{\frac{5}{3}}-10a^4b\,x^{\frac{4}{3}}+16a^3b^2x-32a^2b^3x^{\frac{2}{3}}+128a\,b^4x^{\frac{1}{3}}+256b^5\right)}{21\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^6}$	77

```
[In] int(x^2/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*x*(b+a*x^(1/3))*(7*a^5*x^(5/3)-10*a^4*b*x^(4/3)+16*a^3*b^2*x-32*a^2*b^
3*x^(2/3)+128*a*b^4*x^(1/3)+256*b^5)/(b*x^(2/3)+a*x)^(3/2)/a^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. 2(120) = 240.

Time = 125.66 (sec) , antiderivative size = 1598, normalized size of antiderivative = 9.99

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/21*((3145728*a^3*b^13 + 2621440*a^3*b^12 - 983040*a^3*b^11 - 10192*a^12
+ 196608*(17*a^6 - 3*a^3)*b^10 + 4096*(464*a^6 + 53*a^3)*b^9 - 6144*(246*a^
6 + a^3)*b^8 + 768*(1120*a^9 - 2560*a^6 - 3*a^3)*b^7 - 256*(548*a^9 - 1569*
a^6)*b^6 - 768*(1477*a^9 + 31*a^6)*b^5 - 48*(2304*a^12 + 21176*a^9 + 33*a^6
)*b^4 - 4032*(96*a^12 - 23*a^9)*b^3 - 12*(27648*a^12 + 527*a^9)*b^2 + 3*(39
296*a^12 + 51*a^9)*b)*x^2 + (3145728*b^16 + 2621440*b^15 + 196608*(17*a^3 -
 3)*b^13 - 983040*b^14 + 4096*(464*a^3 + 53)*b^12 - 10192*a^9*b^3 - 6144*(2
46*a^3 + 1)*b^11 + 768*(1120*a^6 - 2560*a^3 - 3)*b^10 - 256*(548*a^6 - 1569
*a^3)*b^9 - 768*(1477*a^6 + 31*a^3)*b^8 - 48*(2304*a^9 + 21176*a^6 + 33*a^3
)*b^7 - 4032*(96*a^9 - 23*a^6)*b^6 - 12*(27648*a^9 + 527*a^6)*b^5 + 3*(3929
6*a^9 + 51*a^6)*b^4)*x - 2*(7*(4096*a^7*b^9 + 6144*a^7*b^8 + 768*a^7*b^7 -
4096*a^13 - 144*a^10*b^2 + 216*a^10*b - 27*a^10 + 256*(16*a^10 - 7*a^7)*b^6
 + 48*(128*a^10 - 3*a^7)*b^5 + 24*(32*a^10 + 9*a^7)*b^4 - (5888*a^10 + 27*a
^7)*b^3)*x^3 - 58*(4096*a^4*b^12 + 6144*a^4*b^11 + 768*a^4*b^10 - 144*a^7*b
^5 + 216*a^7*b^4 + 256*(16*a^7 - 7*a^4)*b^9 + 48*(128*a^7 - 3*a^4)*b^8 + 24
*(32*a^7 + 9*a^4)*b^7 - (5888*a^7 + 27*a^4)*b^6 - (4096*a^10 + 27*a^7)*b^3)
*x^2 - 128*(4096*a*b^15 + 6144*a*b^14 + 768*a*b^13 + 256*(16*a^4 - 7*a)*b^1
2 - 144*a^4*b^8 + 48*(128*a^4 - 3*a)*b^11 + 216*a^4*b^7 + 24*(32*a^4 + 9*a)
*b^10 - (5888*a^4 + 27*a)*b^9 - (4096*a^7 + 27*a^4)*b^6)*x + (1048576*b^16
+ 1572864*b^15 + 65536*(16*a^3 - 7)*b^13 + 196608*b^14 + 12288*(128*a^3 - 3
)*b^12 - 36864*a^3*b^9 + 6144*(32*a^3 + 9)*b^11 + 55296*a^3*b^8 - 256*(5888
*a^3 + 27)*b^10 - 256*(4096*a^6 + 27*a^3)*b^7 - 17*(4096*a^6*b^10 + 6144*a^
6*b^9 + 768*a^6*b^8 - 144*a^9*b^3 + 216*a^9*b^2 + 256*(16*a^9 - 7*a^6)*b^7
+ 48*(128*a^9 - 3*a^6)*b^6 + 24*(32*a^9 + 9*a^6)*b^5 - (5888*a^9 + 27*a^6)*
b^4 - (4096*a^12 + 27*a^9)*b)*x^2 + 176*(4096*a^3*b^13 + 6144*a^3*b^12 + 76
8*a^3*b^11 - 144*a^6*b^6 + 216*a^6*b^5 + 256*(16*a^6 - 7*a^3)*b^10 + 48*(12
8*a^6 - 3*a^3)*b^9 + 24*(32*a^6 + 9*a^3)*b^8 - (5888*a^6 + 27*a^3)*b^7 - (4
096*a^9 + 27*a^6)*b^4)*x)*x^(2/3) + 3*(11*(4096*a^5*b^11 + 6144*a^5*b^10 +
768*a^5*b^9 - 144*a^8*b^4 + 216*a^8*b^3 + 256*(16*a^8 - 7*a^5)*b^8 + 48*(12
8*a^8 - 3*a^5)*b^7 + 24*(32*a^8 + 9*a^5)*b^6 - (5888*a^8 + 27*a^5)*b^5 - (4
096*a^11 + 27*a^8)*b^2)*x^2 + 32*(4096*a^2*b^14 + 6144*a^2*b^13 + 768*a^2*b
^12 - 144*a^5*b^7 + 256*(16*a^5 - 7*a^2)*b^11 + 216*a^5*b^6 + 48*(128*a^5 -
 3*a^2)*b^10 + 24*(32*a^5 + 9*a^2)*b^9 - (5888*a^5 + 27*a^2)*b^8 - (4096*a^
8 + 27*a^5)*b^5)*x)*x^(1/3))*sqrt(a*x + b*x^(2/3)))/((4096*a^9*b^9 + 6144*a
^9*b^8 + 768*a^9*b^7 - 4096*a^15 - 144*a^12*b^2 + 216*a^12*b - 27*a^12 + 25
```

```
6*(16*a^12 - 7*a^9)*b^6 + 48*(128*a^12 - 3*a^9)*b^5 + 24*(32*a^12 + 9*a^9)*
b^4 - (5888*a^12 + 27*a^9)*b^3)*x^2 + (4096*a^6*b^12 + 6144*a^6*b^11 + 768*
a^6*b^10 - 144*a^9*b^5 + 216*a^9*b^4 + 256*(16*a^9 - 7*a^6)*b^9 + 48*(128*a
^9 - 3*a^6)*b^8 + 24*(32*a^9 + 9*a^6)*b^7 - (5888*a^9 + 27*a^6)*b^6 - (4096
*a^12 + 27*a^9)*b^3)*x)
```

Sympy [F]

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x**2/(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{x^2}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{512\,b^{\frac{9}{2}}}{21\,a^6} + \frac{6\,b^5}{\sqrt{ax^{\frac{1}{3}}+b}a^6} + \frac{2\left(7\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{48} - 45\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{48}b + 126\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{48}b^2 - 210\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{48}b^3 + 315\sqrt{ax^{\frac{1}{3}}+b}a}{21\,a^{54}}$$

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
[Out] -512/21*b^(9/2)/a^6 + 6*b^5/(sqrt(a*x^(1/3) + b)*a^6) + 2/21*(7*(a*x^(1/3)
+ b)^(9/2)*a^48 - 45*(a*x^(1/3) + b)^(7/2)*a^48*b + 126*(a*x^(1/3) + b)^(5/
2)*a^48*b^2 - 210*(a*x^(1/3) + b)^(3/2)*a^48*b^3 + 315*sqrt(a*x^(1/3) + b)*
a^48*b^4)/a^54
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x^2}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}} \,dx$$

```
[In] int(x^2/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

3.197 $\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=-\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{16b\sqrt{bx^{2/3}+ax}}{a^3\sqrt[3]{x}}$$

```
[Out] -6*x/a/(b*x^(2/3)+a*x)^(1/2)+8*(b*x^(2/3)+a*x)^(1/2)/a^2-16*b*(b*x^(2/3)+a*
x)^(1/2)/a^3/x^(1/3)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2040, 2027, 2039}

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=-\frac{16b\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}}+\frac{8\sqrt{ax+bx^{2/3}}}{a^2}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

```
[In] Int[x/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x)/(a*Sqrt[b*x^(2/3) + a*x]) + (8*Sqrt[b*x^(2/3) + a*x])/a^2 - (16*b*Sq
rt[b*x^(2/3) + a*x])/(a^3*x^(1/3))
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
```

```
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{4\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{a}\\
&= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{(8b)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{3a^2}\\
&= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{16b\sqrt{bx^{2/3}+ax}}{a^3\sqrt[3]{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

$$
\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{2\left(-8b^2\sqrt[3]{x}-4abx^{2/3}+a^2x\right)}{a^3\sqrt{bx^{2/3}+ax}}
$$

```
[In] Integrate[x/(b*x^(2/3) + a*x)^(3/2),x]
[Out] (2*(-8*b^2*x^(1/3) - 4*a*b*x^(2/3) + a^2*x))/(a^3*Sqrt[b*x^(2/3) + a*x])
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(a^{2}x^{\frac{2}{3}}-4ab\,x^{\frac{1}{3}}-8b^{2}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{3}}$	45
default	$\frac{2x\left(b+a\,x^{\frac{1}{3}}\right)\left(a^{2}x^{\frac{2}{3}}-4ab\,x^{\frac{1}{3}}-8b^{2}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}a^{3}}$	45

```
[In] int(x/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x*(b+a*x^(1/3))*(a^2*x^(2/3)-4*a*b*x^(1/3)-8*b^2)/(b*x^(2/3)+a*x)^(3/2)/a
^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(54) = 108$.

Time = 123.33 (sec) , antiderivative size = 1107, normalized size of antiderivative = 16.28

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] ((98304*a^3*b^10 + 81920*a^3*b^9 - 30720*a^3*b^8 + 1456*a^9 + 6144*(16*a^6
- 3*a^3)*b^7 + 6784*(8*a^6 + a^3)*b^6 - 192*(236*a^6 + a^3)*b^5 + 24*(1536*
a^9 - 2512*a^6 - 3*a^3)*b^4 + 32*(576*a^9 + 379*a^6)*b^3 - 12*(2304*a^9 + 6
1*a^6)*b^2 - 3*(10112*a^9 + 15*a^6)*b)*x^2 + (98304*b^13 + 81920*b^12 + 614
4*(16*a^3 - 3)*b^10 - 30720*b^11 + 6784*(8*a^3 + 1)*b^9 + 1456*a^6*b^3 - 19
2*(236*a^3 + 1)*b^8 + 24*(1536*a^6 - 2512*a^3 - 3)*b^7 + 32*(576*a^6 + 379*
a^3)*b^6 - 12*(2304*a^6 + 61*a^3)*b^5 - 3*(10112*a^6 + 15*a^3)*b^4)*x + 2*(
(4096*a^4*b^9 + 6144*a^4*b^8 + 768*a^4*b^7 - 4096*a^10 - 144*a^7*b^2 + 216*
a^7*b - 27*a^7 + 256*(16*a^7 - 7*a^4)*b^6 + 48*(128*a^7 - 3*a^4)*b^5 + 24*(
32*a^7 + 9*a^4)*b^4 - (5888*a^7 + 27*a^4)*b^3)*x^2 - 3*(4096*a^2*b^11 + 614
4*a^2*b^10 + 768*a^2*b^9 - 144*a^5*b^4 + 256*(16*a^5 - 7*a^2)*b^8 + 216*a^5
*b^3 + 48*(128*a^5 - 3*a^2)*b^7 + 24*(32*a^5 + 9*a^2)*b^6 - (5888*a^5 + 27*
a^2)*b^5 - (4096*a^8 + 27*a^5)*b^2)*x^(4/3) + 4*(4096*a*b^12 + 6144*a*b^11
+ 768*a*b^10 + 256*(16*a^4 - 7*a)*b^9 - 144*a^4*b^5 + 48*(128*a^4 - 3*a)*b^
8 + 216*a^4*b^4 + 24*(32*a^4 + 9*a)*b^7 - (5888*a^4 + 27*a)*b^6 - (4096*a^7
 + 27*a^4)*b^3)*x - (32768*b^13 + 49152*b^12 + 2048*(16*a^3 - 7)*b^10 + 614
4*b^11 + 384*(128*a^3 - 3)*b^9 - 1152*a^3*b^6 + 192*(32*a^3 + 9)*b^8 + 1728
*a^3*b^5 - 8*(5888*a^3 + 27)*b^7 - 8*(4096*a^6 + 27*a^3)*b^4 + 5*(4096*a^3*
```

```
b^10 + 6144*a^3*b^9 + 768*a^3*b^8 - 144*a^6*b^3 + 216*a^6*b^2 + 256*(16*a^6
 - 7*a^3)*b^7 + 48*(128*a^6 - 3*a^3)*b^6 + 24*(32*a^6 + 9*a^3)*b^5 - (5888*
a^6 + 27*a^3)*b^4 - (4096*a^9 + 27*a^6)*b)*x)*x^(2/3))*sqrt(a*x + b*x^(2/3)
))/((4096*a^6*b^9 + 6144*a^6*b^8 + 768*a^6*b^7 - 4096*a^12 - 144*a^9*b^2 +
216*a^9*b - 27*a^9 + 256*(16*a^9 - 7*a^6)*b^6 + 48*(128*a^9 - 3*a^6)*b^5 +
24*(32*a^9 + 9*a^6)*b^4 - (5888*a^9 + 27*a^6)*b^3)*x^2 + (4096*a^3*b^12 + 6
144*a^3*b^11 + 768*a^3*b^10 - 144*a^6*b^5 + 216*a^6*b^4 + 256*(16*a^6 - 7*a
^3)*b^9 + 48*(128*a^6 - 3*a^3)*b^8 + 24*(32*a^6 + 9*a^3)*b^7 - (5888*a^6 +
27*a^3)*b^6 - (4096*a^9 + 27*a^6)*b^3)*x)
```

Sympy [F]

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{x}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x/(a*x + b*x**(2/3))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{x}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x/(a*x + b*x^(2/3))^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{16\,b^{\frac{3}{2}}}{a^3} - \frac{6\,b^2}{\sqrt{ax^{\frac{1}{3}}+b}a^3} + \frac{2\left(\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^6 - 6\,\sqrt{ax^{\frac{1}{3}}+b}a^6b\right)}{a^9}$$

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
[Out] 16*b^(3/2)/a^3 - 6*b^2/(sqrt(a*x^(1/3) + b)*a^3) + 2*((a*x^(1/3) + b)^(3/2)
*a^6 - 6*sqrt(a*x^(1/3) + b)*a^6*b)/a^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{x}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}\,dx$$

```
[In] int(x/(a*x + b*x^(2/3))^(3/2),x)
[Out] int(x/(a*x + b*x^(2/3))^(3/2), x)
```

3.198 $\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}$$

```
[Out] -6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)+6*x^(1/3)/b/(b*x^
(2/3)+a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2031, 2054, 212}

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

```
[In] Int[(b*x^(2/3) + a*x)^(-3/2),x]
```

```
[Out] (6*x^(1/3))/(b*Sqrt[b*x^(2/3) + a*x]) - (6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}+\frac{\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b}\\
&= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$
\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6\sqrt{bx^{2/3}+ax}}{b\left(b+a\sqrt[3]{x}\right)\sqrt[3]{x}}-\frac{6\text{arctanh}\left(\frac{\sqrt{bx^{2/3}+ax}}{\sqrt{b}\sqrt[3]{x}}\right)}{b^{3/2}}
$$

```
[In] Integrate[(b*x^(2/3) + a*x)^(-3/2),x]
```

```
[Out] (6*Sqrt[b*x^(2/3) + a*x])/(b*(b + a*x^(1/3))*x^(1/3)) - (6*ArcTanh[Sqrt[b*x
^(2/3) + a*x]/(Sqrt[b]*x^(1/3))])/b^(3/2)
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{6x\left(b+a\,x^{\frac{1}{3}}\right)\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b\sqrt{b+a\,x^{\frac{1}{3}}}-b^{\frac{3}{2}}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	56
default	$-\frac{6x\left(b+a\,x^{\frac{1}{3}}\right)\left(\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)b\sqrt{b+a\,x^{\frac{1}{3}}}-b^{\frac{3}{2}}\right)}{\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$	56

```
[In] int(1/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6*x*(b+a*x^(1/3))*(arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b*(b+a*x^(1/3))^(1
/2)-b^(3/2))/(b*x^(2/3)+a*x)^(3/2)/b^(5/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(-3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}-\frac{6\left(\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\right)}{\sqrt{-b}b^{\frac{3}{2}}}+\frac{6}{\sqrt{ax^{\frac{1}{3}}+b}b}$$

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) - 6*(sqrt(b)*arctan(sqr
t(b)/sqrt(-b)) + sqrt(-b))/(sqrt(-b)*b^(3/2)) + 6/(sqrt(a*x^(1/3) + b)*b)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx=-\frac{2\,x\,{\left(\frac{b}{a\,x^{1/3}}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{3}{2};\ \frac{5}{2};\ -\frac{b}{a\,x^{1/3}}\right)}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}$$

```
[In] int(1/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] -(2*x*(b/(a*x^(1/3)) + 1)^(3/2)*hypergeom([3/2, 3/2], 5/2, -b/(a*x^(1/3))))
/(a*x + b*x^(2/3))^(3/2)
```

3.199 $\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 146

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x} - \frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}} + \frac{105a^3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{9/2}}$$

```
[Out] 105/8*a^3*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)+6/b/x^(2/3
)/(b*x^(2/3)+a*x)^(1/2)-7*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)+35/4*a*(b*x^(2/
3)+a*x)^(1/2)/b^3/x-105/8*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2048, 2050, 2054, 212}

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{105a^3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{9/2}} - \frac{105a^2\sqrt{ax+bx^{2/3}}}{8b^4x^{2/3}} + \frac{35a\sqrt{ax+bx^{2/3}}}{4b^3x} - \frac{7\sqrt{ax+bx^{2/3}}}{b^2x^{4/3}} + \frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

```
[In] Int[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(2/3)*Sqrt[b*x^(2/3) + a*x]) - (7*Sqrt[b*x^(2/3) + a*x])/(b^2*x^(4/3
)) + (35*a*Sqrt[b*x^(2/3) + a*x])/(4*b^3*x) - (105*a^2*Sqrt[b*x^(2/3) + a*x
```

```
])/(8*b^4*x^(2/3)) + (105*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*
x]])/(8*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}+\frac{7\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}-\frac{(35a)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{6b^2}\\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}+\frac{(35a^2)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{8b^3}\\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}\\
&\quad-\frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}}-\frac{(35a^3)\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{16b^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x} \\
&\quad - \frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}} + \frac{(105a^3)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^4} \\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x} \\
&\quad - \frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}} + \frac{105a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.70 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.75

$$
\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{-\sqrt{b}\left(8b^3-14ab^2\sqrt[3]{x}+35a^2bx^{2/3}+105a^3x\right)+105a^3\sqrt{b+a\sqrt[3]{x}}\,x\,\text{arctanh}\left(\frac{\sqrt{b+a\sqrt[3]{}}}{\sqrt{b}}\right.}{8b^{9/2}x^{2/3}\sqrt{bx^{2/3}+ax}}
$$

```
[In] Integrate[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (-(Sqrt[b]*(8*b^3 - 14*a*b^2*x^(1/3) + 35*a^2*b*x^(2/3) + 105*a^3*x)) + 105
*a^3*Sqrt[b + a*x^(1/3)]*x*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]])/(8*b^(9/2)
*x^(2/3)*Sqrt[b*x^(2/3) + a*x])
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

method	result	size
derivativedivides	$\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(105\ \text{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{3}x+14b^{\frac{5}{2}}\,a\,x^{\frac{1}{3}}-35b^{\frac{3}{2}}\,a^{2}x^{\frac{2}{3}}-105\sqrt{b}\,a^{3}x-8b^{\frac{7}{2}}\right)}{8\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{9}{2}}}$	88
default	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(105\sqrt{b}\,a^{3}x+35b^{\frac{3}{2}}\,a^{2}x^{\frac{2}{3}}-14b^{\frac{5}{2}}\,a\,x^{\frac{1}{3}}-105\ \text{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{3}x+8b^{\frac{7}{2}}\right)}{8\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{9}{2}}}$	88

```
[In] int(1/x/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(b+a*x^(1/3))*(105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*(b+a*x^(1/3))^(
1/2)*a^3*x+14*b^(5/2)*a*x^(1/3)-35*b^(3/2)*a^2*x^(2/3)-105*b^(1/2)*a^3*x-8*
b^(7/2))/(b*x^(2/3)+a*x)^(3/2)/b^(9/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(1/(x*(a*x + b*x**(2/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=-\frac{105\,a^{3}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{8\,\sqrt{-b}b^{4}}-\frac{6\,a^{3}}{\sqrt{ax^{\frac{1}{3}}+b}b^{4}}-\frac{57\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{3}-136\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{3}b+87\sqrt{ax^{\frac{1}{3}}+b}a^{3}b^{2}}{8\,a^{3}b^{4}x}$$

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
[Out] -105/8*a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) - 6*a^3/(sqr
t(a*x^(1/3) + b)*b^4) - 1/8*(57*(a*x^(1/3) + b)^(5/2)*a^3 - 136*(a*x^(1/3)
+ b)^(3/2)*a^3*b + 87*sqrt(a*x^(1/3) + b)*a^3*b^2)/(a^3*b^4*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}} \,dx$$

```
[In] int(1/(x*(a*x + b*x^(2/3))^(3/2)),x)
[Out] int(1/(x*(a*x + b*x^(2/3))^(3/2)), x)
```

3.200 $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 236

$$\begin{aligned}\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}\\ &\quad+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx^{2/3}+ax}}{320b^5x^{4/3}}\\ &\quad-\frac{3003a^4\sqrt{bx^{2/3}+ax}}{256b^6x}+\frac{9009a^5\sqrt{bx^{2/3}+ax}}{512b^7x^{2/3}}-\frac{9009a^6\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{512b^{15/2}}\end{aligned}$$

```
[Out] -9009/512*a^6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)+6/b/x
^(5/3)/(b*x^(2/3)+a*x)^(1/2)-13/2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)+143/20*
a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2-1287/160*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(5
/3)+3003/320*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)-3003/256*a^4*(b*x^(2/3)+
a*x)^(1/2)/b^6/x+9009/512*a^5*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used

$= \{2048, 2050, 2054, 212\}$

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{9009a^6\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{15/2}}+\frac{9009a^5\sqrt{ax+bx^{2/3}}}{512b^7x^{2/3}} - \frac{3003a^4\sqrt{ax+bx^{2/3}}}{256b^6x}+\frac{3003a^3\sqrt{ax+bx^{2/3}}}{320b^5x^{4/3}}-\frac{1287a^2\sqrt{ax+bx^{2/3}}}{160b^4x^{5/3}} + \frac{143a\sqrt{ax+bx^{2/3}}}{20b^3x^2}-\frac{13\sqrt{ax+bx^{2/3}}}{2b^2x^{7/3}}+\frac{6}{bx^{5/3}\sqrt{ax+bx^{2/3}}}$$

```
[In] Int[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(5/3)*Sqrt[b*x^(2/3) + a*x]) - (13*Sqrt[b*x^(2/3) + a*x])/(2*b^2*x^(
7/3)) + (143*a*Sqrt[b*x^(2/3) + a*x])/(20*b^3*x^2) - (1287*a^2*Sqrt[b*x^(2/
3) + a*x])/(160*b^4*x^(5/3)) + (3003*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^5*x^
(4/3)) - (3003*a^4*Sqrt[b*x^(2/3) + a*x])/(256*b^6*x) + (9009*a^5*Sqrt[b*x^
(2/3) + a*x])/(512*b^7*x^(2/3)) - (9009*a^6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[
b*x^(2/3) + a*x]])/(512*b^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}+\frac{13\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}-\frac{(143a)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{12b^2}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}+\frac{\left(429a^2\right)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{40b^3}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}\\
&\quad-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}-\frac{\left(3003a^3\right)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{320b^4}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}\\
&\quad-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx^{2/3}+ax}}{320b^5x^{4/3}}+\frac{\left(1001a^4\right)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{128b^5}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}\\
&\quad+\frac{3003a^3\sqrt{bx^{2/3}+ax}}{320b^5x^{4/3}}-\frac{3003a^4\sqrt{bx^{2/3}+ax}}{256b^6x}-\frac{\left(3003a^5\right)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{512b^6}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}\\
&\quad-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx^{2/3}+ax}}{320b^5x^{4/3}}-\frac{3003a^4\sqrt{bx^{2/3}+ax}}{256b^6x}\\
&\quad+\frac{9009a^5\sqrt{bx^{2/3}+ax}}{512b^7x^{2/3}}+\frac{\left(3003a^6\right)\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{1024b^7}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}\\
&\quad-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx^{2/3}+ax}}{320b^5x^{4/3}}-\frac{3003a^4\sqrt{bx^{2/3}+ax}}{256b^6x}\\
&\quad+\frac{9009a^5\sqrt{bx^{2/3}+ax}}{512b^7x^{2/3}}-\frac{\left(9009a^6\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{512b^7}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}} - \frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}} + \frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2} \\
&\quad - \frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}} + \frac{3003a^3\sqrt{bx^{2/3}+ax}}{320b^5x^{4/3}} - \frac{3003a^4\sqrt{bx^{2/3}+ax}}{256b^6x} \\
&\quad + \frac{9009a^5\sqrt{bx^{2/3}+ax}}{512b^7x^{2/3}} - \frac{9009a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{512b^{15/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.20

$$
\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6a^6\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},7,\frac{1}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{b^7\sqrt{bx^{2/3}+ax}}
$$

```
[In] Integrate[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (6*a^6*x^(1/3)*Hypergeometric2F1[-1/2, 7, 1/2, 1 + (a*x^(1/3))/b])/(b^7*Sqr
t[b*x^(2/3) + a*x])
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.53

method	result
derivativedivides	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(2288b^{\frac{9}{2}}a^2x^{\frac{2}{3}}-3432b^{\frac{7}{2}}a^3x+6006b^{\frac{5}{2}}a^4x^{\frac{4}{3}}-15015b^{\frac{3}{2}}a^5x^{\frac{5}{3}}+1280b^{\frac{13}{2}}-45045a^6x^2\sqrt{b}+45045\sqrt{b+a\,x^{\frac{1}{3}}}\ \mathrm{ar}\right.}{2560x\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{15}{2}}}$
default	$\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(-2288b^{\frac{9}{2}}a^2x^{\frac{2}{3}}-1280b^{\frac{13}{2}}-45045\sqrt{b+a\,x^{\frac{1}{3}}}\ \operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)a^6x^2+3432b^{\frac{7}{2}}a^3x-6006b^{\frac{5}{2}}a^4x^{\frac{4}{3}}+15015\right.}{2560x\left(b\,x^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}b^{\frac{15}{2}}}$

```
[In] int(1/x^2/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2560*(b+a*x^(1/3))*(2288*b^(9/2)*a^2*x^(2/3)-3432*b^(7/2)*a^3*x+6006*b^(
5/2)*a^4*x^(4/3)-15015*b^(3/2)*a^5*x^(5/3)+1280*b^(13/2)-45045*a^6*x^2*b^(1
/2)+45045*(b+a*x^(1/3))^(1/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*a^6*x^2-
1664*b^(11/2)*a*x^(1/3))/x/(b*x^(2/3)+a*x)^(3/2)/b^(15/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2 \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*x + b*x**(2/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{9009\,a^6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{512\sqrt{-b}b^7}+\frac{6\,a^6}{\sqrt{ax^{\frac{1}{3}}+b}b^7}$$
$$+\frac{29685\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^6-163095\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^6b+364194\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^6b^2-416094\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^6b^3+2465}{2560\,a^6b^7x^2}$$

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 9009/512*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + 6*a^6/(s
qrt(a*x^(1/3) + b)*b^7) + 1/2560*(29685*(a*x^(1/3) + b)^(11/2)*a^6 - 163095
*(a*x^(1/3) + b)^(9/2)*a^6*b + 364194*(a*x^(1/3) + b)^(7/2)*a^6*b^2 - 41609
4*(a*x^(1/3) + b)^(5/2)*a^6*b^3 + 246505*(a*x^(1/3) + b)^(3/2)*a^6*b^4 - 62
475*sqrt(a*x^(1/3) + b)*a^6*b^5)/(a^6*b^7*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^2\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}\,dx$$

```
[In] int(1/(x^2*(a*x + b*x^(2/3))^(3/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(2/3))^(3/2)), x)
```

3.201 $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 324

$$\begin{aligned}
\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} \\
&+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}} \\
&-\frac{46189a^4\sqrt{bx^{2/3}+ax}}{5376b^6x^2}+\frac{138567a^5\sqrt{bx^{2/3}+ax}}{14336b^7x^{5/3}}-\frac{46189a^6\sqrt{bx^{2/3}+ax}}{4096b^8x^{4/3}} \\
&+\frac{230945a^7\sqrt{bx^{2/3}+ax}}{16384b^9x}-\frac{692835a^8\sqrt{bx^{2/3}+ax}}{32768b^{10}x^{2/3}}+\frac{692835a^9\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{32768b^{21/2}}
\end{aligned}$$

```
[Out] 692835/32768*a^9*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)+6/
b/x^(8/3)/(b*x^(2/3)+a*x)^(1/2)-19/3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(10/3)+323
/48*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3-1615/224*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/
x^(8/3)+20995/2688*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)-46189/5376*a^4*(b*
x^(2/3)+a*x)^(1/2)/b^6/x^2+138567/14336*a^5*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(5/
3)-46189/4096*a^6*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)+230945/16384*a^7*(b*x^(
2/3)+a*x)^(1/2)/b^9/x-692835/32768*a^8*(b*x^(2/3)+a*x)^(1/2)/b^10/x^(2/3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2048, 2050, 2054, 212}

$$\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{692835a^9\text{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{32768b^{21/2}} - \frac{692835a^8\sqrt{ax+bx^{2/3}}}{32768b^{10}x^{2/3}} + \frac{230945a^7\sqrt{ax+bx^{2/3}}}{16384b^9x} - \frac{46189a^6\sqrt{ax+bx^{2/3}}}{4096b^8x^{4/3}} + \frac{138567a^5\sqrt{ax+bx^{2/3}}}{14336b^7x^{5/3}} - \frac{46189a^4\sqrt{ax+bx^{2/3}}}{5376b^6x^2} + \frac{20995a^3\sqrt{ax+bx^{2/3}}}{2688b^5x^{7/3}} - \frac{1615a^2\sqrt{ax+bx^{2/3}}}{224b^4x^{8/3}} + \frac{323a\sqrt{ax+bx^{2/3}}}{48b^3x^3} - \frac{19\sqrt{ax+bx^{2/3}}}{3b^2x^{10/3}} + \frac{6}{bx^{8/3}\sqrt{ax+bx^{2/3}}}$$

```
[In] Int[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]) - (19*Sqrt[b*x^(2/3) + a*x])/(3*b^2*x^(
10/3)) + (323*a*Sqrt[b*x^(2/3) + a*x])/(48*b^3*x^3) - (1615*a^2*Sqrt[b*x^(2
/3) + a*x])/(224*b^4*x^(8/3)) + (20995*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^5
*x^(7/3)) - (46189*a^4*Sqrt[b*x^(2/3) + a*x])/(5376*b^6*x^2) + (138567*a^5*
Sqrt[b*x^(2/3) + a*x])/(14336*b^7*x^(5/3)) - (46189*a^6*Sqrt[b*x^(2/3) + a*
x])/(4096*b^8*x^(4/3)) + (230945*a^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^9*x) -
 (692835*a^8*Sqrt[b*x^(2/3) + a*x])/(32768*b^10*x^(2/3)) + (692835*a^9*ArcT
anh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(21/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
 !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\text{integral} = \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}+\frac{19\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}-\frac{(323a)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{18b^2}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}+\frac{\left(1615a^2\right)\int \frac{1}{x^{3}\sqrt{bx^{2/3}+ax}}\,dx}{96b^3}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}-\frac{\left(20995a^3\right)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{1344b^4}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}}+\frac{\left(230945a^4\right)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{16128b^5}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}}-\frac{46189a^4\sqrt{bx^{2/3}+ax}}{5376b^6x^2}-\frac{\left(46189a^5\right)\int \frac{1}{x^{2}\sqrt{bx^{2/3}+ax}}\,dx}{3584b^6}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}}-\frac{46189a^4\sqrt{bx^{2/3}+ax}}{5376b^6x^2}+\frac{138567a^5\sqrt{bx^{2/3}+ax}}{14336b^7x^{5/3}}+\frac{\left(46189a^6\right)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{4096b^7}$$

$$
\begin{aligned}
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} \\
&\quad - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} + \frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}} - \frac{46189a^4\sqrt{bx^{2/3}+ax}}{5376b^6x^2} \\
&\quad + \frac{138567a^5\sqrt{bx^{2/3}+ax}}{14336b^7x^{5/3}} - \frac{46189a^6\sqrt{bx^{2/3}+ax}}{4096b^8x^{4/3}} - \frac{(230945a^7)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{24576b^8}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} \\
&\quad + \frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}} - \frac{46189a^4\sqrt{bx^{2/3}+ax}}{5376b^6x^2} + \frac{138567a^5\sqrt{bx^{2/3}+ax}}{14336b^7x^{5/3}} \\
&\quad - \frac{46189a^6\sqrt{bx^{2/3}+ax}}{4096b^8x^{4/3}} + \frac{230945a^7\sqrt{bx^{2/3}+ax}}{16384b^9x} + \frac{(230945a^8)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{32768b^9}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} \\
&\quad - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} + \frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}} - \frac{46189a^4\sqrt{bx^{2/3}+ax}}{5376b^6x^2} \\
&\quad + \frac{138567a^5\sqrt{bx^{2/3}+ax}}{14336b^7x^{5/3}} - \frac{46189a^6\sqrt{bx^{2/3}+ax}}{4096b^8x^{4/3}} + \frac{230945a^7\sqrt{bx^{2/3}+ax}}{16384b^9x} \\
&\quad - \frac{692835a^8\sqrt{bx^{2/3}+ax}}{32768b^{10}x^{2/3}} - \frac{(230945a^9)\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{65536b^{10}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} \\
&\quad - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} + \frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}} - \frac{46189a^4\sqrt{bx^{2/3}+ax}}{5376b^6x^2} \\
&\quad + \frac{138567a^5\sqrt{bx^{2/3}+ax}}{14336b^7x^{5/3}} - \frac{46189a^6\sqrt{bx^{2/3}+ax}}{4096b^8x^{4/3}} + \frac{230945a^7\sqrt{bx^{2/3}+ax}}{16384b^9x} \\
&\quad - \frac{692835a^8\sqrt{bx^{2/3}+ax}}{32768b^{10}x^{2/3}} + \frac{(692835a^9)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{32768b^{10}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}} - \frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}} + \frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3} \\
&\quad - \frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}} + \frac{20995a^3\sqrt{bx^{2/3}+ax}}{2688b^5x^{7/3}} - \frac{46189a^4\sqrt{bx^{2/3}+ax}}{5376b^6x^2} \\
&\quad + \frac{138567a^5\sqrt{bx^{2/3}+ax}}{14336b^7x^{5/3}} - \frac{46189a^6\sqrt{bx^{2/3}+ax}}{4096b^8x^{4/3}} + \frac{230945a^7\sqrt{bx^{2/3}+ax}}{16384b^9x} \\
&\quad - \frac{692835a^8\sqrt{bx^{2/3}+ax}}{32768b^{10}x^{2/3}} + \frac{692835a^9\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{32768b^{21/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx=-\frac{6a^9\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},10,\frac{1}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{b^{10}\sqrt{bx^{2/3}+ax}}$$

```
[In] Integrate[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (-6*a^9*x^(1/3)*Hypergeometric2F1[-1/2, 10, 1/2, 1 + (a*x^(1/3))/b])/(b^10*
Sqrt[b*x^(2/3) + a*x])
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.49

method	result
derivativedivides	$\frac{(b+a\,x^{\frac{1}{3}})\,(-229376b^{\frac{19}{2}}+272384b^{\frac{17}{2}}a\,x^{\frac{1}{3}}-330752b^{\frac{15}{2}}a^2x^{\frac{2}{3}}+413440b^{\frac{13}{2}}a^3x-537472b^{\frac{11}{2}}a^4x^{\frac{4}{3}}+739024b^{\frac{9}{2}}a^5x^{\frac{5}{3}}-11085}{688128x^2\,(b\,x^{\frac{2}{3}}}$
default	$-\frac{(b+a\,x^{\frac{1}{3}})\,(-272384b^{\frac{17}{2}}a\,x^{\frac{1}{3}}+229376b^{\frac{19}{2}}+330752b^{\frac{15}{2}}a^2x^{\frac{2}{3}}-413440b^{\frac{13}{2}}a^3x+537472b^{\frac{11}{2}}a^4x^{\frac{4}{3}}-739024b^{\frac{9}{2}}a^5x^{\frac{5}{3}}+110}{688128x^2\,(b}$

```
[In] int(1/x^3/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/688128*(b+a*x^(1/3))*(-229376*b^(19/2)+272384*b^(17/2)*a*x^(1/3)-330752*b
^(15/2)*a^2*x^(2/3)+413440*b^(13/2)*a^3*x-537472*b^(11/2)*a^4*x^(4/3)+73902
4*b^(9/2)*a^5*x^(5/3)-1108536*b^(7/2)*a^6*x^2+1939938*b^(5/2)*a^7*x^(7/3)-4
849845*b^(3/2)*a^8*x^(8/3)-14549535*a^9*x^3*b^(1/2)+14549535*arctanh((b+a*x
^(1/3))^(1/2)/b^(1/2))*(b+a*x^(1/3))^(1/2)*a^9*x^3)/x^2/(b*x^(2/3)+a*x)^(3/
2)/b^(21/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a*x + b*x**(2/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{692835\,a^9\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{32768\sqrt{-b}b^{10}} - \frac{6\,a^9}{\sqrt{ax^{\frac{1}{3}}+b}b^{10}}$$
$$-\frac{10420767\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^9 - 88937058\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^9b + 334408914\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^9b^2 - 724860666\left(ax^{\frac{1}{3}}+\right.}{}$$

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -692835/32768*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) - 6*
a^9/(sqrt(a*x^(1/3) + b)*b^10) - 1/688128*(10420767*(a*x^(1/3) + b)^(17/2)*
a^9 - 88937058*(a*x^(1/3) + b)^(15/2)*a^9*b + 334408914*(a*x^(1/3) + b)^(13
/2)*a^9*b^2 - 724860666*(a*x^(1/3) + b)^(11/2)*a^9*b^3 + 993296384*(a*x^(1/
3) + b)^(9/2)*a^9*b^4 - 884769030*(a*x^(1/3) + b)^(7/2)*a^9*b^5 + 503730990
*(a*x^(1/3) + b)^(5/2)*a^9*b^6 - 169799070*(a*x^(1/3) + b)^(3/2)*a^9*b^7 +
26738145*sqrt(a*x^(1/3) + b)*a^9*b^8)/(a^9*b^10*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(bx^{2/3}+ax\right)^{3/2}}\,dx=\int \frac{1}{x^3\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}} \,dx$$

```
[In] int(1/(x^3*(a*x + b*x^(2/3))^(3/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(2/3))^(3/2)), x)
```

3.202 $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 412

$$\begin{aligned}\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}\\ &-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}}-\frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3}\\ &+\frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}}-\frac{2414425a^6\sqrt{bx^{2/3}+ax}}{270336b^8x^{7/3}}+\frac{482885a^7\sqrt{bx^{2/3}+ax}}{49152b^9x^2}\\ &-\frac{1448655a^8\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{5/3}}+\frac{3380195a^9\sqrt{bx^{2/3}+ax}}{262144b^{11}x^{4/3}}-\frac{16900975a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^{12}x}\\ &+\frac{50702925a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{13}x^{2/3}}-\frac{50702925a^{12}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{2097152b^{27/2}}\end{aligned}$$

```
[Out] -50702925/2097152*a^12*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(27
/2)+6/b/x^(11/3)/(b*x^(2/3)+a*x)^(1/2)-25/4*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(13
/3)+575/88*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^4-2415/352*a^2*(b*x^(2/3)+a*x)^(1/
2)/b^4/x^(11/3)+15295/2112*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(10/3)-260015/33
792*a^4*(b*x^(2/3)+a*x)^(1/2)/b^6/x^3+185725/22528*a^5*(b*x^(2/3)+a*x)^(1/2
)/b^7/x^(8/3)-2414425/270336*a^6*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(7/3)+482885/4
9152*a^7*(b*x^(2/3)+a*x)^(1/2)/b^9/x^2-1448655/131072*a^8*(b*x^(2/3)+a*x)^(
1/2)/b^10/x^(5/3)+3380195/262144*a^9*(b*x^(2/3)+a*x)^(1/2)/b^11/x^(4/3)-169
00975/1048576*a^10*(b*x^(2/3)+a*x)^(1/2)/b^12/x+50702925/2097152*a^11*(b*x^
(2/3)+a*x)^(1/2)/b^13/x^(2/3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2048, 2050, 2054, 212}

$$\begin{aligned}\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = &-\frac{50702925a^{12}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{2097152b^{27/2}} \\ &+\frac{50702925a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{13}x^{2/3}}-\frac{16900975a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^{12}x}+\frac{3380195a^9\sqrt{ax+bx^{2/3}}}{262144b^{11}x^{4/3}} \\ &-\frac{1448655a^8\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{5/3}}+\frac{482885a^7\sqrt{ax+bx^{2/3}}}{49152b^9x^2}-\frac{2414425a^6\sqrt{ax+bx^{2/3}}}{270336b^8x^{7/3}} \\ &+\frac{185725a^5\sqrt{ax+bx^{2/3}}}{22528b^7x^{8/3}}-\frac{260015a^4\sqrt{ax+bx^{2/3}}}{33792b^6x^3}+\frac{15295a^3\sqrt{ax+bx^{2/3}}}{2112b^5x^{10/3}} \\ &-\frac{2415a^2\sqrt{ax+bx^{2/3}}}{352b^4x^{11/3}}+\frac{575a\sqrt{ax+bx^{2/3}}}{88b^3x^4}-\frac{25\sqrt{ax+bx^{2/3}}}{4b^2x^{13/3}}+\frac{6}{bx^{11/3}\sqrt{ax+bx^{2/3}}}\end{aligned}$$

```
[In] Int[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(11/3)*Sqrt[b*x^(2/3) + a*x]) - (25*Sqrt[b*x^(2/3) + a*x])/(4*b^2*x^
(13/3)) + (575*a*Sqrt[b*x^(2/3) + a*x])/(88*b^3*x^4) - (2415*a^2*Sqrt[b*x^(
2/3) + a*x])/(352*b^4*x^(11/3)) + (15295*a^3*Sqrt[b*x^(2/3) + a*x])/(2112*b
^5*x^(10/3)) - (260015*a^4*Sqrt[b*x^(2/3) + a*x])/(33792*b^6*x^3) + (185725
*a^5*Sqrt[b*x^(2/3) + a*x])/(22528*b^7*x^(8/3)) - (2414425*a^6*Sqrt[b*x^(2/
3) + a*x])/(270336*b^8*x^(7/3)) + (482885*a^7*Sqrt[b*x^(2/3) + a*x])/(49152
*b^9*x^2) - (1448655*a^8*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(5/3)) + (33
80195*a^9*Sqrt[b*x^(2/3) + a*x])/(262144*b^11*x^(4/3)) - (16900975*a^10*Sqr
t[b*x^(2/3) + a*x])/(1048576*b^12*x) + (50702925*a^11*Sqrt[b*x^(2/3) + a*x]
)/(2097152*b^13*x^(2/3)) - (50702925*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*
x^(2/3) + a*x]])/(2097152*b^(27/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
```

```
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}+\frac{25\int \frac{1}{x^{14/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}-\frac{(575a)\int \frac{1}{x^{13/3}\sqrt{bx^{2/3}+ax}}\,dx}{24b^2}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}+\frac{\left(4025a^2\right)\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx}{176b^3}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}\\
&\quad -\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}-\frac{\left(15295a^3\right)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{704b^4}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}\\
&\quad -\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}}+\frac{\left(260015a^4\right)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{12672b^5}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}\\
&\quad +\frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}}-\frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3}-\frac{\left(1300075a^5\right)\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx}{67584b^6}
\end{aligned}
$$

$$\begin{aligned}
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} \\
&\quad - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} + \frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}} - \frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3} \\
&\quad + \frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}} + \frac{(2414425a^6)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{135168b^7}
\end{aligned}$$

$$\begin{aligned}
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} \\
&\quad + \frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}} - \frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3} + \frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}} \\
&\quad - \frac{2414425a^6\sqrt{bx^{2/3}+ax}}{270336b^8x^{7/3}} - \frac{(2414425a^7)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{147456b^8}
\end{aligned}$$

$$\begin{aligned}
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} \\
&\quad + \frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}} - \frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3} + \frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}} \\
&\quad - \frac{2414425a^6\sqrt{bx^{2/3}+ax}}{270336b^8x^{7/3}} + \frac{482885a^7\sqrt{bx^{2/3}+ax}}{49152b^9x^2} + \frac{(482885a^8)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{32768b^9}
\end{aligned}$$

$$\begin{aligned}
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} \\
&\quad - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} + \frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}} - \frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3} \\
&\quad + \frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}} - \frac{2414425a^6\sqrt{bx^{2/3}+ax}}{270336b^8x^{7/3}} + \frac{482885a^7\sqrt{bx^{2/3}+ax}}{49152b^9x^2} \\
&\quad - \frac{1448655a^8\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{5/3}} - \frac{(3380195a^9)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{262144b^{10}}
\end{aligned}$$

$$\begin{aligned}
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} \\
&\quad + \frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}} - \frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3} + \frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}} \\
&\quad - \frac{2414425a^6\sqrt{bx^{2/3}+ax}}{270336b^8x^{7/3}} + \frac{482885a^7\sqrt{bx^{2/3}+ax}}{49152b^9x^2} - \frac{1448655a^8\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{5/3}} \\
&\quad + \frac{3380195a^9\sqrt{bx^{2/3}+ax}}{262144b^{11}x^{4/3}} + \frac{(16900975a^{10})\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{1572864b^{11}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} \\
&\quad -\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}}-\frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3} \\
&\quad +\frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}}-\frac{2414425a^6\sqrt{bx^{2/3}+ax}}{270336b^8x^{7/3}}+\frac{482885a^7\sqrt{bx^{2/3}+ax}}{49152b^9x^2} \\
&\quad -\frac{1448655a^8\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{5/3}}+\frac{3380195a^9\sqrt{bx^{2/3}+ax}}{262144b^{11}x^{4/3}} \\
&\quad -\frac{16900975a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^{12}x}-\frac{(16900975a^{11})\int\frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{2097152b^{12}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} \\
&\quad -\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}}-\frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3} \\
&\quad +\frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}}-\frac{2414425a^6\sqrt{bx^{2/3}+ax}}{270336b^8x^{7/3}}+\frac{482885a^7\sqrt{bx^{2/3}+ax}}{49152b^9x^2} \\
&\quad -\frac{1448655a^8\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{5/3}}+\frac{3380195a^9\sqrt{bx^{2/3}+ax}}{262144b^{11}x^{4/3}}-\frac{16900975a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^{12}x} \\
&\quad +\frac{50702925a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{13}x^{2/3}}+\frac{(16900975a^{12})\int\frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{4194304b^{13}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} \\
&\quad -\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}}-\frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3} \\
&\quad +\frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}}-\frac{2414425a^6\sqrt{bx^{2/3}+ax}}{270336b^8x^{7/3}}+\frac{482885a^7\sqrt{bx^{2/3}+ax}}{49152b^9x^2} \\
&\quad -\frac{1448655a^8\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{5/3}}+\frac{3380195a^9\sqrt{bx^{2/3}+ax}}{262144b^{11}x^{4/3}}-\frac{16900975a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^{12}x} \\
&\quad +\frac{50702925a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{13}x^{2/3}}-\frac{(50702925a^{12})\,\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{2097152b^{13}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} \\
&\quad -\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2/3}+ax}}{2112b^5x^{10/3}}-\frac{260015a^4\sqrt{bx^{2/3}+ax}}{33792b^6x^3} \\
&\quad +\frac{185725a^5\sqrt{bx^{2/3}+ax}}{22528b^7x^{8/3}}-\frac{2414425a^6\sqrt{bx^{2/3}+ax}}{270336b^8x^{7/3}}+\frac{482885a^7\sqrt{bx^{2/3}+ax}}{49152b^9x^2} \\
&\quad -\frac{1448655a^8\sqrt{bx^{2/3}+ax}}{131072b^{10}x^{5/3}}+\frac{3380195a^9\sqrt{bx^{2/3}+ax}}{262144b^{11}x^{4/3}}-\frac{16900975a^{10}\sqrt{bx^{2/3}+ax}}{1048576b^{12}x} \\
&\quad +\frac{50702925a^{11}\sqrt{bx^{2/3}+ax}}{2097152b^{13}x^{2/3}}-\frac{50702925a^{12}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{2097152b^{27/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.12

$$
\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx=\frac{6a^{12}\sqrt[3]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},13,\frac{1}{2},1+\frac{a\sqrt[3]{x}}{b}\right)}{b^{13}\sqrt{bx^{2/3}+ax}}
$$

```
[In] Integrate[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (6*a^12*x^(1/3)*Hypergeometric2F1[-1/2, 13, 1/2, 1 + (a*x^(1/3))/b])/(b^13*
Sqrt[b*x^(2/3) + a*x])
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.47

method	result
derivativedivides	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(17301504b^{\frac{25}{2}}+1673196525\,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{12}x^{4}-1673196525a^{12}x^{4}\sqrt{b}-19660800b^{\frac{23}{2}}\,a\,x\right.}{}$
default	$\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(1673196525a^{12}x^{4}\sqrt{b}-17301504b^{\frac{25}{2}}-1673196525\,\operatorname{arctanh}\left(\frac{\sqrt{b+a\,x^{\frac{1}{3}}}}{\sqrt{b}}\right)\sqrt{b+a\,x^{\frac{1}{3}}}\,a^{12}x^{4}+19660800b^{\frac{23}{2}}\,a\,x^{\frac{1}{3}}\right.}{}$

```
[In] int(1/x^4/(b*x^(2/3)+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/69206016*(b+a*x^(1/3))*(17301504*b^(25/2)+1673196525*arctanh((b+a*x^(1/3
))^(1/2)/b^(1/2))*(b+a*x^(1/3))^(1/2)*a^12*x^4-1673196525*a^12*x^4*b^(1/2)-
```

```
19660800*b^(23/2)*a*x^(1/3)+22609920*b^(21/2)*a^2*x^(2/3)-26378240*b^(19/2)
*a^3*x+31324160*b^(17/2)*a^4*x^(4/3)-38036480*b^(15/2)*a^5*x^(5/3)+47545600
*b^(13/2)*a^6*x^2-61809280*b^(11/2)*a^7*x^(7/3)+84987760*b^(9/2)*a^8*x^(8/3
)-127481640*b^(7/2)*a^9*x^3+223092870*b^(5/2)*a^10*x^(10/3)-557732175*b^(3/
2)*a^11*x^(11/3))/x^3/(b*x^(2/3)+a*x)^(3/2)/b^(27/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a*x + b*x**(2/3))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{50702925\,a^{12}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{2097152\,\sqrt{-b}b^{13}} + \frac{6\,a^{12}}{\sqrt{ax^{\frac{1}{3}}+b}b^{13}}$$

$$+\frac{1257960429\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{12} - 14537792973\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{12}b + 76667241519\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{12}b^2 - 24371761}{}$$

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 50702925/2097152*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^13)
+ 6*a^12/(sqrt(a*x^(1/3) + b)*b^13) + 1/69206016*(1257960429*(a*x^(1/3) + b
)^(23/2)*a^12 - 14537792973*(a*x^(1/3) + b)^(21/2)*a^12*b + 76667241519*(a*
x^(1/3) + b)^(19/2)*a^12*b^2 - 243717614415*(a*x^(1/3) + b)^(17/2)*a^12*b^3
 + 519393101810*(a*x^(1/3) + b)^(15/2)*a^12*b^4 - 780150847218*(a*x^(1/3) +
 b)^(13/2)*a^12*b^5 + 844265343246*(a*x^(1/3) + b)^(11/2)*a^12*b^6 - 659969
685518*(a*x^(1/3) + b)^(9/2)*a^12*b^7 + 366679446705*(a*x^(1/3) + b)^(7/2)*
a^12*b^8 - 138840292305*(a*x^(1/3) + b)^(5/2)*a^12*b^9 + 32660709939*(a*x^(
1/3) + b)^(3/2)*a^12*b^10 - 3724872723*sqrt(a*x^(1/3) + b)*a^12*b^11)/(a^12
*b^13*x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(bx^{2/3}+ax\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

```
[In] int(1/(x^4*(a*x + b*x^(2/3))^(3/2)),x)
```

```
[Out] int(1/(x^4*(a*x + b*x^(2/3))^(3/2)), x)
```

3.203 $\int x^2(ax^2+bx^3)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}$$

```
[Out] 1/5*a*x^5+1/6*b*x^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = $\{14\}$

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}$$

```
[In] Int[x^2*(a*x^2 + b*x^3),x]
```

```
[Out] (a*x^5)/5 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax^4+bx^5\right)\,dx\\&=\frac{ax^5}{5}+\frac{bx^6}{6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}$$

```
[In] Integrate[x^2*(a*x^2 + b*x^3),x]
```

```
[Out] (a*x^5)/5 + (b*x^6)/6
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^{5}\left(5 b x+6 a\right)}{30}$	14
default	$\frac{1}{5} x^{5} a+\frac{1}{6} b \,x^{6}$	14
norman	$\frac{1}{5} x^{5} a+\frac{1}{6} b \,x^{6}$	14
risch	$\frac{1}{5} x^{5} a+\frac{1}{6} b \,x^{6}$	14
parallelrisch	$\frac{1}{5} x^{5} a+\frac{1}{6} b \,x^{6}$	14

```
[In] int(x^2*(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*x^5*(5*b*x+6*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}$$

```
[In] integrate(x**2*(b*x**3+a*x**2),x)
```

```
[Out] a*x**5/5 + b*x**6/6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(ax^2+bx^3\right)\,dx=\frac{x^5\,\left(6\,a+5\,b\,x\right)}{30}$$

```
[In] int(x^2*(a*x^2 + b*x^3),x)
```

```
[Out] (x^5*(6*a + 5*b*x))/30
```

3.204 $\int x(ax^2+bx^3)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 17

$$\int x(ax^2+bx^3)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}$$

```
[Out] 1/4*a*x^4+1/5*b*x^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\int x(ax^2+bx^3)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}$$

```
[In] Int[x*(a*x^2 + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int (ax^3+bx^4)\,dx\\&=\frac{ax^4}{4}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x\left(ax^2+bx^3\right)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}$$

```
[In] Integrate[x*(a*x^2 + b*x^3),x]
[Out] (a*x^4)/4 + (b*x^5)/5
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^{4}\left(4 b x+5 a\right)}{20}$	14
default	$\frac{1}{4} a\,x^{4}+\frac{1}{5} b\,x^{5}$	14
norman	$\frac{1}{4} a\,x^{4}+\frac{1}{5} b\,x^{5}$	14
risch	$\frac{1}{4} a\,x^{4}+\frac{1}{5} b\,x^{5}$	14
parallelrisch	$\frac{1}{4} a\,x^{4}+\frac{1}{5} b\,x^{5}$	14

```
[In] int(x*(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/20*x^4*(4*b*x+5*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(ax^2+bx^3\right)\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="fricas")
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x\left(ax^2+bx^3\right)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}$$

```
[In] integrate(x*(b*x**3+a*x**2),x)
```

```
[Out] a*x**4/4 + b*x**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(ax^2+bx^3\right)\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(ax^2+bx^3\right)\,dx=\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(ax^2+bx^3\right)\,dx=\frac{x^4\,\left(5\,a+4\,b\,x\right)}{20}$$

```
[In] int(x*(a*x^2 + b*x^3),x)
```

```
[Out] (x^4*(5*a + 4*b*x))/20
```

3.205 $\int (ax^2+bx^3)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int (ax^2+bx^3)\,dx=\frac{ax^3}{3}+\frac{bx^4}{4}$$

```
[Out] 1/3*a*x^3+1/4*b*x^4
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (ax^2+bx^3)\,dx=\frac{ax^3}{3}+\frac{bx^4}{4}$$

```
[In] Int[a*x^2 + b*x^3,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Rubi steps

$$\text{integral}=\frac{ax^3}{3}+\frac{bx^4}{4}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(ax^2+bx^3\right)\,dx=\frac{ax^3}{3}+\frac{bx^4}{4}$$

```
[In] Integrate[a*x^2 + b*x^3,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^{3}(3bx+4a)}{12}$	14
default	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}$	14
norman	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}$	14
risch	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}$	14
parallelrisch	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}$	14
parts	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}$	14

```
[In] int(b*x^3+a*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^3*(3*b*x+4*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3\right)\,dx=\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

```
[In] integrate(b*x^3+a*x^2,x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(ax^2+bx^3\right)\,dx=\frac{ax^3}{3}+\frac{bx^4}{4}$$

```
[In] integrate(b*x**3+a*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3\right)\,dx=\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

```
[In] integrate(b*x^3+a*x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3\right)\,dx=\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

```
[In] integrate(b*x^3+a*x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3\right)\,dx=\frac{x^3\,\left(4\,a+3\,b\,x\right)}{12}$$

```
[In] int(a*x^2 + b*x^3,x)
```

```
[Out] (x^3*(4*a + 3*b*x))/12
```

3.206 $\int \frac{ax^2+bx^3}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{ax^2+bx^3}{x} \, dx = \frac{ax^2}{2}+\frac{bx^3}{3}$$

```
[Out] 1/2*a*x^2+1/3*b*x^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{14\}$

$$\int \frac{ax^2+bx^3}{x} \, dx = \frac{ax^2}{2}+\frac{bx^3}{3}$$

```
[In] Int[(a*x^2 + b*x^3)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ax+bx^2\right) \, dx \\ &= \frac{ax^2}{2}+\frac{bx^3}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3}{x}\,dx=\frac{ax^2}{2}+\frac{bx^3}{3}$$

```
[In] Integrate[(a*x^2 + b*x^3)/x,x]
[Out] (a*x^2)/2 + (b*x^3)/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^2(2bx+3a)}{6}$	14
default	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
norman	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
risch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14
parallelrisch	$\frac{1}{2}a\,x^2+\frac{1}{3}b\,x^3$	14

```
[In] int((b*x^3+a*x^2)/x,x,method=_RETURNVERBOSE)
[Out] 1/6*x^2*(2*b*x+3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3}{x}\,dx=\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="fricas")
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{ax^2+bx^3}{x}\,dx=\frac{ax^2}{2}+\frac{bx^3}{3}$$

```
[In] integrate((b*x**3+a*x**2)/x,x)
```

```
[Out] a*x**2/2 + b*x**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3}{x}\,dx=\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3}{x}\,dx=\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3}{x}\,dx=\frac{x^2\,\left(3\,a+2\,b\,x\right)}{6}$$

```
[In] int((a*x^2 + b*x^3)/x,x)
```

```
[Out] (x^2*(3*a + 2*b*x))/6
```

3.207 $\int \frac{ax^2+bx^3}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{ax^2+bx^3}{x^2}\,dx = ax+\frac{bx^2}{2}$$

```
[Out] a*x+1/2*b*x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{14\}$

$$\int \frac{ax^2+bx^3}{x^2}\,dx = ax+\frac{bx^2}{2}$$

```
[In] Int[(a*x^2 + b*x^3)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (a+bx)\,dx\\ &= ax+\frac{bx^2}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3}{x^2}\,dx = ax+\frac{bx^2}{2}$$

```
[In] Integrate[(a*x^2 + b*x^3)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{x(bx+2a)}{2}$	11
default	$ax+\frac{1}{2}b\,x^2$	11
risch	$ax+\frac{1}{2}b\,x^2$	11
parallelrisch	$ax+\frac{1}{2}b\,x^2$	11
parts	$ax+\frac{1}{2}b\,x^2$	11
norman	$\frac{a\,x^2+\frac{1}{2}b\,x^3}{x}$	17

```
[In] int((b*x^3+a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(b*x+2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax^2+bx^3}{x^2}\,dx = \frac{1}{2}\,bx^2+ax$$

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{ax^2+bx^3}{x^2}\,dx=ax+\frac{bx^2}{2}$$

```
[In] integrate((b*x**3+a*x**2)/x**2,x)
```

```
[Out] a*x + b*x**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax^2+bx^3}{x^2}\,dx=\frac{1}{2}\,bx^2+ax$$

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax^2+bx^3}{x^2}\,dx=\frac{1}{2}\,bx^2+ax$$

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{ax^2+bx^3}{x^2}\,dx=\frac{b\,x^2}{2}+a\,x$$

```
[In] int((a*x^2 + b*x^3)/x^2,x)
```

```
[Out] a*x + (b*x^2)/2
```

3.208 $\int x^2(ax^2+bx^3)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int x^2\left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*b^2*x^9
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\int x^2\left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

```
[In] Int[x^2*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^6(a+bx)^2\,dx\\
&= \int \left(a^2x^6+2abx^7+b^2x^8\right)\,dx\\
&= \frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^2\left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

```
[In] Integrate[x^2*(a*x^2 + b*x^3)^2,x]
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^7\left(28b^2x^2+63abx+36a^2\right)}{252}$	25
default	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{1}{9}b^2x^9$	25
norman	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{1}{9}b^2x^9$	25
risch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{1}{9}b^2x^9$	25
parallelrisch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{1}{9}b^2x^9$	25

```
[In] int(x^2*(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/252*x^7*(28*b^2*x^2+63*a*b*x+36*a^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 \left(ax^2+bx^3\right)^2 \, dx = \frac{1}{9}\, b^2 x^9 + \frac{1}{4}\, abx^8 + \frac{1}{7}\, a^2 x^7$$

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 \left(ax^2+bx^3\right)^2 \, dx = \frac{a^2x^7}{7} + \frac{abx^8}{4} + \frac{b^2x^9}{9}$$

```
[In] integrate(x**2*(b*x**3+a*x**2)**2,x)
[Out] a**2*x**7/7 + a*b*x**8/4 + b**2*x**9/9
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 \left(ax^2+bx^3\right)^2 \, dx = \frac{1}{9}\, b^2 x^9 + \frac{1}{4}\, abx^8 + \frac{1}{7}\, a^2 x^7$$

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 \left(ax^2+bx^3\right)^2 \, dx = \frac{1}{9}\, b^2 x^9 + \frac{1}{4}\, abx^8 + \frac{1}{7}\, a^2 x^7$$

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 \left(a x^2+b x^3\right)^2 \, dx=\frac{a^2\,x^7}{7}+\frac{a\,b\,x^8}{4}+\frac{b^2\,x^9}{9}$$

```
[In] int(x^2*(a*x^2 + b*x^3)^2,x)
[Out] (a^2*x^7)/7 + (b^2*x^9)/9 + (a*b*x^8)/4
```

3.209 $\int x(ax^2+bx^3)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int x\left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*b^2*x^8
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 45}

$$\int x\left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

```
[In] Int[x*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^5(a+bx)^2\,dx \\
&= \int \left(a^2x^5+2abx^6+b^2x^7\right)\,dx \\
&= \frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x\left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

```
[In] Integrate[x*(a*x^2 + b*x^3)^2,x]
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^6\left(21b^2x^2+48abx+28a^2\right)}{168}$	25
default	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25
parallelrisch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{8}b^2x^8$	25

```
[In] int(x*(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/168*x^6*(21*b^2*x^2+48*a*b*x+28*a^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax^2+bx^3\right)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x\left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^6}{6}+\frac{2abx^7}{7}+\frac{b^2x^8}{8}$$

```
[In] integrate(x*(b*x**3+a*x**2)**2,x)
[Out] a**2*x**6/6 + 2*a*b*x**7/7 + b**2*x**8/8
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax^2+bx^3\right)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax^2+bx^3\right)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x\left(ax^2+bx^3\right)^2\,dx=\frac{a^2\,x^6}{6}+\frac{2\,a\,b\,x^7}{7}+\frac{b^2\,x^8}{8}$$

```
[In] int(x*(a*x^2 + b*x^3)^2,x)
[Out] (a^2*x^6)/6 + (b^2*x^8)/8 + (2*a*b*x^7)/7
```

3.210 $\int \left(ax^2+bx^3\right)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \left(ax^2+bx^3\right)^2 \, dx=\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*b^2*x^7
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1607, 45}

$$\int \left(ax^2+bx^3\right)^2 \, dx=\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

```
[In] Int[(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int x^4(a+bx)^2\,dx\\
&= \int \left(a^2x^4+2abx^5+b^2x^6\right)\,dx\\
&= \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

```
[In] Integrate[(a*x^2 + b*x^3)^2,x]
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^5(15b^2x^2+35abx+21a^2)}{105}$	25
default	$\frac{1}{5}x^5a^2+\frac{1}{3}ab\,x^6+\frac{1}{7}b^2x^7$	25
norman	$\frac{1}{5}x^5a^2+\frac{1}{3}ab\,x^6+\frac{1}{7}b^2x^7$	25
risch	$\frac{1}{5}x^5a^2+\frac{1}{3}ab\,x^6+\frac{1}{7}b^2x^7$	25
parallelrisch	$\frac{1}{5}x^5a^2+\frac{1}{3}ab\,x^6+\frac{1}{7}b^2x^7$	25

```
[In] int((b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/105*x^5*(15*b^2*x^2+35*a*b*x+21*a^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^5}{5}+\frac{abx^6}{3}+\frac{b^2x^7}{7}$$

```
[In] integrate((b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**5/5 + a*b*x**6/3 + b**2*x**7/7
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{a^2\,x^5}{5}+\frac{a\,b\,x^6}{3}+\frac{b^2\,x^7}{7}$$

```
[In] int((a*x^2 + b*x^3)^2,x)
```

```
[Out] (a^2*x^5)/5 + (b^2*x^7)/7 + (a*b*x^6)/3
```

3.211 $\int \frac{(ax^2+bx^3)^2}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx=\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx=\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

```
[In] Int[(a*x^2 + b*x^3)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^3(a+bx)^2\,dx\\ &= \int \left(a^2x^3+2abx^4+b^2x^5\right)\,dx\\ &= \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx=\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

```
[In] Integrate[(a*x^2 + b*x^3)^2/x,x]
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^4(10b^2x^2+24abx+15a^2)}{60}$	25
default	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
norman	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
risch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25
parallelrisch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{6}b^2x^6$	25

```
[In] int((b*x^3+a*x^2)^2/x,x,method=_RETURNVERBOSE)
[Out] 1/60*x^4*(10*b^2*x^2+24*a*b*x+15*a^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="fricas")
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{b^2x^6}{6}$$

```
[In] integrate((b*x**3+a*x**2)**2/x,x)
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="maxima")
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="giac")
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x}\,dx = \frac{a^2\,x^4}{4}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^6}{6}$$

```
[In] int((a*x^2 + b*x^3)^2/x,x)
[Out] (a^2*x^4)/4 + (b^2*x^6)/6 + (2*a*b*x^5)/5
```

3.212 $\int \frac{(ax^2+bx^3)^2}{x^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{(ax^2+bx^3)^2}{x^2} \, dx = \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\int \frac{(ax^2+bx^3)^2}{x^2} \, dx = \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

```
[In] Int[(a*x^2 + b*x^3)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^2(a+bx)^2\,dx\\ &= \int \left(a^2x^2+2abx^3+b^2x^4\right)\,dx\\ &= \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx = \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

```
[In] Integrate[(a*x^2 + b*x^3)^2/x^2,x]
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^3(6b^2x^2+15abx+10a^2)}{30}$	25
default	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
risch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
parallelrisch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{1}{5}b^2x^5$	25
norman	$\frac{\frac{1}{3}a^2x^4+\frac{1}{5}b^2x^6+\frac{1}{2}ab\,x^5}{x}$	29

```
[In] int((b*x^3+a*x^2)^2/x^2,x,method=_RETURNVERBOSE)
[Out] 1/30*x^3*(6*b^2*x^2+15*a*b*x+10*a^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx = \frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

```
[In] integrate((b*x**3+a*x**2)**2/x**2,x)
[Out] a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{(ax^2+bx^3)^2}{x^2}\,dx=\frac{a^2\,x^3}{3}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^5}{5}$$

```
[In] int((a*x^2 + b*x^3)^2/x^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

3.213 $\int \frac{x^6}{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{x^6}{ax^2+bx^3}\,dx=-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

```
[Out] -a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\int \frac{x^6}{ax^2+bx^3}\,dx=\frac{a^4\log(a+bx)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

```
[In] Int[x^6/(a*x^2 + b*x^3),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^4}{a+bx}\,dx\\
&= \int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx\\
&= -\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{ax^2+bx^3}\,dx = -\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}$$

```
[In] Integrate[x^6/(a*x^2 + b*x^3),x]
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{-\frac{1}{4}b^3x^4+\frac{1}{3}a\,b^2x^3-\frac{1}{2}a^2b\,x^2+a^3x}{b^4}+\frac{a^4\ln(bx+a)}{b^5}$	52
risch	$-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{a\,x^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\ln(bx+a)}{b^5}$	52
parallelrisch	$\frac{3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln(bx+a)-12a^3bx}{12b^5}$	53
norman	$\frac{\frac{x^5}{4b}-\frac{a\,x^4}{3b^2}+\frac{a^2x^3}{2b^3}-\frac{a^3x^2}{b^4}}{x}+\frac{a^4\ln(bx+a)}{b^5}$	59

```
[In] int(x^6/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] -1/b^4*(-1/4*b^3*x^4+1/3*a*b^2*x^3-1/2*a^2*b*x^2+a^3*x)+a^4*ln(b*x+a)/b^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{ax^2+bx^3}\,dx = \frac{3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log\left(bx+a\right)}{12\,b^5}$$

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{ax^2+bx^3}\,dx = \frac{a^4\log\left(a+bx\right)}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

```
[In] integrate(x**6/(b*x**3+a*x**2),x)
```

```
[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2)
 + x**4/(4*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{ax^2+bx^3}\,dx = \frac{a^4\log\left(bx+a\right)}{b^5}+\frac{3\,b^3x^4-4\,ab^2x^3+6\,a^2bx^2-12\,a^3x}{12\,b^4}$$

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3
*x)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{ax^2+bx^3}\,dx = \frac{a^4\log\left(|bx+a|\right)}{b^5} + \frac{3\,b^3x^4-4\,ab^2x^3+6\,a^2bx^2-12\,a^3x}{12\,b^4}$$

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 1
2*a^3*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{ax^2+bx^3}\,dx = \frac{x^4}{4\,b} + \frac{a^4\,\ln\left(a+b\,x\right)}{b^5} - \frac{a\,x^3}{3\,b^2} - \frac{a^3\,x}{b^4} + \frac{a^2\,x^2}{2\,b^3}$$

```
[In] int(x^6/(a*x^2 + b*x^3),x)
```

```
[Out] x^4/(4*b) + (a^4*log(a + b*x))/b^5 - (a*x^3)/(3*b^2) - (a^3*x)/b^4 + (a^2*x
^2)/(2*b^3)
```

3.214 $\int \frac{x^5}{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{x^5}{ax^2+bx^3}\,dx=\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}$$

```
[Out] a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\int \frac{x^5}{ax^2+bx^3}\,dx=-\frac{a^3\log(a+bx)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

```
[In] Int[x^5/(a*x^2 + b*x^3),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^3}{a+bx}\,dx\\ &= \int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx\\ &= \frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{ax^2+bx^3}\,dx=\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}$$

```
[In] Integrate[x^5/(a*x^2 + b*x^3),x]
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-\frac{1}{2}ab\,x^2+a^2x}{b^3}-\frac{a^3\ln(bx+a)}{b^4}$	41
risch	$\frac{a^2x}{b^3}-\frac{a\,x^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\ln(bx+a)}{b^4}$	41
parallelrisch	$-\frac{-2b^3x^3+3a\,b^2x^2+6a^3\ln(bx+a)-6a^2bx}{6b^4}$	42
norman	$\frac{\frac{a^2x^2}{b^3}+\frac{x^4}{3b}-\frac{a\,x^3}{2b^2}}{x}-\frac{a^3\ln(bx+a)}{b^4}$	48

```
[In] int(x^5/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] 1/b^3*(1/3*b^2*x^3-1/2*a*b*x^2+a^2*x)-a^3*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{ax^2+bx^3}\,dx = \frac{2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)}{6\,b^4}$$

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{ax^2+bx^3}\,dx = -\frac{a^3\log\left(a+bx\right)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

```
[In] integrate(x**5/(b*x**3+a*x**2),x)
[Out] -a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{ax^2+bx^3}\,dx = -\frac{a^3\log\left(bx+a\right)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{ax^2+bx^3}\,dx = -\frac{a^3\log\left(|bx+a|\right)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="giac")
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{ax^2+bx^3}\,dx=\frac{x^3}{3\,b}-\frac{a^3\,\ln\left(a+b\,x\right)}{b^4}-\frac{a\,x^2}{2\,b^2}+\frac{a^2\,x}{b^3}$$

```
[In] int(x^5/(a*x^2 + b*x^3),x)
[Out] x^3/(3*b) - (a^3*log(a + b*x))/b^4 - (a*x^2)/(2*b^2) + (a^2*x)/b^3
```

3.215 $\int \frac{x^4}{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{x^4}{ax^2+bx^3}\,dx = -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\int \frac{x^4}{ax^2+bx^3}\,dx = \frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

```
[In] Int[x^4/(a*x^2 + b*x^3),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2}{a+bx}\,dx\\ &= \int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx\\ &= -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{ax^2+bx^3}\,dx = -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}$$

```
[In] Integrate[x^4/(a*x^2 + b*x^3),x]
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{-\frac{1}{2}b\,x^2+ax}{b^2}+\frac{a^2\ln(bx+a)}{b^3}$	30
risch	$-\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\ln(bx+a)}{b^3}$	30
parallelrisch	$\frac{b^2x^2+2a^2\ln(bx+a)-2abx}{2b^3}$	30
norman	$\frac{\frac{x^3}{2b}-\frac{a\,x^2}{b^2}}{x}+\frac{a^2\ln(bx+a)}{b^3}$	37

```
[In] int(x^4/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] -1/b^2*(-1/2*b*x^2+a*x)+a^2*ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax^2+bx^3}\,dx=\frac{b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{ax^2+bx^3}\,dx=\frac{a^2\log\left(a+bx\right)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

```
[In] integrate(x**4/(b*x**3+a*x**2),x)
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax^2+bx^3}\,dx=\frac{a^2\log\left(bx+a\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{ax^2+bx^3}\,dx=\frac{a^2\log\left(\left|bx+a\right|\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="giac")
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{ax^2 + bx^3} \, dx = \frac{2\,a^2\,\ln\left(a + b\,x\right) + b^2\,x^2 - 2\,a\,b\,x}{2\,b^3}$$

```
[In] int(x^4/(a*x^2 + b*x^3),x)
```

```
[Out] (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

3.216 $\int \frac{x^3}{ax^2+bx^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{x^3}{ax^2+bx^3} \, dx = \frac{x}{b} - \frac{a \log(a+bx)}{b^2}$$

```
[Out] x/b-a*ln(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\int \frac{x^3}{ax^2+bx^3} \, dx = \frac{x}{b} - \frac{a \log(a+bx)}{b^2}$$

```
[In] Int[x^3/(a*x^2 + b*x^3),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x}{a+bx}\,dx\\ &= \int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx\\ &= \frac{x}{b}-\frac{a\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax^2+bx^3}\,dx=\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

```
[In] Integrate[x^3/(a*x^2 + b*x^3),x]
[Out] x/b - (a*Log[a + b*x])/b^2
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
norman	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
risch	$\frac{x}{b}-\frac{a\ln(bx+a)}{b^2}$	19
parallelrisch	$-\frac{a\ln(bx+a)-bx}{b^2}$	19

```
[In] int(x^3/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] x/b-a*ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{ax^2+bx^3}\,dx=\frac{bx-a\log(bx+a)}{b^2}$$

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{ax^2+bx^3}\,dx=-\frac{a\log(a+bx)}{b^2}+\frac{x}{b}$$

```
[In] integrate(x**3/(b*x**3+a*x**2),x)
[Out] -a*log(a + b*x)/b**2 + x/b
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax^2+bx^3}\,dx=\frac{x}{b}-\frac{a\log(bx+a)}{b^2}$$

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] x/b - a*log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{ax^2+bx^3}\,dx=\frac{x}{b}-\frac{a\log(|bx+a|)}{b^2}$$

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="giac")
[Out] x/b - a*log(abs(b*x + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{ax^2+bx^3}\,dx = -\frac{a\,\ln\left(a+b\,x\right)-b\,x}{b^2}$$

```
[In] int(x^3/(a*x^2 + b*x^3),x)
[Out] -(a*log(a + b*x) - b*x)/b^2
```

3.217 $\int \frac{x^2}{ax^2+bx^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 10

$$\int \frac{x^2}{ax^2+bx^3} \, dx = \frac{\log(a+bx)}{b}$$

```
[Out] ln(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 31}

$$\int \frac{x^2}{ax^2+bx^3} \, dx = \frac{\log(a+bx)}{b}$$

```
[In] Int[x^2/(a*x^2 + b*x^3),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{a+bx}\,dx\\ &= \frac{\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2+bx^3}\,dx = \frac{\log(a+bx)}{b}$$

```
[In] Integrate[x^2/(a*x^2 + b*x^3),x]
[Out] Log[a + b*x]/b
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\ln(bx+a)}{b}$	11
norman	$\frac{\ln(bx+a)}{b}$	11
risch	$\frac{\ln(bx+a)}{b}$	11
parallelrisch	$\frac{\ln(bx+a)}{b}$	11

```
[In] int(x^2/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] ln(b*x+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2+bx^3}\,dx = \frac{\log\left(bx+a\right)}{b}$$

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] log(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{ax^2+bx^3}\,dx=\frac{\log{(a+bx)}}{b}$$

```
[In] integrate(x**2/(b*x**3+a*x**2),x)
[Out] log(a + b*x)/b
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2+bx^3}\,dx=\frac{\log{(bx+a)}}{b}$$

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] log(b*x + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{ax^2+bx^3}\,dx=\frac{\log{(|bx+a|)}}{b}$$

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="giac")
[Out] log(abs(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2+bx^3}\,dx=\frac{\ln{(a+b\,x)}}{b}$$

```
[In] int(x^2/(a*x^2 + b*x^3),x)
[Out] log(a + b*x)/b
```

3.218 $\int \frac{x}{ax^2+bx^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{x}{ax^2+bx^3} \, dx = \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

```
[Out] ln(x)/a-ln(b*x+a)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1598, 36, 29, 31}

$$\int \frac{x}{ax^2+bx^3} \, dx = \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

```
[In] Int[x/(a*x^2 + b*x^3),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x(a+bx)}\,dx\\ &= \frac{\int \frac{1}{x}\,dx}{a} - \frac{b\int \frac{1}{a+bx}\,dx}{a}\\ &= \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^2+bx^3}\,dx = \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

```
[In] Integrate[x/(a*x^2 + b*x^3),x]
[Out] Log[x]/a - Log[a + b*x]/a
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{\ln(x)-\ln(bx+a)}{a}$	16
default	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
norman	$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$	19
risch	$\frac{\ln(-x)}{a}-\frac{\ln(bx+a)}{a}$	21

```
[In] int(x/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] (ln(x)-ln(b*x+a))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{ax^2+bx^3}\,dx = -\frac{\log\left(bx+a\right)-\log\left(x\right)}{a}$$

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{x}{ax^2+bx^3}\,dx = \frac{\log\left(x\right)-\log\left(\frac{a}{b}+x\right)}{a}$$

```
[In] integrate(x/(b*x**3+a*x**2),x)
[Out] (log(x) - log(a/b + x))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^2+bx^3}\,dx = -\frac{\log\left(bx+a\right)}{a}+\frac{\log\left(x\right)}{a}$$

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] -log(b*x + a)/a + log(x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x}{ax^2+bx^3}\,dx = -\frac{\log\left(|bx+a|\right)}{a}+\frac{\log\left(|x|\right)}{a}$$

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="giac")
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x}{ax^2+bx^3}\,dx=-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a}$$

```
[In] int(x/(a*x^2 + b*x^3),x)
[Out] -(2*atanh((2*b*x)/a + 1))/a
```

3.219 $\int \frac{1}{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{ax^2+bx^3}\,dx=-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1607, 46}

$$\int \frac{1}{ax^2+bx^3}\,dx=-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

```
[In] Int[(a*x^2 + b*x^3)^(-1),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^2(a+bx)}\,dx\\
&= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^3}\,dx = -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(-1),x]
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{b\ln(x)x-b\ln(bx+a)x+a}{a^2x}$	26
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^2}-\frac{b\ln(x)}{a^2}$	32

```
[In] int(1/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] -(b*ln(x)*x-b*ln(b*x+a)*x+a)/a^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{ax^2+bx^3}\,dx=\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{ax^2+bx^3}\,dx=-\frac{1}{ax}+\frac{b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

```
[In] integrate(1/(b*x**3+a*x**2),x)
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^3}\,dx=\frac{b\log(bx+a)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{ax^2+bx^3}\,dx=\frac{b\log(|bx+a|)}{a^2}-\frac{b\log(|x|)}{a^2}-\frac{1}{ax}$$

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="giac")
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{ax^2+bx^3}\,dx=\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

```
[In] int(1/(a*x^2 + b*x^3),x)
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

3.220 $\int \frac{1}{x(ax^2+bx^3)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx=-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 46}

$$\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx=\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[In] Int[1/(x*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^3(a+bx)}\,dx\\
&= \int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx\\
&= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$
\int \frac{1}{x\,(ax^2+bx^3)}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}
$$

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)),x]
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{2a\,x^{2}}+\frac{b}{x\,a^{2}}+\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln(bx+a)}{a^{3}}$	41
norman	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}+\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln(bx+a)}{a^{3}}$	41
risch	$\frac{\frac{bx}{a^{2}}-\frac{1}{2a}}{x^{2}}+\frac{b^{2}\ln(-x)}{a^{3}}-\frac{b^{2}\ln(bx+a)}{a^{3}}$	43
parallelrisch	$\frac{2b^{2}\ln(x)x^{2}-2b^{2}\ln(bx+a)x^{2}+2abx-a^{2}}{2a^{3}x^{2}}$	44

```
[In] int(1/x/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] -1/2/a/x^2+b/x/a^2+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(ax^2+bx^3)}\,dx = -\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log\left(x\right)-2\,abx+a^2}{2\,a^3x^2}$$

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(ax^2+bx^3)}\,dx = \frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

```
[In] integrate(1/x/(b*x**3+a*x**2),x)
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(ax^2+bx^3)}\,dx = -\frac{b^2\log\left(bx+a\right)}{a^3}+\frac{b^2\log\left(x\right)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(ax^2+bx^3)}\,dx = -\frac{b^2\log\left(|bx+a|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="giac")
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\,(ax^2+bx^3)}\,dx = -\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

```
[In] int(1/(x*(a*x^2 + b*x^3)),x)
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

3.221 $\int \frac{1}{x^2(ax^2+bx^3)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx=-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 46}

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx=-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

```
[In] Int[1/(x^2*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^4(a+bx)}\,dx \\
&= \int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx \\
&= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}$$

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3)),x]
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{1}{3a\,x^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln(bx+a)}{a^4}$	53
norman	$\frac{-\frac{1}{3a}+\frac{bx}{2a^2}-\frac{b^2x^2}{a^3}}{x^3}+\frac{b^3\ln(bx+a)}{a^4}-\frac{b^3\ln(x)}{a^4}$	53
parallelrisch	$-\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6a\,b^2x^2-3a^2bx+2a^3}{6a^4x^3}$	55
risch	$\frac{-\frac{1}{3a}+\frac{bx}{2a^2}-\frac{b^2x^2}{a^3}}{x^3}-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln(-bx-a)}{a^4}$	56

```
[In] int(1/x^2/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx=\frac{6\,b^3x^3\log\left(bx+a\right)-6\,b^3x^3\log\left(x\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3}{6\,a^4x^3}$$

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
2*a^3)/(a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx=\frac{-2a^2+3abx-6b^2x^2}{6a^3x^3}+\frac{b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

```
[In] integrate(1/x**2/(b*x**3+a*x**2),x)
```

```
[Out] (-2*a**2 + 3*a*b*x - 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b +
 x))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx=\frac{b^3\log\left(bx+a\right)}{a^4}-\frac{b^3\log\left(x\right)}{a^4}-\frac{6\,b^2x^2-3\,abx+2\,a^2}{6\,a^3x^3}$$

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/(
a^3*x^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (ax^2 + bx^3)} dx = \frac{b^3 \log (|bx + a|)}{a^4} - \frac{b^3 \log (|x|)}{a^4} - \frac{6 ab^2 x^2 - 3 a^2 bx + 2 a^3}{6 a^4 x^3}$$

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2*
b*x + 2*a^3)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2 (ax^2 + bx^3)} dx = \frac{2 b^3 \operatorname{atanh}\left(\frac{2 b x}{a} + 1\right)}{a^4} - \frac{\frac{a^3}{3} - \frac{a^2 b x}{2} + a b^2 x^2}{a^4 x^3}$$

```
[In] int(1/(x^2*(a*x^2 + b*x^3)),x)
```

```
[Out] (2*b^3*atanh((2*b*x)/a + 1))/a^4 - (a^3/3 + a*b^2*x^2 - (a^2*b*x)/2)/(a^4*x
^3)
```

3.222 $\int \frac{x^8}{\left(ax^2+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \frac{x^8}{\left(ax^2+bx^3\right)^2}\,dx=\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}$$

```
[Out] 3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\int \frac{x^8}{\left(ax^2+bx^3\right)^2}\,dx=-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

```
[In] Int[x^8/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^4}{(a+bx)^2}\,dx\\
&= \int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx\\
&= \frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = \frac{9a^2bx-3ab^2x^2+b^3x^3-\frac{3a^4}{a+bx}-12a^3\log(a+bx)}{3b^5}$$

```
[In] Integrate[x^8/(a*x^2 + b*x^3)^2,x]
[Out] (9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*x
])/(3*b^5)
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\frac{1}{3}b^2x^3-ab\,x^2+3a^2x}{b^4}-\frac{4a^3\ln(bx+a)}{b^5}-\frac{a^4}{b^5(bx+a)}$	57
risch	$\frac{3a^2x}{b^4}-\frac{a\,x^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	57
norman	$\frac{\frac{x^7}{3b}-\frac{2a\,x^6}{3b^2}+\frac{2a^2x^5}{b^3}-\frac{4a^4x^3}{b^5}}{x^3(bx+a)}-\frac{4a^3\ln(bx+a)}{b^5}$	67
parallelrisch	$-\frac{-b^4x^4+2a\,b^3x^3+12\ln(bx+a)a^3bx-6a^2b^2x^2+12a^4\ln(bx+a)+12a^4}{3b^5(bx+a)}$	71

```
[In] int(x^8/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/b^4*(1/3*b^2*x^3-a*b*x^2+3*a^2*x)-4*a^3*ln(b*x+a)/b^5-a^4/b^5/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = \frac{b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4-12\,(a^3bx+a^4)\log{(bx+a)}}{3\,(b^6x+ab^5)}$$

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = -\frac{a^4}{ab^5+b^6x} - \frac{4a^3\log{(a+bx)}}{b^5} + \frac{3a^2x}{b^4} - \frac{ax^2}{b^3} + \frac{x^3}{3b^2}$$

```
[In] integrate(x**8/(b*x**3+a*x**2)**2,x)
```

```
[Out] -a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**2
/b**3 + x**3/(3*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = -\frac{a^4}{b^6x+ab^5} - \frac{4\,a^3\log{(bx+a)}}{b^5} + \frac{b^2x^3-3\,abx^2+9\,a^2x}{3\,b^4}$$

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
9*a^2*x)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = -\frac{4\,a^3\log\left(|bx+a|\right)}{b^5} - \frac{a^4}{(bx+a)b^5} + \frac{b^4x^3 - 3\,ab^3x^2 + 9\,a^2b^2x}{3\,b^6}$$

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^3
*x^2 + 9*a^2*b^2*x)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(ax^2+bx^3)^2}\,dx = \frac{x^3}{3\,b^2} - \frac{4\,a^3\,\ln\left(a+b\,x\right)}{b^5} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,x}{b^4} - \frac{a^4}{b\,\left(x\,b^5+a\,b^4\right)}$$

```
[In] int(x^8/(a*x^2 + b*x^3)^2,x)
```

```
[Out] x^3/(3*b^2) - (4*a^3*log(a + b*x))/b^5 - (a*x^2)/b^3 + (3*a^2*x)/b^4 - a^4/
(b*(a*b^4 + b^5*x))
```

3.223 $\int \frac{x^7}{(ax^2+bx^3)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = -\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}$$

```
[Out] -2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

```
[In] Int[x^7/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\frac{x^3}{(a+bx)^2}\,dx\\&=\int\left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx\\&=-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int\frac{x^7}{(ax^2+bx^3)^2}\,dx=\frac{-4abx+b^2x^2+\frac{2a^3}{a+bx}+6a^2\log(a+bx)}{2b^4}$$

```
[In] Integrate[x^7/(a*x^2 + b*x^3)^2,x]
[Out] (-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	45
default	$-\frac{-\frac{1}{2}b\,x^2+2ax}{b^3}+\frac{3a^2\ln(bx+a)}{b^4}+\frac{a^3}{b^4(bx+a)}$	46
norman	$\frac{\frac{3a^3x^3}{b^4}+\frac{x^6}{2b}-\frac{3a\,x^5}{2b^2}}{x^3(bx+a)}+\frac{3a^2\ln(bx+a)}{b^4}$	56
parallelrisch	$\frac{b^3x^3+6\ln(bx+a)a^2bx-3a\,b^2x^2+6a^3\ln(bx+a)+6a^3}{2b^4(bx+a)}$	59

```
[In] int(x^7/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] -2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Fricas [A] (verification not implemented)

none

Time $= 0.25$ (sec) , antiderivative size $= 62$, normalized size of antiderivative $= 1.35$

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\,(a^2bx+a^3)\log(bx+a)}{2\,(b^5x+ab^4)}$$

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

Sympy [A] (verification not implemented)

Time $= 0.09$ (sec) , antiderivative size $= 44$, normalized size of antiderivative $= 0.96$

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{a^3}{ab^4+b^5x}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

```
[In] integrate(x**7/(b*x**3+a*x**2)**2,x)
```

```
[Out] a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b*
*2)
```

Maxima [A] (verification not implemented)

none

Time $= 0.20$ (sec) , antiderivative size $= 47$, normalized size of antiderivative $= 1.02$

$$\int \frac{x^7}{(ax^2+bx^3)^2}\,dx = \frac{a^3}{b^5x+ab^4}+\frac{3\,a^2\log(bx+a)}{b^4}+\frac{bx^2-4\,ax}{2\,b^3}$$

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{x^7}{\left(ax^2+bx^3\right)^2}\,dx=\frac{3\,a^2\log\left(|bx+a|\right)}{b^4}+\frac{a^3}{(bx+a)b^4}+\frac{b^2x^2-4\,abx}{2\,b^4}$$

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x)
/b^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{x^7}{\left(ax^2+bx^3\right)^2}\,dx=\frac{x^2}{2\,b^2}+\frac{3\,a^2\,\ln\left(a+b\,x\right)}{b^4}+\frac{a^3}{b\,\left(x\,b^4+a\,b^3\right)}-\frac{2\,a\,x}{b^3}$$

```
[In] int(x^7/(a*x^2 + b*x^3)^2,x)
```

```
[Out] x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)/
b^3
```

3.224 $\int \frac{x^6}{\left(ax^2+bx^3\right)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^2} \, dx = \frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}$$

```
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^2} \, dx = -\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

```
[In] Int[x^6/(a*x^2 + b*x^3)^2,x]
```

```
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2}{(a+bx)^2}\,dx\\ &= \int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx\\ &= \frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{(ax^2+bx^3)^2}\,dx = \frac{bx-\frac{a^2}{a+bx}-2a\log(a+bx)}{b^3}$$

```
[In] Integrate[x^6/(a*x^2 + b*x^3)^2,x]
[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{x}{b^2}-\frac{a^2}{b^3(bx+a)}-\frac{2a\ln(bx+a)}{b^3}$	34
risch	$\frac{x}{b^2}-\frac{a^2}{b^3(bx+a)}-\frac{2a\ln(bx+a)}{b^3}$	34
norman	$\frac{\frac{x^5}{b}-\frac{2a^2x^3}{b^3}}{x^3(bx+a)}-\frac{2a\ln(bx+a)}{b^3}$	44
parallelrisch	$-\frac{2\ln(bx+a)xab-b^2x^2+2a^2\ln(bx+a)+2a^2}{b^3(bx+a)}$	49

```
[In] int(x^6/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^2}\,dx = \frac{b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)}{b^4x+ab^3}$$

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^2}\,dx = -\frac{a^2}{ab^3+b^4x}-\frac{2a\log\left(a+bx\right)}{b^3}+\frac{x}{b^2}$$

```
[In] integrate(x**6/(b*x**3+a*x**2)**2,x)
[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^2}\,dx = -\frac{a^2}{b^4x+ab^3}+\frac{x}{b^2}-\frac{2\,a\log\left(bx+a\right)}{b^3}$$

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^2}\,dx = \frac{x}{b^2}-\frac{2\,a\log\left(|bx+a|\right)}{b^3}-\frac{a^2}{(bx+a)b^3}$$

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^2}\,dx=\frac{x}{b^2}-\frac{a^2}{x\,b^4+a\,b^3}-\frac{2\,a\,\ln\left(a+b\,x\right)}{b^3}$$

```
[In] int(x^6/(a*x^2 + b*x^3)^2,x)
```

```
[Out] x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

3.225 $\int \frac{x^5}{(ax^2+bx^3)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx=\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

```
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 45}

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx=\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

```
[In] Int[x^5/(a*x^2 + b*x^3)^2,x]
```

```
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x}{(a+bx)^2}\,dx\\ &= \int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx\\ &= \frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx=\frac{\frac{a}{a+bx}+\log(a+bx)}{b^2}$$

```
[In] Integrate[x^5/(a*x^2 + b*x^3)^2,x]
[Out] (a/(a + b*x) + Log[a + b*x])/b^2
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
norman	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
risch	$\frac{a}{b^2(bx+a)}+\frac{\ln(bx+a)}{b^2}$	24
parallelrisch	$\frac{b\ln(bx+a)x+a\ln(bx+a)+a}{b^2(bx+a)}$	31

```
[In] int(x^5/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{a}{ab^2+b^3x} + \frac{\log(a+bx)}{b^2}$$

```
[In] integrate(x**5/(b*x**3+a*x**2)**2,x)
[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{a}{b^3x+ab^2} + \frac{\log(bx+a)}{b^2}$$

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{(ax^2+bx^3)^2}\,dx = \frac{\log(|bx+a|)}{b^2} + \frac{a}{(bx+a)b^2}$$

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^2}\,dx=\frac{\ln\left(a+b\,x\right)}{b^2}+\frac{a}{b^2\,\left(a+b\,x\right)}$$

```
[In] int(x^5/(a*x^2 + b*x^3)^2,x)
```

```
[Out] log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

3.226 $\int \frac{x^4}{\left(ax^2+bx^3\right)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^2} \, dx=-\frac{1}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 32}

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^2} \, dx=-\frac{1}{b(a+bx)}$$

```
[In] Int[x^4/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(a+bx)^2}\,dx\\ &= -\frac{1}{b(a+bx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^2}\,dx = -\frac{1}{b(a+bx)}$$

```
[In] Integrate[x^4/(a*x^2 + b*x^3)^2,x]
[Out] -(1/(b*(a + b*x)))
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{1}{b(bx+a)}$	13
default	$-\frac{1}{b(bx+a)}$	13
norman	$\frac{x}{a(bx+a)}$	13
risch	$-\frac{1}{b(bx+a)}$	13
parallelrisch	$-\frac{1}{b(bx+a)}$	13

```
[In] int(x^4/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/b/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{b^2x+ab}$$

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] -1/(b^2*x + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{ab+b^2x}$$

```
[In] integrate(x**4/(b*x**3+a*x**2)**2,x)
[Out] -1/(a*b + b**2*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{b^2x+ab}$$

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] -1/(b^2*x + a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{(bx+a)b}$$

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] -1/((b*x + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(ax^2+bx^3)^2}\,dx = -\frac{1}{b\,(a+b\,x)}$$

```
[In] int(x^4/(a*x^2 + b*x^3)^2,x)
```

```
[Out] -1/(b*(a + b*x))
```

3.227 $\int \frac{x^3}{(ax^2+bx^3)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int \frac{x^3}{(ax^2+bx^3)^2} \, dx = \frac{1}{a(a+bx)} + \frac{\log(x)}{a^2} - \frac{\log(a+bx)}{a^2}$$

```
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 46}

$$\int \frac{x^3}{(ax^2+bx^3)^2} \, dx = -\frac{\log(a+bx)}{a^2} + \frac{\log(x)}{a^2} + \frac{1}{a(a+bx)}$$

```
[In] Int[x^3/(a*x^2 + b*x^3)^2,x]
```

```
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x(a+bx)^2}\,dx\\ &= \int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx\\ &= \frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\left(ax^2+bx^3\right)^2}\,dx = \frac{\frac{a}{a+bx}+\log(x)-\log(a+bx)}{a^2}$$

```
[In] Integrate[x^3/(a*x^2 + b*x^3)^2,x]
[Out] (a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{1}{a(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	30
risch	$\frac{1}{a(bx+a)}+\frac{\ln(-x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	32
norman	$-\frac{bx}{a^2(bx+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$	33
parallelrisch	$\frac{b\ln(x)x-b\ln(bx+a)x+a\ln(x)-a\ln(bx+a)-bx}{a^2(bx+a)}$	45

```
[In] int(x^3/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = -\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{1}{a^2+abx}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^2}$$

```
[In] integrate(x**3/(b*x**3+a*x**2)**2,x)
[Out] 1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = \frac{1}{abx+a^2}-\frac{\log(bx+a)}{a^2}+\frac{\log(x)}{a^2}$$

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(ax^2+bx^3)^2}\,dx = -\frac{\log(|bx+a|)}{a^2}+\frac{\log(|x|)}{a^2}+\frac{1}{(bx+a)a}$$

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] -log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\left(ax^2+bx^3\right)^2}\,dx=\frac{1}{a^2+b\,x\,a}-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}$$

```
[In] int(x^3/(a*x^2 + b*x^3)^2,x)
[Out] 1/(a^2 + a*b*x) - (2*atanh((2*b*x)/a + 1))/a^2
```

3.228 $\int \frac{x^2}{\left(ax^2+bx^3\right)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^2} \, dx=-\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}$$

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 46}

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^2} \, dx=-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

```
[In] Int[x^2/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^2(a+bx)^2}\,dx\\ &= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^2}\,dx = -\frac{a\left(\frac{1}{x}+\frac{b}{a+bx}\right)+2b\log(x)-2b\log(a+bx)}{a^3}$$

```
[In] Integrate[x^2/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{1}{a^2x}-\frac{b}{a^2(bx+a)}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(bx+a)}{a^3}$	43
risch	$\frac{-\frac{2bx}{a^2}-\frac{1}{a}}{x(bx+a)}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(-bx-a)}{a^3}$	49
norman	$\frac{\frac{2b^2x^4}{a^3}-\frac{x^2}{a}}{x^3(bx+a)}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(bx+a)}{a^3}$	53
parallelrisch	$-\frac{2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2ab\ln(x)x-2\ln(bx+a)xab-2b^2x^2+a^2}{a^3x(bx+a)}$	70

```
[In] int(x^2/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.50

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^2}\,dx=-\frac{2\,abx+a^2-2\,\left(b^2x^2+abx\right)\log\left(bx+a\right)+2\,\left(b^2x^2+abx\right)\log\left(x\right)}{a^3bx^2+a^4x}$$

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^2}\,dx=\frac{-a-2bx}{a^3x+a^2bx^2}+\frac{2b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

```
[In] integrate(x**2/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^2}\,dx=-\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log\left(bx+a\right)}{a^3}-\frac{2\,b\log\left(x\right)}{a^3}$$

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx=\frac{2\,b\log\left(|bx+a|\right)}{a^3}-\frac{2\,b\log\left(|x|\right)}{a^3}-\frac{2\,bx+a}{(bx^2+ax)a^2}$$

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] 2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*x
)*a^2)
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{(ax^2+bx^3)^2}\,dx=\frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}-\frac{\frac{1}{a}+\frac{2\,b\,x}{a^2}}{b\,x^2+a\,x}$$

```
[In] int(x^2/(a*x^2 + b*x^3)^2,x)
[Out] (4*b*atanh((2*b*x)/a + 1))/a^3 - (1/a + (2*b*x)/a^2)/(a*x + b*x^2)
```

3.229 $\int \frac{x}{\left(ax^2+bx^3\right)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{x}{\left(ax^2+bx^3\right)^2} \, dx = -\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}$$

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 46}

$$\int \frac{x}{\left(ax^2+bx^3\right)^2} \, dx = \frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

```
[In] Int[x/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -1/2*1/(a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4
 - (3*b^2*Log[a + b*x])/a^4
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^3(a+bx)^2}\,dx\\ &= \int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x}{(ax^2+bx^3)^2}\,dx = \frac{a\left(-\frac{a}{x^2}+\frac{4b}{x}+\frac{2b^2}{a+bx}\right)+6b^2\log(x)-6b^2\log(a+bx)}{2a^4}$$

```
[In] Integrate[x/(a*x^2 + b*x^3)^2,x]
[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
b*x])/(2*a^4)
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	57
risch	$\frac{\frac{3b^2x^2}{a^3}+\frac{3bx}{2a^2}-\frac{1}{2a}}{x^2(bx+a)}+\frac{3b^2\ln(-x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	63
norman	$\frac{-\frac{3b^3x^4}{a^4}-\frac{x}{2a}+\frac{3b\,x^2}{2a^2}}{x^3(bx+a)}+\frac{3b^2\ln(x)}{a^4}-\frac{3b^2\ln(bx+a)}{a^4}$	64
parallelrisch	$\frac{6b^3\ln(x)x^3-6b^3\ln(bx+a)x^3+6a\,b^2\ln(x)x^2-6\ln(bx+a)x^2a\,b^2-6b^3x^3+3a^2bx-a^3}{2a^4x^2(bx+a)}$	87

```
[In] int(x/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{x}{(ax^2+bx^3)^2}\,dx = \frac{6\,ab^2x^2+3\,a^2bx-a^3-6\,(b^3x^3+ab^2x^2)\log{(bx+a)}+6\,(b^3x^3+ab^2x^2)\log{(x)}}{2\,(a^4bx^3+a^5x^2)}$$

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x}{(ax^2+bx^3)^2}\,dx = \frac{-a^2+3abx+6b^2x^2}{2a^4x^2+2a^3bx^3} + \frac{3b^2\left(\log{(x)}-\log{\left(\frac{a}{b}+x\right)}\right)}{a^4}$$

```
[In] integrate(x/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(log
(x) - log(a/b + x))/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{x}{(ax^2+bx^3)^2}\,dx = \frac{6\,b^2x^2+3\,abx-a^2}{2\,(a^3bx^3+a^4x^2)} - \frac{3\,b^2\log{(bx+a)}}{a^4} + \frac{3\,b^2\log{(x)}}{a^4}$$

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

$$\int \frac{x}{\left(ax^2+bx^3\right)^2}\,dx = -\frac{3\,b^2\log\left(|bx+a|\right)}{a^4} + \frac{3\,b^2\log\left(|x|\right)}{a^4} + \frac{6\,ab^2x^2+3\,a^2bx-a^3}{2\,(bx+a)a^4x^2}$$

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 + 3
*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{x}{\left(ax^2+bx^3\right)^2}\,dx = \frac{\frac{3\,b^2\,x^2}{a^3}-\frac{1}{2\,a}+\frac{3\,b\,x}{2\,a^2}}{b\,x^3+a\,x^2} - \frac{6\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}$$

```
[In] int(x/(a*x^2 + b*x^3)^2,x)
```

```
[Out] ((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*atan
h((2*b*x)/a + 1))/a^4
```

3.230 $\int \frac{1}{\left(ax^2+bx^3\right)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1}{(ax^2+bx^3)^2} \, dx = -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}$$

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1607, 46}

$$\int \frac{1}{(ax^2+bx^3)^2} \, dx = -\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

```
[In] Int[(a*x^2 + b*x^3)^(-2),x]
```

```
[Out] -1/3*1/(a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4
*b^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^4(a+bx)^2}\,dx \\
&= \int \left(\frac{1}{a^2x^4} - \frac{2b}{a^3x^3} + \frac{3b^2}{a^4x^2} - \frac{4b^3}{a^5x} + \frac{b^4}{a^4(a+bx)^2} + \frac{4b^4}{a^5(a+bx)}\right) dx \\
&= -\frac{1}{3a^2x^3} + \frac{b}{a^3x^2} - \frac{3b^2}{a^4x} - \frac{b^3}{a^4(a+bx)} - \frac{4b^3\log(x)}{a^5} + \frac{4b^3\log(a+bx)}{a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = -\frac{\frac{a(a^3-2a^2bx+6ab^2x^2+12b^3x^3)}{x^3(a+bx)} + 12b^3\log(x) - 12b^3\log(a+bx)}{3a^5}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(-2),x]
```

```
[Out] -1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 12
*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	68
norman	$\frac{\frac{4b^4x^4}{a^5}-\frac{1}{3a}+\frac{2bx}{3a^2}-\frac{2b^2x^2}{a^3}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	72
risch	$\frac{-\frac{4b^3x^3}{a^4}-\frac{2b^2x^2}{a^3}+\frac{2bx}{3a^2}-\frac{1}{3a}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(-bx-a)}{a^5}$	75
parallelrisch	$-\frac{12\ln(x)x^4b^4-12\ln(bx+a)x^4b^4+12\ln(x)x^3a\,b^3-12\ln(bx+a)x^3a\,b^3-12b^4x^4+6a^2b^2x^2-2a^3bx+a^4}{3a^5x^3(bx+a)}$	96

```
[In] int(1/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx = -\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4-12\left(b^4x^4+ab^3x^3\right)\log\left(bx+a\right)+12\left(b^4x^4+ab^3x^3\right)\log\left(x\right)}{3\left(a^5bx^4+a^6x^3\right)}$$

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3*
x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx = \frac{-a^3+2a^2bx-6ab^2x^2-12b^3x^3}{3a^5x^3+3a^4bx^4}+\frac{4b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

```
[In] integrate(1/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*b
*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx = -\frac{12\,b^3x^3+6\,ab^2x^2-2\,a^2bx+a^3}{3\left(a^4bx^4+a^5x^3\right)}+\frac{4\,b^3\log\left(bx+a\right)}{a^5}-\frac{4\,b^3\log\left(x\right)}{a^5}$$

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) + 4
*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{4\,b^3\log\left(|bx+a|\right)}{a^5} - \frac{4\,b^3\log\left(|x|\right)}{a^5} - \frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4}{3\,(bx+a)a^5x^3}$$

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 4*b^3*log(abs(b*x + a))/a^5 - 4*b^3*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 6
*a^2*b^2*x^2 - 2*a^3*b*x + a^4)/((b*x + a)*a^5*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{8\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5} - \frac{\frac{1}{3\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}-\frac{2\,b\,x}{3\,a^2}}{b\,x^4+a\,x^3}$$

```
[In] int(1/(a*x^2 + b*x^3)^2,x)
```

```
[Out] (8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3)
/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

3.231 $\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx = -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}$$

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 46}

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx = \frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

```
[In] Int[1/(x*(a*x^2 + b*x^3)^2),x]
```

```
[Out] -1/4*1/(a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x
) + b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^5(a+bx)^2}\,dx\\ &= \int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx = \frac{\frac{a\left(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4\right)}{x^4(a+bx)}+60b^4\log(x)-60b^4\log(a+bx)}{12a^6}$$

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)^2),x]
```

```
[Out] ((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^4
*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{1}{4a^{2}x^{4}}+\frac{2b}{3a^{3}x^{3}}-\frac{3b^{2}}{2a^{4}x^{2}}+\frac{4b^{3}}{a^{5}x}+\frac{b^{4}}{a^{5}(bx+a)}+\frac{5b^{4}\ln(x)}{a^{6}}-\frac{5b^{4}\ln(bx+a)}{a^{6}}$	79
norman	$\frac{-\frac{5b^{5}x^{5}}{a^{6}}-\frac{1}{4a}+\frac{5bx}{12a^{2}}-\frac{5b^{2}x^{2}}{6a^{3}}+\frac{5b^{3}x^{3}}{2a^{4}}}{x^{4}(bx+a)}+\frac{5b^{4}\ln(x)}{a^{6}}-\frac{5b^{4}\ln(bx+a)}{a^{6}}$	83
risch	$\frac{\frac{5b^{4}x^{4}}{a^{5}}+\frac{5b^{3}x^{3}}{2a^{4}}-\frac{5b^{2}x^{2}}{6a^{3}}+\frac{5bx}{12a^{2}}-\frac{1}{4a}}{x^{4}(bx+a)}+\frac{5b^{4}\ln(-x)}{a^{6}}-\frac{5b^{4}\ln(bx+a)}{a^{6}}$	85
parallelrisch	$\frac{60\ln(x)x^{5}b^{5}-60\ln(bx+a)x^{5}b^{5}+60\ln(x)x^{4}a\,b^{4}-60\ln(bx+a)x^{4}a\,b^{4}-60b^{5}x^{5}+30a^{2}b^{3}x^{3}-10a^{3}b^{2}x^{2}+5a^{4}bx-3a^{5}}{12a^{6}x^{4}(bx+a)}$	109

```
[In] int(1/x/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(ax^{2}+bx^{3}\right)^{2}}\,dx = \frac{60\,ab^{4}x^{4}+30\,a^{2}b^{3}x^{3}-10\,a^{3}b^{2}x^{2}+5\,a^{4}bx-3\,a^{5}-60\,(b^{5}x^{5}+ab^{4}x^{4})\log{(bx+a)}+60\,(b^{5}x^{5}+ab^{4}x^{4})\log}{12\,(a^{6}bx^{5}+a^{7}x^{4})}$$

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/(a
^6*b*x^5 + a^7*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(ax^{2}+bx^{3}\right)^{2}}\,dx = \frac{-3a^{4}+5a^{3}bx-10a^{2}b^{2}x^{2}+30ab^{3}x^{3}+60b^{4}x^{4}}{12a^{6}x^{4}+12a^{5}bx^{5}} + \frac{5b^{4}\left(\log{(x)}-\log{\left(\frac{a}{b}+x\right)}\right)}{a^{6}}$$

```
[In] integrate(1/x/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)/
(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\,(ax^2+bx^3)^2}\,dx = \frac{60\,b^4x^4+30\,ab^3x^3-10\,a^2b^2x^2+5\,a^3bx-3\,a^4}{12\,(a^5bx^5+a^6x^4)} - \frac{5\,b^4\log\left(bx+a\right)}{a^6} + \frac{5\,b^4\log\left(x\right)}{a^6}$$

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5*
b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\,(ax^2+bx^3)^2}\,dx = -\frac{5\,b^4\log\left(|bx+a|\right)}{a^6} + \frac{5\,b^4\log\left(|x|\right)}{a^6} + \frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5}{12\,(bx+a)a^6x^4}$$

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -5*b^4*log(abs(b*x + a))/a^6 + 5*b^4*log(abs(x))/a^6 + 1/12*(60*a*b^4*x^4 +
 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5)/((b*x + a)*a^6*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\,(ax^2+bx^3)^2}\,dx = \frac{\frac{5\,b^3\,x^3}{2\,a^4}-\frac{5\,b^2\,x^2}{6\,a^3}-\frac{1}{4\,a}+\frac{5\,b^4\,x^4}{a^5}+\frac{5\,b\,x}{12\,a^2}}{b\,x^5+a\,x^4} - \frac{10\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}$$

```
[In] int(1/(x*(a*x^2 + b*x^3)^2),x)
```

```
[Out] ((5*b^3*x^3)/(2*a^4) - (5*b^2*x^2)/(6*a^3) - 1/(4*a) + (5*b^4*x^4)/a^5 + (5
*b*x)/(12*a^2))/(a*x^4 + b*x^5) - (10*b^4*atanh((2*b*x)/a + 1))/a^6
```

3.232 $\int x^2\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{2(ax^2+bx^3)^{3/2}}{9b}-\frac{32a^3(ax^2+bx^3)^{3/2}}{315b^4x^3}+\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^2}-\frac{4a(ax^2+bx^3)^{3/2}}{21b^2x}$$

```
[Out] 2/9*(b*x^3+a*x^2)^(3/2)/b-32/315*a^3*(b*x^3+a*x^2)^(3/2)/b^4/x^3+16/105*a^2
*(b*x^3+a*x^2)^(3/2)/b^3/x^2-4/21*a*(b*x^3+a*x^2)^(3/2)/b^2/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2041, 2027, 2039}

$$\int x^2\sqrt{ax^2+bx^3}\,dx=-\frac{32a^3(ax^2+bx^3)^{3/2}}{315b^4x^3}+\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^2}-\frac{4a(ax^2+bx^3)^{3/2}}{21b^2x}+\frac{2(ax^2+bx^3)^{3/2}}{9b}$$

```
[In] Int[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(9*b) - (32*a^3*(a*x^2 + b*x^3)^(3/2))/(315*b^4*x
^3) + (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^2) - (4*a*(a*x^2 + b*x^3)^(
3/2))/(21*b^2*x)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
```

```
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(ax^2+bx^3)^{3/2}}{9b} - \frac{(2a)\int x\sqrt{ax^2+bx^3}\,dx}{3b} \\
&= \frac{2(ax^2+bx^3)^{3/2}}{9b} - \frac{4a(ax^2+bx^3)^{3/2}}{21b^2x} + \frac{(8a^2)\int \sqrt{ax^2+bx^3}\,dx}{21b^2} \\
&= \frac{2(ax^2+bx^3)^{3/2}}{9b} + \frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^2} - \frac{4a(ax^2+bx^3)^{3/2}}{21b^2x} - \frac{(16a^3)\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx}{105b^3} \\
&= \frac{2(ax^2+bx^3)^{3/2}}{9b} - \frac{32a^3(ax^2+bx^3)^{3/2}}{315b^4x^3} + \frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^2} - \frac{4a(ax^2+bx^3)^{3/2}}{21b^2x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2(x^2(a+bx))^{3/2}\left(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3\right)}{315b^4x^3}$$

```
[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3)
)/(315*b^4*x^3)
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.30

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}\left(15b^{2}x^{2}-12abx+8a^{2}\right)}{105b^{3}}$	32
gosper	$-\frac{2(bx+a)\left(-35b^{3}x^{3}+30a\,b^{2}x^{2}-24a^{2}bx+16a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315b^{4}x}$	57
default	$-\frac{2(bx+a)\left(-35b^{3}x^{3}+30a\,b^{2}x^{2}-24a^{2}bx+16a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315b^{4}x}$	57
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-35b^{4}x^{4}-5a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}-8a^{3}bx+16a^{4}\right)}{315x\,b^{4}}$	61
trager	$-\frac{2\left(-35b^{4}x^{4}-5a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}-8a^{3}bx+16a^{4}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315b^{4}x}$	63

```
[In] int(x^2*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx^3+ax^2}}{315\,b^4x}$$

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \int x^2\sqrt{x^2\,(a+bx)}\,dx$$

```
[In] integrate(x**2*(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}}{315\,b^4}$$

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{32\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,b^4}+\frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a\mathrm{sgn}(x)}{b^3}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+\right.}{b^3}\right.}{315\,b}$$

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/315*a^(9/2)*sgn(x)/b^4 + 2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2
)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/b^3 + (35*(b*
x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
 a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

$$\int x^2\sqrt{ax^2+bx^3}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(-16\,a^4+8\,a^3\,b\,x-6\,a^2\,b^2\,x^2+5\,a\,b^3\,x^3+35\,b^4\,x^4\right)}{315\,b^4\,x}$$

```
[In] int(x^2*(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(35*b^4*x^4 - 16*a^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2
 + 8*a^3*b*x))/(315*b^4*x)
```

3.233 $\int x\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^3}-\frac{8a(ax^2+bx^3)^{3/2}}{35b^2x^2}+\frac{2(ax^2+bx^3)^{3/2}}{7bx}$$

```
[Out] 16/105*a^2*(b*x^3+a*x^2)^(3/2)/b^3/x^3-8/35*a*(b*x^3+a*x^2)^(3/2)/b^2/x^2+2
/7*(b*x^3+a*x^2)^(3/2)/b/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2041, 2027, 2039}

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^3}-\frac{8a(ax^2+bx^3)^{3/2}}{35b^2x^2}+\frac{2(ax^2+bx^3)^{3/2}}{7bx}$$

```
[In] Int[x*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^3) - (8*a*(a*x^2 + b*x^3)^(3/2))/
(35*b^2*x^2) + (2*(a*x^2 + b*x^3)^(3/2))/(7*b*x)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(ax^2+bx^3)^{3/2}}{7bx} - \frac{(4a)\int\sqrt{ax^2+bx^3}\,dx}{7b}\\ &= -\frac{8a(ax^2+bx^3)^{3/2}}{35b^2x^2} + \frac{2(ax^2+bx^3)^{3/2}}{7bx} + \frac{(8a^2)\int\frac{\sqrt{ax^2+bx^3}}{x}\,dx}{35b^2}\\ &= \frac{16a^2(ax^2+bx^3)^{3/2}}{105b^3x^3} - \frac{8a(ax^2+bx^3)^{3/2}}{35b^2x^2} + \frac{2(ax^2+bx^3)^{3/2}}{7bx}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int x\sqrt{ax^2+bx^3}\,dx = \frac{2(x^2(a+bx))^{3/2}\,(8a^2-12abx+15b^2x^2)}{105b^3x^3}$$

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^3],x]
[Out] (2*(x^2*(a + b*x))^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3*x^3)
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.26

method	result	size
pseudoelliptic	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bx+2a)}{15b^2}$	21
gosper	$\frac{2(bx+a)(15b^2x^2-12abx+8a^2)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$	46
default	$\frac{2(bx+a)(15b^2x^2-12abx+8a^2)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$	46
risch	$\frac{2\sqrt{x^2(bx+a)}\,(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)}{105x\,b^3}$	50
trager	$\frac{2(15b^3x^3+3a\,b^2x^2-4a^2bx+8a^3)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$	52

```
[In] int(x*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int x\sqrt{ax^2+bx^3}\,dx = \frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{105\,b^3x}$$

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2)/(b
^3*x)
```

Sympy [F]

$$\int x\sqrt{ax^2+bx^3}\,dx = \int x\sqrt{x^2\,(a+bx)}\,dx$$

```
[In] integrate(x*(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x*sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}}{105\,b^3}$$

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int x\sqrt{ax^2+bx^3}\,dx=-\frac{16\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{105\,b^3}+\frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a\mathrm{sgn}(x)}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)\mathrm{sgn}(x)}{b^2}\right)}{105\,b}$$

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -16/105*a^(7/2)*sgn(x)/b^3 + 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/
2)*a + 15*sqrt(b*x + a)*a^2)*a*sgn(x)/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x
+ a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int x\sqrt{ax^2+bx^3}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,(8\,a^3-4\,a^2\,b\,x+3\,a\,b^2\,x^2+15\,b^3\,x^3)}{105\,b^3\,x}$$

```
[In] int(x*(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(8*a^3 + 15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x))/(1
05*b^3*x)
```

3.234 $\int \sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \sqrt{ax^2+bx^3}\,dx = -\frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3} + \frac{2(ax^2+bx^3)^{3/2}}{5bx^2}$$

```
[Out] -4/15*a*(b*x^3+a*x^2)^(3/2)/b^2/x^3+2/5*(b*x^3+a*x^2)^(3/2)/b/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2027, 2039}

$$\int \sqrt{ax^2+bx^3}\,dx = \frac{2(ax^2+bx^3)^{3/2}}{5bx^2} - \frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b*
x^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(ax^2+bx^3)^{3/2}}{5bx^2} - \frac{(2a)\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx}{5b} \\ &= -\frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3} + \frac{2(ax^2+bx^3)^{3/2}}{5bx^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \sqrt{ax^2+bx^3}\,dx = \frac{2\sqrt{x^2(a+bx)}(-2a^2+abx+3b^2x^2)}{15b^2x}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3],x]
[Out] (2*Sqrt[x^2*(a + b*x)]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2*x)
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.25

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
gosper	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^3+a\,x^2}}{15b^2x}$	35
default	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^3+a\,x^2}}{15b^2x}$	35
risch	$-\frac{2\sqrt{x^2(bx+a)}\,(-3b^2x^2-abx+2a^2)}{15x\,b^2}$	39
trager	$-\frac{2(-3b^2x^2-abx+2a^2)\sqrt{b\,x^3+a\,x^2}}{15b^2x}$	41

```
[In] int((b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(b*x+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \sqrt{ax^2+bx^3}\,dx = \frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx^3+ax^2}}{15\,b^2x}$$

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x^3 + a*x^2)/(b^2*x)
```

Sympy [F]

$$\int \sqrt{ax^2+bx^3}\,dx = \int \sqrt{ax^2+bx^3}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(1/2),x)
[Out] Integral(sqrt(a*x**2 + b*x**3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

$$\int \sqrt{ax^2+bx^3}\,dx = \frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}}{15\,b^2}$$

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int \sqrt{ax^2+bx^3}\,dx = \frac{4\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,b^2} + \frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a\mathrm{sgn}(x)}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)\mathrm{sgn}(x)}{b}\right)}{15\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*a^(5/2)*sgn(x)/b^2 + 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a*s
gn(x)/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*sgn(x)/b)/b
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(-2\,a^2+a\,b\,x+3\,b^2\,x^2\right)}{15\,b^2\,x}$$

```
[In] int((a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(3*b^2*x^2 - 2*a^2 + a*b*x))/(15*b^2*x)
```

3.235 $\int \frac{\sqrt{ax^2+bx^3}}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{\sqrt{ax^2+bx^3}}{x} \, dx = \frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

```
[Out] 2/3*(b*x^3+a*x^2)^(3/2)/b/x^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2039}

$$\int \frac{\sqrt{ax^2+bx^3}}{x} \, dx = \frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x,x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{2(ax^2+bx^3)^{3/2}}{3bx^3}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=\frac{2\left(x^2(a+bx)\right)^{3/2}}{3bx^3}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x,x]
[Out] (2*(x^2*(a + b*x))^(3/2))/(3*b*x^3)
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{2\sqrt{x^2(bx+a)}\,(bx+a)}{3xb}$	25
gosper	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
default	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
trager	$\frac{2(bx+a)\sqrt{b\,x^3+a\,x^2}}{3bx}$	27
pseudoelliptic	$2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$	28

```
[In] int((b*x^3+a*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2/3*(x^2*(b*x+a))^(1/2)/x*(b*x+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=\frac{2\,\sqrt{bx^3+ax^2}(bx+a)}{3\,bx}$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(b*x + a)/(b*x)
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3}}{x}\,dx = \int \frac{\sqrt{x^2\,(a + bx)}}{x}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{ax^2 + bx^3}}{x}\,dx = \frac{2\,(bx + a)^{\frac{3}{2}}}{3\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(21) = 42.

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{ax^2 + bx^3}}{x}\,dx = -\frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b} + \frac{2\left(3\,\sqrt{bx + a}a\mathrm{sgn}(x) + \left((bx + a)^{\frac{3}{2}} - 3\,\sqrt{bx + a}a\right)\mathrm{sgn}(x)\right)}{3\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2/3*a^(3/2)*sgn(x)/b + 2/3*(3*sqrt(b*x + a)*a*sgn(x) + ((b*x + a)^(3/2) -
3*sqrt(b*x + a)*a)*sgn(x))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=\int \frac{\sqrt{b\,x^3+a\,x^2}}{x}\,dx$$

```
[In] int((a*x^2 + b*x^3)^(1/2)/x,x)
[Out] int((a*x^2 + b*x^3)^(1/2)/x, x)
```

3.236 $\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 51

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

```
[Out] -2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))*a^(1/2)+2*(b*x^3+a*x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2046, 2033, 212}

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx=\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
```

```
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{ax^2+bx^3}}{x} + a\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx \\
&= \frac{2\sqrt{ax^2+bx^3}}{x} - (2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right) \\
&= \frac{2\sqrt{ax^2+bx^3}}{x} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx = \frac{2x\left(a+bx-\sqrt{a}\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^2,x]
[Out] (2*x*(a + b*x - Sqrt[a]*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/Sqrt
[x^2*(a + b*x)]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx+\sqrt{bx+a}\,\sqrt{a}}{x\sqrt{a}}$	36
default	$-\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\sqrt{bx+a}\right)}{x\sqrt{bx+a}}$	52

```
[In] int((b*x^3+a*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(arctanh((b*x+a)^(1/2)/a^(1/2))*b*x+(b*x+a)^(1/2)*a^(1/2))/x/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx = \left[\frac{\sqrt{a}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}}{x}, \frac{2\left(\sqrt{-a}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\right)}{x}\right]$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*sq
rt(b*x^3 + a*x^2))/x, 2*(sqrt(-a)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*
x)) + sqrt(b*x^3 + a*x^2))/x]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{x^2}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx = \int \frac{\sqrt{bx^3+ax^2}}{x^2}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + 2\,\sqrt{bx+a}\operatorname{sgn}(x) - \frac{2\,\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x)
 - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}}{x} + \frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{\frac{1}{x}}\,1\mathrm{i}}{\sqrt{b}}\right)\,\sqrt{b\,x^3+a\,x^2}\,\left(\frac{1}{x}\right)^{3/2}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^2,x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2))/x + (a^(1/2)*asin((a^(1/2)*(1/x)^(1/2)*1i)/b^(1/2
))*(a*x^2 + b*x^3)^(1/2)*(1/x)^(3/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

3.237 $\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx=-\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

```
[Out] -b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(1/2)-(b*x^3+a*x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2045, 2033, 212}

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx=-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{\sqrt{ax^2+bx^3}}{x^2}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/S
qrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
```

```
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{ax^2+bx^3}}{x^2}+\frac{1}{2}b\int\frac{1}{\sqrt{ax^2+bx^3}}\,dx\\ &= -\frac{\sqrt{ax^2+bx^3}}{x^2}-b\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\ &= -\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23

$$\int\frac{\sqrt{ax^2+bx^3}}{x^3}\,dx=-\frac{\sqrt{a+bx}\left(\sqrt{a}\sqrt{a+bx}+bx\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^3,x]
[Out] -((Sqrt[a + b*x]*(Sqrt[a]*Sqrt[a + b*x] + b*x*ArcTanh[Sqrt[a + b*x]/Sqrt[a]
]))/(Sqrt[a]*Sqrt[x^2*(a + b*x)]))
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{2} x^{2}-\left(2 a^{\frac{3}{2}}+\sqrt{a}\, b x \right) \sqrt{bx+a}}{4 a^{\frac{3}{2}} x^{2}}$	50
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b x+\sqrt{bx+a}\,\sqrt{a}\right)}{x^{2} \sqrt{bx+a}\,\sqrt{a}}$	56
risch	$-\frac{\sqrt{x^{2} \left(bx+a\right)}}{x^{2}}-\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{x^{2} \left(bx+a\right)}}{\sqrt{a}\,x \sqrt{bx+a}}$	57

```
[In] int((b*x^3+a*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(arctanh((b*x+a)^(1/2)/a^(1/2))*b^2*x^2-(2*a^(3/2)+a^(1/2)*b*x)*(b*x+a)
^(1/2))/a^(3/2)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx = \left[\frac{\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}a}{2\,ax^2}, \frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)-\sqrt{bx^3+ax^2}a}{ax^2}\right]$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2
) - 2*sqrt(b*x^3 + a*x^2)*a)/(a*x^2), (sqrt(-a)*b*x^2*arctan(sqrt(b*x^3 + a
*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*a)/(a*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}}{x^3}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx=\int \frac{\sqrt{bx^3+ax^2}}{x^3}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^3, x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx=\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}-\frac{\sqrt{bx+a}b\,\mathrm{sgn}(x)}{x}}{b}$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - sqrt(b*x + a)*b*sgn(x
)/x)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx=\int \frac{\sqrt{b\,x^3+a\,x^2}}{x^3}\,dx$$

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^3,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x^3, x)
```

3.238 $\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = -\frac{\sqrt{ax^2+bx^3}}{2x^3} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} + \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}$$

```
[Out] 1/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)-1/2*(b*x^3+a*x^2)^(1
/2)/x^3-1/4*b*(b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 2050, 2033, 212}

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} - \frac{\sqrt{ax^2+bx^3}}{2x^3}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
[Out] -1/2*Sqrt[a*x^2 + b*x^3]/x^3 - (b*Sqrt[a*x^2 + b*x^3])/(4*a*x^2) + (b^2*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{ax^2+bx^3}}{2x^3} + \frac{1}{4}b\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx \\
&= -\frac{\sqrt{ax^2+bx^3}}{2x^3} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} - \frac{b^2\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a} \\
&= -\frac{\sqrt{ax^2+bx^3}}{2x^3} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} + \frac{b^2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a} \\
&= -\frac{\sqrt{ax^2+bx^3}}{2x^3} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} + \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$
\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(2a+bx)+b^2x^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4a^{3/2}x^3\sqrt{a+bx}}
$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
[Out] (Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(2*a + b*x)) + b^2*x^2*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]]))/(4*a^(3/2)*x^3*Sqrt[a + b*x])
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{3} x^{3}-\left(\sqrt{a}\, b^{2} x^{2}-\frac{2 a^{\frac{3}{2}} b x}{3}-\frac{8 a^{\frac{5}{2}}}{3}\right) \sqrt{bx+a}}{8 a^{\frac{5}{2}} x^{3}}$	61
risch	$-\frac{(bx+2a) \sqrt{x^{2} (bx+a)}}{4 x^{3} a}+\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{x^{2} (bx+a)}}{4 a^{\frac{3}{2}} x \sqrt{bx+a}}$	69
default	$-\frac{\sqrt{b\, x^{3}+a\, x^{2}}\, \left((bx+a)^{\frac{3}{2}} a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) a\, b^{2} x^{2}+\sqrt{bx+a}\, a^{\frac{5}{2}}\right)}{4 x^{3} \sqrt{bx+a}\, a^{\frac{5}{2}}}$	73

```
[In] int((b*x^3+a*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a^(5/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b^3*x^3-(a^(1/2)*b^2*x^2-2/3*a
^(3/2)*b*x-8/3*a^(5/2))*(b*x+a)^(1/2))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = \left[\frac{\sqrt{a} b^2 x^3 \log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(abx+2\,a^2)}{8\,a^2x^3},\right.$$
$$\left.-\frac{\sqrt{-a} b^2 x^3 \arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}(abx+2\,a^2)}{4\,a^2x^3}\right]$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x
^2) - 2*sqrt(b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/4*(sqrt(-a)*b^2*
x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(a*b*x
 + 2*a^2))/(a^2*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx=\int \frac{\sqrt{x^2\,(a+bx)}}{x^4}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx=\int \frac{\sqrt{bx^3+ax^2}}{x^4}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^4, x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx=-\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}\left(x\right)}{\sqrt{-a}a}+\frac{(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}\left(x\right)+\sqrt{bx+a}ab^3\mathrm{sgn}\left(x\right)}{ab^2x^2}}{4\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + ((b*x + a)^(
3/2)*b^3*sgn(x) + sqrt(b*x + a)*a*b^3*sgn(x))/(a*b^2*x^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx=\int \frac{\sqrt{b\,x^3+a\,x^2}}{x^4}\,dx$$

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x^4, x)
```

3.239 $\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=-\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}$$

```
[Out] -1/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)-1/3*(b*x^3+a*x^2)^(
1/2)/x^4-1/12*b*(b*x^3+a*x^2)^(1/2)/a/x^3+1/8*b^2*(b*x^3+a*x^2)^(1/2)/a^2/x
^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 2050, 2033, 212}

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=-\frac{b^3\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
[Out] -1/3*Sqrt[a*x^2 + b*x^3]/x^4 - (b*Sqrt[a*x^2 + b*x^3])/(12*a*x^3) + (b^2*Sq
rt[a*x^2 + b*x^3])/(8*a^2*x^2) - (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]])/(8*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{ax^2+bx^3}}{3x^4}+\frac{1}{6}b\int\frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{b^2\int\frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}+\frac{b^3\int\frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(8a^2+2abx-3b^2x^2)+3b^3x^3\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{5/2}x^4\sqrt{a+bx}}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
[Out] -1/24*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 2*a*b*x - 3*b^2*
x^2) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5/2)*x^4*Sqrt[a + b*x
])
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{5\left(-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^4x^4+\sqrt{bx+a}\left(\sqrt{a}\,b^3x^3-\frac{2a^{\frac{3}{2}}b^2x^2}{3}+\frac{8a^{\frac{5}{2}}bx}{15}+\frac{16a^{\frac{7}{2}}}{5}\right)\right)}{64a^{\frac{7}{2}}x^4}$	72
risch	$-\frac{(-3b^2x^2+2abx+8a^2)\sqrt{x^2(bx+a)}}{24x^4a^2}-\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{8a^{\frac{5}{2}}x\sqrt{bx+a}}$	81
default	$\frac{\sqrt{b\,x^3+a\,x^2}\left(3(bx+a)^{\frac{5}{2}}a^{\frac{5}{2}}-8(bx+a)^{\frac{3}{2}}a^{\frac{7}{2}}-3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2b^3x^3-3\sqrt{bx+a}\,a^{\frac{9}{2}}\right)}{24x^4\sqrt{bx+a}\,a^{\frac{9}{2}}}$	89

```
[In] int((b*x^3+a*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -5/64*(-arctanh((b*x+a)^(1/2)/a^(1/2))*b^4*x^4+(b*x+a)^(1/2)*(a^(1/2)*b^3*x
^3-2/3*a^(3/2)*b^2*x^2+8/15*a^(5/2)*b*x+16/5*a^(7/2)))/a^(7/2)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = \left[\frac{3\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(3\,ab^2x^2-2\,a^2bx-8\,a^3)\sqrt{bx^3+ax^2}}{48\,a^3x^4},\ 3\sqrt{-a}b^3x^4\arctan\left(\sqrt{bx^3+}\right.\right.$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)
, 1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (3*
a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=\int \frac{\sqrt{x^2\,(a+bx)}}{x^5}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=\int \frac{\sqrt{bx^3+ax^2}}{x^5}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^5, x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{5}{2}}\,b^4\mathrm{sgn}(x)-8\,(bx+a)^{\frac{3}{2}}\,ab^4\mathrm{sgn}(x)-3\,\sqrt{bx+a}a^2b^4\mathrm{sgn}(x)}{a^2b^3x^3}}{24\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
+ a)^(5/2)*b^4*sgn(x) - 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*a^
2*b^4*sgn(x))/(a^2*b^3*x^3))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx=\int \frac{\sqrt{b\,x^3+a\,x^2}}{x^5}\,dx$$

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x^5, x)
```

3.240 $\int x^2(ax^2+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int x^2\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2(ax^2+bx^3)^{5/2}}{15b}-\frac{512a^5(ax^2+bx^3)^{5/2}}{45045b^6x^5}+\frac{256a^4(ax^2+bx^3)^{5/2}}{9009b^5x^4}-\frac{64a^3(ax^2+bx^3)^{5/2}}{1287b^4x^3}+\frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^2}-\frac{4a(ax^2+bx^3)^{5/2}}{39b^2x}$$

```
[Out] 2/15*(b*x^3+a*x^2)^(5/2)/b-512/45045*a^5*(b*x^3+a*x^2)^(5/2)/b^6/x^5+256/90
09*a^4*(b*x^3+a*x^2)^(5/2)/b^5/x^4-64/1287*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^3+
32/429*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^2-4/39*a*(b*x^3+a*x^2)^(5/2)/b^2/x
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2041, 2027, 2039}

$$\int x^2\left(ax^2+bx^3\right)^{3/2}\,dx=-\frac{512a^5(ax^2+bx^3)^{5/2}}{45045b^6x^5}+\frac{256a^4(ax^2+bx^3)^{5/2}}{9009b^5x^4}-\frac{64a^3(ax^2+bx^3)^{5/2}}{1287b^4x^3}+\frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^2}-\frac{4a(ax^2+bx^3)^{5/2}}{39b^2x}+\frac{2(ax^2+bx^3)^{5/2}}{15b}$$

```
[In] Int[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(15*b) - (512*a^5*(a*x^2 + b*x^3)^(5/2))/(45045*b
^6*x^5) + (256*a^4*(a*x^2 + b*x^3)^(5/2))/(9009*b^5*x^4) - (64*a^3*(a*x^2 +
 b*x^3)^(5/2))/(1287*b^4*x^3) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^2
) - (4*a*(a*x^2 + b*x^3)^(5/2))/(39*b^2*x)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b} - \frac{(2a)\int x\left(ax^2+bx^3\right)^{3/2}\,dx}{3b} \\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b} - \frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x} + \frac{\left(16a^2\right)\int \left(ax^2+bx^3\right)^{3/2}\,dx}{39b^2} \\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b} + \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2} - \frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x} - \frac{\left(32a^3\right)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x}\,dx}{143b^3} \\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b} - \frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3} + \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2} \\
&\quad - \frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x} + \frac{\left(128a^4\right)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx}{1287b^4} \\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b} + \frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4} - \frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3} \\
&\quad + \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2} - \frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x} - \frac{\left(256a^5\right)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx}{9009b^5}
\end{aligned}
$$

$$
= \frac{2(ax^2+bx^3)^{5/2}}{15b} - \frac{512a^5(ax^2+bx^3)^{5/2}}{45045b^6x^5} + \frac{256a^4(ax^2+bx^3)^{5/2}}{9009b^5x^4} - \frac{64a^3(ax^2+bx^3)^{5/2}}{1287b^4x^3} + \frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^2} - \frac{4a(ax^2+bx^3)^{5/2}}{39b^2x}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$
\int x^2(ax^2+bx^3)^{3/2}\,dx = \frac{2x(a+bx)^3(-256a^5+640a^4bx-1120a^3b^2x^2+1680a^2b^3x^3-2310ab^4x^4+3003b^5x^5)}{45045b^6\sqrt{x^2(a+bx)}}
$$

```
[In] Integrate[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(-256*a^5 + 640*a^4*b*x - 1120*a^3*b^2*x^2 + 1680*a^2*b^3*
x^3 - 2310*a*b^4*x^4 + 3003*b^5*x^5))/(45045*b^6*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.20

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{5}{2}}(35b^2x^2-20abx+8a^2)}{315b^3}$	32
gosper	$-\frac{2(bx+a)(-3003b^5x^5+2310a\,b^4x^4-1680a^2b^3x^3+1120a^3b^2x^2-640a^4bx+256a^5)(b\,x^3+a\,x^2)^{\frac{3}{2}}}{45045b^6x^3}$	79
default	$-\frac{2(bx+a)(-3003b^5x^5+2310a\,b^4x^4-1680a^2b^3x^3+1120a^3b^2x^2-640a^4bx+256a^5)(b\,x^3+a\,x^2)^{\frac{3}{2}}}{45045b^6x^3}$	79
risch	$-\frac{2\sqrt{x^2(bx+a)}\,(-3003b^7x^7-3696a\,b^6x^6-63a^2b^5x^5+70a^3b^4x^4-80a^4b^3x^3+96a^5b^2x^2-128a^6bx+256a^7)}{45045x\,b^6}$	94
trager	$-\frac{2(-3003b^7x^7-3696a\,b^6x^6-63a^2b^5x^5+70a^3b^4x^4-80a^4b^3x^3+96a^5b^2x^2-128a^6bx+256a^7)\sqrt{b\,x^3+a\,x^2}}{45045b^6x}$	96

```
[In] int(x^2*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(b*x+a)^(5/2)*(35*b^2*x^2-20*a*b*x+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.59

$$\int x^2 \left(ax^2 + bx^3\right)^{3/2} \, dx = \frac{2 \left(3003 \, b^7 x^7 + 3696 \, a b^6 x^6 + 63 \, a^2 b^5 x^5 - 70 \, a^3 b^4 x^4 + 80 \, a^4 b^3 x^3 - 96 \, a^5 b^2 x^2 + 128 \, a^6 b x - 256}{45045 \, b^6 x}$$

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x^3 + a*x^2
)/(b^6*x)
```

Sympy [F]

$$\int x^2 \left(ax^2 + bx^3\right)^{3/2} \, dx = \int x^2 \left(x^2 (a + bx)\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate(x**2*(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.53

$$\int x^2 \left(ax^2 + bx^3\right)^{3/2} \, dx = \frac{2 \left(3003 \, b^7 x^7 + 3696 \, a b^6 x^6 + 63 \, a^2 b^5 x^5 - 70 \, a^3 b^4 x^4 + 80 \, a^4 b^3 x^3 - 96 \, a^5 b^2 x^2 + 128 \, a^6 b x - 256}{45045 \, b^6}$$

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x + a)/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(137) = 274$.

Time = 0.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.75

$$\int x^2 \left(ax^2+bx^3\right)^{3/2}\,dx = \frac{512\,a^{\frac{15}{2}}\operatorname{sgn}(x)}{45045\,b^6} + \frac{2\left(\frac{65\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}a^5\right)a^2\operatorname{sgn}(x)}{b^5}+\frac{30\left(231\,(bx+a)^{\frac{13}{2}}-1638\right.}{}\right.}{}$$

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 512/45045*a^(15/2)*sgn(x)/b^6 + 2/45045*(65*(63*(b*x + a)^(11/2) - 385*(b*x
 + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(
b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2*sgn(x)/b^5 + 30*(231*(b*x +
 a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x
 + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 300
3*sqrt(b*x + a)*a^6)*a*sgn(x)/b^5 + 7*(429*(b*x + a)^(15/2) - 3465*(b*x + a
)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175
*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^
6 - 6435*sqrt(b*x + a)*a^7)*sgn(x)/b^5)/b
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

$$\int x^2 \left(ax^2+bx^3\right)^{3/2}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2\,\left(256\,a^5-640\,a^4\,b\,x+1120\,a^3\,b^2\,x^2-1680\,a^2\,b^3\,x^3+2310\,a\,b^4\,x^4-3003\,b^5\,x^5\right)}{45045\,b^6\,x}$$

```
[In] int(x^2*(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(256*a^5 - 3003*b^5*x^5 + 2310*a*b^4*
x^4 + 1120*a^3*b^2*x^2 - 1680*a^2*b^3*x^3 - 640*a^4*b*x))/(45045*b^6*x)
```

3.241 $\int x(ax^2+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 136

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}$$

```
[Out] 256/15015*a^4*(b*x^3+a*x^2)^(5/2)/b^5/x^5-128/3003*a^3*(b*x^3+a*x^2)^(5/2)/
b^4/x^4+32/429*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^3-16/143*a*(b*x^3+a*x^2)^(5/2)
/b^2/x^2+2/13*(b*x^3+a*x^2)^(5/2)/b/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2041, 2027, 2039}

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}$$

```
[In] Int[x*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (256*a^4*(a*x^2 + b*x^3)^(5/2))/(15015*b^5*x^5) - (128*a^3*(a*x^2 + b*x^3)^
(5/2))/(3003*b^4*x^4) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^3) - (16*
a*(a*x^2 + b*x^3)^(5/2))/(143*b^2*x^2) + (2*(a*x^2 + b*x^3)^(5/2))/(13*b*x)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(ax^2+bx^3)^{5/2}}{13bx} - \frac{(8a)\int (ax^2+bx^3)^{3/2}\,dx}{13b}\\
&= -\frac{16a(ax^2+bx^3)^{5/2}}{143b^2x^2} + \frac{2(ax^2+bx^3)^{5/2}}{13bx} + \frac{(48a^2)\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx}{143b^2}\\
&= \frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^3} - \frac{16a(ax^2+bx^3)^{5/2}}{143b^2x^2} + \frac{2(ax^2+bx^3)^{5/2}}{13bx} - \frac{(64a^3)\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx}{429b^3}\\
&= -\frac{128a^3(ax^2+bx^3)^{5/2}}{3003b^4x^4} + \frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^3} - \frac{16a(ax^2+bx^3)^{5/2}}{143b^2x^2}\\
&\quad + \frac{2(ax^2+bx^3)^{5/2}}{13bx} + \frac{(128a^4)\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx}{3003b^4}\\
&= \frac{256a^4(ax^2+bx^3)^{5/2}}{15015b^5x^5} - \frac{128a^3(ax^2+bx^3)^{5/2}}{3003b^4x^4}\\
&\quad + \frac{32a^2(ax^2+bx^3)^{5/2}}{429b^3x^3} - \frac{16a(ax^2+bx^3)^{5/2}}{143b^2x^2} + \frac{2(ax^2+bx^3)^{5/2}}{13bx}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2x(a+bx)^3\left(128a^4-320a^3bx+560a^2b^2x^2-840ab^3x^3+1155b^4x^4\right)}{15015b^5\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[x*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(128*a^4 - 320*a^3*b*x + 560*a^2*b^2*x^2 - 840*a*b^3*x^3 +
 1155*b^4*x^4))/(15015*b^5*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.15

method	result	size
pseudoelliptic	$-\frac{2(bx+a)^{\frac{5}{2}}(-5bx+2a)}{35b^2}$	21
gosper	$\frac{2(bx+a)\left(1155b^4x^4-840a\,b^3x^3+560a^2b^2x^2-320a^3bx+128a^4\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{15015x^3b^5}$	68
default	$\frac{2(bx+a)\left(1155b^4x^4-840a\,b^3x^3+560a^2b^2x^2-320a^3bx+128a^4\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{15015x^3b^5}$	68
risch	$\frac{2\sqrt{x^2(bx+a)}\left(1155b^6x^6+1470a\,x^5b^5+35a^2x^4b^4-40a^3x^3b^3+48a^4x^2b^2-64a^5xb+128a^6\right)}{15015x\,b^5}$	83
trager	$\frac{2\left(1155b^6x^6+1470a\,x^5b^5+35a^2x^4b^4-40a^3x^3b^3+48a^4x^2b^2-64a^5xb+128a^6\right)\sqrt{b\,x^3+a\,x^2}}{15015b^5x}$	85

```
[In] int(x*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*(b*x+a)^(5/2)*(-5*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.62

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx^3+ax}}{15015\,b^5x}$$

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

Sympy [F]

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\int x\left(x^2(a+bx)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.55

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\,(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6)\sqrt{bx+a}}{15015\,b^5}$$

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(116) = 232.

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.81

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=-\frac{256\,a^{\frac{13}{2}}\operatorname{sgn}(x)}{15015\,b^5}+\frac{2\Bigg(\frac{143\,\big(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}a^4\big)a^2\operatorname{sgn}(x)}{b^4}+\frac{130\,\big(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+}{}}{}$$

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -256/15015*a^(13/2)*sgn(x)/b^5 + 2/45045*(143*(35*(b*x + a)^(9/2) - 180*(b*
x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sq
rt(b*x + a)*a^4)*a^2*sgn(x)/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^
(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x +
a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a*sgn(x)/b^4 + 15*(231*(b*x + a)^(13/
2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7
/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b
*x + a)*a^6)*sgn(x)/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int x\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2\,\left(128\,a^4-320\,a^3\,b\,x+560\,a^2\,b^2\,x^2-840\,a\,b^3\,x^3+1155\,b^4\,x^4\right)}{15015\,b^5\,x}$$

```
[In] int(x*(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(128*a^4 + 1155*b^4*x^4 - 840*a*b^3*x^
3 + 560*a^2*b^2*x^2 - 320*a^3*b*x))/(15015*b^5*x)
```

3.242 $\int (ax^2+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int (ax^2+bx^3)^{3/2}\,dx = -\frac{32a^3(ax^2+bx^3)^{5/2}}{1155b^4x^5} + \frac{16a^2(ax^2+bx^3)^{5/2}}{231b^3x^4} - \frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2+bx^3)^{5/2}}{11bx^2}$$

```
[Out] -32/1155*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^5+16/231*a^2*(b*x^3+a*x^2)^(5/2)/b^3
/x^4-4/33*a*(b*x^3+a*x^2)^(5/2)/b^2/x^3+2/11*(b*x^3+a*x^2)^(5/2)/b/x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2027, 2041, 2039}

$$\int (ax^2+bx^3)^{3/2}\,dx = -\frac{32a^3(ax^2+bx^3)^{5/2}}{1155b^4x^5} + \frac{16a^2(ax^2+bx^3)^{5/2}}{231b^3x^4} - \frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2+bx^3)^{5/2}}{11bx^2}$$

```
[In] Int[(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-32*a^3*(a*x^2 + b*x^3)^(5/2))/(1155*b^4*x^5) + (16*a^2*(a*x^2 + b*x^3)^(5
/2))/(231*b^3*x^4) - (4*a*(a*x^2 + b*x^3)^(5/2))/(33*b^2*x^3) + (2*(a*x^2 +
 b*x^3)^(5/2))/(11*b*x^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(ax^2+bx^3)^{5/2}}{11bx^2} - \frac{(6a)\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx}{11b} \\
&= -\frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2+bx^3)^{5/2}}{11bx^2} + \frac{(8a^2)\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx}{33b^2} \\
&= \frac{16a^2(ax^2+bx^3)^{5/2}}{231b^3x^4} - \frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2+bx^3)^{5/2}}{11bx^2} - \frac{(16a^3)\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx}{231b^3} \\
&= -\frac{32a^3(ax^2+bx^3)^{5/2}}{1155b^4x^5} + \frac{16a^2(ax^2+bx^3)^{5/2}}{231b^3x^4} - \frac{4a(ax^2+bx^3)^{5/2}}{33b^2x^3} + \frac{2(ax^2+bx^3)^{5/2}}{11bx^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx = \frac{2x(a+bx)^3\left(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3\right)}{1155b^4\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(a + b*x)^3*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(1155
*b^4*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.12

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
gosper	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
default	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)}{1155x\,b^{4}}$	72
trager	$-\frac{2\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{1155b^{4}x}$	74

```
[In] int((b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/5*(b*x+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \left(ax^{2}+bx^{3}\right)^{3/2}\,dx=\frac{2\left(105\,b^{5}x^{5}+140\,ab^{4}x^{4}+5\,a^{2}b^{3}x^{3}-6\,a^{3}b^{2}x^{2}+8\,a^{4}bx-16\,a^{5}\right)\sqrt{bx^{3}+ax^{2}}}{1155\,b^{4}x}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int \left(ax^{2}+bx^{3}\right)^{3/2}\,dx=\int \left(ax^{2}+bx^{3}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(3/2),x)
[Out] Integral((a*x**2 + b*x**3)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx+a}}{1155\,b^4}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(92) = 184.

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.94

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{32\,a^{\frac{11}{2}}\,\mathrm{sgn}(x)}{1155\,b^4}$$
$$+\frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a^2\mathrm{sgn}(x)}{b^3}+\frac{22\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a\right.}{b^3}\right.}{3465}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 32/1155*a^(11/2)*sgn(x)/b^4 + 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^
(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2*sgn(x)/b^3 + 2
2*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 4
20*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a*sgn(x)/b^3 + 5*(63*(b*x +
 a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x +
a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*sgn(x)/b^3
)/b
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=-\frac{2\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2\,\left(16\,a^3-40\,a^2\,b\,x+70\,a\,b^2\,x^2-105\,b^3\,x^3\right)}{1155\,b^4\,x}$$

```
[In] int((a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(16*a^3 - 105*b^3*x^3 + 70*a*b^2*x^2
- 40*a^2*b*x))/(1155*b^4*x)
```

3.243 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x}\,dx=\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{315b^3x^5}-\frac{8a\left(ax^2+bx^3\right)^{5/2}}{63b^2x^4}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}$$

```
[Out] 16/315*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^5-8/63*a*(b*x^3+a*x^2)^(5/2)/b^2/x^4+2
/9*(b*x^3+a*x^2)^(5/2)/b/x^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2039}

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x}\,dx=\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{315b^3x^5}-\frac{8a\left(ax^2+bx^3\right)^{5/2}}{63b^2x^4}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}$$

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x,x]
```

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(5/2))/(315*b^3*x^5) - (8*a*(a*x^2 + b*x^3)^(5/2))/
(63*b^2*x^4) + (2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(ax^2+bx^3)^{5/2}}{9bx^3} - \frac{(4a)\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx}{9b}\\ &= -\frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4} + \frac{2(ax^2+bx^3)^{5/2}}{9bx^3} + \frac{(8a^2)\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx}{63b^2}\\ &= \frac{16a^2(ax^2+bx^3)^{5/2}}{315b^3x^5} - \frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4} + \frac{2(ax^2+bx^3)^{5/2}}{9bx^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2x(a+bx)^3\left(8a^2-20abx+35b^2x^2\right)}{315b^3\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x,x]
[Out] (2*x*(a + b*x)^3*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3*Sqrt[x^2*(a + b*
x)])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 1.82 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.44

method	result	size
pseudoelliptic	$-2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{2\sqrt{bx+a}\,(bx+4a)}{3}$	35
gosper	$\frac{2(bx+a)\left(35b^{2}x^{2}-20abx+8a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315b^{3}x^{3}}$	46
default	$\frac{2(bx+a)\left(35b^{2}x^{2}-20abx+8a^{2}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315b^{3}x^{3}}$	46
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(35b^{4}x^{4}+50a\,b^{3}x^{3}+3a^{2}b^{2}x^{2}-4a^{3}bx+8a^{4}\right)}{315x\,b^{3}}$	61
trager	$\frac{2\left(35b^{4}x^{4}+50a\,b^{3}x^{3}+3a^{2}b^{2}x^{2}-4a^{3}bx+8a^{4}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315b^{3}x}$	63

```
[In] int((b*x^3+a*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2/3*(b*x+a)^(1/2)*(b*x+4*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x}\,dx=\frac{2\,\left(35\,b^{4}x^{4}+50\,ab^{3}x^{3}+3\,a^{2}b^{2}x^{2}-4\,a^{3}bx+8\,a^{4}\right)\sqrt{bx^{3}+ax^{2}}}{315\,b^{3}x}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x}\,dx=\int \frac{\left(x^{2}(a+bx)\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\,(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4)\sqrt{bx+a}}{315\,b^3}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(68) = 136.

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.16

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = -\frac{16\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,b^3} + \frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a^2\mathrm{sgn}(x)}{b^2} + \frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a\mathrm{sgn}(x)}{b^2} + \left(35\,(bx\right.\right.}{315\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -16/315*a^(9/2)*sgn(x)/b^3 + 2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3
/2)*a + 15*sqrt(b*x + a)*a^2)*a^2*sgn(x)/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(
b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/
b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{(ax^2+bx^3)^{3/2}}{x}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,{(a+b\,x)}^2\,(8\,a^2-20\,a\,b\,x+35\,b^2\,x^2)}{315\,b^3\,x}$$

```
[In] int((a*x^2 + b*x^3)^(3/2)/x,x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(8*a^2 + 35*b^2*x^2 - 20*a*b*x))/(315*
b^3*x)
```

3.244 $\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = -\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}+\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}$$

```
[Out] -4/35*a*(b*x^3+a*x^2)^(5/2)/b^2/x^5+2/7*(b*x^3+a*x^2)^(5/2)/b/x^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2039}

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}$$

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*
x^4)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(ax^2+bx^3)^{5/2}}{7bx^4} - \frac{(2a)\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx}{7b}\\ &= -\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5} + \frac{2(ax^2+bx^3)^{5/2}}{7bx^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = \frac{2(x^2(a+bx))^{5/2}(-2a+5bx)}{35b^2x^5}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^2,x]
[Out] (2*(x^2*(a + b*x))^(5/2)*(-2*a + 5*b*x))/(35*b^2*x^5)
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
default	$-\frac{2(bx+a)(-5bx+2a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	35
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3}\right)}{35x\,b^{2}}$	50
trager	$-\frac{2\left(-5b^{3}x^{3}-8a\,b^{2}x^{2}-a^{2}bx+2a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{35b^{2}x}$	52
pseudoelliptic	$\frac{2bx\sqrt{bx+a}\,\sqrt{a}-3\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx-\sqrt{bx+a}\,a^{\frac{3}{2}}}{x\sqrt{a}}$	52

```
[In] int((b*x^3+a*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -2/35*(b*x+a)*(-5*b*x+2*a)*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx=\frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx^3+ax^2}}{35\,b^2x}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*x^2)/(b^2*x
)
```

Sympy [F]

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**2,x)
[Out] Integral((x**2*(a + b*x))**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx=\frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx+a}}{35\,b^2}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(44) = 88.

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.62

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx=\frac{4\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,b^2}$$
$$+\frac{2\Bigg(\frac{35\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a^2\mathrm{sgn}(x)}{b}+\frac{14\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a\mathrm{sgn}(x)}{b}+\frac{3\Big(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2}{b}}{105\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 4/35*a^(7/2)*sgn(x)/b^2 + 2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a
^2*sgn(x)/b + 14*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
a)*a^2)*a*sgn(x)/b + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x
+ a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b)/b
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^2}\,dx = -\frac{2\,(2\,a-5\,b\,x)\,\sqrt{b\,x^3+a\,x^2}\,{(a+b\,x)}^2}{35\,b^2\,x}$$

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^2,x)
```

```
[Out] -(2*(2*a - 5*b*x)*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(35*b^2*x)
```

3.245 $\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

```
[Out] 2/5*(b*x^3+a*x^2)^(5/2)/b/x^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2039}

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5)
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = \frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2(x^2(a+bx))^{5/2}}{5bx^5}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^3,x]
[Out] (2*(x^2*(a + b*x))^(5/2))/(5*b*x^5)
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5b\,x^{3}}$	27
default	$\frac{2(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5b\,x^{3}}$	27
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(b^{2}x^{2}+2abx+a^{2}\right)}{5xb}$	36
trager	$\frac{2\left(b^{2}x^{2}+2abx+a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{5bx}$	38
pseudoelliptic	$\frac{-3\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}-2\sqrt{bx+a}\,a^{\frac{3}{2}}-5bx\sqrt{bx+a}\,\sqrt{a}}{4x^{2}\sqrt{a}}$	56

```
[In] int((b*x^3+a*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
[Out] 2/5*(b*x+a)/b*(b*x^3+a*x^2)^(3/2)/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx^3+ax^2}}{5\,bx}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x^3 + a*x^2)/(b*x)
```

Sympy [F]

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx=\frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx+a}}{5\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(21) = 42.

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.56

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx=-\frac{2\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b}+\frac{2\,\Big(15\,\sqrt{bx+a}a^2\mathrm{sgn}(x)+10\,\Big((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\Big)a\mathrm{sgn}(x)+\Big(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+}}{15\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -2/5*a^(5/2)*sgn(x)/b + 2/15*(15*sqrt(b*x + a)*a^2*sgn(x) + 10*((b*x + a)^(
3/2) - 3*sqrt(b*x + a)*a)*a*sgn(x) + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2
)*a + 15*sqrt(b*x + a)*a^2)*sgn(x))/b
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2}{5\,b\,x}$$

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^3,x)
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(5*b*x)
```

3.246 $\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \frac{2a\sqrt{ax^2+bx^3}}{x} + \frac{2(ax^2+bx^3)^{3/2}}{3x^3} - 2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

```
[Out] 2/3*(b*x^3+a*x^2)^(3/2)/x^3-2*a^(3/2)*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2)
)+2*a*(b*x^3+a*x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2046, 2033, 212}

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = -2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right) + \frac{2a\sqrt{ax^2+bx^3}}{x} + \frac{2(ax^2+bx^3)^{3/2}}{3x^3}$$

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (2*a*Sqrt[a*x^2 + b*x^3])/x + (2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) - 2*a^(3/2)
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(ax^2+bx^3)^{3/2}}{3x^3}+a\int\frac{\sqrt{ax^2+bx^3}}{x^2}\,dx\\ &= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}+a^2\int\frac{1}{\sqrt{ax^2+bx^3}}\,dx\\ &= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}-\left(2a^2\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\ &= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int\frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx=\frac{2x\sqrt{a+bx}\left(\sqrt{a+bx}(4a+bx)-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (2*x*Sqrt[a + b*x]*(Sqrt[a + b*x]*(4*a + b*x) - 3*a^(3/2)*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]]))/(3*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^3 x^3-\sqrt{bx+a}\left(\sqrt{a}\, b^2 x^2+\frac{14 a^{\frac{3}{2}} b x}{3}+\frac{8 a^{\frac{5}{2}}}{3}\right)}{8 a^{\frac{3}{2}} x^3}$	61
default	$-\frac{2\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(3 a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-(bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)}{3 x^3 (bx+a)^{\frac{3}{2}}}$	63

```
[In] int((b*x^3+a*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^(3/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b^3*x^3-(b*x+a)^(1/2)*(a^(1/2)*
b^2*x^2+14/3*a^(3/2)*b*x+8/3*a^(5/2)))/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.76

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx = \Bigg[\frac{3\,a^{\frac{3}{2}} x \log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(bx+4\,a)}{3\,x}, 2\,\Big(3\,\sqrt{-a}ax\arctan\Big(\sqrt{b}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)
+ 2*sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x, 2/3*(3*sqrt(-a)*a*x*arctan(sqrt(b*x
^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x]
```

Sympy [F]

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^4, x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \frac{2\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2}{3}\,(bx+a)^{\frac{3}{2}}\operatorname{sgn}(x) + 2\,\sqrt{bx+a}a\operatorname{sgn}(x) - \frac{2\left(3\,a^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + 4\,\sqrt{-a}a^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3\,\sqrt{-a}}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*(b*x + a)^(3/2)*
sgn(x) + 2*sqrt(b*x + a)*a*sgn(x) - 2/3*(3*a^2*arctan(sqrt(a)/sqrt(-a)) + 4
*sqrt(-a)*a^(3/2))*sgn(x)/sqrt(-a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^4}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^4}\,dx$$

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^4, x)
```

3.247 $\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \frac{3b\sqrt{ax^2+bx^3}}{x} - \frac{(ax^2+bx^3)^{3/2}}{x^4} - 3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

```
[Out] -(b*x^3+a*x^2)^(3/2)/x^4-3*b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))*a^(1/2)
+3*b*(b*x^3+a*x^2)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 2046, 2033, 212}

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = -3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right) + \frac{3b\sqrt{ax^2+bx^3}}{x} - \frac{(ax^2+bx^3)^{3/2}}{x^4}$$

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (3*b*Sqrt[a*x^2 + b*x^3])/x - (a*x^2 + b*x^3)^(3/2)/x^4 - 3*Sqrt[a]*b*ArcTa
nh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2046

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*
(n - j)*(p/(c^j*(m + n*p + 1))), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}+\frac{1}{2}(3b)\int\frac{\sqrt{ax^2+bx^3}}{x^2}\,dx\\
&= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}+\frac{1}{2}(3ab)\int\frac{1}{\sqrt{ax^2+bx^3}}\,dx\\
&= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}-(3ab)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\
&= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}-3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx=-\frac{\sqrt{a+bx}\left((a-2bx)\sqrt{a+bx}+3\sqrt{a}bx\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
[Out] -((Sqrt[a + b*x]*((a - 2*b*x)*Sqrt[a + b*x] + 3*Sqrt[a]*b*x*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]]))/Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{a\sqrt{x^{2}(bx+a)}}{x^{2}}+\frac{b\left(4\sqrt{bx+a}-6\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\sqrt{x^{2}(bx+a)}}{2x\sqrt{bx+a}}$	70
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(-2bx\sqrt{bx+a}\,\sqrt{a}+3\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abx+\sqrt{bx+a}\,a^{\frac{3}{2}}\right)}{x^{4}(bx+a)^{\frac{3}{2}}\sqrt{a}}$	72
pseudoelliptic	$\frac{-\frac{3\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{4}x^{4}}{64}+\frac{3\sqrt{bx+a}\,\left(\sqrt{a}\,b^{3}x^{3}-\frac{2a^{\frac{3}{2}}b^{2}x^{2}}{3}-8a^{\frac{5}{2}}bx-\frac{16a^{\frac{7}{2}}}{3}\right)}{64}}{a^{\frac{5}{2}}x^{4}}$	72

```
[In] int((b*x^3+a*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x^2*(x^2*(b*x+a))^(1/2)+1/2*b*(4*(b*x+a)^(1/2)-6*a^(1/2)*arctanh((b*x+a)
^(1/2)/a^(1/2)))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.86

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \left[\frac{3\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(2\,bx-a)}{2\,x^2}, \frac{3\,\sqrt{-a}bx^2\arctan\left(\sqrt{b\right.}}{}\right.$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x
^2) + 2*sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2, (3*sqrt(-a)*b*x^2*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^5}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^5, x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \frac{\frac{3\,ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} + 2\sqrt{bx+a}ab^2\mathrm{sgn}(x) - \frac{\sqrt{bx+a}ab\mathrm{sgn}(x)}{x}}{b}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*b
^2*sgn(x) - sqrt(b*x + a)*a*b*sgn(x)/x)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^5}\,dx$$

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^5, x)
```

3.248 $\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}-\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}$$

```
[Out] -1/2*(b*x^3+a*x^2)^(3/2)/x^5-3/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))
/a^(1/2)-3/4*b*(b*x^3+a*x^2)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2045, 2033, 212}

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = -\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}-\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(4*x^2) - (a*x^2 + b*x^3)^(3/2)/(2*x^5) - (3*b^2
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*Sqrt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ax^2+bx^3)^{3/2}}{2x^5}+\frac{1}{4}(3b)\int\frac{\sqrt{ax^2+bx^3}}{x^3}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}+\frac{1}{8}\left(3b^2\right)\int\frac{1}{\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}-\frac{1}{4}\left(3b^2\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int\frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx=-\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(2a+5bx)+3b^2x^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4\sqrt{a}x^3\sqrt{a+bx}}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
[Out] -1/4*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(2*a + 5*b*x) + 3*b^2*x^2*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(Sqrt[a]*x^3*Sqrt[a + b*x])
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{(5bx+2a)\sqrt{x^2(bx+a)}}{4x^3}-\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{4\sqrt{a}\,x\sqrt{bx+a}}$	67
default	$-\frac{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2+5\sqrt{a}\,(bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a^{\frac{3}{2}}\right)}{4x^5(bx+a)^{\frac{3}{2}}\sqrt{a}}$	74
pseudoelliptic	$-\frac{-\frac{15\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x^5b^5}{128}+\sqrt{bx+a}\left(\frac{15\sqrt{a}\,b^4x^4}{128}-\frac{5a^{\frac{3}{2}}b^3x^3}{64}+\frac{a^{\frac{5}{2}}b^2x^2}{16}+\frac{11a^{\frac{7}{2}}bx}{8}+a^{\frac{9}{2}}\right)}{5a^{\frac{7}{2}}x^5}$	82

```
[In] int((b*x^3+a*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(5*b*x+2*a)/x^3*(x^2*(b*x+a))^(1/2)-3/4*b^2/a^(1/2)*arctanh((b*x+a)^(1
/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.90

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx=\left[\frac{3\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(5\,abx+2\,a^2)}{8\,ax^3},\ \frac{3\,\sqrt{-a}b^2x^3\operatorname{arcta}}{}\right.$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) - 2*sqrt(b*x^3 + a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3), 1/4*(3*sqrt(-a)*b
^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*(5*
a*b*x + 2*a^2))/(a*x^3)]
```

Sympy [F]

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^6}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^6, x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} - \frac{5\,(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x) - 3\,\sqrt{bx+a}ab^3\mathrm{sgn}(x)}{b^2x^2}}{4\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - (5*(b*x + a)^(3
/2)*b^3*sgn(x) - 3*sqrt(b*x + a)*a*b^3*sgn(x))/(b^2*x^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(b\,x^3+a\,x^2)^{3/2}}{x^6}\,dx$$

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^6,x)
[Out] int((a*x^2 + b*x^3)^(3/2)/x^6, x)
```

3.249 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx=-\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}+\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}$$

```
[Out] -1/3*(b*x^3+a*x^2)^(3/2)/x^6+1/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))
/a^(3/2)-1/4*b*(b*x^3+a*x^2)^(1/2)/x^3-1/8*b^2*(b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 2050, 2033, 212}

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx=\frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
[Out] -1/4*(b*Sqrt[a*x^2 + b*x^3])/x^3 - (b^2*Sqrt[a*x^2 + b*x^3])/(8*a*x^2) - (a
*x^2 + b*x^3)^(3/2)/(3*x^6) + (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/(8*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ax^2+bx^3)^{3/2}}{3x^6}+\frac{1}{2}b\int\frac{\sqrt{ax^2+bx^3}}{x^4}\,dx\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{(ax^2+bx^3)^{3/2}}{3x^6}+\frac{1}{8}b^2\int\frac{1}{x\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{(ax^2+bx^3)^{3/2}}{3x^6}-\frac{b^3\int\frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a}\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{(ax^2+bx^3)^{3/2}}{3x^6}+\frac{b^3\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a}\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{(ax^2+bx^3)^{3/2}}{3x^6}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(8a^2+14abx+3b^2x^2)+3b^3x^3\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{24a^{3/2}x^4\sqrt{a+bx}}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(8*a^2 + 14*a*b*x + 3*b^2*x^2
)) + 3*b^3*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(24*a^(3/2)*x^4*Sqrt[a + b*
x])
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{(3b^2x^2+14abx+8a^2)\sqrt{x^2(bx+a)}}{24x^4a}+\frac{b^3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{8a^{\frac{3}{2}}x\sqrt{bx+a}}$	81
default	$-\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\left(3a^{\frac{3}{2}}(bx+a)^{\frac{5}{2}}-3\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^3x^3+8a^{\frac{5}{2}}(bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a^{\frac{7}{2}}\right)}{24x^6(bx+a)^{\frac{3}{2}}a^{\frac{5}{2}}}$	87
pseudoelliptic	$-\frac{13\left(\frac{105\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^6x^6}{1664}+\sqrt{bx+a}\left(-\frac{105\sqrt{a}\,b^5x^5}{1664}+\frac{35a^{\frac{3}{2}}b^4x^4}{832}-\frac{7a^{\frac{5}{2}}b^3x^3}{208}+\frac{3b^2x^2a^{\frac{7}{2}}}{104}+a^{\frac{9}{2}}bx+\frac{10a^{\frac{11}{2}}}{13}\right)\right)}{60a^{\frac{9}{2}}x^6}$	94

```
[In] int((b*x^3+a*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(3*b^2*x^2+14*a*b*x+8*a^2)/x^4/a*(x^2*(b*x+a))^(1/2)+1/8*b^3/a^(3/2)*
arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.61

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = \left[\frac{3\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{48\,a^2x^4},\right.$$
$$\left.-\frac{3\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+(3\,ab^2x^2+14\,a^2bx+8\,a^3)\sqrt{bx^3+ax^2}}{24\,a^2x^4}\right]$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) - 2*(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4
), -1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4)]
```

Sympy [F]

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**7, x)
```

Maxima [F]

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^7, x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx=-\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}\left(x\right)}{\sqrt{-a}a}+\frac{3\,(bx+a)^{\frac{5}{2}}b^4\mathrm{sgn}\left(x\right)+8\,(bx+a)^{\frac{3}{2}}ab^4\mathrm{sgn}\left(x\right)-3\,\sqrt{bx+a}a^2b^4\mathrm{sgn}\left(x\right)}{ab^3x^3}}{24\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + (3*(b*x +
 a)^(5/2)*b^4*sgn(x) + 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*a^2
*b^4*sgn(x))/(a*b^3*x^3))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx=\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^7}\,dx$$

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^7,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^7, x)
```

3.250 $\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}-\frac{3b^4\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}$$

```
[Out] -1/4*(b*x^3+a*x^2)^(3/2)/x^7-3/64*b^4*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2)
)/a^(5/2)-1/8*b*(b*x^3+a*x^2)^(1/2)/x^4-1/32*b^2*(b*x^3+a*x^2)^(1/2)/a/x^3+
3/64*b^3*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 2050, 2033, 212}

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = -\frac{3b^4\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}-\frac{b\sqrt{ax^2+bx^3}}{8x^4}$$

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
[Out] -1/8*(b*Sqrt[a*x^2 + b*x^3])/x^4 - (b^2*Sqrt[a*x^2 + b*x^3])/(32*a*x^3) + (
3*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^2) - (a*x^2 + b*x^3)^(3/2)/(4*x^7) - (
3*b^4*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ax^2+bx^3)^{3/2}}{4x^7}+\frac{1}{8}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx\\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}+\frac{1}{16}b^2\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}-\frac{(3b^3)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{64a}\\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{(ax^2+bx^3)^{3/2}}{4x^7}+\frac{(3b^4)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{128a^2}\\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}\\
&\quad-\frac{(ax^2+bx^3)^{3/2}}{4x^7}-\frac{(3b^4)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{64a^2}
\end{aligned}
$$

$$= -\frac{b\sqrt{ax^2+bx^3}}{8x^4} - \frac{b^2\sqrt{ax^2+bx^3}}{32ax^3} + \frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2} - \frac{(ax^2+bx^3)^{3/2}}{4x^7} - \frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(16a^3+24a^2bx+2ab^2x^2-3b^3x^3)+3b^4x^4\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{64a^{5/2}x^5\sqrt{a+bx}}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
[Out] -1/64*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(16*a^3 + 24*a^2*b*x + 2*
a*b^2*x^2 - 3*b^3*x^3) + 3*b^4*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5/2
)*x^5*Sqrt[a + b*x])
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.67

method	result	si
risch	$-\frac{(-3b^3x^3+2a\,b^2x^2+24a^2bx+16a^3)\sqrt{x^2(bx+a)}}{64x^5a^2}-\frac{3b^4\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{64a^{\frac{5}{2}}x\sqrt{bx+a}}$	9
default	$\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\left(3(bx+a)^{\frac{7}{2}}a^{\frac{5}{2}}-11(bx+a)^{\frac{5}{2}}a^{\frac{7}{2}}-3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2x^4b^4-11(bx+a)^{\frac{3}{2}}a^{\frac{9}{2}}+3\sqrt{bx+a}\,a^{\frac{11}{2}}\right)}{64x^7(bx+a)^{\frac{3}{2}}a^{\frac{9}{2}}}$	1
pseudoelliptic	$-\frac{5\left(-\frac{63\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x^7b^7}{1280}+\sqrt{bx+a}\left(\frac{63\sqrt{a}\,b^6x^6}{1280}-\frac{21a^{\frac{3}{2}}b^5x^5}{640}+\frac{21a^{\frac{5}{2}}b^4x^4}{800}-\frac{9a^{\frac{7}{2}}b^3x^3}{400}+\frac{a^{\frac{9}{2}}b^2x^2}{50}+a^{\frac{11}{2}}bx+\frac{4a^{\frac{13}{2}}}{5}\right)\right)}{28a^{\frac{11}{2}}x^7}$	1

```
[In] int((b*x^3+a*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(-3*b^3*x^3+2*a*b^2*x^2+24*a^2*b*x+16*a^3)/x^5/a^2*(x^2*(b*x+a))^(1/2
)-3/64*b^4/a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*
x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.44

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \left[\frac{3\sqrt{a}b^4x^5\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(3\,ab^3x^3-2\,a^2b^2x^2-24\,a^3bx-16\,a^4)\sqrt{bx^3}}{128\,a^3x^5}\right.$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/128*(3*sqrt(a)*b^4*x^5*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) + 2*(3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*x^3
+ a*x^2))/(a^3*x^5), 1/64*(3*sqrt(-a)*b^4*x^5*arctan(sqrt(b*x^3 + a*x^2)*sq
rt(-a)/(a*x)) + (3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*
x^3 + a*x^2))/(a^3*x^5)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^8}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^8}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^8, x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx=\frac{\frac{3\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{7}{2}}b^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{5}{2}}ab^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{3}{2}}a^2b^5\mathrm{sgn}(x)+3\,\sqrt{bx+a}a^3b^5\mathrm{s}}{a^2b^4x^4}}{64\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/64*(3*b^5*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
+ a)^(7/2)*b^5*sgn(x) - 11*(b*x + a)^(5/2)*a*b^5*sgn(x) - 11*(b*x + a)^(3/2
)*a^2*b^5*sgn(x) + 3*sqrt(b*x + a)*a^3*b^5*sgn(x))/(a^2*b^4*x^4))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx=\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^8} \,d x$$

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^8,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^8, x)
```

3.251 $\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 165

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = -\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} -\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}-\frac{(ax^2+bx^3)^{3/2}}{5x^8}+\frac{3b^5\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}$$

```
[Out] -1/5*(b*x^3+a*x^2)^(3/2)/x^8+3/128*b^5*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2
))/a^(7/2)-3/40*b*(b*x^3+a*x^2)^(1/2)/x^5-1/80*b^2*(b*x^3+a*x^2)^(1/2)/a/x^
4+1/64*b^3*(b*x^3+a*x^2)^(1/2)/a^2/x^3-3/128*b^4*(b*x^3+a*x^2)^(1/2)/a^3/x^
2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 2050, 2033, 212}

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \frac{3b^5\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} +\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}-\frac{(ax^2+bx^3)^{3/2}}{5x^8}-\frac{3b\sqrt{ax^2+bx^3}}{40x^5}$$

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(40*x^5) - (b^2*Sqrt[a*x^2 + b*x^3])/(80*a*x^4)
+ (b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^3) - (3*b^4*Sqrt[a*x^2 + b*x^3])/(128
```

```
*a^3*x^2) - (a*x^2 + b*x^3)^(3/2)/(5*x^8) + (3*b^5*ArcTanh[(Sqrt[a]*x)/Sqrt
[a*x^2 + b*x^3]])/(128*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2045

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Dist[b*p
*((n - j)/(c^n*(m + j*p + 1))), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ax^2+bx^3)^{3/2}}{5x^8}+\frac{1}{10}(3b)\int\frac{\sqrt{ax^2+bx^3}}{x^6}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{(ax^2+bx^3)^{3/2}}{5x^8}+\frac{1}{80}\left(3b^2\right)\int\frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}-\frac{(ax^2+bx^3)^{3/2}}{5x^8}-\frac{b^3\int\frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{32a}\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}-\frac{(ax^2+bx^3)^{3/2}}{5x^8}+\frac{(3b^4)\int\frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{128a^2}
\end{aligned}$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} - \frac{(3b^5)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{256a^3}$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} + \frac{(3b^5)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{128a^3}$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{(ax^2+bx^3)^{3/2}}{5x^8} + \frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.70

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \frac{\sqrt{x^2(a+bx)}\,(-\sqrt{a}\sqrt{a+bx}(128a^4+176a^3bx+8a^2b^2x^2-10ab^3x^3+15b^4x^4)+15b^5x}{640a^{7/2}x^6\sqrt{a+bx}}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

```
[Out] (Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(128*a^4 + 176*a^3*b*x + 8*a^
2*b^2*x^2 - 10*a*b^3*x^3 + 15*b^4*x^4)) + 15*b^5*x^5*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]]))/(640*a^(7/2)*x^6*Sqrt[a + b*x])
```

Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.62

method	result
risch	$-\frac{(15b^4x^4-10a\,b^3x^3+8a^2b^2x^2+176a^3bx+128a^4)\sqrt{x^2(bx+a)}}{640x^6a^3}+\frac{3b^5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{128a^{\frac{7}{2}}x\sqrt{bx+a}}$
default	$-\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\,(15(bx+a)^{\frac{9}{2}}a^{\frac{7}{2}}-70(bx+a)^{\frac{7}{2}}a^{\frac{9}{2}}+128(bx+a)^{\frac{5}{2}}a^{\frac{11}{2}}-15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^3x^5b^5+70(bx+a)^{\frac{3}{2}}a^{\frac{13}{2}}-15\sqrt{bx+a}}{640x^8(bx+a)^{\frac{3}{2}}a^{\frac{13}{2}}}$
pseudoelliptic	$-\frac{\frac{693\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^8x^8}{256}+\sqrt{bx+a}\,(-\frac{693\sqrt{a}\,b^7x^7}{256}+\frac{231a^{\frac{3}{2}}b^6x^6}{128}-\frac{231a^{\frac{5}{2}}b^5x^5}{160}+\frac{99a^{\frac{7}{2}}b^4x^4}{80}-\frac{11a^{\frac{9}{2}}b^3x^3}{10}+a^{\frac{11}{2}}b^2x^2+68a^{\frac{13}{2}}bx+}{448a^{\frac{13}{2}}x^8}$

```
[In] int((b*x^3+a*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/640*(15*b^4*x^4-10*a*b^3*x^3+8*a^2*b^2*x^2+176*a^3*b*x+128*a^4)/x^6/a^3*
(x^2*(b*x+a))^(1/2)+3/128*b^5/a^(7/2)*arctanh((b*x+a)^(1/2)/a^(1/2))/x/(b*x
+a)^(1/2)*(x^2*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.33

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \left[\frac{15\,\sqrt{a}b^5x^6\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) - 2\,(15\,ab^4x^4 - 10\,a^2b^3x^3 + 8\,a^3b^2x^2 + 176\,a}{1280\,a^4x^6}\right.$$

$$\left. -\frac{15\,\sqrt{-a}b^5x^6\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right) + (15\,ab^4x^4 - 10\,a^2b^3x^3 + 8\,a^3b^2x^2 + 176\,a^4bx + 128\,a^5)\sqrt{bx^3+ax^2}}{640\,a^4x^6}\right]$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*sqrt(a)*b^5*x^6*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt
(a))/x^2) - 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8*a^3*b^2*x^2 + 176*a^4*b*x
+ 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6), -1/640*(15*sqrt(-a)*b^5*x^6*arct
an(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8
*a^3*b^2*x^2 + 176*a^4*b*x + 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \int \frac{(x^2(a+bx))^{\frac{3}{2}}}{x^9}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**9, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^9}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}}{x^9}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^9, x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.76

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx =$$

$$-\frac{\frac{15\,b^{6}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^{3}}+\frac{15\,(bx+a)^{\frac{9}{2}}b^{6}\mathrm{sgn}(x)-70\,(bx+a)^{\frac{7}{2}}ab^{6}\mathrm{sgn}(x)+128\,(bx+a)^{\frac{5}{2}}a^{2}b^{6}\mathrm{sgn}(x)+70\,(bx+a)^{\frac{3}{2}}a^{3}b^{6}\mathrm{sgn}(x)-15\,\sqrt{bx+a}a^{4}b^{6}\mathrm{sg}}{a^{3}b^{5}x^{5}}}{640\,b}$$

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/640*(15*b^6*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^3) + (15*(
b*x + a)^(9/2)*b^6*sgn(x) - 70*(b*x + a)^(7/2)*a*b^6*sgn(x) + 128*(b*x + a)
^(5/2)*a^2*b^6*sgn(x) + 70*(b*x + a)^(3/2)*a^3*b^6*sgn(x) - 15*sqrt(b*x + a
)*a^4*b^6*sgn(x))/(a^3*b^5*x^5))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx=\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^9} \,d x$$

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^9,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^9, x)
```

3.252 $\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx=\frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

```
[Out] 16/35*a^2*(b*x^3+a*x^2)^(1/2)/b^3-32/35*a^3*(b*x^3+a*x^2)^(1/2)/b^4/x-12/35
*a*x*(b*x^3+a*x^2)^(1/2)/b^2+2/7*x^2*(b*x^3+a*x^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 1602}

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx=-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}+\frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

```
[In] Int[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^3])/(35*b^3) - (32*a^3*Sqrt[a*x^2 + b*x^3])/(35*b^
4*x) - (12*a*x*Sqrt[a*x^2 + b*x^3])/(35*b^2) + (2*x^2*Sqrt[a*x^2 + b*x^3])/
(7*b)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
```

```
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2\sqrt{ax^2+bx^3}}{7b} - \frac{(6a)\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx}{7b} \\
&= -\frac{12ax\sqrt{ax^2+bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2+bx^3}}{7b} + \frac{(24a^2)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{35b^2} \\
&= \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3} - \frac{12ax\sqrt{ax^2+bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2+bx^3}}{7b} - \frac{(16a^3)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{35b^3} \\
&= \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3} - \frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x} - \frac{12ax\sqrt{ax^2+bx^3}}{35b^2} + \frac{2x^2\sqrt{ax^2+bx^3}}{7b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3)}{35b^4x}$$

```
[In] Integrate[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35
*b^4*x)
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.50

method	result	size
trager	$-\frac{2\left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) \sqrt{b \,x^{3}+a \,x^{2}}}{35 b^{4} x}$	52
risch	$-\frac{2 x \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right)}{35 \sqrt{x^{2} \left(b x +a \right)}\, b^{4}}$	53
pseudoelliptic	$\frac{2 \sqrt{b x +a}\, \left(35 b^{4} x^{4}-40 a \,b^{3} x^{3}+48 a^{2} b^{2} x^{2}-64 a^{3} b x +128 a^{4}\right)}{315 b^{5}}$	54
gosper	$-\frac{2 \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) x}{35 b^{4} \sqrt{b \,x^{3}+a \,x^{2}}}$	55
default	$-\frac{2 \left(b x +a \right) \left(-5 b^{3} x^{3}+6 a \,b^{2} x^{2}-8 a^{2} b x +16 a^{3}\right) x}{35 b^{4} \sqrt{b \,x^{3}+a \,x^{2}}}$	55

```
[In] int(x^4/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)/b^4/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\,(5\,b^3x^3-6\,ab^2x^2+8\,a^2bx-16\,a^3)\sqrt{bx^3+ax^2}}{35\,b^4x}$$

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^
4*x)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{x^4}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
[In] integrate(x**4/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4)}{35\,\sqrt{bx+a}b^4}$$

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*x
 + a)*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = \frac{32\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,b^4} + \frac{2\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)}{35\,b^4\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/35*a^(7/2)*sgn(x)/b^4 + 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a +
 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/(b^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}\,(16\,a^3-8\,a^2\,b\,x+6\,a\,b^2\,x^2-5\,b^3\,x^3)}{35\,b^4\,x}$$

```
[In] int(x^4/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 - 5*b^3*x^3 + 6*a*b^2*x^2 - 8*a^2*b*x))/(
35*b^4*x)
```

3.253 $\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx=-\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}+\frac{2x\sqrt{ax^2+bx^3}}{5b}$$

```
[Out] -8/15*a*(b*x^3+a*x^2)^(1/2)/b^2+16/15*a^2*(b*x^3+a*x^2)^(1/2)/b^3/x+2/5*x*(
b*x^3+a*x^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 1602}

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx=\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}-\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{2x\sqrt{ax^2+bx^3}}{5b}$$

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-8*a*Sqrt[a*x^2 + b*x^3])/(15*b^2) + (16*a^2*Sqrt[a*x^2 + b*x^3])/(15*b^3*
x) + (2*x*Sqrt[a*x^2 + b*x^3])/(5*b)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2x\sqrt{ax^2+bx^3}}{5b} - \frac{(4a)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{5b}\\ &= -\frac{8a\sqrt{ax^2+bx^3}}{15b^2} + \frac{2x\sqrt{ax^2+bx^3}}{5b} + \frac{(8a^2)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{15b^2}\\ &= -\frac{8a\sqrt{ax^2+bx^3}}{15b^2} + \frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x} + \frac{2x\sqrt{ax^2+bx^3}}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}(8a^2-4abx+3b^2x^2)}{15b^3x}$$

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3],x]
[Out] (2*Sqrt[x^2*(a + b*x)]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3*x)
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
trager	$\frac{2\left(3b^2x^2-4abx+8a^2\right)\sqrt{b\,x^3+a\,x^2}}{15b^3x}$	41
risch	$\frac{2x(bx+a)\left(3b^2x^2-4abx+8a^2\right)}{15\sqrt{x^2(bx+a)}\,b^3}$	42
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,\left(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3\right)}{35b^4}$	43
gosper	$\frac{2(bx+a)\left(3b^2x^2-4abx+8a^2\right)x}{15b^3\sqrt{b\,x^3+a\,x^2}}$	44
default	$\frac{2(bx+a)\left(3b^2x^2-4abx+8a^2\right)x}{15b^3\sqrt{b\,x^3+a\,x^2}}$	44

```
[In] int(x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\,(3\,b^2x^2-4\,abx+8\,a^2)\sqrt{bx^3+ax^2}}{15\,b^3x}$$

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{x^3}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
[In] integrate(x**3/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x**3/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\,(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3)}{15\,\sqrt{bx+a}b^3}$$

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = -\frac{16\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,b^3} + \frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)}{15\,b^3\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -16/15*a^(5/2)*sgn(x)/b^3 + 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a
+ 15*sqrt(b*x + a)*a^2)/(b^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^2-4\,a\,b\,x+3\,b^2\,x^2\right)}{15\,b^3\,x}$$

```
[In] int(x^3/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 + 3*b^2*x^2 - 4*a*b*x))/(15*b^3*x)
```

3.254 $\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

```
[Out] 2/3*(b*x^3+a*x^2)^(1/2)/b-4/3*a*(b*x^3+a*x^2)^(1/2)/b^2/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 1602}

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{ax^2+bx^3}}{3b} - \frac{(2a)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{3b} \\
&= \frac{2\sqrt{ax^2+bx^3}}{3b} - \frac{4a\sqrt{ax^2+bx^3}}{3b^2x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx = \frac{2(-2a+bx)\sqrt{x^2(a+bx)}}{3b^2x}$$

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3],x]
[Out] (2*(-2*a + b*x)*Sqrt[x^2*(a + b*x)])/(3*b^2*x)
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

method	result	size
trager	$-\frac{2(-bx+2a)\sqrt{b\,x^3+a\,x^2}}{3b^2x}$	30
risch	$-\frac{2x(bx+a)(-bx+2a)}{3\sqrt{x^2(bx+a)}\,b^2}$	31
pseudoelliptic	$\frac{2\sqrt{bx+a}\,(3b^2x^2-4abx+8a^2)}{15b^3}$	32
gosper	$-\frac{2(bx+a)(-bx+2a)x}{3b^2\sqrt{b\,x^3+a\,x^2}}$	33
default	$-\frac{2(bx+a)(-bx+2a)x}{3b^2\sqrt{b\,x^3+a\,x^2}}$	33

```
[In] int(x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(-b*x+2*a)/b^2/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time $=0.26$ (sec) , antiderivative size $=28$, normalized size of antiderivative $=0.57$

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{bx^3+ax^2}(bx-2\,a)}{3\,b^2x}$$

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(b*x - 2*a)/(b^2*x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{x^2}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
[In] integrate(x**2/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x**2/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time $=0.25$ (sec) , antiderivative size $=30$, normalized size of antiderivative $=0.61$

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\,(b^2x^2-abx-2\,a^2)}{3\sqrt{bx+a}b^2}$$

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)
```

Giac [A] (verification not implemented)

none

Time $=0.27$ (sec) , antiderivative size $=38$, normalized size of antiderivative $=0.78$

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx=\frac{4\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b^2}+\frac{2\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)}{3\,b^2\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 4/3*a^(3/2)*sgn(x)/b^2 + 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/(b^2*sgn
(x))
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx=-\frac{\left(\frac{4\,a}{3\,b^2}-\frac{2\,x}{3\,b}\right)\,\sqrt{b\,x^3+a\,x^2}}{x}$$

```
[In] int(x^2/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] -(((4*a)/(3*b^2) - (2*x)/(3*b))*(a*x^2 + b*x^3)^(1/2))/x
```

3.255 $\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{ax^2+bx^3}}{bx}$$

```
[Out] 2*(b*x^3+a*x^2)^(1/2)/b/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1602}

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{ax^2+bx^3}}{bx}$$

```
[In] Int[x/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/(b*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{2\sqrt{ax^2+bx^3}}{bx}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{x^2(a+bx)}}{bx}$$

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^3],x]
[Out] (2*Sqrt[x^2*(a + b*x)])/(b*x)
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,(-bx+2a)}{3b^2}$	21
trager	$\frac{2\sqrt{b\,x^3+a\,x^2}}{bx}$	22
risch	$\frac{2x(bx+a)}{\sqrt{x^2(bx+a)}\,b}$	23
gosper	$\frac{2x(bx+a)}{b\sqrt{b\,x^3+a\,x^2}}$	25
default	$\frac{2x(bx+a)}{b\sqrt{b\,x^3+a\,x^2}}$	25

```
[In] int(x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\,\sqrt{bx^3+ax^2}}{bx}$$

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(b*x^3 + a*x^2)/(b*x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x}{\sqrt{x^2\left(a+bx\right)}}\,dx$$

```
[In] integrate(x/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}}{b}$$

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{a}\mathrm{sgn}(x)}{b} + \frac{2\sqrt{bx+a}}{b\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*sgn(x)/b + 2*sqrt(b*x + a)/(b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\left|x\right|\sqrt{a+b\,x}}{b\,x}$$

```
[In] int(x/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*abs(x)*(a + b*x)^(1/2))/(b*x)
```

3.256 $\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2033, 212}

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

```
[In] Int[1/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
```

```
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2x\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^3],x]
[Out] (-2*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a +
b*x)])
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.43

method	result	size
pseudoelliptic	$\frac{2\sqrt{bx+a}}{b}$	13
default	$-\frac{2x\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{b\,x^3+a\,x^2}\,\sqrt{a}}$	39

```
[In] int(1/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.47

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)}{\sqrt{a}},\ \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)}{a}\right]$$

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)/sqrt(a), 2*sqrt(-
a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$$

```
[In] integrate(1/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}}\,dx$$

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\,\mathrm{sgn}(x)}{\sqrt{-a}}+\frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\mathrm{sgn}\,(x)}$$

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(sqrt(b*x + a)/sqrt(-
a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
[In] int(1/(a*x^2 + b*x^3)^(1/2),x)
[Out] int(1/(a*x^2 + b*x^3)^(1/2), x)
```

3.257 $\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

```
[Out] b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)-(b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2050, 2033, 212}

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]
])/a^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
```

```
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{ax^2+bx^3}}{ax^2}-\frac{b\int\frac{1}{\sqrt{ax^2+bx^3}}\,dx}{2a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int\frac{1}{x\sqrt{ax^2+bx^3}}\,dx=\frac{-\sqrt{a}(a+bx)+bx\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-(Sqrt[a]*(a + b*x)) + b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(
a^(3/2)*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.33

method	result	size
pseudoelliptic	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{\sqrt{bx+a}\,\left(\sqrt{bx+a}\,a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a b x\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,a^{\frac{5}{2}}}$	55
risch	$-\frac{bx+a}{a\sqrt{x^{2}(bx+a)}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{a^{\frac{3}{2}}\sqrt{x^{2}(bx+a)}}$	59

```
[In] int(1/x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.35

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}a}{2\,a^2x^2},\right.$$
$$\left.-\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}a}{a^2x^2}\right]$$

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2
) - 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2), -(sqrt(-a)*b*x^2*arctan(sqrt(b*x^3
+ a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x\sqrt{x^2\,(a+bx)}}\,dx$$

```
[In] integrate(1/x/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+ax^2}x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx=-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx+a}b}{ax}}{b\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
(b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{x\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
[In] int(1/(x*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x*(a*x^2 + b*x^3)^(1/2)), x)
```

3.258 $\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx=-\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}$$

```
[Out] -3/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)-1/2*(b*x^3+a*x^2)^(
1/2)/a/x^3+3/4*b*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2050, 2033, 212}

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx=-\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^2) -
(3*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}-\frac{(3b)\int\frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{4a}\\ &= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}+\frac{(3b^2)\int\frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a^2}\\ &= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{(3b^2)\,\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a^2}\\ &= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int\frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx=\frac{\sqrt{a}(-2a^2+abx+3b^2x^2)-3b^2x^2\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (Sqrt[a]*(-2*a^2 + a*b*x + 3*b^2*x^2) - 3*b^2*x^2*Sqrt[a + b*x]*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]])/(4*a^(5/2)*x*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-\sqrt{bx+a}\,\sqrt{a}}{a^{\frac{3}{2}}x}$	36
risch	$-\frac{(bx+a)(-3bx+2a)}{4a^2x\sqrt{x^2(bx+a)}}-\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{4a^{\frac{5}{2}}\sqrt{x^2(bx+a)}}$	73
default	$-\frac{\sqrt{bx+a}\left(-3a^{\frac{3}{2}}bx\sqrt{bx+a}+3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^2x^2+2\sqrt{bx+a}\,a^{\frac{5}{2}}\right)}{4x\sqrt{b\,x^3+a\,x^2}\,a^{\frac{7}{2}}}$	77

```
[In] int(1/x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^(3/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b*x-(b*x+a)^(1/2)*a^(1/2))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \left[\frac{3\,\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(3\,abx-2\,a^2)}{8\,a^3x^3}, \frac{3\,\sqrt{-a}b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+}{4\,a^3x^3}\right.$$

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) + 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/4*(3*sqrt(-a)
*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(
3*a*b*x - 2*a^2))/(a^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^2\sqrt{x^2\,(a+bx)}}\,dx$$

```
[In] integrate(1/x**2/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,(bx+a)^{\frac{3}{2}}b^3 - 5\,\sqrt{bx+a}ab^3}{a^2b^2x^2}}{4\,b\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/
2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/(b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,\sqrt{\frac{a}{b\,x}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a}{b\,x}\right)}{5\,x\,\sqrt{b\,x^3+a\,x^2}}$$

```
[In] int(1/(x^2*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] -(2*(a/(b*x) + 1)^(1/2)*hypergeom([1/2, 5/2], 7/2, -a/(b*x)))/(5*x*(a*x^2 +
 b*x^3)^(1/2))
```

3.259 $\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3} -\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b^3\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}$$

```
[Out] 5/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(7/2)-1/3*(b*x^3+a*x^2)^(1
/2)/a/x^4+5/12*b*(b*x^3+a*x^2)^(1/2)/a^2/x^3-5/8*b^2*(b*x^3+a*x^2)^(1/2)/a^
3/x^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2050, 2033, 212}

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \frac{5b^3\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

```
[In] Int[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) + (5*b*Sqrt[a*x^2 + b*x^3])/(12*a^2*x^3) -
 (5*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^2) + (5*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[
a*x^2 + b*x^3]])/(8*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}-\frac{(5b)\int\frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{6a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}+\frac{(5b^2)\int\frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}-\frac{(5b^3)\int\frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^3}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{(5b^3)\,\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a^3}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int\frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx=\frac{-\sqrt{a}(8a^3-2a^2bx+5ab^2x^2+15b^3x^3)+15b^3x^3\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{7/2}x^2\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-(Sqrt[a]*(8*a^3 - 2*a^2*b*x + 5*a*b^2*x^2 + 15*b^3*x^3)) + 15*b^3*x^3*Sqr
t[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(7/2)*x^2*Sqrt[x^2*(a + b*
x)])
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$\frac{-3 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^{2} x^{2}-2 \sqrt{bx+a}\, a^{\frac{3}{2}}+3 b x \sqrt{bx+a}\, \sqrt{a}}{4 x^{2} a^{\frac{5}{2}}}$	56
risch	$-\frac{(bx+a)\left(15 b^{2} x^{2}-10 a b x+8 a^{2}\right)}{24 a^{3} x^{2} \sqrt{x^{2}(bx+a)}}+\frac{5 b^{3} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) \sqrt{bx+a}\, x}{8 a^{\frac{7}{2}} \sqrt{x^{2}(bx+a)}}$	84
default	$-\frac{\sqrt{bx+a}\left(15 a^{\frac{3}{2}} b^{2} x^{2} \sqrt{bx+a}-15 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) a\, b^{3} x^{3}-10 a^{\frac{5}{2}} b x \sqrt{bx+a}+8 \sqrt{bx+a}\, a^{\frac{7}{2}}\right)}{24 x^{2} \sqrt{b\, x^{3}+a\, x^{2}}\, a^{\frac{9}{2}}}$	95

```
[In] int(1/x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-3*arctanh((b*x+a)^(1/2)/a^(1/2))*b^2*x^2-2*(b*x+a)^(1/2)*a^(3/2)+3*b*
x*(b*x+a)^(1/2)*a^(1/2))/x^2/a^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^{3}\sqrt{ax^{2}+bx^{3}}}\,dx = \left[\frac{15\,\sqrt{a}b^{3}x^{4}\log\left(\frac{bx^{2}+2\,ax+2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)-2\,(15\,ab^{2}x^{2}-10\,a^{2}bx+8\,a^{3})\sqrt{bx^{3}+ax^{2}}}{48\,a^{4}x^{4}},\right.$$
$$\left.-\frac{15\,\sqrt{-a}b^{3}x^{4}\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-a}}{ax}\right)+(15\,ab^{2}x^{2}-10\,a^{2}bx+8\,a^{3})\sqrt{bx^{3}+ax^{2}}}{24\,a^{4}x^{4}}\right]$$

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) - 2*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*x
^4), -1/24*(15*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))
+ (15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^3\sqrt{x^2\,(a+bx)}}\,dx$$

```
[In] integrate(1/x**3/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} + \frac{15\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+33\,\sqrt{bx+a}a^2b^4}{a^3b^3x^3}}{24\,b\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a)
^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^3*
x^3))/(b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
[In] int(1/(x^3*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x^2 + b*x^3)^(1/2)), x)
```

3.260 $\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{2x^4}{b\sqrt{ax^2+bx^3}}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}$$

```
[Out] -2*x^4/b/(b*x^3+a*x^2)^(1/2)-16/5*a*(b*x^3+a*x^2)^(1/2)/b^3+32/5*a^2*(b*x^3
+a*x^2)^(1/2)/b^4/x+12/5*x*(b*x^3+a*x^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2040, 2041, 1602}

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

```
[In] Int[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^4)/(b*Sqrt[a*x^2 + b*x^3]) - (16*a*Sqrt[a*x^2 + b*x^3])/(5*b^3) + (32
*a^2*Sqrt[a*x^2 + b*x^3])/(5*b^4*x) + (12*x*Sqrt[a*x^2 + b*x^3])/(5*b^2)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}+\frac{6\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx}{b}\\
&= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}-\frac{(24a)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{5b^2}\\
&= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}+\frac{(16a^2)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{5b^3}\\
&= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.51

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2x(16a^3+8a^2bx-2ab^2x^2+b^3x^3)}{5b^4\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[x^2*(a + b*x
)])
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{2(bx+a)\left(b^{3}x^{3}-2a\,b^{2}x^{2}+8a^{2}bx+16a^{3}\right)x^{3}}{5b^{4}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	56
default	$\frac{2(bx+a)\left(b^{3}x^{3}-2a\,b^{2}x^{2}+8a^{2}bx+16a^{3}\right)x^{3}}{5b^{4}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	56
trager	$\frac{2\left(b^{3}x^{3}-2a\,b^{2}x^{2}+8a^{2}bx+16a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{5(bx+a)b^{4}x}$	58
risch	$\frac{2\left(b^{2}x^{2}-3abx+11a^{2}\right)(bx+a)x}{5b^{4}\sqrt{x^{2}(bx+a)}}+\frac{2a^{3}x}{b^{4}\sqrt{x^{2}(bx+a)}}$	62
pseudoelliptic	$\frac{\frac{2}{11}b^{6}x^{6}-\frac{8}{33}a\,x^{5}b^{5}+\frac{80}{231}a^{2}x^{4}b^{4}-\frac{128}{231}a^{3}x^{3}b^{3}+\frac{256}{231}a^{4}x^{2}b^{2}-\frac{1024}{231}a^{5}xb-\frac{2048}{231}a^{6}}{b^{7}\sqrt{bx+a}}$	76

```
[In] int(x^6/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(b*x+a)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)*x^3/b^4/(b*x^3+a*x^2)^(3
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2\left(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3\right)\sqrt{bx^3+ax^2}}{5\left(b^5x^2+ab^4x\right)}$$

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^5*x
^2 + a*b^4*x)
```

Sympy [F]

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{x^6}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**6/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3)}{5\,\sqrt{bx+a}b^4}$$

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)/(sqrt(b*x + a)*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{32\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b^4} + \frac{2\,a^3}{\sqrt{bx+a}b^4\mathrm{sgn}\,(x)} + \frac{2\left((bx+a)^{\frac{5}{2}}b^{16}-5\,(bx+a)^{\frac{3}{2}}ab^{16}+15\,\sqrt{bx+a}a^2b^{16}\right)}{5\,b^{20}\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -32/5*a^(5/2)*sgn(x)/b^4 + 2*a^3/(sqrt(b*x + a)*b^4*sgn(x)) + 2/5*((b*x + a
)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/(b^20*
sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{x^6}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,(16\,a^3+8\,a^2\,b\,x-2\,a\,b^2\,x^2+b^3\,x^3)}{5\,b^4\,x\,(a+b\,x)}$$

```
[In] int(x^6/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 + b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x))/(5*b
^4*x*(a + b*x))
```

3.261 $\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{2x^3}{b\sqrt{ax^2+bx^3}}+\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}$$

```
[Out] -2*x^3/b/(b*x^3+a*x^2)^(1/2)+8/3*(b*x^3+a*x^2)^(1/2)/b^2-16/3*a*(b*x^3+a*x^
2)^(1/2)/b^3/x
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2040, 2041, 1602}

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}+\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

```
[In] Int[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^3)/(b*Sqrt[a*x^2 + b*x^3]) + (8*Sqrt[a*x^2 + b*x^3])/(3*b^2) - (16*a*
Sqrt[a*x^2 + b*x^3])/(3*b^3*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2x^3}{b\sqrt{ax^2+bx^3}}+\frac{4\int\frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{b}\\ &= -\frac{2x^3}{b\sqrt{ax^2+bx^3}}+\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{(8a)\int\frac{x}{\sqrt{ax^2+bx^3}}\,dx}{3b^2}\\ &= -\frac{2x^3}{b\sqrt{ax^2+bx^3}}+\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int\frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx=\frac{2x(-8a^2-4abx+b^2x^2)}{3b^3\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[x^5/(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	46
default	$-\frac{2(bx+a)\left(-b^{2}x^{2}+4abx+8a^{2}\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	46
trager	$-\frac{2\left(-b^{2}x^{2}+4abx+8a^{2}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{3(bx+a)b^{3}x}$	48
risch	$-\frac{2(-bx+5a)(bx+a)x}{3b^{3}\sqrt{x^{2}(bx+a)}}-\frac{2a^{2}x}{b^{3}\sqrt{x^{2}(bx+a)}}$	52
pseudoelliptic	$\frac{\frac{2}{9}b^{5}x^{5}-\frac{20}{63}a\,b^{4}x^{4}+\frac{32}{63}a^{2}b^{3}x^{3}-\frac{64}{63}a^{3}b^{2}x^{2}+\frac{256}{63}a^{4}bx+\frac{512}{63}a^{5}}{b^{6}\sqrt{bx+a}}$	65

```
[In] int(x^5/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*x+a)*(-b^2*x^2+4*a*b*x+8*a^2)*x^3/b^3/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx=\frac{2\,(b^2x^2-4\,abx-8\,a^2)\sqrt{bx^3+ax^2}}{3\,(b^4x^2+ab^3x)}$$

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^4*x^2 + a*b^3*x)
```

Sympy [F]

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx=\int \frac{x^5}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**5/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^2x^2-4\,abx-8\,a^2)}{3\,\sqrt{bx+a}b^3}$$

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)/(sqrt(b*x + a)*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = \frac{16\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b^3} - \frac{2\,a^2}{\sqrt{bx+a}b^3\mathrm{sgn}\,(x)} + \frac{2\left((bx+a)^{\frac{3}{2}}b^6-6\,\sqrt{bx+a}ab^6\right)}{3\,b^9\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 16/3*a^(3/2)*sgn(x)/b^3 - 2*a^2/(sqrt(b*x + a)*b^3*sgn(x)) + 2/3*((b*x + a)
^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b^6)/(b^9*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}\,(8\,a^2+4\,a\,b\,x-b^2\,x^2)}{3\,b^3\,x\,\left(a+b\,x\right)}$$

```
[In] int(x^5/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 - b^2*x^2 + 4*a*b*x))/(3*b^3*x*(a + b*x))
```

3.262 $\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{4\sqrt{ax^2+bx^3}}{b^2x}$$

```
[Out] -2*x^2/b/(b*x^3+a*x^2)^(1/2)+4*(b*x^3+a*x^2)^(1/2)/b^2/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2040, 1602}

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

```
[In] Int[x^4/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^2)/(b*Sqrt[a*x^2 + b*x^3]) + (4*Sqrt[a*x^2 + b*x^3])/(b^2*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2040

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n
 - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{2\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{b}\\ &= -\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{4\sqrt{ax^2+bx^3}}{b^2x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2x(2a+bx)}{b^2\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[x^4/(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(2*a + b*x))/(b^2*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2(bx+a)(bx+2a)x^3}{b^2(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	34
default	$\frac{2(bx+a)(bx+2a)x^3}{b^2(b\,x^3+a\,x^2)^{\frac{3}{2}}}$	34
trager	$\frac{2(bx+2a)\sqrt{b\,x^3+a\,x^2}}{(bx+a)b^2x}$	36
risch	$\frac{2(bx+a)x}{b^2\sqrt{x^2(bx+a)}}+\frac{2ax}{b^2\sqrt{x^2(bx+a)}}$	42
pseudoelliptic	$\frac{\frac{2}{7}b^4x^4-\frac{16}{35}a\,b^3x^3+\frac{32}{35}a^2b^2x^2-\frac{128}{35}a^3bx-\frac{256}{35}a^4}{b^5\sqrt{bx+a}}$	54

```
[In] int(x^4/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*(b*x+a)*(b*x+2*a)*x^3/b^2/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2\sqrt{bx^3+ax^2}(bx+2\,a)}{b^3x^2+ab^2x}$$

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] 2*sqrt(b*x^3 + a*x^2)*(b*x + 2*a)/(b^3*x^2 + a*b^2*x)
```

Sympy [F]

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{x^4}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**4/(b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**4/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.40

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2\,(bx+2\,a)}{\sqrt{bx+a}b^2}$$

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] 2*(b*x + 2*a)/(sqrt(b*x + a)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2\left(\frac{\sqrt{bx+a}}{b\mathrm{sgn}\left(x\right)}+\frac{a}{\sqrt{bx+a}b\mathrm{sgn}\left(x\right)}\right)}{b}-\frac{4\sqrt{a}\mathrm{sgn}\left(x\right)}{b^2}$$

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] 2*(sqrt(b*x + a)/(b*sgn(x)) + a/(sqrt(b*x + a)*b*sgn(x)))/b - 4*sqrt(a)*sgn
(x)/b^2
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(2\,a+b\,x)\,\sqrt{b\,x^3+a\,x^2}}{b^2\,x\,\left(a+b\,x\right)}$$

```
[In] int(x^4/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] (2*(2*a + b*x)*(a*x^2 + b*x^3)^(1/2))/(b^2*x*(a + b*x))
```

3.263 $\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx=-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

```
[Out] -2*x/b/(b*x^3+a*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1602}

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx=-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

```
[In] Int[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[a*x^2 + b*x^3])
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{2x}{b\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[x^3/(a*x^2 + b*x^3)^(3/2),x]
[Out] (-2*x)/(b*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
gosper	$-\frac{2(bx+a)x^{3}}{b\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	27
default	$-\frac{2(bx+a)x^{3}}{b\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	27
trager	$-\frac{2\sqrt{b\,x^{3}+a\,x^{2}}}{(bx+a)bx}$	29
pseudoelliptic	$\frac{\frac{2}{5}b^{3}x^{3}-\frac{4}{5}a\,b^{2}x^{2}+\frac{16}{5}a^{2}bx+\frac{32}{5}a^{3}}{b^{4}\sqrt{bx+a}}$	42

```
[In] int(x^3/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*(b*x+a)/b*x^3/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{2\sqrt{bx^3+ax^2}}{b^2x^2+abx}$$

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x^3 + a*x^2)/(b^2*x^2 + a*b*x)
```

Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^3}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**3/(b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**3/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2}{\sqrt{bx+a}b}$$

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(b*x + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,\mathrm{sgn}(x)}{\sqrt{a}b} - \frac{2}{\sqrt{bx+a}b\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] 2*sgn(x)/(sqrt(a)*b) - 2/(sqrt(b*x + a)*b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{b\,x^3+a\,x^2}}{b\,x\,\left(a+b\,x\right)}$$

```
[In] int(x^3/(a*x^2 + b*x^3)^(3/2),x)
[Out] -(2*(a*x^2 + b*x^3)^(1/2))/(b*x*(a + b*x))
```

3.264 $\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}} \, dx=\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

```
[Out] -2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)+2*x/a/(b*x^3+a*x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2048, 2033, 212}

$$\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}} \, dx=\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

```
[In] Int[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/a^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x}{a\sqrt{ax^2+bx^3}} + \frac{\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{a} \\
&= \frac{2x}{a\sqrt{ax^2+bx^3}} - \frac{2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{a} \\
&= \frac{2x}{a\sqrt{ax^2+bx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$
\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2x\left(\sqrt{a}-\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{3/2}\sqrt{x^2(a+bx)}}
$$

```
[In] Integrate[x^2/(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(Sqrt[a] - Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(3/2)*Sqr
t[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$\frac{\frac{2}{3}b^2x^2-\frac{8}{3}abx-\frac{16}{3}a^2}{b^3\sqrt{bx+a}}$	31
default	$-\frac{2x^3(bx+a)\left(\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\sqrt{bx+a}-a^{\frac{3}{2}}\right)}{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}a^{\frac{5}{2}}}$	54

```
[In] int(x^2/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b^2*x^2-4*a*b*x-8*a^2)/b^3/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.00

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{(bx^2+ax)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}a}{a^2bx^2+a^3x},\ \frac{2\left((bx^2+ax)\sqrt{-a}\,\mathrm{arct}\right.}{}\right.$$

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x^2 + a*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) + 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*b*x^2 + a^3*x), 2*((b*x^2 + a*x)*sqrt
(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a
^2*b*x^2 + a^3*x)]
```

Sympy [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.48

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^{\frac{3}{2}}} + \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\,(x)} + \frac{2}{\sqrt{bx+a}a\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(a)*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a))*sgn(x)/(sqrt(-a)*a^(3/2))
+ 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a*sgn(x)) + 2/(sqrt(b*x + a)*a
*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2}{(b\,x^3+a\,x^2)^{3/2}}\,dx$$

```
[In] int(x^2/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] int(x^2/(a*x^2 + b*x^3)^(3/2), x)
```

3.265 $\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2}{a\sqrt{ax^2+bx^3}}-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{3b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}$$

```
[Out] 3*b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)+2/a/(b*x^3+a*x^2)^(1/2)-
3*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2048, 2050, 2033, 212}

$$\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{3b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

```
[In] Int[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] 2/(a*Sqrt[a*x^2 + b*x^3]) - (3*Sqrt[a*x^2 + b*x^3])/(a^2*x^2) + (3*b*ArcTan
h[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{a\sqrt{ax^2+bx^3}} + \frac{3\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{a} \\
&= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} - \frac{(3b)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{2a^2} \\
&= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} + \frac{(3b)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{a^2} \\
&= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \frac{-\sqrt{a}(a+3bx)+3bx\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*(a + 3*b*x)) + 3*b*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]
])/(a^(5/2)*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.27

method	result	size
pseudoelliptic	$\frac{2bx+4a}{b^2\sqrt{bx+a}}$	20
default	$\frac{x^2(bx+a)\left(3\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-3\sqrt{a}\,bx-a^{\frac{3}{2}}\right)}{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}a^{\frac{5}{2}}}$	62
risch	$-\frac{bx+a}{a^2\sqrt{x^2(bx+a)}}-\frac{b\left(\frac{4}{\sqrt{bx+a}}-\frac{6\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{2a^2\sqrt{x^2(bx+a)}}$	75

```
[In] int(x/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*b*x+4*a)/b^2/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.52

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{3\,(b^2x^3+abx^2)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}(3\,abx+a^2)}{2\,(a^3bx^3+a^4x^2)},\right.$$
$$\left.-\frac{3\,(b^2x^3+abx^2)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}(3\,abx+a^2)}{a^3bx^3+a^4x^2}\right]$$

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^3 + a*b*x^2)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x
^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4
*x^2), -(3*(b^2*x^3 + a*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)
/(a*x)) + sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4*x^2)]
```

Sympy [F]

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{x}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(x\right)} - \frac{3\left(bx+a\right)b-2\,ab}{\left(\left(bx+a\right)^{\frac{3}{2}}-\sqrt{bx+a}a\right)a^2\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(x)) - (3*(b*x + a)*b
- 2*a*b)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{x}{{\left(b\,x^3+a\,x^2\right)}^{3/2}} \,dx$$

```
[In] int(x/(a*x^2 + b*x^3)^(3/2),x)
[Out] int(x/(a*x^2 + b*x^3)^(3/2), x)
```

3.266 $\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2}{ax\sqrt{ax^2+bx^3}} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} + \frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2} - \frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}$$

```
[Out] -15/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(7/2)+2/a/x/(b*x^3+a*x^2
)^(1/2)-5/2*(b*x^3+a*x^2)^(1/2)/a^2/x^3+15/4*b*(b*x^3+a*x^2)^(1/2)/a^3/x^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2031, 2050, 2033, 212}

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}} + \frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} + \frac{2}{ax\sqrt{ax^2+bx^3}}$$

```
[In] Int[(a*x^2 + b*x^3)^(-3/2),x]
```

```
[Out] 2/(a*x*Sqrt[a*x^2 + b*x^3]) - (5*Sqrt[a*x^2 + b*x^3])/(2*a^2*x^3) + (15*b*S
qrt[a*x^2 + b*x^3])/(4*a^3*x^2) - (15*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 +
b*x^3]])/(4*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2031

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{ax\sqrt{ax^2+bx^3}} + \frac{5\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{a} \\
&= \frac{2}{ax\sqrt{ax^2+bx^3}} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} - \frac{(15b)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{4a^2} \\
&= \frac{2}{ax\sqrt{ax^2+bx^3}} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} + \frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2} + \frac{(15b^2)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a^3} \\
&= \frac{2}{ax\sqrt{ax^2+bx^3}} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} + \frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2} - \frac{(15b^2)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a^3} \\
&= \frac{2}{ax\sqrt{ax^2+bx^3}} - \frac{5\sqrt{ax^2+bx^3}}{2a^2x^3} + \frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2} - \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{\sqrt{a}\left(-2a^2+5abx+15b^2x^2\right)-15b^2x^2\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}x\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[(a*x^2 + b*x^3)^(-3/2),x]
```

```
[Out] (Sqrt[a]*(-2*a^2 + 5*a*b*x + 15*b^2*x^2) - 15*b^2*x^2*Sqrt[a + b*x]*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2)*x*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.12

method	result	size
pseudoelliptic	$-\frac{2}{b\sqrt{bx+a}}$	13
default	$-\frac{x(bx+a)\left(15\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^2x^2-5a^{\frac{3}{2}}bx-15\sqrt{a}\,b^2x^2+2a^{\frac{5}{2}}\right)}{4\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}a^{\frac{7}{2}}}$	76
risch	$-\frac{(bx+a)(-7bx+2a)}{4a^3x\sqrt{x^2(bx+a)}}+\frac{b^2\left(\frac{16}{\sqrt{bx+a}}-\frac{30\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{8a^3\sqrt{x^2(bx+a)}}$	88

```
[In] int(1/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.99

$$\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\left[\frac{15\left(b^3x^4+ab^2x^3\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(15\,ab^2x^2+5\,a^2bx-2\,a^3\right)\sqrt{bx^3}}{8\left(a^4bx^4+a^5x^3\right)}\right.$$

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 +
a*x^2)*sqrt(a))/x^2) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*
x^2))/(a^4*b*x^4 + a^5*x^3), 1/4*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(-a)*arctan(
sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sq
rt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3)]
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(-3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3\mathrm{sgn}\left(x\right)} + \frac{2\,b^2}{\sqrt{bx+a}a^3\mathrm{sgn}\left(x\right)} + \frac{7\,(bx+a)^{\frac{3}{2}}b^2 - 9\,\sqrt{bx+a}ab^2}{4\,a^3b^2x^2\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*sgn(x)) + 2*b^2/(sqrt
(b*x + a)*a^3*sgn(x)) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)
/(a^3*b^2*x^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.38

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,x\,\left(\frac{a}{b\,x}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a}{b\,x}\right)}{7\,\left(b\,x^3+a\,x^2\right)^{3/2}}$$

```
[In] int(1/(a*x^2 + b*x^3)^(3/2),x)
[Out] -(2*x*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -a/(b*x)))/(7*(a*x^2 +
 b*x^3)^(3/2))
```

3.267 $\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 138

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b^3\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}$$

```
[Out] 35/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(9/2)+2/a/x^2/(b*x^3+a*x^
2)^(1/2)-7/3*(b*x^3+a*x^2)^(1/2)/a^2/x^4+35/12*b*(b*x^3+a*x^2)^(1/2)/a^3/x^
3-35/8*b^2*(b*x^3+a*x^2)^(1/2)/a^4/x^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2048, 2050, 2033, 212}

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{35b^3\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

```
[In] Int[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] 2/(a*x^2*Sqrt[a*x^2 + b*x^3]) - (7*Sqrt[a*x^2 + b*x^3])/(3*a^2*x^4) + (35*b
*Sqrt[a*x^2 + b*x^3])/(12*a^3*x^3) - (35*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^4*x^
2) + (35*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(8*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{ax^2\sqrt{ax^2+bx^3}}+\frac{7\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx}{a}\\
&= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}-\frac{(35b)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{6a^2}\\
&= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}+\frac{(35b^2)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a^3}\\
&= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}-\frac{(35b^3)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2}{ax^2\sqrt{ax^2+bx^3}} - \frac{7\sqrt{ax^2+bx^3}}{3a^2x^4} + \frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3} \\
&\quad - \frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2} + \frac{(35b^3)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a^4} \\
&= \frac{2}{ax^2\sqrt{ax^2+bx^3}} - \frac{7\sqrt{ax^2+bx^3}}{3a^2x^4} + \frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3} \\
&\quad - \frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2} + \frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$
\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{-\sqrt{a}(8a^3-14a^2bx+35ab^2x^2+105b^3x^3)+105b^3x^3\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{9/2}x^2\sqrt{x^2(a+bx)}}
$$

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
[Out] (-(Sqrt[a]*(8*a^3 - 14*a^2*b*x + 35*a*b^2*x^2 + 105*b^3*x^3)) + 105*b^3*x^3
*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(9/2)*x^2*Sqrt[x^2*(a
+ b*x)])
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

method	result	size
pseudoelliptic	$-\frac{2\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{a\sqrt{bx+a}}$	31
default	$-\frac{(bx+a)\left(-105\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,b^3x^3-14a^{\frac{5}{2}}bx+35a^{\frac{3}{2}}b^2x^2+105\sqrt{a}\,b^3x^3+8a^{\frac{7}{2}}\right)}{24(b\,x^3+a\,x^2)^{\frac{3}{2}}a^{\frac{9}{2}}}$	86
risch	$-\frac{(bx+a)(57b^2x^2-22abx+8a^2)}{24a^4x^2\sqrt{x^2(bx+a)}}-\frac{b^3\left(\frac{32}{\sqrt{bx+a}}-\frac{70\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{16a^4\sqrt{x^2(bx+a)}}$	99

```
[In] int(1/x/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2/a/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.75

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \left[\frac{105\left(b^4x^5+ab^3x^4\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\left(105\,ab^3x^3+35\,a^2b^2x^2-14\,a\right.}{48\left(a^5bx^5+a^6x^4\right)}\right.$$

$$\left.-\frac{105\left(b^4x^5+ab^3x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\left(105\,ab^3x^3+35\,a^2b^2x^2-14\,a^3bx+8\,a^4\right)\sqrt{bx^3+ax^2}}{24\left(a^5bx^5+a^6x^4\right)}\right]$$

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3
+ a*x^2)*sqrt(a))/x^2) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8
*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4), -1/24*(105*(b^4*x^5 + a*b
^3*x^4)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (105*a*b^3*x^
3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 +
a^6*x^4)]
```

Sympy [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{1}{x\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{35\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^4\mathrm{sgn}\left(x\right)} - \frac{2\,b^3}{\sqrt{bx+a}a^4\mathrm{sgn}\left(x\right)} - \frac{57\,(bx+a)^{\frac{5}{2}}b^3 - 136\,(bx+a)^{\frac{3}{2}}ab^3 + 87\,\sqrt{bx+a}a^2b^3}{24\,a^4b^3x^3\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -35/8*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4*sgn(x)) - 2*b^3/(sqr
t(b*x + a)*a^4*sgn(x)) - 1/24*(57*(b*x + a)^(5/2)*b^3 - 136*(b*x + a)^(3/2)
*a*b^3 + 87*sqrt(b*x + a)*a^2*b^3)/(a^4*b^3*x^3*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{1}{x\,{\left(b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

```
[In] int(1/(x*(a*x^2 + b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x*(a*x^2 + b*x^3)^(3/2)), x)
```

3.268 $\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 166

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2} - \frac{315b^4\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}}$$

```
[Out] -315/64*b^4*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(11/2)+2/a/x^3/(b*x^3+
a*x^2)^(1/2)-9/4*(b*x^3+a*x^2)^(1/2)/a^2/x^5+21/8*b*(b*x^3+a*x^2)^(1/2)/a^3
/x^4-105/32*b^2*(b*x^3+a*x^2)^(1/2)/a^4/x^3+315/64*b^3*(b*x^3+a*x^2)^(1/2)/
a^5/x^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2048, 2050, 2033, 212}

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{315b^4\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}} + \frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

```
[In] Int[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] 2/(a*x^3*Sqrt[a*x^2 + b*x^3]) - (9*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^5) + (21*b
*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^4) - (105*b^2*Sqrt[a*x^2 + b*x^3])/(32*a^4*x
```

```
^3) + (315*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^5*x^2) - (315*b^4*ArcTanh[(Sqrt[a
]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2048

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
-1]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{ax^3\sqrt{ax^2+bx^3}} + \frac{9\int \frac{1}{x^4\sqrt{ax^2+bx^3}}\,dx}{a} \\
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} - \frac{(63b)\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx}{8a^2} \\
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} + \frac{(105b^2)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{16a^3} \\
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} \\
&\quad - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} - \frac{(315b^3)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{64a^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} \\
&\quad - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2} + \frac{(315b^4)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{128a^5} \\
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} \\
&\quad + \frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2} - \frac{(315b^4)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{64a^5} \\
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} \\
&\quad - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2} - \frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.64

$$
\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{\sqrt{a}(-16a^4+24a^3bx-42a^2b^2x^2+105ab^3x^3+315b^4x^4)-315b^4x^4\sqrt{a+bx}\,\mathrm{arctanh}}{64a^{11/2}x^3\sqrt{x^2(a+bx)}}
$$

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-16*a^4 + 24*a^3*b*x - 42*a^2*b^2*x^2 + 105*a*b^3*x^3 + 315*b^4*x
^4) - 315*b^4*x^4*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(11/2
)*x^3*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.31

method	result	size
pseudoelliptic	$\frac{3\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bx-3\sqrt{a}\,bx-a^{\frac{3}{2}}}{x\,a^{\frac{5}{2}}\sqrt{bx+a}}$	51
default	$-\frac{(bx+a)\left(315\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^4x^4-24a^{\frac{7}{2}}bx+42a^{\frac{5}{2}}b^2x^2-105a^{\frac{3}{2}}b^3x^3-315b^4x^4\sqrt{a}+16a^{\frac{9}{2}}\right)}{64x(b\,x^3+a\,x^2)^{\frac{3}{2}}a^{\frac{11}{2}}}$	100
risch	$-\frac{(bx+a)(-187b^3x^3+82a\,b^2x^2-40a^2bx+16a^3)}{64a^5x^3\sqrt{x^2(bx+a)}}+\frac{b^4\left(\frac{256}{\sqrt{bx+a}}-\frac{630\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{128a^5\sqrt{x^2(bx+a)}}$	110

```
[In] int(1/x^2/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (3*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*b*x-3*a^(1/2)*b*x-a^(3/2))/
x/a^(5/2)/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^2 \left(ax^2+bx^3\right)^{3/2}}\,dx = \left[\frac{315\left(b^5x^6+ab^4x^5\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(315\,ab^4x^4+105\,a^2b^3x^3-\right.}{128\left(a^6bx^6+a^7x^5\right)}\right.$$

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3
 + a*x^2)*sqrt(a))/x^2) + 2*(315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x
^2 + 24*a^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5), 1/64*
(315*(b^5*x^6 + a*b^4*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*
x)) + (315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x^2 + 24*a^4*b*x - 16*a
^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5)]
```

Sympy [F]

$$\int \frac{1}{x^2 \left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{1}{x^2 \left(x^2 \left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{315\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{64\,\sqrt{-a}a^5\mathrm{sgn}\left(x\right)} + \frac{2\,b^4}{\sqrt{bx+a}a^5\mathrm{sgn}\left(x\right)} + \frac{187\,(bx+a)^{\frac{7}{2}}b^4 - 643\,(bx+a)^{\frac{5}{2}}ab^4 + 765\,(bx+a)^{\frac{3}{2}}a^2b^4 - 325\,\sqrt{bx+a}a^3b^4}{64\,a^5b^4x^4\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 315/64*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5*sgn(x)) + 2*b^4/(sq
rt(b*x + a)*a^5*sgn(x)) + 1/64*(187*(b*x + a)^(7/2)*b^4 - 643*(b*x + a)^(5/
2)*a*b^4 + 765*(b*x + a)^(3/2)*a^2*b^4 - 325*sqrt(b*x + a)*a^3*b^4)/(a^5*b^
4*x^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.89 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2\left(\frac{a}{b\,x}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{2};\ \frac{13}{2};\ -\frac{a}{b\,x}\right)}{11\,x\left(b\,x^3+a\,x^2\right)^{3/2}}$$

```
[In] int(1/(x^2*(a*x^2 + b*x^3)^(3/2)),x)
```

```
[Out] -(2*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 11/2], 13/2, -a/(b*x)))/(11*x*(a*x^
2 + b*x^3)^(3/2))
```

3.269 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\begin{aligned}\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx &= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}}-\frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2}\\ &\quad+\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}\end{aligned}$$

```
[Out] -5/8*a^3*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(7/2)+1/3*x^(3/2)*(
b*x^3+a*x^2)^(1/2)/b+5/8*a^2*(b*x^3+a*x^2)^(1/2)/b^3/x^(1/2)-5/12*a*x^(1/2)
*(b*x^3+a*x^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2049, 2054, 212}

$$\begin{aligned}\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx &= -\frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}\\ &\quad+\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}}-\frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2}+\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}\end{aligned}$$

```
[In] Int[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (5*a^2*Sqrt[a*x^2 + b*x^3])/(8*b^3*Sqrt[x]) - (5*a*Sqrt[x]*Sqrt[a*x^2 + b*x
^3])/(12*b^2) + (x^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a^3*ArcTanh[(Sqrt[
b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(8*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{(5a)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx}{6b}\\
&= -\frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} + \frac{(5a^2)\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx}{8b^2}\\
&= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{(5a^3)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{16b^3}\\
&= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}\\
&\quad - \frac{(5a^3)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^3}\\
&= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{b}x^{3/2}(15a^3+5a^2bx-2ab^2x^2+8b^3x^3)+30a^3x\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{24b^{7/2}\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[b]*x^(3/2)*(15*a^3 + 5*a^2*b*x - 2*a*b^2*x^2 + 8*b^3*x^3) + 30*a^3*x*
Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(24*b^(
7/2)*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{(8b^2x^2-10abx+15a^2)x^{\frac{3}{2}}(bx+a)}{24b^3\sqrt{x^2(bx+a)}}-\frac{5a^3\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)\sqrt{x}\,\sqrt{x(bx+a)}}{16b^{\frac{7}{2}}\sqrt{x^2(bx+a)}}$	100
default	$\frac{\sqrt{x}\left(16b^{\frac{9}{2}}x^4-4b^{\frac{7}{2}}a\,x^3+10b^{\frac{5}{2}}a^2x^2+30a^3b^{\frac{3}{2}}x-15\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^3b\right)}{48\sqrt{b\,x^3+a\,x^2}\,b^{\frac{9}{2}}}$	103

```
[In] int(x^(7/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(8*b^2*x^2-10*a*b*x+15*a^2)*x^(3/2)*(b*x+a)/b^3/(x^2*(b*x+a))^(1/2)-5/
16*a^3/b^(7/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/(x^2*(b*x+a))^(1/2
)*x^(1/2)*(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.44

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{15\,a^3\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)+2\,(8\,b^3x^2-10\,ab^2x+15\,a^2b)\sqrt{bx^3+ax^2}\sqrt{x}}{48\,b^4x}\right.$$

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*
sqrt(x))/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*sqr
t(x))/(b^4*x), 1/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/
```

```
(b*x^(3/2))) + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*sqrt
(x))/(b^4*x)]
```

Sympy [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\left(a+bx\right)}}\,dx$$

```
[In] integrate(x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = -\frac{5\,a^3\log\left(|a|\right)\mathrm{sgn}(x)}{16\,b^{\frac{7}{2}}} + \frac{\sqrt{bx+a}\left(2\,x\left(\frac{4\,x}{b}-\frac{5\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)\sqrt{x}+\frac{15\,a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{7}{2}}}}{24\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(a))*sgn(x)/b^(7/2) + 1/24*(sqrt(b*x + a)*(2*x*(4*x/b - 5*
a/b^2) + 15*a^2/b^3)*sqrt(x) + 15*a^3*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x +
 a)))/b^(7/2))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{x^{7/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
[In] int(x^(7/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(7/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.270 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx=-\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}+\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}$$

```
[Out] 3/4*a^2*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(5/2)-3/4*a*(b*x^3+a
*x^2)^(1/2)/b^2/x^(1/2)+1/2*x^(1/2)*(b*x^3+a*x^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2049, 2054, 212}

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx=\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}$$

```
[In] Int[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-3*a*Sqrt[a*x^2 + b*x^3])/(4*b^2*Sqrt[x]) + (Sqrt[x]*Sqrt[a*x^2 + b*x^3])/
(2*b) + (3*a^2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(4*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b} - \frac{(3a)\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx}{4b} \\
&= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b} + \frac{(3a^2)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{8b^2} \\
&= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b} + \frac{(3a^2)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^2} \\
&= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{b}x^{3/2}(-3a^2-abx+2b^2x^2)+6a^2x\sqrt{a+bx}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{5/2}\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[b]*x^(3/2)*(-3*a^2 - a*b*x + 2*b^2*x^2) + 6*a^2*x*Sqrt[a + b*x]*ArcTa
nh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(4*b^(5/2)*Sqrt[x^2*(a +
b*x)])
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

method	result	size
risch	$-\frac{(-2bx+3a)x^{\frac{3}{2}}(bx+a)}{4b^{2}\sqrt{x^{2}(bx+a)}}+\frac{3a^{2}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x}\,\sqrt{x(bx+a)}}{8b^{\frac{5}{2}}\sqrt{x^{2}(bx+a)}}$	89
default	$\frac{\sqrt{x}\left(4b^{\frac{7}{2}}x^{3}-2b^{\frac{5}{2}}a\,x^{2}-6a^{2}b^{\frac{3}{2}}x+3\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{2}b\right)}{8\sqrt{b\,x^{3}+a\,x^{2}}\,b^{\frac{7}{2}}}$	92

```
[In] int(x^(5/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*b*x+3*a)*x^(3/2)*(b*x+a)/b^2/(x^2*(b*x+a))^(1/2)+3/8*a^2/b^(5/2)*l
n((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/(x^2*(b*x+a))^(1/2)*x^(1/2)*(x*(b*
x+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.67

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx=\left[\frac{3\,a^{2}\sqrt{b}x\log\left(\frac{2\,bx^{2}+ax+2\,\sqrt{bx^{3}+ax^{2}}\sqrt{b}\sqrt{x}}{x}\right)+2\,\sqrt{bx^{3}+ax^{2}}(2\,b^{2}x-3\,ab)\sqrt{x}}{8\,b^{3}x},\right.$$
$$\left.-\frac{3\,a^{2}\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-b}}{bx^{\frac{3}{2}}}\right)-\sqrt{bx^{3}+ax^{2}}(2\,b^{2}x-3\,ab)\sqrt{x}}{4\,b^{3}x}\right]$$

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sq
rt(x))/x) + 2*sqrt(b*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x), -1/4*
(3*a^2*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) - sqrt(b
*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x)]
```

Sympy [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
[In] integrate(x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x**(5/2)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(5/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{3\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{8\,b^{\frac{5}{2}}} + \frac{\sqrt{bx+a}\sqrt{x}\left(\frac{2\,x}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{5}{2}}}}{4\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 3/8*a^2*log(abs(a))*sgn(x)/b^(5/2) + 1/4*(sqrt(b*x + a)*sqrt(x)*(2*x/b - 3*
a/b^2) - 3*a^2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(5/2))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^{5/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
[In] int(x^(5/2)/(a*x^2 + b*x^3)^(1/2),x)
[Out] int(x^(5/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.271 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

```
[Out] -a*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(3/2)+(b*x^3+a*x^2)^(1/2)
/b/x^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2049, 2054, 212}

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

```
[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 +
 b*x^3]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{2b} \\
&= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b} \\
&= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{b}x^{3/2}(a+bx)+2ax\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{b^{3/2}\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
[Out] (Sqrt[b]*x^(3/2)*(a + b*x) + 2*a*x*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/
(Sqrt[a] - Sqrt[a + b*x])])/(b^(3/2)*Sqrt[x^2*(a + b*x)])
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{x^{\frac{3}{2}}(bx+a)}{b\sqrt{x^{2}(bx+a)}}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)\sqrt{x}\,\sqrt{x(bx+a)}}{2b^{\frac{3}{2}}\sqrt{x^{2}(bx+a)}}$	78
default	$\frac{\sqrt{x}\left(2b^{\frac{5}{2}}x^{2}+2b^{\frac{3}{2}}ax-a\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b\right)}{2\sqrt{b\,x^{3}+a\,x^{2}}\,b^{\frac{5}{2}}}$	79

```
[In] int(x^(3/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*x^(3/2)*(b*x+a)/(x^2*(b*x+a))^(1/2)-1/2*a/b^(3/2)*ln((1/2*a+b*x)/b^(1/2
)+(b*x^2+a*x)^(1/2))/(x^2*(b*x+a))^(1/2)*x^(1/2)*(x*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.18

$$\int\frac{x^{3/2}}{\sqrt{ax^{2}+bx^{3}}}\,dx=\left[\frac{a\sqrt{b}x\log\left(\frac{2\,bx^{2}+ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{b}\sqrt{x}}{x}\right)+2\,\sqrt{bx^{3}+ax^{2}}b\sqrt{x}}{2\,b^{2}x},\frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-b}}{bx^{\frac{3}{2}}}\right)}{b^{2}x}\right.$$

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x
))/x) + 2*sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) + sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2
*x)]
```

Sympy [F]

$$\int\frac{x^{3/2}}{\sqrt{ax^{2}+bx^{3}}}\,dx=\int\frac{x^{\frac{3}{2}}}{\sqrt{x^{2}\,(a+bx)}}\,dx$$

```
[In] integrate(x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{x^{\frac{3}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx=-\frac{a\log\left(|a|\right)\operatorname{sgn}(x)}{2\,b^{\frac{3}{2}}}+\frac{\frac{a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{\sqrt{bx+a}\sqrt{x}}{b}}{\operatorname{sgn}\left(x\right)}$$

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*log(abs(a))*sgn(x)/b^(3/2) + (a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x
+ a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x)/b)/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{x^{3/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
[In] int(x^(3/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.272 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2054, 212}

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

```
[In] Int[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2x\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x^2(a+bx)}}$$

```
[In] Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*x*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt[
x^2*(a + b*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(26) = 52.

Time = 1.77 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

method	result	size
default	$\frac{\sqrt{x}\,\sqrt{x(bx+a)}\,\ln\left(\frac{2\sqrt{b\,x^2+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)}{\sqrt{b\,x^3+a\,x^2}\,\sqrt{b}}$	58

```
[In] int(x^(1/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*x^3+a*x^2)^(1/2)*x^(1/2)*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)
*b^(1/2)+2*b*x+a)/b^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}}{bx^{\frac{3}{2}}}\right)}{b}\right]$$

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x))/x)/sqrt(b), -2
*sqrt(-b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2)))/b]
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x^2\,(a+bx)}}\,dx$$

```
[In] integrate(x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(sqrt(x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}\,dx$$

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\log\left(|a|\right)\operatorname{sgn}(x)}{\sqrt{b}} - \frac{2\,\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}\operatorname{sgn}\left(x\right)}$$

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] log(abs(a))*sgn(x)/sqrt(b) - 2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/(
sqrt(b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{\sqrt{x}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
[In] int(x^(1/2)/(a*x^2 + b*x^3)^(1/2),x)
[Out] int(x^(1/2)/(a*x^2 + b*x^3)^(1/2), x)
```

3.273 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

```
[Out] -2*(b*x^3+a*x^2)^(1/2)/a/x^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2039}

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

```
[In] Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(a*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{x^2(a+bx)}}{ax^{3/2}}$$

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[x^2*(a + b*x)])/(a*x^(3/2))
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{2\sqrt{x}\,(bx+a)}{\sqrt{x^2(bx+a)}\,a}$	25
gosper	$-\frac{2\sqrt{x}\,(bx+a)}{a\sqrt{b\,x^3+a\,x^2}}$	27
default	$-\frac{2\sqrt{x}\,(bx+a)}{a\sqrt{b\,x^3+a\,x^2}}$	27

```
[In] int(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(x^2*(b*x+a))^(1/2)*x^(1/2)/a*(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,\sqrt{bx^3+ax^2}}{ax^{\frac{3}{2}}}$$

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x^3 + a*x^2)/(a*x^(3/2))
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{\sqrt{x}\sqrt{x^2(a+bx)}}\,dx$$

```
[In] integrate(1/x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+ax^2}\sqrt{x}}\,dx$$

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx=\frac{4\sqrt{b}}{\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)\operatorname{sgn}(x)}$$

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4*sqrt(b)/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx=\int \frac{1}{\sqrt{x}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
[In] int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.274 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx=-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}+\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}$$

```
[Out] -2/3*(b*x^3+a*x^2)^(1/2)/a/x^(5/2)+4/3*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx=\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

```
[In] Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(3*a*x^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*x
^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}-\frac{(2b)\int\frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{3a}\\&=-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}+\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.55

$$\int\frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx=-\frac{2(a-2bx)\sqrt{x^2(a+bx)}}{3a^2x^{5/2}}$$

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
[Out] (-2*(a - 2*b*x)*Sqrt[x^2*(a + b*x)])/(3*a^2*x^(5/2))
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.55

method	result	size
risch	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x^2(bx+a)}\,\sqrt{x}\,a^2}$	31
gosper	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x}\,a^2\sqrt{b\,x^3+a\,x^2}}$	33
default	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x}\,a^2\sqrt{b\,x^3+a\,x^2}}$	33

```
[In] int(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3/(x^2*(b*x+a))^(1/2)/x^(1/2)*(b*x+a)*(-2*b*x+a)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+ax^2}(2\,bx-a)}{3\,a^2x^{\frac{5}{2}}}$$

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(2*b*x - a)/(a^2*x^(5/2))
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

```
[In] integrate(1/x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(3/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(\frac{2\,(bx+a)b^3}{a^2}-\frac{3\,b^3}{a}\right)\sqrt{bx+a}b}{3\left((bx+a)b-ab\right)^{\frac{3}{2}}|b|\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*(b*x + a)*b^3/a^2 - 3*b^3/a)*sqrt(b*x + a)*b/(((b*x + a)*b - a*b)^(3
/2)*abs(b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{3/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
[In] int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)),x)
[Out] int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.275 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx=-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}$$

```
[Out] -2/5*(b*x^3+a*x^2)^(1/2)/a/x^(7/2)+8/15*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(5/2)-1
6/15*b^2*(b*x^3+a*x^2)^(1/2)/a^3/x^(3/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx=-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

```
[In] Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(5*a*x^(7/2)) + (8*b*Sqrt[a*x^2 + b*x^3])/(15*a^2*
x^(5/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(15*a^3*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}-\frac{(4b)\int\frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}+\frac{(8b^2)\int\frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{15a^2}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

$$\int\frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx=-\frac{2\sqrt{x^2(a+bx)}\left(3a^2-4abx+8b^2x^2\right)}{15a^3x^{7/2}}$$

```
[In] Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
[Out] (-2*Sqrt[x^2*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(7/2))
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{2(bx+a)\left(8b^2x^2-4abx+3a^2\right)}{15\sqrt{x^2(bx+a)}\,x^{\frac{3}{2}}\,a^3}$	44
gosper	$-\frac{2(bx+a)\left(8b^2x^2-4abx+3a^2\right)}{15x^{\frac{3}{2}}\,a^3\sqrt{b\,x^3+a\,x^2}}$	46
default	$-\frac{2(bx+a)\left(8b^2x^2-4abx+3a^2\right)}{15x^{\frac{3}{2}}\,a^3\sqrt{b\,x^3+a\,x^2}}$	46

```
[In] int(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/15/(x^2*(b*x+a))^(1/2)/x^(3/2)*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,(8\,b^2x^2-4\,abx+3\,a^2)\sqrt{bx^3+ax^2}}{15\,a^3x^{\frac{7}{2}}}$$

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x^3 + a*x^2)/(a^3*x^(7/2))
```

Sympy [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

```
[In] integrate(1/x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(5/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \frac{32\left(10\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4-5\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2+a^2\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5\operatorname{sgn}\left(x\right)}$$

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^2 + a^2)*b^(5/2)/(((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5*sg
n(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{5/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
[In] int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)),x)
[Out] int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.276 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}} + \frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}} - \frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}} + \frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}$$

```
[Out] -2/7*(b*x^3+a*x^2)^(1/2)/a/x^(9/2)+12/35*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(7/2)-
16/35*b^2*(b*x^3+a*x^2)^(1/2)/a^3/x^(5/2)+32/35*b^3*(b*x^3+a*x^2)^(1/2)/a^4
/x^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}} - \frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}} + \frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}} - \frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

```
[In] Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(7*a*x^(9/2)) + (12*b*Sqrt[a*x^2 + b*x^3])/(35*a^2
*x^(7/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(35*a^3*x^(5/2)) + (32*b^3*Sqrt[a*
x^2 + b*x^3])/(35*a^4*x^(3/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
```

```
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}-\frac{(6b)\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx}{7a}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}+\frac{(24b^2)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx}{35a^2}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}-\frac{(16b^3)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{35a^3}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx=\frac{2\sqrt{x^2(a+bx)}(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3)}{35a^4x^{9/2}}$$

```
[In] Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
[Out] (2*Sqrt[x^2*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35
*a^4*x^(9/2))
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

method	result	size
risch	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35\sqrt{x^{2}(bx+a)}\,x^{\frac{5}{2}}\,a^{4}}$	55
gosper	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35x^{\frac{5}{2}}\,a^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	57
default	$-\frac{2(bx+a)\left(-16b^{3}x^{3}+8a\,b^{2}x^{2}-6a^{2}bx+5a^{3}\right)}{35x^{\frac{5}{2}}\,a^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	57

```
[In] int(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35/(x^2*(b*x+a))^(1/2)/x^(5/2)*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*
x+5*a^3)/a^4
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3\right)\sqrt{bx^3+ax^2}}{35\,a^4x^{\frac{9}{2}}}$$

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^3 + a*x^2)/(a^
4*x^(9/2))
```

Sympy [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\left(a+bx\right)}}\,dx$$

```
[In] integrate(1/x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(7/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{64\left(35\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^6-21\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4+7\,a^2\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right.\right.}{35\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^7\operatorname{sgn}\left(x\right)}$$

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/((
(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{x^{7/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

```
[In] int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

3.277 $\int x^{1-3n}(ax^2+bx^3)^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx=\frac{x^{2-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\,\text{Hypergeometric2F1}\left(2-n,-n,3-n,-\frac{bx}{a}\right)}{2-n}$$

```
[Out] x^(2-3*n)*(b*x^3+a*x^2)^n*hypergeom([-n, 2-n],[3-n],-b*x/a)/(2-n)/((1+b*x/a
)^n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2057, 68, 66}

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx=\frac{x^{2-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\,\text{Hypergeometric2F1}\left(2-n,-n,3-n,-\frac{bx}{a}\right)}{2-n}$$

```
[In] Int[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] (x^(2 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)/
a)])/((2 - n)*(1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
```

```
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{1-n}(a+bx)^n\,dx\\ &= \left(x^{-2n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{1-n}\left(1+\frac{bx}{a}\right)^n\,dx\\ &= \frac{x^{2-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx = \frac{x^{2-3n}(x^2(a+bx))^n\left(1+\frac{bx}{a}\right)^{-n}\text{Hypergeometric2F1}\left(2-n,-n,3-n,-\frac{bx}{a}\right)}{2-n}$$

```
[In] Integrate[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] (x^(2 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)/
a)])/((2 - n)*(1 + (b*x)/a)^n)
```

Maple [F]

$$\int x^{1-3n}\left(b\,x^{3}+a\,x^{2}\right)^{n}dx$$

```
[In] int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)
[Out] int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)
```

Fricas [F]

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n+1}\,dx$$

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
[Out] integral((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

Sympy [F]

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx=\int x^{1-3n}\left(x^2(a+bx)\right)^n\,dx$$

```
[In] integrate(x**(1-3*n)*(b*x**3+a*x**2)**n,x)
[Out] Integral(x**(1 - 3*n)*(x**2*(a + b*x))**n, x)
```

Maxima [F]

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n+1}\,dx$$

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

Giac [F]

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n+1}\,dx$$

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx=\int x^{1-3\,n}\,\left(b\,x^3+a\,x^2\right)^n\,dx$$

```
[In] int(x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x)
[Out] int(x^(1 - 3*n)*(a*x^2 + b*x^3)^n, x)
```

3.278 $\int x^{-3n}(ax^2+bx^3)^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx=\frac{x^{-1-3n}(ax^2+bx^3)^{1+n}\,\text{Hypergeometric2F1}\left(1,2,2-n,-\frac{bx}{a}\right)}{a(1-n)}$$

```
[Out] x^(-1-3*n)*(b*x^3+a*x^2)^(1+n)*hypergeom([1, 2],[2-n],-b*x/a)/a/(1-n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2057, 68, 66}

$$\begin{gathered}\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx\\ =\frac{x^{1-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\,\text{Hypergeometric2F1}\left(1-n,-n,2-n,-\frac{bx}{a}\right)}{1-n}\end{gathered}$$

```
[In] Int[(a*x^2 + b*x^3)^n/x^(3*n),x]
```

```
[Out] (x^(1 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[1 - n, -n, 2 - n, -((b*x)/
a)])/((1 - n)*(1 + (b*x)/a)^n)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-n}(a+bx)^n\,dx\\ &= \left(x^{-2n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-n}\left(1+\frac{bx}{a}\right)^n\,dx\\ &= \frac{x^{1-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(1-n,-n;2-n;-\frac{bx}{a}\right)}{1-n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx = \frac{x^{1-3n}\left(x^2(a+bx)\right)^n\left(1+\frac{bx}{a}\right)^{-n}\text{Hypergeometric2F1}\left(1-n,-n,2-n,-\frac{bx}{a}\right)}{1-n}$$

```
[In] Integrate[(a*x^2 + b*x^3)^n/x^(3*n),x]
```

```
[Out] (x^(1 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[1 - n, -n, 2 - n, -((b*x)/
a)])/((1 - n)*(1 + (b*x)/a)^n)
```

Maple [F]

$$\int \left(b\,x^{3}+a\,x^{2}\right)^{n} x^{-3n}dx$$

```
[In] int((b*x^3+a*x^2)^n/(x^(3*n)),x)
```

```
[Out] int((b*x^3+a*x^2)^n/(x^(3*n)),x)
```

Fricas [F]

$$\int x^{-3n}\left(ax^{2}+bx^{3}\right)^{n}\,dx=\int \frac{\left(bx^{3}+ax^{2}\right)^{n}}{x^{3\,n}}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x^2)^n/x^(3*n), x)
```

Sympy [F]

$$\int x^{-3n}\left(ax^{2}+bx^{3}\right)^{n}\,dx=\int x^{-3n}\left(x^{2}(a+bx)\right)^{n}\,dx$$

```
[In] integrate((b*x**3+a*x**2)**n/(x**(3*n)),x)
```

```
[Out] Integral((x**2*(a + b*x))**n/x**(3*n), x)
```

Maxima [F]

$$\int x^{-3n}\left(ax^{2}+bx^{3}\right)^{n}\,dx=\int \frac{\left(bx^{3}+ax^{2}\right)^{n}}{x^{3\,n}}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n/x^(3*n), x)
```

Giac [F]

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx=\int\frac{\left(bx^3+ax^2\right)^n}{x^{3\,n}}\,dx$$

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n/x^(3*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx=\int\frac{\left(b\,x^3+a\,x^2\right)^n}{x^{3\,n}}\,dx$$

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n),x)
```

```
[Out] int((a*x^2 + b*x^3)^n/x^(3*n), x)
```

3.279 $\int x^{-1-3n}\left(ax^2+bx^3\right)^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int x^{-1-3n}\left(ax^2+bx^3\right)^n\,dx = -\frac{x^{-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n \text{Hypergeometric2F1}\left(-n,-n,1-n,-\frac{bx}{a}\right)}{n}$$

```
[Out] -(b*x^3+a*x^2)^n*hypergeom([-n, -n],[1-n],-b*x/a)/n/(x^(3*n))/((1+b*x/a)^n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2057, 68, 66}

$$\int x^{-1-3n}\left(ax^2+bx^3\right)^n\,dx = -\frac{x^{-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n \text{Hypergeometric2F1}\left(-n,-n,1-n,-\frac{bx}{a}\right)}{n}$$

```
[In] Int[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -(((a*x^2 + b*x^3)^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3*
n)*(1 + (b*x)/a)^n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
```

```
&& !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0]))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-1-n}(a+bx)^n\,dx \\
&= \left(x^{-2n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-1-n}\left(1+\frac{bx}{a}\right)^n\,dx \\
&= -\frac{x^{-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\begin{aligned}
&\int x^{-1-3n}\left(ax^2+bx^3\right)^n\,dx \\
&= -\frac{x^{-3n}\left(x^2(a+bx)\right)^n\left(1+\frac{bx}{a}\right)^{-n}\text{Hypergeometric2F1}\left(-n,-n,1-n,-\frac{bx}{a}\right)}{n}
\end{aligned}$$

```
[In] Integrate[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]
[Out] -(((x^2*(a + b*x))^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3*
n)*(1 + (b*x)/a)^n))
```

Maple [F]

$$\int x^{-1-3n}\left(b\,x^{3}+a\,x^{2}\right)^{n}dx$$

```
[In] int(x^(-1-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] int(x^(-1-3*n)*(b*x^3+a*x^2)^n,x)
```

Fricas [F]

$$\int x^{-1-3n}\left(ax^{2}+bx^{3}\right)^{n}\,dx=\int\left(bx^{3}+ax^{2}\right)^{n}x^{-3\,n-1}\,dx$$

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

Sympy [F]

$$\int x^{-1-3n}\left(ax^{2}+bx^{3}\right)^{n}\,dx=\int x^{-3n-1}\left(x^{2}(a+bx)\right)^{n}\,dx$$

```
[In] integrate(x**(-1-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Integral(x**(-3*n - 1)*(x**2*(a + b*x))**n, x)
```

Maxima [F]

$$\int x^{-1-3n}\left(ax^{2}+bx^{3}\right)^{n}\,dx=\int\left(bx^{3}+ax^{2}\right)^{n}x^{-3\,n-1}\,dx$$

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

Giac [F]

$$\int x^{-1-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n-1}\,dx$$

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-3n}\left(ax^2+bx^3\right)^n\,dx=\int \frac{\left(b\,x^3+a\,x^2\right)^n}{x^{3\,n+1}}\,dx$$

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 1),x)
[Out] int((a*x^2 + b*x^3)^n/x^(3*n + 1), x)
```

3.280 $\int x^{-2-3n}(ax^2+bx^3)^n \, dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n \, dx = -\frac{x^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a(1+n)}$$

```
[Out] -(b*x^3+a*x^2)^(1+n)/a/(1+n)/(x^(3+3*n))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2039}

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n \, dx = -\frac{x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a(n+1)}$$

```
[In] Int[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((a*x^2 + b*x^3)^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = -\frac{x^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a(1+n)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=-\frac{x^{-3(1+n)}\left(x^2(a+bx)\right)^{1+n}}{a(1+n)}$$

```
[In] Integrate[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
[Out] -((x^2*(a + b*x))^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

method	result	size
gosper	$-\frac{x^{-1-3n}(bx+a)\left(b\,x^3+a\,x^2\right)^n}{a(1+n)}$	36
parallelrisch	$-\frac{x^2x^{-2-3n}\left(x^2(bx+a)\right)^nb+x\,x^{-2-3n}\left(x^2(bx+a)\right)^na}{a(1+n)}$	56

```
[In] int(x^(-2-3*n)*(b*x^3+a*x^2)^n,x,method=_RETURNVERBOSE)
[Out] -x^(-1-3*n)/a/(1+n)*(b*x+a)*(b*x^3+a*x^2)^n
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=-\frac{(bx^2+ax)(bx^3+ax^2)^nx^{-3\,n-2}}{an+a}$$

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
[Out] -(b*x^2 + a*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 2)/(a*n + a)
```

Sympy [F]

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=\int x^{-3n-2}\left(x^2(a+bx)\right)^n\,dx$$

```
[In] integrate(x**(-2-3*n)*(b*x**3+a*x**2)**n,x)
[Out] Integral(x**(-3*n - 2)*(x**2*(a + b*x))**n, x)
```

Maxima [F]

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n-2}\,dx$$

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

Giac [F]

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n-2}\,dx$$

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=-\left(b\,x^3+a\,x^2\right)^n\left(\frac{x}{x^{3\,n+2}\,\left(n+1\right)}+\frac{b\,x^2}{a\,x^{3\,n+2}\,\left(n+1\right)}\right)$$

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 2),x)
[Out] -(a*x^2 + b*x^3)^n*(x/(x^(3*n + 2)*(n + 1)) + (b*x^2)/(a*x^(3*n + 2)*(n + 1
)))
```

3.281 $\int x^{-3-3n}(ax^2+bx^3)^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=-\frac{x^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a(2+n)}+\frac{bx^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a^2(1+n)(2+n)}$$

```
[Out] -x^(-4-3*n)*(b*x^3+a*x^2)^(1+n)/a/(2+n)+b*(b*x^3+a*x^2)^(1+n)/a^2/(1+n)/(2+
n)/(x^(3+3*n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=\frac{bx^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a(n+2)}$$

```
[In] Int[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-4 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(2 + n))) + (b*(a*x^2 + b*x^3)^
(1 + n))/(a^2*(1 + n)*(2 + n)*x^(3*(1 + n)))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^{-4-3n}(ax^2+bx^3)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-3n}(ax^2+bx^3)^n\,dx}{a(2+n)}\\ &= -\frac{x^{-4-3n}(ax^2+bx^3)^{1+n}}{a(2+n)}+\frac{bx^{-3(1+n)}(ax^2+bx^3)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int x^{-3-3n}(ax^2+bx^3)^n\,dx=-\frac{x^{-4-3n}(a+an-bx)\left(x^2(a+bx)\right)^{1+n}}{a^2(1+n)(2+n)}$$

```
[In] Integrate[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
[Out] -((x^(-4 - 3*n)*(a + a*n - b*x)*(x^2*(a + b*x))^(1 + n))/(a^2*(1 + n)*(2 +
n)))
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{x^{-2-3n}(bx+a)\left(b\,x^3+a\,x^2\right)^n(an-bx+a)}{a^2(1+n)(2+n)}$	50

```
[In] int(x^(-3-3*n)*(b*x^3+a*x^2)^n,x,method=_RETURNVERBOSE)
[Out] -x^(-2-3*n)/a^2/(1+n)/(2+n)*(b*x+a)*(b*x^3+a*x^2)^n*(a*n-b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=-\frac{\left(abnx^2-b^2x^3+\left(a^2n+a^2\right)x\right)\left(bx^3+ax^2\right)^nx^{-3\,n-3}}{a^2n^2+3\,a^2n+2\,a^2}$$

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 3)/(a^
2*n^2 + 3*a^2*n + 2*a^2)
```

Sympy [F]

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=\int x^{-3n-3}\left(x^2(a+bx)\right)^n\,dx$$

```
[In] integrate(x**(-3-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Integral(x**(-3*n - 3)*(x**2*(a + b*x))**n, x)
```

Maxima [F]

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^nx^{-3\,n-3}\,dx$$

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

Giac [F]

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^nx^{-3\,n-3}\,dx$$

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx=-\left(b\,x^3+a\,x^2\right)^n\left(\frac{x\,(n+1)}{x^{3\,n+3}\,\left(n^2+3\,n+2\right)}-\frac{b^2\,x^3}{a^2\,x^{3\,n+3}\,\left(n^2+3\,n+2\right)}+\frac{b\,n\,x^2}{a\,x^{3\,n+3}\,\left(n^2+3\,n+2\right)}\right)$$

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 3),x)
```

```
[Out] -(a*x^2 + b*x^3)^n*((x*(n + 1))/(x^(3*n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(
a^2*x^(3*n + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(3*n + 3)*(3*n + n^2 + 2)
))
```

3.282 $\int x^{-4-3n}(ax^2+bx^3)^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx=-\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)}+\frac{2bx^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a^2(2+n)(3+n)}-\frac{2b^2x^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a^3(1+n)(2+n)(3+n)}$$

```
[Out] -x^(-5-3*n)*(b*x^3+a*x^2)^(1+n)/a/(3+n)+2*b*x^(-4-3*n)*(b*x^3+a*x^2)^(1+n)/
a^2/(2+n)/(3+n)-2*b^2*(b*x^3+a*x^2)^(1+n)/a^3/(2+n)/(n^2+4*n+3)/(x^(3+3*n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx=-\frac{2b^2x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^3(n+1)(n+2)(n+3)}+\frac{2bx^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+2)(n+3)}-\frac{x^{-3n-5}\left(ax^2+bx^3\right)^{n+1}}{a(n+3)}$$

```
[In] Int[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-5 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(3 + n))) + (2*b*x^(-4 - 3*n)*(
a*x^2 + b*x^3)^(1 + n))/(a^2*(2 + n)*(3 + n)) - (2*b^2*(a*x^2 + b*x^3)^(1 +
 n))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n)))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^{-5-3n}(ax^2+bx^3)^{1+n}}{a(3+n)} - \frac{(2b)\int x^{-3-3n}(ax^2+bx^3)^n\,dx}{a(3+n)}\\
&= -\frac{x^{-5-3n}(ax^2+bx^3)^{1+n}}{a(3+n)} + \frac{2bx^{-4-3n}(ax^2+bx^3)^{1+n}}{a^2(2+n)(3+n)} + \frac{(2b^2)\int x^{-2-3n}(ax^2+bx^3)^n\,dx}{a^2(2+n)(3+n)}\\
&= -\frac{x^{-5-3n}(ax^2+bx^3)^{1+n}}{a(3+n)} + \frac{2bx^{-4-3n}(ax^2+bx^3)^{1+n}}{a^2(2+n)(3+n)} - \frac{2b^2x^{-3(1+n)}(ax^2+bx^3)^{1+n}}{a^3(1+n)(2+n)(3+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int x^{-4-3n}(ax^2+bx^3)^n\,dx = -\frac{x^{-3(1+n)}(a+bx)\,(x^2(a+bx))^n\,(a^2(2+3n+n^2)-2ab(1+n)x+2b^2x^2)}{a^3(1+n)(2+n)(3+n)}$$

```
[In] Integrate[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -(((a + b*x)*(x^2*(a + b*x))^n*(a^2*(2 + 3*n + n^2) - 2*a*b*(1 + n)*x + 2*b
^2*x^2))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n))))
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

method	result	size
gosper	$-\frac{x^{-3-3n}(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{n}\left(a^{2}n^{2}-2abnx+2b^{2}x^{2}+3a^{2}n-2abx+2a^{2}\right)}{a^{3}(1+n)(2+n)(3+n)}$	84

```
[In] int(x^(-4-3*n)*(b*x^3+a*x^2)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -x^(-3-3*n)/a^3/(1+n)/(2+n)/(3+n)*(b*x+a)*(b*x^3+a*x^2)^n*(a^2*n^2-2*a*b*n*
x+2*b^2*x^2+3*a^2*n-2*a*b*x+2*a^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int x^{-4-3n}\left(ax^{2}+bx^{3}\right)^{n}\,dx = \frac{\left(2\,ab^{2}nx^{3}-2\,b^{3}x^{4}-(a^{2}bn^{2}+a^{2}bn)x^{2}-(a^{3}n^{2}+3\,a^{3}n+2\,a^{3})x\right)\left(bx^{3}+ax^{2}\right)^{n}x^{-3\,n-4}}{a^{3}n^{3}+6\,a^{3}n^{2}+11\,a^{3}n+6\,a^{3}}$$

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] (2*a*b^2*n*x^3 - 2*b^3*x^4 - (a^2*b*n^2 + a^2*b*n)*x^2 - (a^3*n^2 + 3*a^3*n
 + 2*a^3)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 4)/(a^3*n^3 + 6*a^3*n^2 + 11*a^3*n
 + 6*a^3)
```

Sympy [F]

$$\int x^{-4-3n}\left(ax^{2}+bx^{3}\right)^{n}\,dx = \int x^{-3n-4}\left(x^{2}(a+bx)\right)^{n}\,dx$$

```
[In] integrate(x**(-4-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Integral(x**(-3*n - 4)*(x**2*(a + b*x))**n, x)
```

Maxima [F]

$$\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n-4}\,dx$$

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

Giac [F]

$$\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx=\int \left(bx^3+ax^2\right)^n x^{-3\,n-4}\,dx$$

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.35

$$\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx=-\left(b\,x^3+a\,x^2\right)^n\left(\frac{x\,\left(n^2+3\,n+2\right)}{x^{3\,n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,b^3\,x^4}{a^3\,x^{3\,n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,b^2\,n\,x^3}{a^2\,x^{3\,n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{b\,n\,x^2\,\left(n+1\right)}{a\,x^{3\,n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}\right)$$

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 4),x)
[Out] -(a*x^2 + b*x^3)^n*((x*(3*n + n^2 + 2))/(x^(3*n + 4)*(11*n + 6*n^2 + n^3 +
6)) + (2*b^3*x^4)/(a^3*x^(3*n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (2*b^2*n*x^3
)/(a^2*x^(3*n + 4)*(11*n + 6*n^2 + n^3 + 6)) + (b*n*x^2*(n + 1))/(a*x^(3*n
+ 4)*(11*n + 6*n^2 + n^3 + 6)))
```

3.283 $\int \frac{x^{11}}{(ax^2+bx^5)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{x^{11}}{(ax^2+bx^5)^3} dx = \frac{x^6}{6a(a+bx^3)^2}$$

```
[Out] 1/6*x^6/a/(b*x^3+a)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 270}

$$\int \frac{x^{11}}{(ax^2+bx^5)^3} dx = \frac{x^6}{6a(a+bx^3)^2}$$

```
[In] Int[x^11/(a*x^2 + b*x^5)^3,x]
```

```
[Out] x^6/(6*a*(a + b*x^3)^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^5}{(a+bx^3)^3}\,dx\\ &= \frac{x^6}{6a\,(a+bx^3)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{a+2bx^3}{6b^2\,(a+bx^3)^2}$$

```
[In] Integrate[x^11/(a*x^2 + b*x^5)^3,x]
[Out] -1/6*(a + 2*b*x^3)/(b^2*(a + b*x^3)^2)
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2b\,x^3+a}{6(b\,x^3+a)^2b^2}$	23
parallelrisch	$\frac{-2b\,x^3-a}{6b^2(b\,x^3+a)^2}$	25
risch	$\frac{-\frac{x^3}{3b}-\frac{a}{6b^2}}{(b\,x^3+a)^2}$	26
default	$\frac{a}{6b^2(b\,x^3+a)^2}-\frac{1}{3b^2(b\,x^3+a)}$	31
norman	$\frac{-\frac{x^8}{3b}-\frac{a\,x^5}{6b^2}}{x^5(b\,x^3+a)^2}$	32

```
[In] int(x^11/(b*x^5+a*x^2)^3,x,method=_RETURNVERBOSE)
[Out] -1/6*(2*b*x^3+a)/(b*x^3+a)^2/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{2\,bx^3+a}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = \frac{-a-2bx^3}{6a^2b^2+12ab^3x^3+6b^4x^6}$$

```
[In] integrate(x**11/(b*x**5+a*x**2)**3,x)
```

```
[Out] (-a - 2*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{2\,bx^3+a}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{2\,bx^3+a}{6\,(bx^3+a)^2b^2}$$

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="giac")
[Out] -1/6*(2*b*x^3 + a)/((b*x^3 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{x^{11}}{(ax^2+bx^5)^3}\,dx = -\frac{\frac{a}{6\,b^2}+\frac{x^3}{3\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

```
[In] int(x^11/(a*x^2 + b*x^5)^3,x)
[Out] -(a/(6*b^2) + x^3/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

3.284 $\int \frac{x^9}{\sqrt{ax^2+bx^5}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}} \, dx = \frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x} - \frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2} + \frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

```
[Out] 16/45*a^2*(b*x^5+a*x^2)^(1/2)/b^3/x-8/45*a*x^2*(b*x^5+a*x^2)^(1/2)/b^2+2/15
*x^5*(b*x^5+a*x^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 1602}

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}} \, dx = \frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x} - \frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2} + \frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

```
[In] Int[x^9/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^5])/(45*b^3*x) - (8*a*x^2*Sqrt[a*x^2 + b*x^5])/(45
*b^2) + (2*x^5*Sqrt[a*x^2 + b*x^5])/(15*b)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^5\sqrt{ax^2+bx^5}}{15b} - \frac{(4a)\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx}{5b} \\
&= -\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2} + \frac{2x^5\sqrt{ax^2+bx^5}}{15b} + \frac{(8a^2)\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx}{15b^2} \\
&= \frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x} - \frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2} + \frac{2x^5\sqrt{ax^2+bx^5}}{15b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{x^2(a+bx^3)}(8a^2-4abx^3+3b^2x^6)}{45b^3x}$$

```
[In] Integrate[x^9/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*Sqrt[x^2*(a + b*x^3)]*(8*a^2 - 4*a*b*x^3 + 3*b^2*x^6))/(45*b^3*x)
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.54

method	result	size
trager	$\frac{2(3b^2x^6-4ab\,x^3+8a^2)\sqrt{b\,x^5+a\,x^2}}{45b^3x}$	43
gosper	$\frac{2(b\,x^3+a)(3b^2x^6-4ab\,x^3+8a^2)x}{45b^3\sqrt{b\,x^5+a\,x^2}}$	48
default	$\frac{2(b\,x^3+a)(3b^2x^6-4ab\,x^3+8a^2)x}{45b^3\sqrt{b\,x^5+a\,x^2}}$	48
risch	$\frac{2x(b\,x^3+a)(3b^2x^6-4ab\,x^3+8a^2)}{45\sqrt{x^2(b\,x^3+a)}\,b^3}$	48

```
[In] int(x^9/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*(3*b^2*x^6-4*a*b*x^3+8*a^2)/b^3/x*(b*x^5+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,(3\,b^2x^6-4\,abx^3+8\,a^2)\sqrt{bx^5+ax^2}}{45\,b^3x}$$

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*b^2*x^6 - 4*a*b*x^3 + 8*a^2)*sqrt(b*x^5 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^9}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

```
[In] integrate(x**9/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**9/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,(3\,b^3x^9-ab^2x^6+4\,a^2bx^3+8\,a^3)}{45\,\sqrt{bx^3+a}b^3}$$

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/45*(3*b^3*x^9 - a*b^2*x^6 + 4*a^2*b*x^3 + 8*a^3)/(sqrt(b*x^3 + a)*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx=-\frac{16\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{45\,b^3}+\frac{2\,\sqrt{bx^3+a}a^2}{3\,b^3\mathrm{sgn}\left(x\right)}+\frac{2\,\left(3\,(bx^3+a)^{\frac{5}{2}}-10\,(bx^3+a)^{\frac{3}{2}}a\right)}{45\,b^3\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -16/45*a^(5/2)*sgn(x)/b^3 + 2/3*sqrt(b*x^3 + a)*a^2/(b^3*sgn(x)) + 2/45*(3*
(b*x^3 + a)^(5/2) - 10*(b*x^3 + a)^(3/2)*a)/(b^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\,\sqrt{b\,x^5+a\,x^2}\,\left(8\,a^2-4\,a\,b\,x^3+3\,b^2\,x^6\right)}{45\,b^3\,x}$$

```
[In] int(x^9/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^5)^(1/2)*(8*a^2 + 3*b^2*x^6 - 4*a*b*x^3))/(45*b^3*x)
```

3.285 $\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx=-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}+\frac{2x^2\sqrt{ax^2+bx^5}}{9b}$$

```
[Out] -4/9*a*(b*x^5+a*x^2)^(1/2)/b^2/x+2/9*x^2*(b*x^5+a*x^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 1602}

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx=\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

```
[In] Int[x^6/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-4*a*Sqrt[a*x^2 + b*x^5])/(9*b^2*x) + (2*x^2*Sqrt[a*x^2 + b*x^5])/(9*b)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2x^2\sqrt{ax^2+bx^5}}{9b} - \frac{(2a)\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx}{3b}\\ &= -\frac{4a\sqrt{ax^2+bx^5}}{9b^2x} + \frac{2x^2\sqrt{ax^2+bx^5}}{9b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = \frac{2(-2a+bx^3)\sqrt{x^2(a+bx^3)}}{9b^2x}$$

```
[In] Integrate[x^6/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*(-2*a + b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*b^2*x)
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

method	result	size
trager	$-\frac{2(-b\,x^3+2a)\sqrt{b\,x^5+a\,x^2}}{9b^2x}$	32
gosper	$-\frac{2(b\,x^3+a)(-b\,x^3+2a)x}{9b^2\sqrt{b\,x^5+a\,x^2}}$	37
default	$-\frac{2(b\,x^3+a)(-b\,x^3+2a)x}{9b^2\sqrt{b\,x^5+a\,x^2}}$	37
risch	$-\frac{2x(b\,x^3+a)(-b\,x^3+2a)}{9\sqrt{x^2(b\,x^3+a)}\,b^2}$	37

```
[In] int(x^6/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/9*(-b*x^3+2*a)/b^2/x*(b*x^5+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\sqrt{bx^5+ax^2}(bx^3-2\,a)}{9\,b^2x}$$

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 - 2*a)/(b^2*x)
```

Sympy [F]

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^6}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

```
[In] integrate(x**6/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**6/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.65

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\,(b^2x^6-abx^3-2\,a^2)}{9\,\sqrt{bx^3+a}b^2}$$

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/9*(b^2*x^6 - a*b*x^3 - 2*a^2)/(sqrt(b*x^3 + a)*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx=\frac{4\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b^2}+\frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b^2\mathrm{sgn}\,(x)}-\frac{2\,\sqrt{bx^3+a}a}{3\,b^2\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] 4/9*a^(3/2)*sgn(x)/b^2 + 2/9*(b*x^3 + a)^(3/2)/(b^2*sgn(x)) - 2/3*sqrt(b*x^
3 + a)*a/(b^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{b\,x^5+a\,x^2}\,\left(\frac{4\,a}{9\,b^2}-\frac{2\,x^3}{9\,b}\right)}{x}$$

```
[In] int(x^6/(a*x^2 + b*x^5)^(1/2),x)
[Out] -((a*x^2 + b*x^5)^(1/2)*((4*a)/(9*b^2) - (2*x^3)/(9*b)))/x
```

3.286 $\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

```
[Out] 2/3*(b*x^5+a*x^2)^(1/2)/b/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1602}

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^5])/(3*b*x)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{x^2\left(a+bx^3\right)}}{3bx}$$

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*Sqrt[x^2*(a + b*x^3)])/(3*b*x)
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
trager	$\frac{2\sqrt{b\,x^5+a\,x^2}}{3bx}$	22
gosper	$\frac{2x\left(b\,x^3+a\right)}{3b\sqrt{b\,x^5+a\,x^2}}$	27
default	$\frac{2x\left(b\,x^3+a\right)}{3b\sqrt{b\,x^5+a\,x^2}}$	27
risch	$\frac{2x\left(b\,x^3+a\right)}{3\sqrt{x^2\left(b\,x^3+a\right)}\,b}$	27

```
[In] int(x^3/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(b*x^5+a*x^2)^(1/2)/b/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\sqrt{bx^5+ax^2}}{3\,bx}$$

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x^5 + a*x^2)/(b*x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

```
[In] integrate(x**3/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**3/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\sqrt{bx^3+a}}{3\,b}$$

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(b*x^3 + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2\,\sqrt{a}\mathrm{sgn}(x)}{3\,b} + \frac{2\,\sqrt{bx^3+a}}{3\,b\mathrm{sgn}\,(x)}$$

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] -2/3*sqrt(a)*sgn(x)/b + 2/3*sqrt(b*x^3 + a)/(b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\sqrt{b\,x^5+a\,x^2}}{3\,b\,x}$$

```
[In] int(x^3/(a*x^2 + b*x^5)^(1/2),x)
[Out] (2*(a*x^2 + b*x^5)^(1/2))/(3*b*x)
```

3.287 $\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

```
[Out] -2/3*arctanh(x*a^(1/2)/(b*x^5+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2033, 212}

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

```
[In] Int[1/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
```

```
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^5}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2x\sqrt{a+bx^3}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-2*x*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*Sqrt[x^2
*(a + b*x^3)])
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{2x\sqrt{b\,x^{3}+a}\;\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{b\,x^{5}+a\,x^{2}}\;\sqrt{a}}$	43

```
[In] int(1/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(b*x^5+a*x^2)^(1/2)*x*(b*x^3+a)^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))
/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx = \left[\frac{\log\left(\frac{bx^4+2\,ax-2\,\sqrt{bx^5+ax^2}\sqrt{a}}{x^4}\right)}{3\,\sqrt{a}}, \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{bx^5+ax^2}\sqrt{-a}}{ax}\right)}{3\,a}\right]$$

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] [1/3*log((b*x^4 + 2*a*x - 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4)/sqrt(a), 2/3*
sqrt(-a)*arctan(sqrt(b*x^5 + a*x^2)*sqrt(-a)/(a*x))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$$

```
[In] integrate(1/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/sqrt(a*x**2 + b*x**5), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*x^5 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{3\,\sqrt{-a}} + \frac{2\,\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] -2/3*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*arctan(sqrt(b*x^3 + a)/
sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(1/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(1/(a*x^2 + b*x^5)^(1/2), x)
```

3.288 $\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx=-\frac{\sqrt{ax^2+bx^5}}{3ax^4}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}$$

```
[Out] 1/3*b*arctanh(x*a^(1/2)/(b*x^5+a*x^2)^(1/2))/a^(3/2)-1/3*(b*x^5+a*x^2)^(1/2
)/a/x^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2050, 2033, 212}

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx=\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

```
[In] Int[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -1/3*Sqrt[a*x^2 + b*x^5]/(a*x^4) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
5]])/(3*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{ax^2+bx^5}}{3ax^4} - \frac{b\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx}{2a} \\
&= -\frac{\sqrt{ax^2+bx^5}}{3ax^4} + \frac{b\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^5}}\right)}{3a} \\
&= -\frac{\sqrt{ax^2+bx^5}}{3ax^4} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \frac{-\sqrt{a}(a+bx^3)+bx^3\sqrt{a+bx^3}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}x^2\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-(Sqrt[a]*(a + b*x^3)) + b*x^3*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqr
t[a]])/(3*a^(3/2)*x^2*Sqrt[x^2*(a + b*x^3)])
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\sqrt{b\,x^{3}+a}\,\left(b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)a\,x^{3}-\sqrt{b\,x^{3}+a}\,a^{\frac{3}{2}}\right)}{3x^{2}\sqrt{b\,x^{5}+a\,x^{2}}\,a^{\frac{5}{2}}}$	66
risch	$-\frac{b\,x^{3}+a}{3a\,x^{2}\sqrt{x^{2}\left(b\,x^{3}+a\right)}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\sqrt{b\,x^{3}+a}\,x}{3a^{\frac{3}{2}}\sqrt{x^{2}\left(b\,x^{3}+a\right)}}$	73

```
[In] int(1/x^3/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/x^2*(b*x^3+a)^(1/2)*(b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a*x^3-(b*x^3+a)
^(1/2)*a^(3/2))/(b*x^5+a*x^2)^(1/2)/a^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.15

$$\int \frac{1}{x^{3}\sqrt{ax^{2}+bx^{5}}}\,dx=\left[\frac{\sqrt{a}bx^{4}\log\left(\frac{bx^{4}+2\,ax+2\,\sqrt{bx^{5}+ax^{2}}\sqrt{a}}{x^{4}}\right)-2\,\sqrt{bx^{5}+ax^{2}}a}{6\,a^{2}x^{4}},\ -\frac{\sqrt{-a}bx^{4}\arctan\left(\frac{\sqrt{bx^{5}+ax^{2}}\sqrt{-a}}{ax}\right)+\sqrt{bx^{5}+ax^{2}}a}{3\,a^{2}x^{4}}\right]$$

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*b*x^4*log((b*x^4 + 2*a*x + 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4
) - 2*sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4), -1/3*(sqrt(-a)*b*x^4*arctan(sqrt(b*
x^5 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^{3}\sqrt{ax^{2}+bx^{5}}}\,dx=\int \frac{1}{x^{3}\sqrt{x^{2}\left(a+bx^{3}\right)}}\,dx$$

```
[In] integrate(1/x**3/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2x^3}}\,dx$$

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = -\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\sqrt{bx^3+a}b}{ax^3}}{3\,b\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^3 + a)*b
/(a*x^3))/(b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(1/(x^3*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x^2 + b*x^5)^(1/2)), x)
```

3.289 $\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 238

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{4\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] 2/5*(b*x^5+a*x^2)^(1/2)/b-4/15*a*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6
^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used

$= \{2049, 2057, 224\}$

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{4\sqrt{2+\sqrt{3}}ax\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[In] Int[x^4/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^5])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*x*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a*x^2 + b*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{(2a)\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx}{5b} \\
&= \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{\left(2ax\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{5b\sqrt{ax^2+bx^5}} \\
&= \frac{2\sqrt{ax^2+bx^5}}{5b} \\
&\quad - \frac{4\sqrt{2+\sqrt{3}}ax\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.29

$$
\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x^2\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{5b\sqrt{x^2\left(a+bx^3\right)}}
$$

```
[In] Integrate[x^4/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*x^2*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)]))/(5*b*Sqrt[x^2*(a + b*x^3)])
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.04

method	result
default	$\frac{2x\left(ia\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\,\sqrt{-\frac{i\left(i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F\left(\frac{\sqrt{3}\,\sqrt{2}}{}\right.\right.}{15\sqrt{b\,x^{5}+a\,x^{2}}\,b^{2}}$
risch	$\frac{2x^{2}\left(b\,x^{3}+a\right)}{5b\sqrt{x^{2}\left(b\,x^{3}+a\right)}}+\frac{4ia\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(\right.}{}\right.}{\left(-a\,b^{2}\right)^{}}}}{15b^{2}\sqrt{x^{2}\left(b\,x^{3}+\right.}}$

```
[In] int(x^4/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*x*(I*a*3^(1/2)*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2
)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^
(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))+3*b^2*x^4+3*a*b*x)/(b*x^5+a*x^2)^(1/
2)/b^2
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx=-\frac{2\left(2\,a\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\sqrt{bx^5+ax^2}b\right)}{5\,b^2}$$

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*(2*a*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - sqrt(b*x^5 + a*x^2)*b
)/b^2
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^4}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(x**4/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**4/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^4}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^4}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^4}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(x^4/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^4/(a*x^2 + b*x^5)^(1/2), x)
```

3.290 $\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 212

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] 2/3*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/b^(1/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2057, 224}

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\sqrt{2+\sqrt{3}}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[In] Int[x/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.25

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sq
rt[x^2*(a + b*x^3)]
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.09

method	result
default	$-\frac{ix\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}-2bx-\left(-a\,b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}F(\sqrt{3}\sqrt{2}}{3\sqrt{b\,x^5+a\,x^2}\,b}$

```
[In] int(x/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/(b*x^5+a*x^2)^(1/2)*x*3^(1/2)/b*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*
b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2
)*2^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b
^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.07

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)}{\sqrt{b}}$$

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2*weierstrassPInverse(0, -4*a/b, x)/sqrt(b)
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

```
[In] integrate(x/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(x/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x/(a*x^2 + b*x^5)^(1/2), x)
```

3.291 $\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 243

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{ax^2+bx^5}}{2ax^3} - \frac{\sqrt{2+\sqrt{3}}\,b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] -1/2*(b*x^5+a*x^2)^(1/2)/a/x^3-1/6*b^(2/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/(b*x^5+a*x^2)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

$= \{2050, 2057, 224\}$

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx =$$

$$-\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -1/2*Sqrt[a*x^2 + b*x^5]/(a*x^3) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*x*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a*x^2 + b*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{ax^2+bx^5}}{2ax^3} - \frac{b\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx}{4a} \\
&= -\frac{\sqrt{ax^2+bx^5}}{2ax^3} - \frac{\left(bx\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{4a\sqrt{ax^2+bx^5}} \\
&= -\frac{\sqrt{ax^2+bx^5}}{2ax^3} \\
&\quad - \frac{\sqrt{2+\sqrt{3}}\,b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.23

$$
\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\ \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x\sqrt{x^2\left(a+bx^3\right)}}
$$

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
[Out] -1/2*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/
(x*Sqrt[x^2*(a + b*x^3)])
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.02

method	result
default	$\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,F\left(\sqrt{3}\,\sqrt{2}\right.}{12x\sqrt{b\,x^{5}+a\,x^{2}}\,a}$ $i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}$
risch	$-\frac{b\,x^{3}+a}{2ax\sqrt{x^{2}\left(b\,x^{3}+a\right)}}+$

[In] int(1/x^2/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/12/x*(I*3^(1/2)*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x^2-6*b*x^3-6*a)/(b*x^5+a*x^2)^(1/2)/a

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{b}x^3\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{bx^5+ax^2}}{2\,ax^3}$$

[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")

[Out] -1/2*(sqrt(b)*x^3*weierstrassPInverse(0, -4*a/b, x) + sqrt(b*x^5 + a*x^2))/(a*x^3)

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^2\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(1/x**2/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{\frac{a}{b\,x^3}+1}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\ \frac{13}{6};\ -\frac{a}{b\,x^3}\right)}{7\,x\,\sqrt{b\,x^5+a\,x^2}}$$

```
[In] int(1/(x^2*(a*x^2 + b*x^5)^(1/2)),x)
[Out] -(2*(a/(b*x^3) + 1)^(1/2)*hypergeom([1/2, 7/6], 13/6, -a/(b*x^3)))/(7*x*(a*
x^2 + b*x^5)^(1/2))
```

3.292 $\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 514

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = -\frac{8ax(a+bx^3)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{2x\sqrt{ax^2+bx^5}}{7b}$$

$$+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{8\sqrt{2}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] -8/7*a*x*(b*x^3+a)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(1
/2)+2/7*x*(b*x^5+a*x^2)^(1/2)/b-8/21*a^(4/3)*x*(a^(1/3)+b^(1/3)*x)*Elliptic
F((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/7*3^(1/4)*a^(4/3)*x*(
a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(
b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2049, 2057, 309, 224, 1891}

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx =$$

$$-\frac{8\sqrt{2}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{8ax(a+bx^3)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{2x\sqrt{ax^2+bx^5}}{7b}$$

```
[In] Int[x^5/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-8*a*x*(a + b*x^3))/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*
x^2 + b*x^5]) + (2*x*Sqrt[a*x^2 + b*x^5])/(7*b) + (4*3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - (8*Sqrt[2]*a^(4/3)*x*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*b^(5/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a*x^2 + b*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{(4a)\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx}{7b} \\
&= \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{\left(4ax\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{7b\sqrt{ax^2+bx^5}}
\end{aligned}
$$

$$= \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{\left(4ax\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}\sqrt{ax^2+bx^5}} + \frac{\left(4\left(1-\sqrt{3}\right)a^{4/3}x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}\sqrt{ax^2+bx^5}}$$

$$= -\frac{8ax(a+bx^3)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} + \frac{2x\sqrt{ax^2+bx^5}}{7b} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Bigg(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{}}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} - \frac{8\sqrt{2}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.13

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x^3\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{7b\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[x^5/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3,
-((b*x^3)/a)]))/(7*b*Sqrt[x^2*(a + b*x^3)])
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.05

method	result
pseudoelliptic	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-b\,x^{3}+2a\right)}{9b^{2}}$
risch	$\frac{2x^{3}\left(b\,x^{3}+a\right)}{7b\sqrt{x^{2}\left(b\,x^{3}+a\right)}}+\frac{8ia\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{}}}{}$
default	$-\frac{2x\left(3i\left(-a\,b^{2}\right)^{\frac{2}{3}}\sqrt{3}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\right.}{}$

```
[In] int(x^5/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/9*(b*x^3+a)^(1/2)*(-b*x^3+2*a)/b^2
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.09

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \frac{2\left(\sqrt{bx^5+ax^2}bx+4\,a\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)\right)}{7\,b^2}$$

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/7*(sqrt(b*x^5 + a*x^2)*b*x + 4*a*sqrt(b)*weierstrassZeta(0, -4*a/b, weier
strassPInverse(0, -4*a/b, x)))/b^2
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(x**5/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**5/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^5}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(x^5/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^5/(a*x^2 + b*x^5)^(1/2), x)
```

3.293 $\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 484

$$\begin{aligned}\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2x(a+bx^3)}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} \\ &- \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} \\ &+ \frac{2\sqrt{2}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}\end{aligned}$$

```
[Out] 2*x*(b*x^3+a)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(1/2)+2
/3*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/
3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2057, 309, 224, 1891}

$$
\begin{aligned}
&\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx \\
&= \frac{2\sqrt{2}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} \\
&\quad -\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} \\
&\quad +\frac{2x(a+bx^3)}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}
\end{aligned}
$$

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x*(a + b*x^3))/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 +
 b*x^5]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (
2*Sqrt[2]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}$$

$$= \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}\sqrt{ax^2+bx^5}} - \frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}\sqrt{ax^2+bx^5}}$$

$$
\begin{aligned}
&= \frac{2x(a+bx^3)}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} \\
&\quad - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} \\
&\quad + \frac{2\sqrt{2}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$
\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^3\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{x^2(a+bx^3)}}
$$

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(2
*Sqrt[x^2*(a + b*x^3)])
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.03

method	result
pseudoelliptic	$\frac{2\sqrt{b\,x^3+a}}{3b}$
default	$-\frac{i x\sqrt{3}\left(-a\,b^2\right)^{\frac{2}{3}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}-2bx-\left(-a\,b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+2bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{}$

```
[In] int(x^2/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(b*x^3+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.05

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{\sqrt{b}}$$

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))/sqrt(b)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^2}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

```
[In] integrate(x**2/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**2/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^2}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^2}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^2}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(x^2/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^2/(a*x^2 + b*x^5)^(1/2), x)
```

3.294 $\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 510

$$
\begin{aligned}
\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx &= \frac{\sqrt[3]{b}x(a+bx^3)}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{ax^2} \\
&\quad -\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} \\
&\quad +\frac{\sqrt{2}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

```
[Out] b^(1/3)*x*(b*x^3+a)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(1/2)-(
b*x^5+a*x^2)^(1/2)/a/x^2+1/3*b^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^
(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/a^(2/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*3^(1/4)*b^(1/3)*x*(a^(1/3)+
b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^5+a*
x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2050, 2057, 309, 224, 1891}

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$$

$$= \frac{\sqrt{2}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{\sqrt{ax^2+bx^5}}{ax^2}+\frac{\sqrt[3]{b}x(a+bx^3)}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}$$

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (b^(1/3)*x*(a + b*x^3))/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 +
 b*x^5]) - Sqrt[a*x^2 + b*x^5]/(a*x^2) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)
*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (Sqrt[2]*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b
*x^5])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{ax^2+bx^5}}{ax^2}+\frac{b\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx}{2a}\\ &= -\frac{\sqrt{ax^2+bx^5}}{ax^2}+\frac{\left(bx\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt{ax^2+bx^5}}\end{aligned}$$

$$= -\frac{\sqrt{ax^2+bx^5}}{ax^2} + \frac{\left(b^{2/3}x\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt{ax^2+bx^5}} - \frac{\left(\left(1-\sqrt{3}\right)b^{2/3}x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{2a^{2/3}\sqrt{ax^2+bx^5}}$$

$$= \frac{\sqrt[3]{b}x(a+bx^3)}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} - \frac{\sqrt{ax^2+bx^5}}{ax^2} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} + \frac{\sqrt{2}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.10

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^5]),x]
[Out] -((Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/Sqr
t[x^2*(a + b*x^3)])
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.04

method	result
pseudoelliptic	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$
risch	$i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right.}{}}$ $-\frac{b\,x^{3}+a}{a\sqrt{x^{2}\left(b\,x^{3}+a\right)}}-$
default	$3i\left(-a\,b^{2}\right)^{\frac{2}{3}}\sqrt{3}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,E$

```
[In] int(1/x/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{b}x^2\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\sqrt{bx^5+ax^2}}{ax^2}$$

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -(sqrt(b)*x^2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
 + sqrt(b*x^5 + a*x^2))/(a*x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{x\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(1/x/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{bx^5+ax^2}x}\,dx$$

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{bx^5+ax^2}x}\,dx$$

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{x\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(1/(x*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x*(a*x^2 + b*x^5)^(1/2)), x)
```

3.295 $\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 265

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}}+\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] 1/5*x^(5/2)*(b*x^5+a*x^2)^(1/2)/b-7/20*a*(b*x^5+a*x^2)^(1/2)/b^2/x^(1/2)+7/
120*a^(5/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/
(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a
^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^
2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/b^2/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2049, 2057, 335, 231}

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\Bigg(\right.}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}}+\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}$$

```
[In] Int[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-7*a*Sqrt[a*x^2 + b*x^5])/(20*b^2*Sqrt[x]) + (x^(5/2)*Sqrt[a*x^2 + b*x^5])
/(5*b) + (7*a^(5/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(40*3^(1/4)*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x)
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
```

```
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} - \frac{(7a)\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx}{10b} \\
&= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{(7a^2)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{40b^2} \\
&= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{\left(7a^2x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{40b^2\sqrt{ax^2+bx^5}} \\
&= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{\left(7a^2x\sqrt{a+bx^3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{20b^2\sqrt{ax^2+bx^5}} \\
&= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} \\
&\quad + \frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.32

$$
\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{3/2}\left(-7a^2-3abx^3+4b^2x^6+7a^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{20b^2\sqrt{x^2\left(a+bx^3\right)}}
$$

```
[In] Integrate[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(-7*a^2 - 3*a*b*x^3 + 4*b^2*x^6 + 7*a^2*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(20*b^2*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.80

method	result
risch	$-\frac{\left(-4b\,x^{3}+7a\right)x^{\frac{3}{2}}\left(b\,x^{3}+a\right)}{20b^{2}\sqrt{x^{2}\left(b\,x^{3}+a\right)}}+\frac{7a^{2}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right.}{20b\left(\right.}$
default	Expression too large to display

```
[In] int(x^(13/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(-4*b*x^3+7*a)/b^2*x^(3/2)*(b*x^3+a)/(x^2*(b*x^3+a))^(1/2)+7/20*a^2/b
*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1
/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a
*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)
^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))/(x^2*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+a))^(1/2)
```

Fricas [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(9/2)/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(x**(13/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**(13/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{13/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(x^(13/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(13/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.296 $\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 525

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = -\frac{5\left(1+\sqrt{3}\right)ax^{3/2}(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}$$
$$+\frac{5\sqrt[4]{3}a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$
$$+\frac{5\left(1-\sqrt{3}\right)a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] -5/8*a*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)/(b*x^5+a*x^2)^(1/2)+1/4*x^(3/2)*(b*x^5+a*x^2)^(1/2)/b+5/8*3^(1/4)*a^(4/3)
*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*
x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2)+5/48*a^(4/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*
(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*
```

```
(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*
x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2
^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(
a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2049, 2057, 335, 314, 231, 1895}

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{5\left(1-\sqrt{3}\right)a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right.\right.}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$+\frac{5\sqrt[4]{3}a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{5\left(1+\sqrt{3}\right)ax^{3/2}\left(a+bx^3\right)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}$$

```
[In] Int[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-5*(1 + Sqrt[3])*a*x^(3/2)*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) + (x^(3/2)*Sqrt[a*x^2 + b*x^5])/(4*b) + (
5*3^(1/4)*a^(4/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcC
os[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)]
, (2 + Sqrt[3])/4])/(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (5*(1 - Sqrt[3])*a
^(4/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3)
 + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt
[3])/4])/(16*3^(1/4)*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
```

```
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} - \frac{(5a)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{8b} \\
&= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} - \frac{\left(5ax\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{8b\sqrt{ax^2+bx^5}} \\
&= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} - \frac{\left(5ax\sqrt{a+bx^3}\right)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{4b\sqrt{ax^2+bx^5}} \\
&= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} + \frac{\left(5ax\sqrt{a+bx^3}\right)\operatorname{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{8b^{5/3}\sqrt{ax^2+bx^5}} \\
&\quad + \frac{\left(5\left(1-\sqrt{3}\right)a^{5/3}x\sqrt{a+bx^3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{8b^{5/3}\sqrt{ax^2+bx^5}} \\
&= -\frac{5\left(1+\sqrt{3}\right)ax^{3/2}(a+bx^3)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} + \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} \\
&\quad + \frac{5\sqrt[4]{3}a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} \\
&\quad + \frac{5\left(1-\sqrt{3}\right)a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.13

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{7/2}\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{4b\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(7/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/
6, -((b*x^3)/a)]))/(4*b*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.12

method	result	size
risch	Expression too large to display	1115
default	Expression too large to display	2586

```
[In] int(x^(11/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*x^(7/2)*(b*x^3+a)/(x^2*(b*x^3+a))^(1/2)-5/8*a/b*(x*(x+1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-
a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-
1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
```

```
^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*x*(x-1/b*(-a
*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)/(x^2*(b*x^3+a))^(1
/2)*x^(1/2)*(x*(b*x^3+a))^(1/2)
```

Fricas [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(7/2)/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{11}{2}}}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

```
[In] integrate(x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**(11/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{11/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(x^(11/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(11/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.297 $\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

```
[Out] -1/3*a*arctanh(x^(5/2)*b^(1/2)/(b*x^5+a*x^2)^(1/2))/b^(3/2)+1/3*x^(1/2)*(b*
x^5+a*x^2)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2049, 2054, 212}

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

```
[In] Int[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a*x^2 + b*x^5])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x
^2 + b*x^5]])/(3*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b} - \frac{a\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx}{2b}\\ &= \frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b} - \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b}\\ &= \frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\sqrt{b}x^{5/2}(a+bx^3) - ax\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3b^{3/2}\sqrt{x^2(a+bx^3)}}$$

```
[In] Integrate[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
[Out] (Sqrt[b]*x^(5/2)*(a + b*x^3) - a*x*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + Sq
rt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{x^{\frac{3}{2}}\left(b\,x^{3}+a\right)\left(x\sqrt{b}\,\sqrt{x(b\,x^{3}+a)}-\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)a\right)}{3\sqrt{b\,x^{5}+a\,x^{2}}\,\sqrt{x(b\,x^{3}+a)}\,b^{\frac{3}{2}}}$	79
risch	$\frac{x^{\frac{5}{2}}\left(b\,x^{3}+a\right)}{3b\sqrt{x^{2}(b\,x^{3}+a)}}-\frac{a\ \operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)\sqrt{x}\,\sqrt{x(b\,x^{3}+a)}}{3b^{\frac{3}{2}}\,\sqrt{x^{2}(b\,x^{3}+a)}}$	82

```
[In] int(x^(9/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(3/2)*(b*x^3+a)*(x*b^(1/2)*(x*(b*x^3+a))^(1/2)-arctanh(1/x^2*(x*(b*x^
3+a))^(1/2)/b^(1/2))*a)/(b*x^5+a*x^2)^(1/2)/(x*(b*x^3+a))^(1/2)/b^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.28

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \Bigg[\frac{a\sqrt{b}\log\left(-8\,b^2x^6-8\,abx^3+4\,\sqrt{bx^5+ax^2}(2\,bx^3+a)\sqrt{b}\sqrt{x}-a^2\right)+4\,\sqrt{bx^5+ax^2}b}{12\,b^2}$$

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 + 4*sqrt(b*x^5 + a*x^2)*(2*b*x^
3 + a)*sqrt(b)*sqrt(x) - a^2) + 4*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2, 1/6*(
a*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(-b)*sqrt(x)/(2*b*x^3 + a)) + 2
*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2]
```

Sympy [F]

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{9}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(9/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{9}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{\sqrt{bx^3+a}\,x^{\frac{3}{2}}}{3\,b\text{sgn}\left(x\right)}+\frac{a\log\left(\left|-\sqrt{b}x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\,b^{\frac{3}{2}}\text{sgn}\left(x\right)}$$

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^3 + a)*x^(3/2)/(b*sgn(x)) + 1/3*a*log(abs(-sqrt(b)*x^(3/2) + s
qrt(b*x^3 + a)))/(b^(3/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{9/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(x^(9/2)/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^(9/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.298 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 237

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] 1/2*(b*x^5+a*x^2)^(1/2)/b/x^(1/2)-1/12*a^(2/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*
((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF(
(1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x
*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

$= \{2049,\ 2057,\ 335,\ 231\}$

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[In] Int[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] Sqrt[a*x^2 + b*x^5]/(2*b*Sqrt[x]) - (a^(2/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 +
 b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{a\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{4b} \\
&= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{\left(ax\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{4b\sqrt{ax^2+bx^5}} \\
&= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{\left(ax\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{2b\sqrt{ax^2+bx^5}} \\
&= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} \\
&\quad - \frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.30

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{3/2}\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{2b\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6
, -((b*x^3)/a)]))/(2*b*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 727, normalized size of antiderivative = 3.07

method	result
risch	$\frac{x^{\frac{3}{2}}\left(b\,x^{3}+a\right)}{2b\sqrt{x^{2}\left(b\,x^{3}+a\right)}}-\frac{a\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b\left(-\frac{\left(-a\,b^{2}\right)}{2b}\right)}}}{2\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\right)}$
default	Expression too large to display

```
[In] int(x^(7/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*x^(3/2)*(b*x^3+a)/(x^2*(b*x^3+a))^(1/2)-1/2*a*(1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b
*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(
1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x*(x-1
/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/(x^2*(b*x
^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+a))^(1/2)
```

Fricas [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(3/2)/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**(7/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{7/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(x^(7/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(7/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.299 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 492

$$
\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{\left(1+\sqrt{3}\right)x^{3/2}(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} \\
&- \frac{\sqrt[4]{3}\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} \\
&- \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

```
[Out] x^(3/2)*(b*x^3+a)*(1+3^(1/2))/b^(2/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^
5+a*x^2)^(1/2)-3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1
/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*
x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*a^(1/3)*x^(3/2)*(a^(1/3)+b^
(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*E
```

```
llipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*
x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2057, 335, 314, 231, 1895}

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = -\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} - \frac{\sqrt[4]{3}\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} + \frac{\left(1+\sqrt{3}\right)x^{3/2}\left(a+bx^3\right)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}$$

```
[In] Int[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] ((1 + Sqrt[3])*x^(3/2)*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/
3)*x)*Sqrt[a*x^2 + b*x^5]) - (3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a
^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*
x^5]) - ((1 - Sqrt[3])*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}$$

$$= \frac{\left(2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax^2+bx^5}}$$

$$= -\frac{\left(x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax^2+bx^5}} - \frac{\left(\left(1-\sqrt{3}\right)a^{2/3}x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax^2+bx^5}}$$

$$= \frac{\left(1+\sqrt{3}\right)x^{3/2}(a+bx^3)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} - \frac{\sqrt[4]{3}\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.12

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x^{7/2}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{5\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/
a)])/(5*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 2374, normalized size of antiderivative = 4.83

method	result	size
default	Expression too large to display	2374

```
[In] int(x^(5/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^2*(-I*(-(I*3^(1/2)-3)*x*b/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elli
pticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^
(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3
^(1/2)*b*x^2+2*I*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3
^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*x-2*(-(I*3^(1/2)-3)*x*b/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(
(I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1
/3)*b*x^2+3*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*b*x^2+I*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
E((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a*b+4*(-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*(-a*b^2)^(2/3)*x-6*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(
```

```
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x-I*3^(1/2)*b^2*x^3+2*(-(I*
3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*a*b-3*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1
)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*a*b-I*(-a*b^2)^(1/3)*3^(1/2)*b*x^2-I*3^(1/2)*x*(-a*b^2
)^(2/3)+3*b^2*x^3+3*(-a*b^2)^(1/3)*b*x^2+3*x*(-a*b^2)^(2/3))/(x*(b*x^3+a))^
(1/2)/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1
/2)
```

Fricas [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^3 + a), x)
```

Sympy [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\,(a+bx^3)}}\,dx$$

```
[In] integrate(x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx=\int \frac{x^{5/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(x^(5/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(5/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.300 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

```
[Out] 2/3*arctanh(x^(5/2)*b^(1/2)/(b*x^5+a*x^2)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2054, 212}

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

```
[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.67

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3\sqrt{b}\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(3*Sqrt[b]*Sqr
t[x^2*(a + b*x^3)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(26) = 52.

Time = 1.78 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

method	result	size
default	$\frac{2x^{\frac{3}{2}}\left(b\,x^{3}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{b\,x^{5}+a\,x^{2}}\,\sqrt{x\left(b\,x^{3}+a\right)}\,\sqrt{b}}$	59

```
[In] int(x^(3/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/(x*(b*x^3+a))^(1/2)/b^(1/2)*arcta
nh(1/x^2*(x*(b*x^3+a))^(1/2)/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.81

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-4\,\sqrt{bx^5+ax^2}(2\,bx^3+a)\sqrt{b}\sqrt{x}-a^2\right)}{6\,\sqrt{b}},\right.$$
$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{bx^5+ax^2}\sqrt{-b}\sqrt{x}}{2\,bx^3+a}\right)}{3\,b}\right]$$

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*sqrt(b*x^5 + a*x^2)*(2*b*x^3 + a)*sqrt(
b)*sqrt(x) - a^2)/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(
-b)*sqrt(x)/(2*b*x^3 + a))/b]
```

Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(b*x^5 + a*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\log\left(|a|\right)\mathrm{sgn}(x)}{3\sqrt{b}} - \frac{2\log\left(\left|-\sqrt{b}x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\sqrt{b}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(a))*sgn(x)/sqrt(b) - 2/3*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3
+ a)))/(sqrt(b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{x^{3/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(x^(3/2)/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.301 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 203

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] 1/3*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b
^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
a^(1/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{2057, 335, 231\}$

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx$$

$$= \frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[In] Int[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1
- Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4
])/(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}$$

$$= \frac{\left(2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax^2+bx^5}}$$

$$= \frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x^{3/2}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^(3/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a
)])/Sqrt[x^2*(a + b*x^3)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.15

method	result
default	$-\frac{4x^{\frac{3}{2}}\left(b\,x^{3}+a\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)xb}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+2bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-2bx-\left(-a\,b^{2}\right)^{\frac{1}{3}}}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\,F\left(\sqrt{-\frac{(i\sqrt{3}}{\left(i\sqrt{3}-1\right)(-b}}\right.}{\sqrt{b\,x^{5}+a\,x^{2}}\,b\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{x\left(b\,x^{3}+a\right)}\left(i\sqrt{3}-3\right)\sqrt{\frac{x\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\left(i\sqrt{3}\right.}{}}}$

```
[In] int(x^(1/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b/(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x
*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*
b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(I*3^(1
/2)*b^2*x^2-2*I*(-a*b^2)^(1/3)*3^(1/2)*b*x+I*3^(1/2)*(-a*b^2)^(2/3)-b^2*x^2
+2*(-a*b^2)^(1/3)*b*x-(-a*b^2)^(2/3))/(x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1/
b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.08

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = -\frac{2\,\mathrm{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)}{\sqrt{a}}$$

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*weierstrassPInverse(0, -4*b/a, 1/x)/sqrt(a)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{\sqrt{x}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x**2*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}\,dx$$

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = \int \frac{\sqrt{x}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(x^(1/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(1/2)/(a*x^2 + b*x^5)^(1/2), x)
```

3.302 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 519

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x^{3/2}(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} - \frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}} - \frac{2\sqrt[4]{3}\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] 2*b^(1/3)*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/a/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(
b*x^5+a*x^2)^(1/2)-2*(b*x^5+a*x^2)^(1/2)/a/x^(3/2)-2*3^(1/4)*b^(1/3)*x^(3/2
)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^5+a*x
^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2)-1/3*b^(1/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/
```

```
2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(
1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*
(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+
b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2050, 2057, 335, 314, 231, 1895}

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx =$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Bigg(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}(2+\sqrt{3}}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{2\sqrt[4]{3}\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}+\frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x^{3/2}(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}$$

```
[In] Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (2*(1 + Sqrt[3])*b^(1/3)*x^(3/2)*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(a*x^(3/2)) - (2*3
^(1/4)*b^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[
(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (
2 + Sqrt[3])/4])/(a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - ((1 - Sqrt[3])*b^(1/3)*
x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 -
 Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4]
)/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
```

```
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}+\frac{(2b)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{a}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}+\frac{\left(2bx\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{a\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}+\frac{\left(4bx\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}-\frac{\left(2\sqrt[3]{b}x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax^2+bx^5}} - \frac{\left(2\left(1-\sqrt{3}\right)\sqrt[3]{b}x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{\sqrt[3]{a}\sqrt{ax^2+bx^5}}$$

$$= \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x^{3/2}(a+bx^3)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}} - \frac{2\sqrt[4]{3}\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} - \frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{x}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3)
/a)])/Sqrt[x^2*(a + b*x^3)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.15

method	result	size
risch	Expression too large to display	1115
default	Expression too large to display	2860

```
[In] int(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*x^3+a)/a/(x^2*(b*x^3+a))^(1/2)*x^(1/2)+2*b/a*(x*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b
^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b
*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
```

```
^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/3)))/(b*x*(x-1/b*(-a*b^
2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)/(x^2*(b*x^3+a))^(1/2)
*x^(1/2)*(x*(b*x^3+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,b}{a},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)}{\sqrt{a}}$$

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x))/sqrt(a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(1/x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}\sqrt{x}}\,dx$$

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{bx^5+ax^2}\sqrt{x}}\,dx$$

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{\sqrt{x}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(1/(x^(1/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(1/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.303 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

```
[Out] -2/3*(b*x^5+a*x^2)^(1/2)/a/x^(5/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2039}

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

```
[In] Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(3*a*x^(5/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral}=-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{x^2\left(a+bx^3\right)}}{3ax^{5/2}}$$

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
[Out] (-2*Sqrt[x^2*(a + b*x^3)])/(3*a*x^(5/2))
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

method	result	size
gosper	$-\frac{2\left(b\,x^{3}+a\right)}{3\sqrt{x}\,a\sqrt{b\,x^{5}+a\,x^{2}}}$	29
default	$-\frac{2\left(b\,x^{3}+a\right)}{3\sqrt{x}\,a\sqrt{b\,x^{5}+a\,x^{2}}}$	29
risch	$-\frac{2\left(b\,x^{3}+a\right)}{3a\sqrt{x^{2}\left(b\,x^{3}+a\right)}\,\sqrt{x}}$	29

```
[In] int(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3/x^(1/2)*(b*x^3+a)/a/(b*x^5+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\,\sqrt{bx^5+ax^2}}{3\,ax^{\frac{5}{2}}}$$

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(b*x^5 + a*x^2)/(a*x^(5/2))
```

Sympy [F]

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(1/x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\left(bx^4+ax\right)}{3\sqrt{bx^3+a}ax^{\frac{5}{2}}}$$

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\left(\frac{\sqrt{b+\frac{a}{x^3}}}{a}-\frac{\sqrt{b}}{a}\right)}{3\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(sqrt(b + a/x^3)/a - sqrt(b)/a)/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{x^{3/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.304 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 235

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}} - \frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] -2/5*(b*x^5+a*x^2)^(1/2)/a/x^(7/2)-2/15*b*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(
1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(
1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/
4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*
x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

$= \{2050, 2057, 335, 231\}$

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx =$$

$$-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

```
[In] Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(5*a*x^(7/2)) - (2*b*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^
(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[
a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{(2b)\int\frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{\left(2bx\sqrt{a+bx^3}\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{5a\sqrt{ax^2+bx^5}}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{\left(4bx\sqrt{a+bx^3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{5a\sqrt{ax^2+bx^5}}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}\\
&\quad-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.24

$$\int\frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{5x^{3/2}\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(5
*x^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 732, normalized size of antiderivative = 3.11

method	result
risch	$-\frac{2\left(b\,x^{3}+a\right)}{5a\,x^{\frac{3}{2}}\sqrt{x^{2}\left(b\,x^{3}+a\right)}}-\frac{4b^{2}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\,\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{}}{5a\left(-\frac{3\left(-\right.}{}\right.}$
default	Expression too large to display

```
[In] int(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/a*(b*x^3+a)/x^(3/2)/(x^2*(b*x^3+a))^(1/2)-4/5*b^2/a*(1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b
^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*x
*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*Ellipti
cF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/(x^2
*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx=\frac{2\left(2\sqrt{a}bx^4\mathrm{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^5+ax^2}a\sqrt{x}\right)}{5\,a^2x^4}$$

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(2*sqrt(a)*b*x^4*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^5 + a*x
^2)*a*sqrt(x))/(a^2*x^4)
```

Sympy [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(1/x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{5/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(1/(x^(5/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^(5/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.305 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 555

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3/2}(a+bx^3)}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}$$

$$-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}+\frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}}$$

$$+\frac{8\sqrt[4]{3}b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$+\frac{4\left(1-\sqrt{3}\right)b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] -8/7*b^(4/3)*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/a^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))/(b*x^5+a*x^2)^(1/2)-2/7*(b*x^5+a*x^2)^(1/2)/a/x^(9/2)+8/7*b*(b*x^5+a*x^
2)^(1/2)/a^2/x^(3/2)+8/7*3^(1/4)*b^(4/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1
/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1
/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^
(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4
*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3
```

```
)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+4/21*b^(4/3)*x^(3/2)*
(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
a^(5/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2050, 2057, 335, 314, 231, 1895}

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = \frac{4\left(1-\sqrt{3}\right)b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{7\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$+\frac{8\sqrt[4]{3}b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$-\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3/2}(a+bx^3)}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}}$$

```
[In] Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-8*(1 + Sqrt[3])*b^(4/3)*x^(3/2)*(a + b*x^3))/(7*a^2*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(7*a*x^(9/2))
 + (8*b*Sqrt[a*x^2 + b*x^5])/(7*a^2*x^(3/2)) + (8*3^(1/4)*b^(4/3)*x^(3/2)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])
*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(7*a^(5
/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*Sqrt[a*x^2 + b*x^5]) + (4*(1 - Sqrt[3])*b^(4/3)*x^(3/2)*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(7*3^(1/4)*a^(5/3)
```

```
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} - \frac{(4b)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx}{7a} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{(8b^2)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{7a^2} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{\left(8b^2x\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{7a^2\sqrt{ax^2+bx^5}} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{\left(16b^2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{7a^2\sqrt{ax^2+bx^5}} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} \\
&\quad + \frac{\left(8b^{4/3}x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{7a^2\sqrt{ax^2+bx^5}} \\
&\quad + \frac{\left(8\left(1-\sqrt{3}\right)b^{4/3}x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{7a^{4/3}\sqrt{ax^2+bx^5}} \\
&= -\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3/2}(a+bx^3)}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} - \frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} \\
&\quad + \frac{8\sqrt[4]{3}b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}} \\
&\quad + \frac{4\left(1-\sqrt{3}\right)b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},-\frac{bx^3}{a}\right)}{7x^{5/2}\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-7/6, 1/2, -1/6, -((b*x^3)/a)])/(
7*x^(5/2)*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.28 (sec) , antiderivative size = 1125, normalized size of antiderivative = 2.03

method	result	size
risch	Expression too large to display	1125
default	Expression too large to display	3048

```
[In] int(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7*(b*x^3+a)*(-4*b*x^3+a)/a^2/x^(5/2)/(x^2*(b*x^3+a))^(1/2)-8/7*b^2/a^2*(
x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x
/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/
3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)
*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1
/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1
```

```
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*b/(-a*b^2)^(1/
3)))/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)/(x^2*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\left(4\sqrt{a}bx^5\text{weierstrassZeta}\left(0,-\frac{4b}{a},\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)+\sqrt{bx^5+ax^2}a\sqrt{x}\right)}{7a^2x^5}$$

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/7*(4*sqrt(a)*b*x^5*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4*
b/a, 1/x)) + sqrt(b*x^5 + a*x^2)*a*sqrt(x))/(a^2*x^5)
```

Sympy [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(1/x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{7/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(1/(x^(7/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.306 $\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}+\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}$$

```
[Out] -2/9*(b*x^5+a*x^2)^(1/2)/a/x^(11/2)+4/9*b*(b*x^5+a*x^2)^(1/2)/a^2/x^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2041, 2039}

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = \frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

```
[In] Int[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(9*a*x^(11/2)) + (4*b*Sqrt[a*x^2 + b*x^5])/(9*a^2*
x^(5/2))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
```

```
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}-\frac{(2b)\int\frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}+\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int\frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2(a-2bx^3)\sqrt{x^2(a+bx^3)}}{9a^2x^{11/2}}$$

```
[In] Integrate[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
[Out] (-2*(a - 2*b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*a^2*x^(11/2))
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2(b\,x^3+a)\,(-2b\,x^3+a)}{9x^{\frac{7}{2}}\,a^2\sqrt{b\,x^5+a\,x^2}}$	37
default	$-\frac{2(b\,x^3+a)\,(-2b\,x^3+a)}{9x^{\frac{7}{2}}\,a^2\sqrt{b\,x^5+a\,x^2}}$	37
risch	$-\frac{2(b\,x^3+a)\,(-2b\,x^3+a)}{9\sqrt{x^2\,(b\,x^3+a)}\,x^{\frac{7}{2}}\,a^2}$	37

```
[In] int(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/9*(b*x^3+a)*(-2*b*x^3+a)/x^(7/2)/a^2/(b*x^5+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = \frac{2\,\sqrt{bx^5+ax^2}(2\,bx^3-a)}{9\,a^2x^{\frac{11}{2}}}$$

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(2*b*x^3 - a)/(a^2*x^(11/2))
```

Sympy [F]

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{9}{2}}\sqrt{x^2\,(a+bx^3)}}\,dx$$

```
[In] integrate(1/x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(9/2)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = \frac{2\,(2\,b^2x^7+abx^4-a^2x)}{9\,\sqrt{bx^3+a}a^2x^{\frac{11}{2}}}$$

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\left(\frac{\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}}{a^2}-\frac{3\sqrt{b+\frac{a}{x^3}}\,b}{a^2}+\frac{2\,b^{\frac{3}{2}}}{a^2}\right)}{9\,\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] -2/9*((b + a/x^3)^(3/2)/a^2 - 3*sqrt(b + a/x^3)*b/a^2 + 2*b^(3/2)/a^2)/sgn(
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx=\int \frac{1}{x^{9/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.307 $\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 265

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}} + \frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55\sqrt[4]{3}a^{7/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] -2/11*(b*x^5+a*x^2)^(1/2)/a/x^(13/2)+16/55*b*(b*x^5+a*x^2)^(1/2)/a^2/x^(7/2
)+16/165*b^2*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2
/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))
^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
*3^(3/4)/a^(7/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2050, 2057, 335, 231}

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = \frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right.\right.}{55\sqrt[4]{3}a^{7/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

$$+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

```
[In] Int[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(11*a*x^(13/2)) + (16*b*Sqrt[a*x^2 + b*x^5])/(55*a
^2*x^(7/2)) + (16*b^2*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[
ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)], (2 + Sqrt[3])/4])/(55*3^(1/4)*a^(7/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2050

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}} - \frac{(8b)\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx}{11a} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}} + \frac{(16b^2)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{55a^2} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}} + \frac{\left(16b^2x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{55a^2\sqrt{ax^2+bx^5}} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}} + \frac{\left(32b^2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{55a^2\sqrt{ax^2+bx^5}} \\
&= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}} + \frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}} \\
&\quad + \frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55\sqrt[4]{3}a^{7/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{1}{2},-\frac{5}{6},-\frac{bx^3}{a}\right)}{11x^{9/2}\sqrt{x^2\left(a+bx^3\right)}}$$

```
[In] Integrate[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-11/6, 1/2, -5/6, -((b*x^3)/a)])/
(11*x^(9/2)*Sqrt[x^2*(a + b*x^3)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.80

method	result
risch	$-\frac{2\left(b\,x^{3}+a\right)\left(-8b\,x^{3}+5a\right)}{55a^{2}x^{\frac{9}{2}}\sqrt{x^{2}\left(b\,x^{3}+a\right)}}+\frac{32b^{3}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}{55a^{2}\left(\right.}$
default	Expression too large to display

```
[In] int(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/55*(b*x^3+a)*(-8*b*x^3+5*a)/a^2/x^(9/2)/(x^2*(b*x^3+a))^(1/2)+32/55*b^3/
a^2*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2
*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1
/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(-a*b^2)^(1/3)/(b*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2))/(x^2*(b*x^3+a))^(1/2)*x^(1/2)*(x*(b*x^3+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\left(16\sqrt{a}b^2x^7\mathrm{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^5+ax^2}(8\,abx^3-5\,a^2)\sqrt{x}\right)}{55\,a^3x^7}$$

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/55*(16*sqrt(a)*b^2*x^7*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^5
+ a*x^2)*(8*a*b*x^3 - 5*a^2)*sqrt(x))/(a^3*x^7)
```

Sympy [F]

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{\frac{11}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

```
[In] integrate(1/x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(11/2)*sqrt(x**2*(a + b*x**3))), x)
```

Maxima [F]

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{11}{2}}}\,dx$$

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

Giac [F]

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{11}{2}}}\,dx$$

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx = \int \frac{1}{x^{11/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

```
[In] int(1/(x^(11/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^(11/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

3.308 $\int \frac{x}{ax^3+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{x}{ax^3+bx^4}\,dx=-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 46}

$$\int \frac{x}{ax^3+bx^4}\,dx=-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

```
[In] Int[x/(a*x^3 + b*x^4),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^2(a+bx)}\,dx \\
&= \int \left(\frac{1}{ax^2} - \frac{b}{a^2x} + \frac{b^2}{a^2(a+bx)}\right) dx \\
&= -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx)}{a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^3+bx^4}\,dx = -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx)}{a^2}$$

```
[In] Integrate[x/(a*x^3 + b*x^4),x]
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{b\ln(x)x-b\ln(bx+a)x+a}{a^2x}$	26
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^2}-\frac{b\ln(x)}{a^2}$	32

```
[In] int(x/(b*x^4+a*x^3),x,method=_RETURNVERBOSE)
[Out] -(b*ln(x)*x-b*ln(b*x+a)*x+a)/a^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x}{ax^3+bx^4}\,dx = \frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{x}{ax^3+bx^4}\,dx = -\frac{1}{ax}+\frac{b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

```
[In] integrate(x/(b*x**4+a*x**3),x)
```

```
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^3+bx^4}\,dx = \frac{b\log(bx+a)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x}{ax^3+bx^4}\,dx = \frac{b\log(|bx+a|)}{a^2}-\frac{b\log(|x|)}{a^2}-\frac{1}{ax}$$

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time $= 0.05$ (sec) , antiderivative size $= 25$, normalized size of antiderivative $= 0.89$

$$\int \frac{x}{ax^3+bx^4}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

```
[In] int(x/(a*x^3 + b*x^4),x)
```

```
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

3.309 $\int \frac{1}{ax^3+bx^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{1}{ax^3+bx^4}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}$$

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1607, 46}

$$\int \frac{1}{ax^3+bx^4}\,dx = \frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[In] Int[(a*x^3 + b*x^4)^(-1),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{x^3(a+bx)}\,dx \\
&= \int \left(\frac{1}{ax^3} - \frac{b}{a^2x^2} + \frac{b^2}{a^3x} - \frac{b^3}{a^3(a+bx)}\right) dx \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b^2\log(x)}{a^3} - \frac{b^2\log(a+bx)}{a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^3+bx^4}\,dx = -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b^2\log(x)}{a^3} - \frac{b^2\log(a+bx)}{a^3}$$

```
[In] Integrate[(a*x^3 + b*x^4)^(-1),x]
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{1}{2a\,x^2}+\frac{b}{x\,a^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	41
norman	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}+\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	41
risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}+\frac{b^2\ln(-x)}{a^3}-\frac{b^2\ln(bx+a)}{a^3}$	43
parallelrisch	$\frac{2b^2\ln(x)x^2-2b^2\ln(bx+a)x^2+2abx-a^2}{2a^3x^2}$	44

```
[In] int(1/(b*x^4+a*x^3),x,method=_RETURNVERBOSE)
[Out] -1/2/a/x^2+b/x/a^2+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{ax^3 + bx^4}\, dx = -\frac{2\, b^2 x^2 \log\left(bx + a\right) - 2\, b^2 x^2 \log\left(x\right) - 2\, abx + a^2}{2\, a^3 x^2}$$

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="fricas")
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{1}{ax^3 + bx^4}\, dx = \frac{-a + 2bx}{2a^2x^2} + \frac{b^2\left(\log\left(x\right) - \log\left(\frac{a}{b} + x\right)\right)}{a^3}$$

```
[In] integrate(1/(b*x**4+a*x**3),x)
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{ax^3 + bx^4}\, dx = -\frac{b^2 \log\left(bx + a\right)}{a^3} + \frac{b^2 \log\left(x\right)}{a^3} + \frac{2\, bx - a}{2\, a^2 x^2}$$

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="maxima")
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{ax^3 + bx^4}\, dx = -\frac{b^2 \log\left(|bx + a|\right)}{a^3} + \frac{b^2 \log\left(|x|\right)}{a^3} + \frac{2\, abx - a^2}{2\, a^3 x^2}$$

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="giac")
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{1}{ax^3+bx^4}\,dx=-\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

```
[In] int(1/(a*x^3 + b*x^4),x)
```

```
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

3.310 $\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = -\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}+\frac{x\sqrt{ax^3+bx^4}}{3b}-\frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}$$

```
[Out] -5/8*a^3*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(7/2)-5/12*a*(b*x^4+a*x
^3)^(1/2)/b^2+5/8*a^2*(b*x^4+a*x^3)^(1/2)/b^3/x+1/3*x*(b*x^4+a*x^3)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2049, 2054, 212}

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = -\frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}+\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}-\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{x\sqrt{ax^3+bx^4}}{3b}$$

```
[In] Int[x^4/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-5*a*Sqrt[a*x^3 + b*x^4])/(12*b^2) + (5*a^2*Sqrt[a*x^3 + b*x^4])/(8*b^3*x)
 + (x*Sqrt[a*x^3 + b*x^4])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3
+ b*x^4]])/(8*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{(5a)\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx}{6b}\\
&= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{x\sqrt{ax^3+bx^4}}{3b} + \frac{(5a^2)\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx}{8b^2}\\
&= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{(5a^3)\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{16b^3}\\
&= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{(5a^3)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^3}\\
&= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$
\begin{aligned}
&\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx\\
&= \frac{\sqrt{b}x^2(15a^3+5a^2bx-2ab^2x^2+8b^3x^3)+30a^3x^{3/2}\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{24b^{7/2}\sqrt{x^3(a+bx)}}
\end{aligned}
$$

```
[In] Integrate[x^4/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(15*a^3 + 5*a^2*b*x - 2*a*b^2*x^2 + 8*b^3*x^3) + 30*a^3*x^(3/2
)*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/(24*b
^(7/2)*Sqrt[x^3*(a + b*x)])
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{8b^{\frac{5}{2}}x^{2}\sqrt{x^{3}(bx+a)}-10a\,b^{\frac{3}{2}}x\sqrt{x^{3}(bx+a)}-15\,\operatorname{arctanh}\left(\frac{\sqrt{x^{3}(bx+a)}}{x^{2}\sqrt{b}}\right)a^{3}x+15a^{2}\sqrt{x^{3}(bx+a)}\,\sqrt{b}}{24b^{\frac{7}{2}}x}$	91
risch	$\frac{(8b^{2}x^{2}-10abx+15a^{2})x^{2}(bx+a)}{24b^{3}\sqrt{x^{3}(bx+a)}}-\frac{5a^{3}\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)x\sqrt{x(bx+a)}}{16b^{\frac{7}{2}}\sqrt{x^{3}(bx+a)}}$	98
default	$\frac{x\sqrt{x(bx+a)}\left(16x^{2}\sqrt{b\,x^{2}+ax}\,b^{\frac{7}{2}}-20\sqrt{b\,x^{2}+ax}\,b^{\frac{5}{2}}ax+30\sqrt{b\,x^{2}+ax}\,b^{\frac{3}{2}}a^{2}-15\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)a^{3}b\right)}{48\sqrt{b\,x^{4}+a\,x^{3}}\,b^{\frac{9}{2}}}$	120

```
[In] int(x^4/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(8*b^(5/2)*x^2*(x^3*(b*x+a))^(1/2)-10*a*b^(3/2)*x*(x^3*(b*x+a))^(1/2)-
15*arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))*a^3*x+15*a^2*(x^3*(b*x+a))^(1/2
)*b^(1/2))/b^(7/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.53

$$\int \frac{x^{4}}{\sqrt{ax^{3}+bx^{4}}}\,dx$$

$$= \left[\frac{15\,a^{3}\sqrt{b}x\log\left(\frac{2\,bx^{2}+ax-2\,\sqrt{bx^{4}+ax^{3}}\sqrt{b}}{x}\right)+2\,(8\,b^{3}x^{2}-10\,ab^{2}x+15\,a^{2}b)\sqrt{bx^{4}+ax^{3}}}{48\,b^{4}x},\ 15\,a^{3}\sqrt{-b}x\arctan\left(\right.\right.$$

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))
/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x), 1
/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) + (8*b^
3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x)]
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^4}{\sqrt{x^3\left(a+bx\right)}}\,dx$$

```
[In] integrate(x**4/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(x**4/sqrt(x**3*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^4}{\sqrt{bx^4+ax^3}}\,dx$$

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(b*x^4 + a*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \frac{1}{24}\sqrt{bx^2+ax}\left(2\,x\left(\frac{4\,x}{b\mathrm{sgn}\left(x\right)}-\frac{5\,a}{b^2\mathrm{sgn}\left(x\right)}\right)+\frac{15\,a^2}{b^3\mathrm{sgn}\left(x\right)}\right) - \frac{5\,a^3\log\left(|a|\right)\mathrm{sgn}(x)}{16\,b^{\frac{7}{2}}} + \frac{5\,a^3\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{16\,b^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] 1/24*sqrt(b*x^2 + a*x)*(2*x*(4*x/(b*sgn(x)) - 5*a/(b^2*sgn(x))) + 15*a^2/(b
^3*sgn(x))) - 5/16*a^3*log(abs(a))*sgn(x)/b^(7/2) + 5/16*a^3*log(abs(2*(sqr
t(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a))/(b^(7/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx=\int \frac{x^4}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

```
[In] int(x^4/(a*x^3 + b*x^4)^(1/2),x)
[Out] int(x^4/(a*x^3 + b*x^4)^(1/2), x)
```

3.311 $\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx=\frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}$$

```
[Out] 3/4*a^2*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(5/2)+1/2*(b*x^4+a*x^3)^
(1/2)/b-3/4*a*(b*x^4+a*x^3)^(1/2)/b^2/x
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2049, 2054, 212}

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx=\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\sqrt{ax^3+bx^4}}{2b}$$

```
[In] Int[x^3/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*Sqrt[a*x^3 + b*x^4])/(4*b^2*x) + (3*a^2*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/(4*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{ax^3+bx^4}}{2b} - \frac{(3a)\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx}{4b} \\
&= \frac{\sqrt{ax^3+bx^4}}{2b} - \frac{3a\sqrt{ax^3+bx^4}}{4b^2x} + \frac{(3a^2)\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{8b^2} \\
&= \frac{\sqrt{ax^3+bx^4}}{2b} - \frac{3a\sqrt{ax^3+bx^4}}{4b^2x} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^2} \\
&= \frac{\sqrt{ax^3+bx^4}}{2b} - \frac{3a\sqrt{ax^3+bx^4}}{4b^2x} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \frac{\sqrt{b}x^2(-3a^2-abx+2b^2x^2)+6a^2x^{3/2}\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{5/2}\sqrt{x^3(a+bx)}}$$

```
[In] Integrate[x^3/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(-3*a^2 - a*b*x + 2*b^2*x^2) + 6*a^2*x^(3/2)*Sqrt[a + b*x]*Arc
Tanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(4*b^(5/2)*Sqrt[x^3*(a
+ b*x)])
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{x^{3}\left(bx+a\right)}}{x^{2}\sqrt{b}}\right) a^{2} x+2 b^{\frac{3}{2}} x \sqrt{x^{3}\left(bx+a\right)}-3 a \sqrt{b}\, \sqrt{x^{3}\left(bx+a\right)}}{4 b^{\frac{5}{2}} x}$	69
risch	$-\frac{\left(-2 b x+3 a\right) x^{2} \left(bx+a\right)}{4 b^{2} \sqrt{x^{3}\left(bx+a\right)}}+\frac{3 a^{2} \ln\left(\frac{\frac{a}{2}+b x}{\sqrt{b}}+\sqrt{b\,x^{2}+a x}\right) x \sqrt{x\left(bx+a\right)}}{8 b^{\frac{5}{2}} \sqrt{x^{3}\left(bx+a\right)}}$	87
default	$\frac{x \sqrt{x\left(bx+a\right)}\, \left(4 x \sqrt{b\,x^{2}+a x}\, b^{\frac{5}{2}}-6 \sqrt{b\,x^{2}+a x}\, b^{\frac{3}{2}} a+3 \ln\left(\frac{2 \sqrt{b\,x^{2}+a x}\, \sqrt{b}+2 b x+a}{2 \sqrt{b}}\right) a^{2} b\right)}{8 \sqrt{b\,x^{4}+a\,x^{3}}\, b^{\frac{7}{2}}}$	98

```
[In] int(x^3/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^(5/2)*(3*arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))*a^2*x+2*b^(3/2)*x*(
x^3*(b*x+a))^(1/2)-3*a*b^(1/2)*(x^3*(b*x+a))^(1/2))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.74

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \left[\frac{3\,a^2\sqrt{b}x\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}(2\,b^2x-3\,ab)}{8\,b^3x},\right.$$
$$\left.-\frac{3\,a^2\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right)-\sqrt{bx^4+ax^3}(2\,b^2x-3\,ab)}{4\,b^3x}\right]$$

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x
) + 2*sqrt(b*x^4 + a*x^3)*(2*b^2*x - 3*a*b))/(b^3*x), -1/4*(3*a^2*sqrt(-b)*
x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) - sqrt(b*x^4 + a*x^3)*(2*b^2
*x - 3*a*b))/(b^3*x)]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^3}{\sqrt{x^3\left(a+bx\right)}}\,dx$$

```
[In] integrate(x**3/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(x**3/sqrt(x**3*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^3}{\sqrt{bx^4+ax^3}}\,dx$$

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(b*x^4 + a*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \frac{1}{4}\,\sqrt{bx^2+ax}\left(\frac{2\,x}{b\mathrm{sgn}\left(x\right)} - \frac{3\,a}{b^2\mathrm{sgn}\left(x\right)}\right) + \frac{3\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{8\,b^{\frac{5}{2}}} - \frac{3\,a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x/(b*sgn(x)) - 3*a/(b^2*sgn(x))) + 3/8*a^2*log(abs
(a))*sgn(x)/b^(5/2) - 3/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqr
t(b) + a))/(b^(5/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^3}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

```
[In] int(x^3/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x^3/(a*x^3 + b*x^4)^(1/2), x)
```

3.312 $\int \frac{x^2}{\sqrt{ax^3+bx^4}} dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx=\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

```
[Out] -a*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(3/2)+(b*x^4+a*x^3)^(1/2)/b/x
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2049, 2054, 212}

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx=\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

```
[In] Int[x^2/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/
b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
```

```
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{2b}\\ &= \frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{b}\\ &= \frac{\sqrt{ax^3+bx^4}}{bx} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \frac{\sqrt{b}x^2(a+bx) + 2ax^{3/2}\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}-\sqrt{a+bx}}\right)}{b^{3/2}\sqrt{x^3(a+bx)}}$$

```
[In] Integrate[x^2/Sqrt[a*x^3 + b*x^4],x]
[Out] (Sqrt[b]*x^2*(a + b*x) + 2*a*x^(3/2)*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x]
)/(Sqrt[a] - Sqrt[a + b*x])])/(b^(3/2)*Sqrt[x^3*(a + b*x)])
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\operatorname{arctanh}\left(\frac{\sqrt{x^{3}\left(bx+a\right)}}{x^{2}\sqrt{b}}\right)ax+\sqrt{b}\,\sqrt{x^{3}\left(bx+a\right)}}{b^{\frac{3}{2}}x}$	47
risch	$\frac{x^{2}\left(bx+a\right)}{b\sqrt{x^{3}\left(bx+a\right)}}-\frac{a\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)x\sqrt{x\left(bx+a\right)}}{2b^{\frac{3}{2}}\,\sqrt{x^{3}\left(bx+a\right)}}$	76
default	$\frac{x\sqrt{x\left(bx+a\right)}\,\left(2\sqrt{b\,x^{2}+ax}\,b^{\frac{3}{2}}-a\ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)b\right)}{2\sqrt{b\,x^{4}+a\,x^{3}}\,b^{\frac{5}{2}}}$	78

```
[In] int(x^2/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))*a*x+b^(1/2)*(x^3*(b*x+a))^(1/2))
/b^(3/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.18

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx$$
$$=\left[\frac{a\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}b}{2\,b^2x}, \frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right)+\sqrt{bx^4+ax^3}b}{b^2x}\right]$$

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x) +
2*sqrt(b*x^4 + a*x^3)*b)/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*
sqrt(-b)/(b*x^2)) + sqrt(b*x^4 + a*x^3)*b)/(b^2*x)]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx=\int \frac{x^2}{\sqrt{x^3\left(a+bx\right)}}\,dx$$

```
[In] integrate(x**2/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**3*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+ax^3}}\,dx$$

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(b*x^4 + a*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = -\frac{a\log\left(|a|\right)\operatorname{sgn}(x)}{2\,b^{\frac{3}{2}}} + \frac{a\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{2\,b^{\frac{3}{2}}\operatorname{sgn}\left(x\right)} + \frac{\sqrt{bx^2+ax}}{b\operatorname{sgn}\left(x\right)}$$

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] -1/2*a*log(abs(a))*sgn(x)/b^(3/2) + 1/2*a*log(abs(2*(sqrt(b)*x - sqrt(b*x^2
 + a*x))*sqrt(b) + a))/(b^(3/2)*sgn(x)) + sqrt(b*x^2 + a*x)/(b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x^2}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

```
[In] int(x^2/(a*x^3 + b*x^4)^(1/2),x)
[Out] int(x^2/(a*x^3 + b*x^4)^(1/2), x)
```

3.313 $\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 32

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2054, 212}

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

```
[In] Int[x/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = -\frac{2x^{3/2}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x^3(a+bx)}}$$

```
[In] Integrate[x/Sqrt[a*x^3 + b*x^4],x]
[Out] (-2*x^(3/2)*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x^3*(a + b*x)])
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{2\ \operatorname{arctanh}\left(\frac{\sqrt{x^{3}(bx+a)}}{x^{2}\sqrt{b}}\right)}{\sqrt{b}}$	25
default	$\frac{x\sqrt{x(bx+a)}\ \ln\left(\frac{2\sqrt{b\,x^{2}+ax}\,\sqrt{b}+2bx+a}{2\sqrt{b}}\right)}{\sqrt{b\,x^{4}+a\,x^{3}}\,\sqrt{b}}$	56

```
[In] int(x/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b^(1/2)*arctanh((x^3*(b*x+a))^(1/2)/x^2/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.31

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = \left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right)}{b}\right]$$

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x)/sqrt(b), -2*sqrt(-b
)*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2))/b]
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x}{\sqrt{x^3\,(a+bx)}}\,dx$$

```
[In] integrate(x/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(x/sqrt(x**3*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = \int \frac{x}{\sqrt{bx^4+ax^3}}\,dx$$

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(b*x^4 + a*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx = \frac{\log\left(|a|\right)\operatorname{sgn}(x)}{\sqrt{b}} - \frac{\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{\sqrt{b}\operatorname{sgn}\left(x\right)}$$

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] log(abs(a))*sgn(x)/sqrt(b) - log(abs(2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt
(b) + a))/(sqrt(b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx=\int \frac{x}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

```
[In] int(x/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x/(a*x^3 + b*x^4)^(1/2), x)
```

3.314 $\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

```
[Out] -2*(b*x^4+a*x^3)^(1/2)/a/x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2025}

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

```
[In] Int[1/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(a*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\text{integral}=-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{x^3(a+bx)}}{ax^2}$$

```
[In] Integrate[1/Sqrt[a*x^3 + b*x^4],x]
[Out] (-2*Sqrt[x^3*(a + b*x)])/(a*x^2)
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x^3(bx+a)}}{a\,x^2}$	20
trager	$-\frac{2\sqrt{b\,x^4+a\,x^3}}{a\,x^2}$	22
risch	$-\frac{2x(bx+a)}{\sqrt{x^3(bx+a)}\,a}$	23
gosper	$-\frac{2x(bx+a)}{a\sqrt{b\,x^4+a\,x^3}}$	25
default	$-\frac{2\sqrt{x(bx+a)}\,\sqrt{b\,x^2+ax}}{\sqrt{b\,x^4+a\,x^3}\,a}$	39

```
[In] int(1/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(x^3*(b*x+a))^(1/2)/a/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=-\frac{2\,\sqrt{bx^4+ax^3}}{ax^2}$$

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x^4 + a*x^3)/(a*x^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$$

```
[In] integrate(1/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(1/sqrt(a*x**3 + b*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+ax^3}}\,dx$$

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*x^4 + a*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=\frac{2}{\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\operatorname{sgn}(x)}$$

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] 2/((sqrt(b)*x - sqrt(b*x^2 + a*x))*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{b\,x^4+a\,x^3}}{a\,x^2}$$

```
[In] int(1/(a*x^3 + b*x^4)^(1/2),x)
[Out] -(2*(a*x^3 + b*x^4)^(1/2))/(a*x^2)
```

3.315 $\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}+\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}$$

```
[Out] -2/3*(b*x^4+a*x^3)^(1/2)/a/x^3+4/3*b*(b*x^4+a*x^3)^(1/2)/a^2/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2025}

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx=\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

```
[In] Int[1/(x*Sqrt[a*x^3 + b*x^4]),x]
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}-\frac{(2b)\int\frac{1}{\sqrt{ax^3+bx^4}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}+\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int\frac{1}{x\sqrt{ax^3+bx^4}}\,dx=-\frac{2(a-2bx)(a+bx)}{3a^2\sqrt{x^3(a+bx)}}$$

```
[In] Integrate[1/(x*Sqrt[a*x^3 + b*x^4]),x]
[Out] (-2*(a - 2*b*x)*(a + b*x))/(3*a^2*Sqrt[x^3*(a + b*x)])
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.50

method	result	size
pseudoelliptic	$-\frac{2(-2bx+a)\sqrt{x^3(bx+a)}}{3a^2x^3}$	26
trager	$-\frac{2(-2bx+a)\sqrt{b\,x^4+a\,x^3}}{3a^2x^3}$	28
risch	$-\frac{2(bx+a)(-2bx+a)}{3\sqrt{x^3(bx+a)}\,a^2}$	28
gosper	$-\frac{2(bx+a)(-2bx+a)}{3a^2\sqrt{b\,x^4+a\,x^3}}$	30
default	$-\frac{2\sqrt{x(bx+a)}\,\sqrt{b\,x^2+ax}\,(-2bx+a)}{3x\sqrt{b\,x^4+a\,x^3}\,a^2}$	48

```
[In] int(1/x/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(-2*b*x+a)/a^2/x^3*(x^3*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx=\frac{2\sqrt{bx^4+ax^3}(2\,bx-a)}{3\,a^2x^3}$$

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x^4 + a*x^3)*(2*b*x - a)/(a^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx=\int \frac{1}{x\sqrt{x^3\,(a+bx)}}\,dx$$

```
[In] integrate(1/x/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(1/(x*sqrt(x**3*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+ax^3}x}\,dx$$

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx=\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right)}{3\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3\operatorname{sgn}\left(x\right)}$$

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) + a)/((sqrt(b)*x - sqrt(b*x^
2 + a*x))^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx = -\frac{2\,a\,\sqrt{b\,x^4+a\,x^3}-4\,b\,x\,\sqrt{b\,x^4+a\,x^3}}{3\,a^2\,x^3}$$

```
[In] int(1/(x*(a*x^3 + b*x^4)^(1/2)),x)
[Out] -(2*a*(a*x^3 + b*x^4)^(1/2) - 4*b*x*(a*x^3 + b*x^4)^(1/2))/(3*a^2*x^3)
```

3.316 $\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{5ax^4} + \frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3} - \frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}$$

```
[Out] -2/5*(b*x^4+a*x^3)^(1/2)/a/x^4+8/15*b*(b*x^4+a*x^3)^(1/2)/a^2/x^3-16/15*b^2
*(b*x^4+a*x^3)^(1/2)/a^3/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2025}

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = -\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2} + \frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3} - \frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

```
[In] Int[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(5*a*x^4) + (8*b*Sqrt[a*x^3 + b*x^4])/(15*a^2*x^3)
 - (16*b^2*Sqrt[a*x^3 + b*x^4])/(15*a^3*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}-\frac{(4b)\int\frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{5a}\\&=-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}+\frac{(8b^2)\int\frac{1}{\sqrt{ax^3+bx^4}}\,dx}{15a^2}\\&=-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int\frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{x^3(a+bx)}(3a^2-4abx+8b^2x^2)}{15a^3x^4}$$

```
[In] Integrate[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^4)
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$-\frac{2\left(8b^2x^2-4abx+3a^2\right)\sqrt{x^3(bx+a)}}{15a^3x^4}$	39
trager	$-\frac{2\left(8b^2x^2-4abx+3a^2\right)\sqrt{b\,x^4+a\,x^3}}{15a^3x^4}$	41
risch	$-\frac{2(bx+a)\left(8b^2x^2-4abx+3a^2\right)}{15x\sqrt{x^3(bx+a)}\,a^3}$	44
gosper	$-\frac{2(bx+a)\left(8b^2x^2-4abx+3a^2\right)}{15x\,a^3\sqrt{b\,x^4+a\,x^3}}$	46
default	$-\frac{2\sqrt{x(bx+a)}\,\sqrt{b\,x^2+ax}\,\left(8b^2x^2-4abx+3a^2\right)}{15x^2\sqrt{b\,x^4+a\,x^3}\,a^3}$	61

```
[In] int(1/x^2/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(8*b^2*x^2-4*a*b*x+3*a^2)/a^3/x^4*(x^3*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{bx^4+ax^3}(8\,b^2x^2-4\,abx+3\,a^2)}{15\,a^3x^4}$$

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] -2/15*sqrt(b*x^4 + a*x^3)*(8*b^2*x^2 - 4*a*b*x + 3*a^2)/(a^3*x^4)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx=\int \frac{1}{x^2\sqrt{x^3\,(a+bx)}}\,dx$$

```
[In] integrate(1/x**2/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(x**3*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+ax^3}x^2}\,dx$$

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx=\frac{2\left(20\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2b+15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a\sqrt{b}+3\,a^2\right)}{15\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^5\operatorname{sgn}(x)}$$

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] 2/15*(20*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*b + 15*(sqrt(b)*x - sqrt(b*x^2 +
 a*x))*a*sqrt(b) + 3*a^2)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^5*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx = -\frac{2\,\sqrt{b\,x^4+a\,x^3}\,\left(3\,a^2-4\,a\,b\,x+8\,b^2\,x^2\right)}{15\,a^3\,x^4}$$

```
[In] int(1/(x^2*(a*x^3 + b*x^4)^(1/2)),x)
[Out] -(2*(a*x^3 + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^2 - 4*a*b*x))/(15*a^3*x^4)
```

3.317 $\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}$$

```
[Out] -2/7*(b*x^4+a*x^3)^(1/2)/a/x^5+12/35*b*(b*x^4+a*x^3)^(1/2)/a^2/x^4-16/35*b^
2*(b*x^4+a*x^3)^(1/2)/a^3/x^3+32/35*b^3*(b*x^4+a*x^3)^(1/2)/a^4/x^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2025}

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

```
[In] Int[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(7*a*x^5) + (12*b*Sqrt[a*x^3 + b*x^4])/(35*a^2*x^4
) - (16*b^2*Sqrt[a*x^3 + b*x^4])/(35*a^3*x^3) + (32*b^3*Sqrt[a*x^3 + b*x^4]
)/(35*a^4*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}-\frac{(6b)\int\frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx}{7a}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}+\frac{(24b^2)\int\frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{35a^2}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}-\frac{(16b^3)\int\frac{1}{\sqrt{ax^3+bx^4}}\,dx}{35a^3}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.49

$$\int\frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx=\frac{2\sqrt{x^3(a+bx)}(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3)}{35a^4x^5}$$

```
[In] Integrate[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
[Out] (2*Sqrt[x^3*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35
*a^4*x^5)
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.46

method	result	size
pseudoelliptic	$-\frac{2\left(-16 b^{3} x^{3}+8 a \,b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right) \sqrt{x^{3} \left(b x +a \right)}}{35 a^{4} x^{5}}$	50
trager	$-\frac{2\left(-16 b^{3} x^{3}+8 a \,b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right) \sqrt{b \,x^{4}+a \,x^{3}}}{35 a^{4} x^{5}}$	52
risch	$-\frac{2 \left(b x +a \right) \left(-16 b^{3} x^{3}+8 a \,b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right)}{35 x^{2} \sqrt{x^{3} \left(b x +a \right)}\, a^{4}}$	55
gosper	$-\frac{2 \left(b x +a \right) \left(-16 b^{3} x^{3}+8 a \,b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right)}{35 x^{2} a^{4} \sqrt{b \,x^{4}+a \,x^{3}}}$	57
default	$-\frac{2 \sqrt{x \left(b x +a \right)}\, \sqrt{b \,x^{2}+a x}\, \left(-16 b^{3} x^{3}+8 a \,b^{2} x^{2}-6 a^{2} b x +5 a^{3}\right)}{35 x^{3} \sqrt{b \,x^{4}+a \,x^{3}}\, a^{4}}$	72

```
[In] int(1/x^3/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/a^4/x^5*(x^3*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \frac{2\,(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3)\sqrt{bx^4+ax^3}}{35\,a^4x^5}$$

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^4 + a*x^3)/(a^
4*x^5)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{x^3\sqrt{x^3\,(a+bx)}}\,dx$$

```
[In] integrate(1/x**3/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**3*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+ax^3}x^3}\,dx$$

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx$$
$$=\frac{2\left(70\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3b^{\frac{3}{2}}+84\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2ab+35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)a^2\sqrt{b}+5\,a^3\right)}{35\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^7\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(70*(sqrt(b)*x - sqrt(b*x^2 + a*x))^3*b^(3/2) + 84*(sqrt(b)*x - sqrt(b
*x^2 + a*x))^2*a*b + 35*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^2*sqrt(b) + 5*a^3
)/((sqrt(b)*x - sqrt(b*x^2 + a*x))^7*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx=\frac{12\,b\,\sqrt{b\,x^4+a\,x^3}}{35\,a^2\,x^4}-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{7\,a\,x^5}-\frac{16\,b^2\,\sqrt{b\,x^4+a\,x^3}}{35\,a^3\,x^3}+\frac{32\,b^3\,\sqrt{b\,x^4+a\,x^3}}{35\,a^4\,x^2}$$

```
[In] int(1/(x^3*(a*x^3 + b*x^4)^(1/2)),x)
```

```
[Out] (12*b*(a*x^3 + b*x^4)^(1/2))/(35*a^2*x^4) - (2*(a*x^3 + b*x^4)^(1/2))/(7*a*
x^5) - (16*b^2*(a*x^3 + b*x^4)^(1/2))/(35*a^3*x^3) + (32*b^3*(a*x^3 + b*x^4
)^(1/2))/(35*a^4*x^2)
```

3.318 $\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}$$

```
[Out] -2/9*(b*x^4+a*x^3)^(1/2)/a/x^6+16/63*b*(b*x^4+a*x^3)^(1/2)/a^2/x^5-32/105*b
^2*(b*x^4+a*x^3)^(1/2)/a^3/x^4+128/315*b^3*(b*x^4+a*x^3)^(1/2)/a^4/x^3-256/
315*b^4*(b*x^4+a*x^3)^(1/2)/a^5/x^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2041, 2025}

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx=-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

```
[In] Int[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(9*a*x^6) + (16*b*Sqrt[a*x^3 + b*x^4])/(63*a^2*x^5
) - (32*b^2*Sqrt[a*x^3 + b*x^4])/(105*a^3*x^4) + (128*b^3*Sqrt[a*x^3 + b*x^
4])/(315*a^4*x^3) - (256*b^4*Sqrt[a*x^3 + b*x^4])/(315*a^5*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6} - \frac{(8b)\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx}{9a} \\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5} + \frac{(16b^2)\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx}{21a^2} \\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5} - \frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4} - \frac{(64b^3)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{105a^3} \\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5} - \frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4} \\
&\quad + \frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3} + \frac{(128b^4)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{315a^4} \\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6} + \frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5} - \frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4} + \frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3} - \frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = -\frac{2\sqrt{x^3(a+bx)}(35a^4-40a^3bx+48a^2b^2x^2-64ab^3x^3+128b^4x^4)}{315a^5x^6}$$

```
[In] Integrate[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)]*(35*a^4 - 40*a^3*b*x + 48*a^2*b^2*x^2 - 64*a*b^3*x^
3 + 128*b^4*x^4))/(315*a^5*x^6)
```

Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

method	result	size
pseudoelliptic	$-\frac{2\left(128 b^{4} x^{4}-64 a\, b^{3} x^{3}+48 a^{2} b^{2} x^{2}-40 a^{3} b x +35 a^{4}\right) \sqrt{x^{3} \left(b x +a \right)}}{315 a^{5} x^{6}}$	61
trager	$-\frac{2\left(128 b^{4} x^{4}-64 a\, b^{3} x^{3}+48 a^{2} b^{2} x^{2}-40 a^{3} b x +35 a^{4}\right) \sqrt{b\, x^{4}+a\, x^{3}}}{315 a^{5} x^{6}}$	63
risch	$-\frac{2 \left(b x +a \right) \left(128 b^{4} x^{4}-64 a\, b^{3} x^{3}+48 a^{2} b^{2} x^{2}-40 a^{3} b x +35 a^{4}\right)}{315 x^{3} \sqrt{x^{3} \left(b x +a \right)}\, a^{5}}$	66
gosper	$-\frac{2 \left(b x +a \right) \left(128 b^{4} x^{4}-64 a\, b^{3} x^{3}+48 a^{2} b^{2} x^{2}-40 a^{3} b x +35 a^{4}\right)}{315 x^{3} a^{5} \sqrt{b\, x^{4}+a\, x^{3}}}$	68
default	$-\frac{2 \sqrt{x \left(b x +a \right)}\, \sqrt{b\, x^{2}+a x}\, \left(128 b^{4} x^{4}-64 a\, b^{3} x^{3}+48 a^{2} b^{2} x^{2}-40 a^{3} b x +35 a^{4}\right)}{315 x^{4} \sqrt{b\, x^{4}+a\, x^{3}}\, a^{5}}$	83

```
[In] int(1/x^4/(b*x^4+a*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*(128*b^4*x^4-64*a*b^3*x^3+48*a^2*b^2*x^2-40*a^3*b*x+35*a^4)/a^5/x^6*
(x^3*(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = -\frac{2\,(128\,b^4x^4-64\,ab^3x^3+48\,a^2b^2x^2-40\,a^3bx+35\,a^4)\sqrt{bx^4+ax^3}}{315\,a^5x^6}$$

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(128*b^4*x^4 - 64*a*b^3*x^3 + 48*a^2*b^2*x^2 - 40*a^3*b*x + 35*a^4)*
sqrt(b*x^4 + a*x^3)/(a^5*x^6)
```

Sympy [F]

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{x^4\sqrt{x^3\,(a+bx)}}\,dx$$

```
[In] integrate(1/x**4/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x**3*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax^3}x^4}\,dx$$

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^4), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = \frac{2\left(1008\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^4b^2+1680\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^3ab^{\frac{3}{2}}+1080\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^2a^2b+315}{315\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)^9\operatorname{sgn}(x)}$$

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(1008*(sqrt(b)*x - sqrt(b*x^2 + a*x))^4*b^2 + 1680*(sqrt(b)*x - sqrt(
b*x^2 + a*x))^3*a*b^(3/2) + 1080*(sqrt(b)*x - sqrt(b*x^2 + a*x))^2*a^2*b +
315*(sqrt(b)*x - sqrt(b*x^2 + a*x))*a^3*sqrt(b) + 35*a^4)/((sqrt(b)*x - sqr
t(b*x^2 + a*x))^9*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx = \frac{16\,b\,\sqrt{b\,x^4+a\,x^3}}{63\,a^2\,x^5}-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{9\,a\,x^6}-\frac{32\,b^2\,\sqrt{b\,x^4+a\,x^3}}{105\,a^3\,x^4}+\frac{128\,b^3\,\sqrt{b\,x^4+a\,x^3}}{315\,a^4\,x^3}-\frac{256\,b^4\,\sqrt{b\,x^4+a\,x^3}}{315\,a^5\,x^2}$$

```
[In] int(1/(x^4*(a*x^3 + b*x^4)^(1/2)),x)
```

```
[Out] (16*b*(a*x^3 + b*x^4)^(1/2))/(63*a^2*x^5) - (2*(a*x^3 + b*x^4)^(1/2))/(9*a*
x^6) - (32*b^2*(a*x^3 + b*x^4)^(1/2))/(105*a^3*x^4) + (128*b^3*(a*x^3 + b*x
^4)^(1/2))/(315*a^4*x^3) - (256*b^4*(a*x^3 + b*x^4)^(1/2))/(315*a^5*x^2)
```

3.319 $\int \frac{1}{x^3+bx^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{1}{x^3+bx^5}\,dx=-\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)$$

```
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1607, 272, 46}

$$\int \frac{1}{x^3+bx^5}\,dx=\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

```
[In] Int[(x^3 + b*x^5)^(-1),x]
```

```
[Out] -1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x^3(1+bx^2)}\,dx \\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+bx)}\,dx, x, x^2\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right)dx, x, x^2\right) \\ &= -\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3+bx^5}\,dx = -\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)$$

```
[In] Integrate[(x^3 + b*x^5)^(-1),x]
[Out] -1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{2x^2}-b\ln(x)+\frac{b\ln(bx^2+1)}{2}$	23
norman	$-\frac{1}{2x^2}-b\ln(x)+\frac{b\ln(bx^2+1)}{2}$	23
risch	$-\frac{1}{2x^2}-b\ln(x)+\frac{b\ln(-bx^2-1)}{2}$	24
meijerg	$\frac{b\left(-\frac{1}{x^2b}-2\ln(x)-\ln(b)+\ln(bx^2+1)\right)}{2}$	29
parallelrisch	$-\frac{2b\ln(x)x^2-b\ln(bx^2+1)x^2+1}{2x^2}$	30

```
[In] int(1/(b*x^5+x^3),x,method=_RETURNVERBOSE)
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3+bx^5}\,dx=\frac{bx^2\log\left(bx^2+1\right)-2\,bx^2\log\left(x\right)-1}{2\,x^2}$$

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="fricas")
[Out] 1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3+bx^5}\,dx=-b\log\left(x\right)+\frac{b\log\left(x^2+\frac{1}{b}\right)}{2}-\frac{1}{2x^2}$$

```
[In] integrate(1/(b*x**5+x**3),x)
[Out] -b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3+bx^5}\,dx=\frac{1}{2}\,b\log\left(bx^2+1\right)-b\log\left(x\right)-\frac{1}{2\,x^2}$$

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="maxima")
[Out] 1/2*b*log(b*x^2 + 1) - b*log(x) - 1/2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3+bx^5}\,dx = -\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2+1\right|\right)+\frac{bx^2-1}{2\,x^2}$$

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="giac")
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3+bx^5}\,dx = \frac{b\,\ln\left(b\,x^2+1\right)}{2}-b\,\ln\left(x\right)-\frac{1}{2\,x^2}$$

```
[In] int(1/(b*x^5 + x^3),x)
[Out] (b*log(b*x^2 + 1))/2 - b*log(x) - 1/(2*x^2)
```

3.320 $\int \frac{1}{-x^3+bx^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{1}{-x^3+bx^5}\,dx = \frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1-bx^2\right)$$

```
[Out] 1/2/x^2-b*ln(x)+1/2*b*ln(-b*x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1607, 272, 46}

$$\int \frac{1}{-x^3+bx^5}\,dx = \frac{1}{2}b\log\left(1-bx^2\right) - b\log(x) + \frac{1}{2x^2}$$

```
[In] Int[(-x^3 + b*x^5)^(-1),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x^3\left(-1+bx^2\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-1+bx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{-1+bx}\right)dx, x, x^2\right) \\
&= \frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{-x^3+bx^5}\,dx = \frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)$$

```
[In] Integrate[(-x^3 + b*x^5)^(-1),x]
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{1}{2x^2}-b\ln\left(x\right)+\frac{b\ln\left(b\,x^2-1\right)}{2}$	23
norman	$\frac{1}{2x^2}-b\ln\left(x\right)+\frac{b\ln\left(b\,x^2-1\right)}{2}$	23
risch	$\frac{1}{2x^2}-b\ln\left(x\right)+\frac{b\ln\left(-b\,x^2+1\right)}{2}$	24
parallelrisch	$-\frac{2b\ln(x)x^2-b\ln\left(b\,x^2-1\right)x^2-1}{2x^2}$	30
meijerg	$\frac{b\left(\frac{1}{x^2b}-2\ln(x)-\ln(-b)+\ln\left(-b\,x^2+1\right)\right)}{2}$	31

```
[In] int(1/(b*x^5-x^3),x,method=_RETURNVERBOSE)
[Out] 1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{1}{-x^3+bx^5}\,dx=\frac{bx^2\log\left(bx^2-1\right)-2\,bx^2\log\left(x\right)+1}{2\,x^2}$$

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="fricas")
[Out] 1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{-x^3+bx^5}\,dx=-b\log\left(x\right)+\frac{b\log\left(x^2-\frac{1}{b}\right)}{2}+\frac{1}{2x^2}$$

```
[In] integrate(1/(b*x**5-x**3),x)
[Out] -b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{-x^3+bx^5}\,dx=\frac{1}{2}\,b\log\left(bx^2-1\right)-b\log\left(x\right)+\frac{1}{2\,x^2}$$

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="maxima")
[Out] 1/2*b*log(b*x^2 - 1) - b*log(x) + 1/2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1}{-x^3+bx^5}\,dx = -\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2-1\right|\right)+\frac{bx^2+1}{2\,x^2}$$

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="giac")
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{-x^3+bx^5}\,dx = \frac{b\,\ln\left(b\,x^2-1\right)}{2}-b\,\ln\left(x\right)+\frac{1}{2\,x^2}$$

```
[In] int(1/(b*x^5 - x^3),x)
[Out] (b*log(b*x^2 - 1))/2 - b*log(x) + 1/(2*x^2)
```

3.321 $\int \frac{1}{ax+bx}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{1}{ax+bx}\,dx=\frac{\log(x)}{a+b}$$

```
[Out] ln(x)/(a+b)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 29}

$$\int \frac{1}{ax+bx}\,dx=\frac{\log(x)}{a+b}$$

```
[In] Int[(a*x + b*x)^(-1),x]
```

```
[Out] Log[x]/(a + b)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(a+b)x}\,dx\\ &= \frac{\int \frac{1}{x}\,dx}{a+b}\\ &= \frac{\log(x)}{a+b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{ax+bx}\,dx = \frac{\log(ax+bx)}{a+b}$$

```
[In] Integrate[(a*x + b*x)^(-1),x]
[Out] Log[a*x + b*x]/(a + b)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{\ln(x)}{a+b}$	9
norman	$\frac{\ln(x)}{a+b}$	9
risch	$\frac{\ln(x)}{a+b}$	9
parallelrisch	$\frac{\ln(x)}{a+b}$	9

```
[In] int(1/(a*x+b*x),x,method=_RETURNVERBOSE)
[Out] ln(x)/(a+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx}\,dx=\frac{\log\left(x\right)}{a+b}$$

```
[In] integrate(1/(a*x+b*x),x, algorithm="fricas")
```

```
[Out] log(x)/(a + b)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{ax+bx}\,dx=\frac{\log\left(x\right)}{a+b}$$

```
[In] integrate(1/(a*x+b*x),x)
```

```
[Out] log(x)/(a + b)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{ax+bx}\,dx=\frac{\log\left(ax+bx\right)}{a+b}$$

```
[In] integrate(1/(a*x+b*x),x, algorithm="maxima")
```

```
[Out] log(a*x + b*x)/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1}{ax+bx}\,dx=\frac{\log\left(\left|ax+bx\right|\right)}{a+b}$$

```
[In] integrate(1/(a*x+b*x),x, algorithm="giac")
```

```
[Out] log(abs(a*x + b*x))/(a + b)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx}\,dx=\frac{\ln\left(x\right)}{a+b}$$

```
[In] int(1/(a*x + b*x),x)
[Out] log(x)/(a + b)
```

3.322 $\int \frac{1}{(ax+bx)^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{1}{(ax+bx)^2} \, dx = -\frac{1}{(a+b)^2 x}$$

```
[Out] -1/(a+b)^2/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 30}

$$\int \frac{1}{(ax+bx)^2} \, dx = -\frac{1}{x(a+b)^2}$$

```
[In] Int[(a*x + b*x)^(-2),x]
```

```
[Out] -(1/((a + b)^2*x))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(a+b)^2x^2}\,dx\\ &= \frac{\int \frac{1}{x^2}\,dx}{(a+b)^2}\\ &= -\frac{1}{(a+b)^2x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(a+b)^2x}$$

```
[In] Integrate[(a*x + b*x)^(-2),x]
[Out] -(1/((a + b)^2*x))
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gosper	$-\frac{1}{(a+b)^2x}$	11
default	$-\frac{1}{(a+b)^2x}$	11
norman	$-\frac{1}{(a+b)^2x}$	11
risch	$-\frac{1}{(a+b)^2x}$	11
parallelrisch	$-\frac{1}{(a+b)^2x}$	11

```
[In] int(1/(a*x+b*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/(a+b)^2/x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(a^2+2\,ab+b^2)x}$$

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="fricas")
[Out] -1/((a^2 + 2*a*b + b^2)*x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{x\,(a^2+2ab+b^2)}$$

```
[In] integrate(1/(a*x+b*x)**2,x)
[Out] -1/(x*(a**2 + 2*a*b + b**2))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(ax+bx)(a+b)}$$

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="maxima")
[Out] -1/((a*x + b*x)*(a + b))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{(ax+bx)(a+b)}$$

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="giac")
[Out] -1/((a*x + b*x)*(a + b))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax+bx)^2}\,dx = -\frac{1}{x\,(a+b)^2}$$

```
[In] int(1/(a*x + b*x)^2,x)
[Out] -1/(x*(a + b)^2)
```

3.323 $\int \frac{1}{(ax+bx)^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2(a+b)^3x^2}$$

```
[Out] -1/2/(a+b)^3/x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 30}

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2x^2(a+b)^3}$$

```
[In] Int[(a*x + b*x)^(-3),x]
```

```
[Out] -1/2*1/((a + b)^3*x^2)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{(a+b)^3x^3}\,dx \\
&= \frac{\int \frac{1}{x^3}\,dx}{(a+b)^3} \\
&= -\frac{1}{2(a+b)^3x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2(a+b)^3x^2}$$

```
[In] Integrate[(a*x + b*x)^(-3),x]
[Out] -1/2*1/((a + b)^3*x^2)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{1}{2(a+b)^3x^2}$	11
default	$-\frac{1}{2(a+b)^3x^2}$	11
norman	$-\frac{1}{2(a+b)^3x^2}$	11
risch	$-\frac{1}{2(a+b)^3x^2}$	11
parallelrisch	$-\frac{1}{2(a+b)^3x^2}$	11

```
[In] int(1/(a*x+b*x)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/(a+b)^3/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)x^2}$$

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="fricas")
[Out] -1/2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2x^2\,(a^3+3a^2b+3ab^2+b^3)}$$

```
[In] integrate(1/(a*x+b*x)**3,x)
[Out] -1/(2*x**2*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2\,(ax+bx)^2(a+b)}$$

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="maxima")
[Out] -1/2/((a*x + b*x)^2*(a + b))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2\,(ax+bx)^2(a+b)}$$

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="giac")
```

```
[Out] -1/2/((a*x + b*x)^2*(a + b))
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{(ax+bx)^3}\,dx = -\frac{1}{2\,x^2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}$$

```
[In] int(1/(a*x + b*x)^3,x)
```

```
[Out] -1/(2*x^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))
```

3.324 $\int \frac{1}{ax^2+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{1}{ax^2+bx^2}\,dx = -\frac{1}{(a+b)x}$$

```
[Out] -1/(a+b)/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\int \frac{1}{ax^2+bx^2}\,dx = -\frac{1}{x(a+b)}$$

```
[In] Int[(a*x^2 + b*x^2)^(-1),x]
```

```
[Out] -(1/((a + b)*x))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(a+b)x^2}\,dx\\ &= \frac{\int \frac{1}{x^2}\,dx}{a+b}\\ &= -\frac{1}{(a+b)x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^2}\,dx = -\frac{1}{(a+b)x}$$

```
[In] Integrate[(a*x^2 + b*x^2)^(-1),x]
[Out] -(1/((a + b)*x))
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gosper	$-\frac{1}{(a+b)x}$	11
default	$-\frac{1}{(a+b)x}$	11
norman	$-\frac{1}{(a+b)x}$	11
risch	$-\frac{1}{(a+b)x}$	11
parallelrisch	$-\frac{1}{(a+b)x}$	11

```
[In] int(1/(a*x^2+b*x^2),x,method=_RETURNVERBOSE)
[Out] -1/(a+b)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^2}\,dx=-\frac{1}{(a+b)x}$$

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="fricas")
[Out] -1/((a + b)*x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{ax^2+bx^2}\,dx=-\frac{1}{x\,(a+b)}$$

```
[In] integrate(1/(a*x**2+b*x**2),x)
[Out] -1/(x*(a + b))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^2}\,dx=-\frac{1}{(a+b)x}$$

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="maxima")
[Out] -1/((a + b)*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^2}\,dx=-\frac{1}{(a+b)x}$$

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="giac")
[Out] -1/((a + b)*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^2}\,dx=-\frac{1}{x\,(a+b)}$$

```
[In] int(1/(a*x^2 + b*x^2),x)
[Out] -1/(x*(a + b))
```

3.325 $\int \frac{1}{ax^n+bx^n}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{1}{ax^n+bx^n}\,dx=\frac{x^{1-n}}{(a+b)(1-n)}$$

```
[Out] x^(1-n)/(a+b)/(1-n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\int \frac{1}{ax^n+bx^n}\,dx=\frac{x^{1-n}}{(1-n)(a+b)}$$

```
[In] Int[(a*x^n + b*x^n)^(-1),x]
```

```
[Out] x^(1 - n)/((a + b)*(1 - n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-n}}{a+b}\,dx\\ &= \frac{\int x^{-n}\,dx}{a+b}\\ &= \frac{x^{1-n}}{(a+b)(1-n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^n+bx^n}\,dx = \frac{x^{1-n}}{(a+b)(1-n)}$$

```
[In] Integrate[(a*x^n + b*x^n)^(-1),x]
[Out] x^(1 - n)/((a + b)*(1 - n))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{x\,x^{-n}}{(-1+n)(a+b)}$	19
risch	$-\frac{x\,x^{-n}}{(-1+n)(a+b)}$	19
parallelrisch	$-\frac{x\,x^{-n}}{(-1+n)(a+b)}$	19
norman	$-\frac{x\,\mathrm{e}^{-n\ln(x)}}{an+bn-a-b}$	26

```
[In] int(1/(a*x^n+b*x^n),x,method=_RETURNVERBOSE)
[Out] -x/(-1+n)/(x^n)/(a+b)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{ax^n + bx^n}\,dx = -\frac{x}{((a+b)n - a - b)x^n}$$

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="fricas")
```

```
[Out] -x/(((a + b)*n - a - b)*x^n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(10) = 20$.

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{1}{ax^n + bx^n}\,dx = \begin{cases} -\frac{x}{anx^n - ax^n + bnx^n - bx^n} & \text{for } n \neq 1 \\ \frac{\log(x)}{a+b} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x**n+b*x**n),x)
```

```
[Out] Piecewise((-x/(a*n*x**n - a*x**n + b*n*x**n - b*x**n), Ne(n, 1)), (log(x)/(
a + b), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{ax^n + bx^n}\,dx = -\frac{x}{(a(n-1) + b(n-1))x^n}$$

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="maxima")
```

```
[Out] -x/((a*(n - 1) + b*(n - 1))*x^n)
```

Giac [F]

$$\int \frac{1}{ax^n + bx^n}\,dx = \int \frac{1}{ax^n + bx^n}\,dx$$

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x^n + b*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{ax^n + bx^n}\,dx = -\frac{x^{1-n}}{(a+b)\,(n-1)}$$

```
[In] int(1/(a*x^n + b*x^n),x)
```

```
[Out] -x^(1 - n)/((a + b)*(n - 1))
```

3.326 $\int \frac{1}{(ax^n+bx^n)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx=\frac{x^{1-2n}}{(a+b)^2(1-2n)}$$

```
[Out] x^(1-2*n)/(a+b)^2/(1-2*n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx=\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

```
[In] Int[(a*x^n + b*x^n)^(-2),x]
```

```
[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-2n}}{(a+b)^2}\,dx\\ &= \frac{\int x^{-2n}\,dx}{(a+b)^2}\\ &= \frac{x^{1-2n}}{(a+b)^2(1-2n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx = \frac{x^{1-2n}}{(a+b)^2(1-2n)}$$

```
[In] Integrate[(a*x^n + b*x^n)^(-2),x]
[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$-\frac{x\,x^{-2n}}{(-1+2n)(a+b)^2}$	21
parallelrisch	$-\frac{x\,x^{-2n}}{(-1+2n)(a+b)^2}$	21
risch	$-\frac{x\,x^{-2n}}{(a^2+2ab+b^2)(-1+2n)}$	29
norman	$-\frac{x\,\mathrm{e}^{-2n\ln(x)}}{(2an+2bn-a-b)(a+b)}$	33

```
[In] int(1/(a*x^n+b*x^n)^2,x,method=_RETURNVERBOSE)
[Out] -x/(-1+2*n)/(x^n)^2/(a+b)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx=\frac{x}{(a^2+2\,ab+b^2-2\,(a^2+2\,ab+b^2)n)x^{2\,n}}$$

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] x/((a^2 + 2*a*b + b^2 - 2*(a^2 + 2*a*b + b^2)*n)*x^(2*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(15) = 30.

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.10

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx=\begin{cases}-\frac{x}{2a^2nx^{2n}-a^2x^{2n}+4abnx^{2n}-2abx^{2n}+2b^2nx^{2n}-b^2x^{2n}} & \text{for } n\neq\frac{1}{2}\\ \frac{\log{(x)}}{a^2+2ab+b^2} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x**n+b*x**n)**2,x)
```

```
[Out] Piecewise((-x/(2*a**2*n*x**(2*n) - a**2*x**(2*n) + 4*a*b*n*x**(2*n) - 2*a*b
*x**(2*n) + 2*b**2*n*x**(2*n) - b**2*x**(2*n)), Ne(n, 1/2)), (log(x)/(a**2
+ 2*a*b + b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx=-\frac{x}{(a^2(2\,n-1)+2\,ab(2\,n-1)+b^2(2\,n-1))x^{2\,n}}$$

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -x/((a^2*(2*n - 1) + 2*a*b*(2*n - 1) + b^2*(2*n - 1))*x^(2*n))
```

Giac [F]

$$\int \frac{1}{\left(ax^{n}+bx^{n}\right)^{2}}\, dx = \int \frac{1}{\left(ax^{n}+bx^{n}\right)^{2}}\, dx$$

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="giac")
[Out] integrate((a*x^n + b*x^n)^(-2), x)
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(ax^{n}+bx^{n}\right)^{2}}\, dx = -\frac{x^{1-2\,n}}{{\left(a+b\right)}^2\,\left(2\,n-1\right)}$$

```
[In] int(1/(a*x^n + b*x^n)^2,x)
[Out] -x^(1 - 2*n)/((a + b)^2*(2*n - 1))
```

3.327 $\int \frac{1}{(ax^n+bx^n)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx = \frac{x^{1-3n}}{(a+b)^3(1-3n)}$$

```
[Out] x^(1-3*n)/(a+b)^3/(1-3*n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx = \frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

```
[In] Int[(a*x^n + b*x^n)^(-3),x]
```

```
[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-3n}}{(a+b)^3}\,dx\\ &= \frac{\int x^{-3n}\,dx}{(a+b)^3}\\ &= \frac{x^{1-3n}}{(a+b)^3(1-3n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx = \frac{x^{1-3n}}{(a+b)^3(1-3n)}$$

```
[In] Integrate[(a*x^n + b*x^n)^(-3),x]
[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$-\frac{x\,x^{-3n}}{(-1+3n)(a+b)^3}$	21
parallelrisch	$-\frac{x\,x^{-3n}}{(-1+3n)(a+b)^3}$	21
norman	$-\frac{x\,\mathrm{e}^{-3n\ln(x)}}{(3an+3bn-a-b)(a+b)^2}$	33
risch	$-\frac{x\,x^{-3n}}{(a^3+3a^2b+3a\,b^2+b^3)(-1+3n)}$	37

```
[In] int(1/(a*x^n+b*x^n)^3,x,method=_RETURNVERBOSE)
[Out] -x/(-1+3*n)/(x^n)^3/(a+b)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(21) = 42.

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx = \frac{x}{(a^3+3\,a^2b+3\,ab^2+b^3-3\,(a^3+3\,a^2b+3\,ab^2+b^3)n)x^{3\,n}}$$

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] x/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*n)*x^
(3*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(15) = 30.

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 5.95

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx = \begin{cases} -\frac{x}{3a^3nx^{3n}-a^3x^{3n}+9a^2bnx^{3n}-3a^2bx^{3n}+9ab^2nx^{3n}-3ab^2x^{3n}+3b^3nx^{3n}-b^3x^{3n}} & \text{for } n\neq\frac{1}{3} \\ \frac{\log{(x)}}{a^3+3a^2b+3ab^2+b^3} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x**n+b*x**n)**3,x)
```

```
[Out] Piecewise((-x/(3*a**3*n*x**(3*n) - a**3*x**(3*n) + 9*a**2*b*n*x**(3*n) - 3*
a**2*b*x**(3*n) + 9*a*b**2*n*x**(3*n) - 3*a*b**2*x**(3*n) + 3*b**3*n*x**(3*
n) - b**3*x**(3*n)), Ne(n, 1/3)), (log(x)/(a**3 + 3*a**2*b + 3*a*b**2 + b**
3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(21) = 42.

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx = -\frac{x}{(a^3(3\,n-1)+3\,a^2b(3\,n-1)+3\,ab^2(3\,n-1)+b^3(3\,n-1))x^{3\,n}}$$

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -x/((a^3*(3*n - 1) + 3*a^2*b*(3*n - 1) + 3*a*b^2*(3*n - 1) + b^3*(3*n - 1))
*x^(3*n))
```

Giac [F]

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx=\int \frac{1}{(ax^n+bx^n)^3}\,dx$$

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x^n + b*x^n)^(-3), x)
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{(ax^n+bx^n)^3}\,dx=-\frac{x^{1-3\,n}}{(a+b)^3\,\left(3\,n-1\right)}$$

```
[In] int(1/(a*x^n + b*x^n)^3,x)
```

```
[Out] -x^(1 - 3*n)/((a + b)^3*(3*n - 1))
```

3.328 $\int \left(ax+bx^{14}\right)^{12} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \left(ax+bx^{14}\right)^{12} \, dx = \frac{\left(a+bx^{13}\right)^{13}}{169b}$$

```
[Out] 1/169*(b*x^13+a)^13/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 267}

$$\int \left(ax+bx^{14}\right)^{12} \, dx = \frac{\left(a+bx^{13}\right)^{13}}{169b}$$

```
[In] Int[(a*x + b*x^14)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^{12}\left(a+bx^{13}\right)^{12}\,dx \\
&= \frac{\left(a+bx^{13}\right)^{13}}{169b}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. $2(16) = 32$.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$
\begin{aligned}
\int \left(ax+bx^{14}\right)^{12}\,dx = {}& \frac{a^{12}x^{13}}{13} + \frac{6}{13}a^{11}bx^{26} + \frac{22}{13}a^{10}b^2x^{39} + \frac{55}{13}a^9b^3x^{52} + \frac{99}{13}a^8b^4x^{65} \\
&+ \frac{132}{13}a^7b^5x^{78} + \frac{132}{13}a^6b^6x^{91} + \frac{99}{13}a^5b^7x^{104} + \frac{55}{13}a^4b^8x^{117} \\
&+ \frac{22}{13}a^3b^9x^{130} + \frac{6}{13}a^2b^{10}x^{143} + \frac{1}{13}ab^{11}x^{156} + \frac{b^{12}x^{169}}{169}
\end{aligned}
$$

```
[In] Integrate[(a*x + b*x^14)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 1.79 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{1}{169}b^{12}x^{169} + \frac{1}{13}a\,b^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91}$
parallelrisch	$\frac{1}{169}b^{12}x^{169} + \frac{1}{13}a\,b^{11}x^{156} + \frac{6}{13}a^2b^{10}x^{143} + \frac{22}{13}a^3b^9x^{130} + \frac{55}{13}a^4b^8x^{117} + \frac{99}{13}a^5b^7x^{104} + \frac{132}{13}a^6b^6x^{91}$
gosper	$\frac{x^{13}\left(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+1716a^7b^5x^{65}+1287a^8b^4x\right.}{169}$
risch	$\frac{b^{12}x^{169}}{169} + \frac{a\,b^{11}x^{156}}{13} + \frac{6a^2b^{10}x^{143}}{13} + \frac{22a^3b^9x^{130}}{13} + \frac{55a^4b^8x^{117}}{13} + \frac{99a^5b^7x^{104}}{13} + \frac{132a^6b^6x^{91}}{13} + \frac{132a^7b^5x^{78}}{13} + 99$

```
[In] int((b*x^14+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*b*a^11*
x^26+1/13*a^12*x^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12}\,dx = \frac{1}{169}\,b^{12}x^{169} + \frac{1}{13}\,ab^{11}x^{156} + \frac{6}{13}\,a^{2}b^{10}x^{143} + \frac{22}{13}\,a^{3}b^{9}x^{130} + \frac{55}{13}\,a^{4}b^{8}x^{117} + \frac{99}{13}\,a^{5}b^{7}x^{104} + \frac{132}{13}\,a^{6}b^{6}x^{91} + \frac{132}{13}\,a^{7}b^{5}x^{78} + \frac{99}{13}\,a^{8}b^{4}x^{65} + \frac{55}{13}\,a^{9}b^{3}x^{52} + \frac{22}{13}\,a^{10}b^{2}x^{39} + \frac{6}{13}\,a^{11}bx^{26} + \frac{1}{13}\,a^{12}x^{13}$$

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int \left(ax+bx^{14}\right)^{12}\,dx = \frac{a^{12}x^{13}}{13} + \frac{6a^{11}bx^{26}}{13} + \frac{22a^{10}b^{2}x^{39}}{13} + \frac{55a^{9}b^{3}x^{52}}{13} + \frac{99a^{8}b^{4}x^{65}}{13} + \frac{132a^{7}b^{5}x^{78}}{13} + \frac{132a^{6}b^{6}x^{91}}{13} + \frac{99a^{5}b^{7}x^{104}}{13} + \frac{55a^{4}b^{8}x^{117}}{13} + \frac{22a^{3}b^{9}x^{130}}{13} + \frac{6a^{2}b^{10}x^{143}}{13} + \frac{ab^{11}x^{156}}{13} + \frac{b^{12}x^{169}}{169}$$

```
[In] integrate((b*x**14+a*x)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12}\,dx &= \frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}\\&\quad+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}\\&\quad+\frac{99}{13}\,a^{8}b^{4}x^{65}+\frac{55}{13}\,a^{9}b^{3}x^{52}+\frac{22}{13}\,a^{10}b^{2}x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}\end{aligned}$$

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12}\,dx &= \frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}\\&\quad+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}\\&\quad+\frac{99}{13}\,a^{8}b^{4}x^{65}+\frac{55}{13}\,a^{9}b^{3}x^{52}+\frac{22}{13}\,a^{10}b^{2}x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}\end{aligned}$$

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12}\,dx=\frac{a^{12}\,x^{13}}{13}+\frac{6\,a^{11}\,b\,x^{26}}{13}+\frac{22\,a^{10}\,b^2\,x^{39}}{13}+\frac{55\,a^9\,b^3\,x^{52}}{13}+\frac{99\,a^8\,b^4\,x^{65}}{13}+\frac{132\,a^7\,b^5\,x^{78}}{13}+\frac{132\,a^6\,b^6\,x^{91}}{13}+\frac{99\,a^5\,b^7\,x^{104}}{13}+\frac{55\,a^4\,b^8\,x^{117}}{13}+\frac{22\,a^3\,b^9\,x^{130}}{13}+\frac{6\,a^2\,b^{10}\,x^{143}}{13}+\frac{a\,b^{11}\,x^{156}}{13}+\frac{b^{12}\,x^{169}}{169}$$

```
[In] int((a*x + b*x^14)^12,x)
```

```
[Out] (a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13
+ (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (13
2*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55*a^
4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13
```

3.329 $\int x^{12}(ax+bx^{26})^{12}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 267}

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[In] Int[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{24}\left(a+bx^{25}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{25}\right)^{13}}{325b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. $2(16) = 32$.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx = {} & \frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}\\ & +\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}\\ & +\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}\end{aligned}$$

```
[In] Integrate[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 1.96 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}+$
parallelrisch	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}+$
gosper	$\frac{x^{25}\left(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a^8b^4\right.}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{22a^3b^9x^{250}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{99a^5b^7x^{200}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{132a^7b^5x^{150}}{25}+$ 99

```
[In] int(x^12*(b*x^26+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 99/25*a^8*b^4*x^125+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+11/5*a^4*b^8*x^
225+1/325*b^12*x^325+99/25*a^5*b^7*x^200+132/25*a^7*b^5*x^150+132/25*a^6*b^
6*x^175+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*b*a^11*x^50+1/25*a*b^11
*x^300+1/25*a^12*x^25
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}+\frac{99}{25}\,a^{8}b^{4}x^{125}+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

```
[In] integrate(x**12*(b*x**26+a*x)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx = {} & \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250} \\ & +\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150} \\ & +\frac{99}{25}\,a^{8}b^{4}x^{125}+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}\end{aligned}$$

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx = {} & \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250} \\ & +\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150} \\ & +\frac{99}{25}\,a^{8}b^{4}x^{125}+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}\end{aligned}$$

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

```
[In] int(x^12*(a*x + b*x^26)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

3.330 $\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 267}

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[In] Int[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{36}\left(a+bx^{37}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = \frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

```
[In] Integrate[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 2.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
parallelrisch	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
gosper	$x^{37}\big(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a^8b$ / 481
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{132a^7b^5x^{222}}{37}+$

```
[In] int(x^24*(b*x^38+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 6/37*b*a^11*x^74+1/481*b^12*x^481+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+1
32/37*a^7*b^5*x^222+132/37*a^6*b^6*x^259+22/37*a^3*b^9*x^370+1/37*a^12*x^37
+99/37*a^8*b^4*x^185+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/37*a*b^11*x
^444+6/37*a^2*b^10*x^407
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = {} & \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}\\ & +\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}\\ & +\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}\end{aligned}$$

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = {} & \frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}\\ & +\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}\\ & +\frac{22a^{3}b^{9}x^{370}}{37}+\frac{6a^{2}b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}\end{aligned}$$

```
[In] integrate(x**24*(b*x**38+a*x)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

```
[In] int(x^24*(a*x + b*x^38)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.331 $\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx=\frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}$$

```
[Out] 1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1598, 267}

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx=\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

```
[In] Int[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx$$
$$= \frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx = \frac{\left(a+bx^{1+12m}\right)^{13}}{13b+156bm}$$

```
[In] Integrate[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(25) = 50.

Time = 53.75 (sec) , antiderivative size = 338, normalized size of antiderivative = 12.52

method	result
parallelrisch	$13a^{12}x^{-12+12m}x^{13}+78b\,a^{11}x^{-12+12m}x^{2+12m}x^{12}+286a^{10}b^{2}x^{-12+12m}x^{4+24m}x^{11}+715a^{9}b^{3}x^{-12+12m}x^{6+36m}x^{10}+1287a^{8}b^{4}x$
risch	$\frac{b^{12}x^{26+156m}}{13(1+12m)x^{13}}+\frac{a\,b^{11}x^{24+144m}}{(1+12m)x^{12}}+\frac{6a^{2}b^{10}x^{22+132m}}{(1+12m)x^{11}}+\frac{22a^{3}b^{9}x^{20+120m}}{(1+12m)x^{10}}+\frac{55a^{4}b^{8}x^{18+108m}}{(1+12m)x^{9}}+\frac{99a^{5}b^{7}x^{16+96m}}{(1+12m)x^{8}}+132$

```
[In] int(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*(13*a^12*x^(-12+12*m)*x^13+78*b*a^11*x^(-12+12*m)*x^(2+12*m)*x^12+286*
a^10*b^2*x^(-12+12*m)*(x^(2+12*m))^2*x^11+715*a^9*b^3*x^(-12+12*m)*(x^(2+12
*m))^3*x^10+1287*a^8*b^4*x^(-12+12*m)*(x^(2+12*m))^4*x^9+1716*a^7*b^5*x^(-1
2+12*m)*(x^(2+12*m))^5*x^8+1716*a^6*b^6*x^(-12+12*m)*(x^(2+12*m))^6*x^7+128
7*a^5*b^7*x^(-12+12*m)*(x^(2+12*m))^7*x^6+715*a^4*b^8*x^(-12+12*m)*(x^(2+12
*m))^8*x^5+286*a^3*b^9*x^(-12+12*m)*(x^(2+12*m))^9*x^4+78*a^2*b^10*x^(-12+1
2*m)*(x^(2+12*m))^10*x^3+13*a*b^11*x^(-12+12*m)*(x^(2+12*m))^11*x^2+b^12*x^
(-12+12*m)*(x^(2+12*m))^12*x)/(1+12*m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 231, normalized size of antiderivative = 8.56

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$$
$$= \frac{13\,a^{12}x^{12}x^{12\,m+2} + 78\,a^{11}bx^{11}x^{24\,m+4} + 286\,a^{10}b^2x^{10}x^{36\,m+6} + 715\,a^9b^3x^9x^{48\,m+8} + 1287\,a^8b^4x^8x^{60\,m+10} + 1}{}$$

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(13*a^12*x^12*x^(12*m + 2) + 78*a^11*b*x^11*x^(24*m + 4) + 286*a^10*b^
2*x^10*x^(36*m + 6) + 715*a^9*b^3*x^9*x^(48*m + 8) + 1287*a^8*b^4*x^8*x^(60
*m + 10) + 1716*a^7*b^5*x^7*x^(72*m + 12) + 1716*a^6*b^6*x^6*x^(84*m + 14)
+ 1287*a^5*b^7*x^5*x^(96*m + 16) + 715*a^4*b^8*x^4*x^(108*m + 18) + 286*a^3
*b^9*x^3*x^(120*m + 20) + 78*a^2*b^10*x^2*x^(132*m + 22) + 13*a*b^11*x*x^(1
44*m + 24) + b^12*x^(156*m + 26))/((12*m + 1)*x^13)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(19) = 38$.

Time = 6.30 (sec) , antiderivative size = 520, normalized size of antiderivative = 19.26

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$$
$$= \begin{cases} \frac{13a^{12}x^{13}x^{12m-12}}{156m+13} + \frac{78a^{11}bx^{12}x^{12m-12}x^{12m+2}}{156m+13} + \frac{286a^{10}b^2x^{11}x^{12m-12}x^{24m+4}}{156m+13} + \frac{715a^9b^3x^{10}x^{12m-12}x^{36m+6}}{156m+13} + \frac{1287a^8b^4x^9x^{12m-1}}{156m+13} \\ a^{12}\log(x) + 12a^{11}b\log(x) + 66a^{10}b^2\log(x) + 220a^9b^3\log(x) + 495a^8b^4\log(x) + 792a^7b^5\log(x) + 924 \end{cases}$$

```
[In] integrate(x**(-12+12*m)*(a*x+b*x**(2+12*m))**12,x)
```

```
[Out] Piecewise((13*a**12*x**13*x**(12*m - 12)/(156*m + 13) + 78*a**11*b*x**12*x*
*(12*m - 12)*x**(12*m + 2)/(156*m + 13) + 286*a**10*b**2*x**11*x**(12*m - 1
2)*x**(24*m + 4)/(156*m + 13) + 715*a**9*b**3*x**10*x**(12*m - 12)*x**(36*m
 + 6)/(156*m + 13) + 1287*a**8*b**4*x**9*x**(12*m - 12)*x**(48*m + 8)/(156*
m + 13) + 1716*a**7*b**5*x**8*x**(12*m - 12)*x**(60*m + 10)/(156*m + 13) +
1716*a**6*b**6*x**7*x**(12*m - 12)*x**(72*m + 12)/(156*m + 13) + 1287*a**5*
b**7*x**6*x**(12*m - 12)*x**(84*m + 14)/(156*m + 13) + 715*a**4*b**8*x**5*x
**(12*m - 12)*x**(96*m + 16)/(156*m + 13) + 286*a**3*b**9*x**4*x**(12*m - 1
2)*x**(108*m + 18)/(156*m + 13) + 78*a**2*b**10*x**3*x**(12*m - 12)*x**(120
*m + 20)/(156*m + 13) + 13*a*b**11*x**2*x**(12*m - 12)*x**(132*m + 22)/(156
*m + 13) + b**12*x*x**(12*m - 12)*x**(144*m + 24)/(156*m + 13), Ne(m, -1/12
)), (a**12*log(x) + 12*a**11*b*log(x) + 66*a**10*b**2*log(x) + 220*a**9*b**
3*log(x) + 495*a**8*b**4*log(x) + 792*a**7*b**5*log(x) + 924*a**6*b**6*log(
x) + 792*a**5*b**7*log(x) + 495*a**4*b**8*log(x) + 220*a**3*b**9*log(x) + 6
6*a**2*b**10*log(x) + 12*a*b**11*log(x) + b**12*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(25) = 50$.

Time = 0.23 (sec) , antiderivative size = 275, normalized size of antiderivative = 10.19

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx=\frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)}+\frac{ab^{11}x^{144\,m+12}}{12\,m+1}+\frac{6\,a^2b^{10}x^{132\,m+11}}{12\,m+1}+\frac{22\,a^3b^9x^{120\,m+10}}{12\,m+1}+\frac{55\,a^4b^8x^{108\,m+9}}{12\,m+1}+\frac{99\,a^5b^7x^{96\,m+8}}{12\,m+1}+\frac{132\,a^6b^6x^{84\,m+7}}{12\,m+1}+\frac{132\,a^7b^5x^{72\,m+6}}{12\,m+1}+\frac{99\,a^8b^4x^{60\,m+5}}{12\,m+1}+\frac{55\,a^9b^3x^{48\,m+4}}{12\,m+1}+\frac{22\,a^{10}b^2x^{36\,m+3}}{12\,m+1}+\frac{6\,a^{11}bx^{24\,m+2}}{12\,m+1}+\frac{a^{12}x^{12\,m+1}}{12\,m+1}$$

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6*
a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1) +
 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1) +
 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m + 1)
+ 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m + 1) +
 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m + 1) + a
^12*x^(12*m + 1)/(12*m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(25) = 50$.

Time = 0.40 (sec) , antiderivative size = 285, normalized size of antiderivative = 10.56

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$$

$$=13\,a^{12}x^{12}e^{(12\,m\log(x)+2\,\log(x))}+78\,a^{11}bx^{11}e^{(24\,m\log(x)+4\,\log(x))}+286\,a^{10}b^2x^{10}e^{(36\,m\log(x)+6\,\log(x))}+715\,a^9b^3x^9e$$

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="giac")
```

```
[Out] 1/13*(13*a^12*x^12*e^(12*m*log(x) + 2*log(x)) + 78*a^11*b*x^11*e^(24*m*log(
x) + 4*log(x)) + 286*a^10*b^2*x^10*e^(36*m*log(x) + 6*log(x)) + 715*a^9*b^3
*x^9*e^(48*m*log(x) + 8*log(x)) + 1287*a^8*b^4*x^8*e^(60*m*log(x) + 10*log(
x)) + 1716*a^7*b^5*x^7*e^(72*m*log(x) + 12*log(x)) + 1716*a^6*b^6*x^6*e^(84
*m*log(x) + 14*log(x)) + 1287*a^5*b^7*x^5*e^(96*m*log(x) + 16*log(x)) + 715
*a^4*b^8*x^4*e^(108*m*log(x) + 18*log(x)) + 286*a^3*b^9*x^3*e^(120*m*log(x)
 + 20*log(x)) + 78*a^2*b^10*x^2*e^(132*m*log(x) + 22*log(x)) + 13*a*b^11*x*
e^(144*m*log(x) + 24*log(x)) + b^12*e^(156*m*log(x) + 26*log(x)))/(12*m*x^1
3 + x^13)
```

Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 287, normalized size of antiderivative = 10.63

$$\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx=\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{13\,a^{12}\,x\,x^{12\,m}}{156\,m+13}+\frac{78\,a^{11}\,b\,x^{24\,m}\,x^2}{156\,m+13}+\frac{13\,a\,b^{11}\,x^{144\,m}\,x^{12}}{156\,m+13}+\frac{286\,a^{10}\,b^2\,x^{36\,m}\,x^3}{156\,m+13}+\frac{715\,a^9\,b^3\,x^{48\,m}\,x^4}{156\,m+13}+\frac{1287\,a^8\,b^4\,x^{60\,m}\,x^5}{156\,m+13}+\frac{1716\,a^7\,b^5\,x^{72\,m}\,x^6}{156\,m+13}+\frac{1716\,a^6\,b^6\,x^{84\,m}\,x^7}{156\,m+13}+\frac{1287\,a^5\,b^7\,x^{96\,m}\,x^8}{156\,m+13}+\frac{715\,a^4\,b^8\,x^{108\,m}\,x^9}{156\,m+13}+\frac{286\,a^3\,b^9\,x^{120\,m}\,x^{10}}{156\,m+13}+\frac{78\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{156\,m+13}$$

```
[In] int(x^(12*m - 12)*(a*x + b*x^(12*m + 2))^12,x)
```

```
[Out] (b^12*x^(156*m)*x^13)/(156*m + 13) + (13*a^12*x*x^(12*m))/(156*m + 13) + (7
8*a^11*b*x^(24*m)*x^2)/(156*m + 13) + (13*a*b^11*x^(144*m)*x^12)/(156*m + 1
3) + (286*a^10*b^2*x^(36*m)*x^3)/(156*m + 13) + (715*a^9*b^3*x^(48*m)*x^4)/
(156*m + 13) + (1287*a^8*b^4*x^(60*m)*x^5)/(156*m + 13) + (1716*a^7*b^5*x^(
72*m)*x^6)/(156*m + 13) + (1716*a^6*b^6*x^(84*m)*x^7)/(156*m + 13) + (1287*
a^5*b^7*x^(96*m)*x^8)/(156*m + 13) + (715*a^4*b^8*x^(108*m)*x^9)/(156*m + 1
3) + (286*a^3*b^9*x^(120*m)*x^10)/(156*m + 13) + (78*a^2*b^10*x^(132*m)*x^1
1)/(156*m + 13)
```

3.332 $\int (ax + bx^{14})^{12} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int (ax + bx^{14})^{12} \, dx = \frac{(a + bx^{13})^{13}}{169b}$$

```
[Out] 1/169*(b*x^13+a)^13/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 267}

$$\int (ax + bx^{14})^{12} \, dx = \frac{(a + bx^{13})^{13}}{169b}$$

```
[In] Int[(a*x + b*x^14)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^{12}\left(a+bx^{13}\right)^{12}\,dx \\
&= \frac{\left(a+bx^{13}\right)^{13}}{169b}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$
\begin{aligned}
\int \left(ax+bx^{14}\right)^{12}\,dx = {} & \frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65} \\
& +\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117} \\
& +\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}
\end{aligned}
$$

```
[In] Integrate[(a*x + b*x^14)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 1.82 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}$
parallelrisch	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}$
gosper	$x^{13}(b^{12}x^{156}+13a\,b^{11}x^{143}+78a^2b^{10}x^{130}+286a^3b^9x^{117}+715a^4b^8x^{104}+1287a^5b^7x^{91}+1716a^6b^6x^{78}+1716a^7b^5x^{65}+1287a^8b^4x$ / 169
risch	$\frac{b^{12}x^{169}}{169}+\frac{a\,b^{11}x^{156}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{22a^3b^9x^{130}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{132a^7b^5x^{78}}{13}+$

```
[In] int((b*x^14+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*b*a^11*
x^26+1/13*a^12*x^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12}\,dx &= \frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}\\ &\quad+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}\\ &\quad+\frac{99}{13}\,a^{8}b^{4}x^{65}+\frac{55}{13}\,a^{9}b^{3}x^{52}+\frac{22}{13}\,a^{10}b^{2}x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}\end{aligned}$$

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12}\,dx &= \frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^{2}x^{39}}{13}+\frac{55a^{9}b^{3}x^{52}}{13}+\frac{99a^{8}b^{4}x^{65}}{13}\\ &\quad+\frac{132a^{7}b^{5}x^{78}}{13}+\frac{132a^{6}b^{6}x^{91}}{13}+\frac{99a^{5}b^{7}x^{104}}{13}+\frac{55a^{4}b^{8}x^{117}}{13}\\ &\quad+\frac{22a^{3}b^{9}x^{130}}{13}+\frac{6a^{2}b^{10}x^{143}}{13}+\frac{ab^{11}x^{156}}{13}+\frac{b^{12}x^{169}}{169}\end{aligned}$$

```
[In] integrate((b*x**14+a*x)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12}\,dx &= \frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}\\&\quad+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}\\&\quad+\frac{99}{13}\,a^{8}b^{4}x^{65}+\frac{55}{13}\,a^{9}b^{3}x^{52}+\frac{22}{13}\,a^{10}b^{2}x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}\end{aligned}$$

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12}\,dx &= \frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}\\&\quad+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}\\&\quad+\frac{99}{13}\,a^{8}b^{4}x^{65}+\frac{55}{13}\,a^{9}b^{3}x^{52}+\frac{22}{13}\,a^{10}b^{2}x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}\end{aligned}$$

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax+bx^{14}\right)^{12}\,dx=\frac{a^{12}\,x^{13}}{13}+\frac{6\,a^{11}\,b\,x^{26}}{13}+\frac{22\,a^{10}\,b^2\,x^{39}}{13}+\frac{55\,a^9\,b^3\,x^{52}}{13}+\frac{99\,a^8\,b^4\,x^{65}}{13}+\frac{132\,a^7\,b^5\,x^{78}}{13}+\frac{132\,a^6\,b^6\,x^{91}}{13}+\frac{99\,a^5\,b^7\,x^{104}}{13}+\frac{55\,a^4\,b^8\,x^{117}}{13}+\frac{22\,a^3\,b^9\,x^{130}}{13}+\frac{6\,a^2\,b^{10}\,x^{143}}{13}+\frac{a\,b^{11}\,x^{156}}{13}+\frac{b^{12}\,x^{169}}{169}$$

```
[In] int((a*x + b*x^14)^12,x)
```

```
[Out] (a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13
+ (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (13
2*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55*a^
4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13
```

3.333 $\int (ax^2+bx^{27})^{12}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \left(ax^2+bx^{27}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1607, 267}

$$\int \left(ax^2+bx^{27}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[In] Int[(a*x^2 + b*x^27)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^{24}\left(a+bx^{25}\right)^{12}\,dx\\
&= \frac{\left(a+bx^{25}\right)^{13}}{325b}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$
\begin{aligned}
\int \left(ax^2+bx^{27}\right)^{12}\,dx = {}& \frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}\\
&+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}\\
&+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}
\end{aligned}
$$

```
[In] Integrate[(a*x^2 + b*x^27)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 1.99 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}+$
parallelrisch	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}+$
gosper	$\frac{x^{25}(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a^8b^4}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{22a^3b^9x^{250}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{99a^5b^7x^{200}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{132a^7b^5x^{150}}{25}+$

```
[In] int((b*x^27+a*x^2)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 99/25*a^8*b^4*x^125+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+11/5*a^4*b^8*x^
225+1/325*b^12*x^325+99/25*a^5*b^7*x^200+132/25*a^7*b^5*x^150+132/25*a^6*b^
6*x^175+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*b*a^11*x^50+1/25*a*b^11
*x^300+1/25*a^12*x^25
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int \left(ax^2+bx^{27}\right)^{12}\,dx &= \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}\\ &\quad+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}\\ &\quad+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}\end{aligned}$$

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="fricas")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int \left(ax^2+bx^{27}\right)^{12}\,dx &= \frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^2x^{75}}{25}+\frac{11a^9b^3x^{100}}{5}+\frac{99a^8b^4x^{125}}{25}\\ &\quad+\frac{132a^7b^5x^{150}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{99a^5b^7x^{200}}{25}+\frac{11a^4b^8x^{225}}{5}\\ &\quad+\frac{22a^3b^9x^{250}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}\end{aligned}$$

```
[In] integrate((b*x**27+a*x**2)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int \left(ax^2+bx^{27}\right)^{12}\,dx &= \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}\\&\quad+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}\\&\quad+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}\end{aligned}$$

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int \left(ax^2+bx^{27}\right)^{12}\,dx &= \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}\\&\quad+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}\\&\quad+\frac{99}{25}\,a^8b^4x^{125}+\frac{11}{5}\,a^9b^3x^{100}+\frac{22}{25}\,a^{10}b^2x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}\end{aligned}$$

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax^2+bx^{27}\right)^{12} dx = \frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

```
[In] int((a*x^2 + b*x^27)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

3.334 $\int (ax^3 + bx^{40})^{12}\, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int (ax^3 + bx^{40})^{12}\, dx = \frac{(a + bx^{37})^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1607, 267}

$$\int (ax^3 + bx^{40})^{12}\, dx = \frac{(a + bx^{37})^{13}}{481b}$$

```
[In] Int[(a*x^3 + b*x^40)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x^{36}\left(a+bx^{37}\right)^{12}\,dx$$
$$=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int \left(ax^3+bx^{40}\right)^{12}\,dx &= \frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}\\ &\quad+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}\\ &\quad+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}\end{aligned}$$

```
[In] Integrate[(a*x^3 + b*x^40)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 2.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
parallelrisch	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
gosper	$\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a^8b}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{132a^7b^5x^{222}}{37}+$

```
[In] int((b*x^40+a*x^3)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 6/37*b*a^11*x^74+1/481*b^12*x^481+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+1
32/37*a^7*b^5*x^222+132/37*a^6*b^6*x^259+22/37*a^3*b^9*x^370+1/37*a^12*x^37
+99/37*a^8*b^4*x^185+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/37*a*b^11*x
^444+6/37*a^2*b^10*x^407
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time $= 0.24$ (sec) , antiderivative size $= 134$, normalized size of antiderivative $= 8.38$

$$\int \left(ax^3+bx^{40}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}+\frac{22}{37}\,a^3b^9x^{370}+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}+\frac{132}{37}\,a^6b^6x^{259}+\frac{132}{37}\,a^7b^5x^{222}+\frac{99}{37}\,a^8b^4x^{185}+\frac{55}{37}\,a^9b^3x^{148}+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="fricas")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time $= 0.05$ (sec) , antiderivative size $= 160$, normalized size of antiderivative $= 10.00$

$$\int \left(ax^3+bx^{40}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^2x^{111}}{37}+\frac{55a^9b^3x^{148}}{37}+\frac{99a^8b^4x^{185}}{37}+\frac{132a^7b^5x^{222}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

```
[In] integrate((b*x**40+a*x**3)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int \left(ax^3+bx^{40}\right)^{12}\,dx &= \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}+\frac{22}{37}\,a^3b^9x^{370}\\&\quad+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}+\frac{132}{37}\,a^6b^6x^{259}+\frac{132}{37}\,a^7b^5x^{222}\\&\quad+\frac{99}{37}\,a^8b^4x^{185}+\frac{55}{37}\,a^9b^3x^{148}+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}\end{aligned}$$

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int \left(ax^3+bx^{40}\right)^{12}\,dx &= \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}+\frac{22}{37}\,a^3b^9x^{370}\\&\quad+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}+\frac{132}{37}\,a^6b^6x^{259}+\frac{132}{37}\,a^7b^5x^{222}\\&\quad+\frac{99}{37}\,a^8b^4x^{185}+\frac{55}{37}\,a^9b^3x^{148}+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}\end{aligned}$$

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int \left(ax^3+bx^{40}\right)^{12}\,dx=\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

```
[In] int((a*x^3 + b*x^40)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.335 $\int \left(ax^m+bx^{1+13m}\right)^{12} dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \left(ax^m+bx^{1+13m}\right)^{12} dx=\frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}$$

```
[Out] 1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1607, 267}

$$\int \left(ax^m+bx^{1+13m}\right)^{12} dx=\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

```
[In] Int[(a*x^m + b*x^(1 + 13*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx \\ &= \frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 7.15

$$\int \left(ax^m+bx^{1+13m}\right)^{12}\,dx$$
$$=\frac{x^{1+12m}(13a^{12}+78a^{11}bx^{1+12m}+286a^{10}b^2x^{2+24m}+715a^9b^3x^{3+36m}+1287a^8b^4x^{4+48m}+1716a^7b^5x^{5+60m}+1}{}$$

```
[In] Integrate[(a*x^m + b*x^(1 + 13*m))^12,x]
```

```
[Out] (x^(1 + 12*m)*(13*a^12 + 78*a^11*b*x^(1 + 12*m) + 286*a^10*b^2*x^(2 + 24*m)
 + 715*a^9*b^3*x^(3 + 36*m) + 1287*a^8*b^4*x^(4 + 48*m) + 1716*a^7*b^5*x^(5
 + 60*m) + 1716*a^6*b^6*x^(6 + 72*m) + 1287*a^5*b^7*x^(7 + 84*m) + 715*a^4*
b^8*x^(8 + 96*m) + 286*a^3*b^9*x^(9 + 108*m) + 78*a^2*b^10*x^(10 + 120*m) +
 13*a*b^11*x^(11 + 132*m) + b^12*x^(12 + 144*m)))/(13 + 156*m)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(25) = 50.

Time = 46.02 (sec) , antiderivative size = 281, normalized size of antiderivative = 10.41

method	result
parallelrisch	$\frac{13a^{12}x^{12m}x+78b\,a^{11}x^{1+13m}x^{11m}x+286a^{10}b^2x^{2+26m}x^{10m}x+715a^9b^3x^{3+39m}x^{9m}x+1287a^8b^4x^{4+52m}x^{8m}x+1716a^7b^5x^{5+65m}}{}$
risch	$\frac{b^{12}x^{13}x^{156m}}{13+156m}+\frac{a\,b^{11}x^{12}x^{144m}}{1+12m}+\frac{6a^2b^{10}x^{11}x^{132m}}{1+12m}+\frac{22a^3b^9x^{10}x^{120m}}{1+12m}+\frac{55a^4b^8x^9x^{108m}}{1+12m}+\frac{99a^5b^7x^8x^{96m}}{1+12m}+\frac{132a^6b^6}{1+1}$

```
[In] int((x^m*a+b*x^(1+13*m))^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*(13*a^12*(x^m)^12*x+78*b*a^11*x^(1+13*m)*(x^m)^11*x+286*a^10*b^2*(x^(1
+13*m))^2*(x^m)^10*x+715*a^9*b^3*(x^(1+13*m))^3*(x^m)^9*x+1287*a^8*b^4*(x^(
1+13*m))^4*(x^m)^8*x+1716*a^7*b^5*(x^(1+13*m))^5*(x^m)^7*x+1716*a^6*b^6*(x^
(1+13*m))^6*(x^m)^6*x+1287*a^5*b^7*(x^(1+13*m))^7*(x^m)^5*x+715*a^4*b^8*(x^
(1+13*m))^8*(x^m)^4*x+286*a^3*b^9*(x^(1+13*m))^9*(x^m)^3*x+78*a^2*b^10*(x^(
1+13*m))^10*(x^m)^2*x+13*a*b^11*(x^(1+13*m))^11*x^m*x+b^12*(x^(1+13*m))^12*
x)/(1+12*m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 7.59

$$\int \left(ax^m + bx^{1+13m}\right)^{12} dx$$
$$= \frac{b^{12}x^{13}x^{156\,m} + 13\,ab^{11}x^{12}x^{144\,m} + 78\,a^2b^{10}x^{11}x^{132\,m} + 286\,a^3b^9x^{10}x^{120\,m} + 715\,a^4b^8x^9x^{108\,m} + 1287\,a^5b^7x^8x}{}$$

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(
132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*
b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) +
1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(
36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(19) = 38$.

Time = 5.03 (sec) , antiderivative size = 471, normalized size of antiderivative = 17.44

$$\int \left(ax^m + bx^{1+13m}\right)^{12} dx$$
$$= \begin{cases} \frac{13a^{12}xx^{12m}}{156m+13} + \frac{78a^{11}bxx^{11m}x^{13m+1}}{156m+13} + \frac{286a^{10}b^2xx^{10m}x^{26m+2}}{156m+13} + \frac{715a^9b^3xx^{9m}x^{39m+3}}{156m+13} + \frac{1287a^8b^4xx^{8m}x^{52m+4}}{156m+13} + \frac{1716a^7b^5xx^{7m}x}{156m+13} \\ a^{12}\log{(x)} + 12a^{11}b\log{(x)} + 66a^{10}b^2\log{(x)} + 220a^9b^3\log{(x)} + 495a^8b^4\log{(x)} + 792a^7b^5\log{(x)} + 924 \end{cases}$$

```
[In] integrate((a*x**m+b*x**(1+13*m))**12,x)
```

```
[Out] Piecewise((13*a**12*x*x**(12*m)/(156*m + 13) + 78*a**11*b*x*x**(11*m)*x**(1
3*m + 1)/(156*m + 13) + 286*a**10*b**2*x*x**(10*m)*x**(26*m + 2)/(156*m + 1
3) + 715*a**9*b**3*x*x**(9*m)*x**(39*m + 3)/(156*m + 13) + 1287*a**8*b**4*x
*x**(8*m)*x**(52*m + 4)/(156*m + 13) + 1716*a**7*b**5*x*x**(7*m)*x**(65*m +
 5)/(156*m + 13) + 1716*a**6*b**6*x*x**(6*m)*x**(78*m + 6)/(156*m + 13) + 1
287*a**5*b**7*x*x**(5*m)*x**(91*m + 7)/(156*m + 13) + 715*a**4*b**8*x*x**(4
*m)*x**(104*m + 8)/(156*m + 13) + 286*a**3*b**9*x*x**(3*m)*x**(117*m + 9)/(
156*m + 13) + 78*a**2*b**10*x*x**(2*m)*x**(130*m + 10)/(156*m + 13) + 13*a*
b**11*x*x**m*x**(143*m + 11)/(156*m + 13) + b**12*x*x**(156*m + 12)/(156*m
+ 13), Ne(m, -1/12)), (a**12*log(x) + 12*a**11*b*log(x) + 66*a**10*b**2*log
(x) + 220*a**9*b**3*log(x) + 495*a**8*b**4*log(x) + 792*a**7*b**5*log(x) +
924*a**6*b**6*log(x) + 792*a**5*b**7*log(x) + 495*a**4*b**8*log(x) + 220*a*
*3*b**9*log(x) + 66*a**2*b**10*log(x) + 12*a*b**11*log(x) + b**12*log(x), T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(25) = 50$.

Time = 0.21 (sec) , antiderivative size = 275, normalized size of antiderivative = 10.19

$$\int \left(ax^m+bx^{1+13m}\right)^{12}\,dx = \frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)}+\frac{ab^{11}x^{144\,m+12}}{12\,m+1}+\frac{6\,a^2b^{10}x^{132\,m+11}}{12\,m+1}+\frac{22\,a^3b^9x^{120\,m+10}}{12\,m+1}+\frac{55\,a^4b^8x^{108\,m+9}}{12\,m+1}+\frac{99\,a^5b^7x^{96\,m+8}}{12\,m+1}+\frac{132\,a^6b^6x^{84\,m+7}}{12\,m+1}+\frac{132\,a^7b^5x^{72\,m+6}}{12\,m+1}+\frac{99\,a^8b^4x^{60\,m+5}}{12\,m+1}+\frac{55\,a^9b^3x^{48\,m+4}}{12\,m+1}+\frac{22\,a^{10}b^2x^{36\,m+3}}{12\,m+1}+\frac{6\,a^{11}bx^{24\,m+2}}{12\,m+1}+\frac{a^{12}x^{12\,m+1}}{12\,m+1}$$

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6*
a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1) +
 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1) +
 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m + 1)
+ 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m + 1) +
 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m + 1) + a
^12*x^(12*m + 1)/(12*m + 1)
```

Giac [F(-1)]

Timed out.

$$\int \left(ax^m+bx^{1+13m}\right)^{12}\,dx = \text{Timed out}$$

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 9.94 (sec) , antiderivative size = 285, normalized size of antiderivative = 10.56

$$\int \left(ax^m+bx^{1+13m}\right)^{12}\,dx = \frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{a^{12}\,x\,x^{12\,m}}{12\,m+1}+\frac{6\,a^{11}\,b\,x^{24\,m}\,x^2}{12\,m+1}+\frac{a\,b^{11}\,x^{144\,m}\,x^{12}}{12\,m+1}+\frac{22\,a^{10}\,b^2\,x^{36\,m}\,x^3}{12\,m+1}+\frac{55\,a^9\,b^3\,x^{48\,m}\,x^4}{12\,m+1}+\frac{99\,a^8\,b^4\,x^{60\,m}\,x^5}{12\,m+1}+\frac{132\,a^7\,b^5\,x^{72\,m}\,x^6}{12\,m+1}+\frac{132\,a^6\,b^6\,x^{84\,m}\,x^7}{12\,m+1}+\frac{99\,a^5\,b^7\,x^{96\,m}\,x^8}{12\,m+1}+\frac{55\,a^4\,b^8\,x^{108\,m}\,x^9}{12\,m+1}+\frac{22\,a^3\,b^9\,x^{120\,m}\,x^{10}}{12\,m+1}+\frac{6\,a^2\,b^{10}\,x^{132\,m}\,x^{11}}{12\,m+1}$$

```
[In] int((a*x^m + b*x^(13*m + 1))^12,x)
```

```
[Out] (b^12*x^(156*m)*x^13)/(156*m + 13) + (a^12*x*x^(12*m))/(12*m + 1) + (6*a^11
*b*x^(24*m)*x^2)/(12*m + 1) + (a*b^11*x^(144*m)*x^12)/(12*m + 1) + (22*a^10
*b^2*x^(36*m)*x^3)/(12*m + 1) + (55*a^9*b^3*x^(48*m)*x^4)/(12*m + 1) + (99*
a^8*b^4*x^(60*m)*x^5)/(12*m + 1) + (132*a^7*b^5*x^(72*m)*x^6)/(12*m + 1) +
(132*a^6*b^6*x^(84*m)*x^7)/(12*m + 1) + (99*a^5*b^7*x^(96*m)*x^8)/(12*m + 1
) + (55*a^4*b^8*x^(108*m)*x^9)/(12*m + 1) + (22*a^3*b^9*x^(120*m)*x^10)/(12
*m + 1) + (6*a^2*b^10*x^(132*m)*x^11)/(12*m + 1)
```

3.336 $\int \left(ax^m+bx^{1+6m}\right)^5 dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \left(ax^m+bx^{1+6m}\right)^5 dx=\frac{\left(a+bx^{1+5m}\right)^6}{6b(1+5m)}$$

```
[Out] 1/6*(a+b*x^(1+5*m))^6/b/(1+5*m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1607, 267}

$$\int \left(ax^m+bx^{1+6m}\right)^5 dx=\frac{\left(a+bx^{5m+1}\right)^6}{6b(5m+1)}$$

```
[In] Int[(a*x^m + b*x^(1 + 6*m))^5,x]
```

```
[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int x^{5m}\left(a+bx^{1+5m}\right)^5\,dx \\ &= \frac{\left(a+bx^{1+5m}\right)^6}{6b(1+5m)} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 88 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \left(ax^m+bx^{1+6m}\right)^5\,dx \\ = \frac{x^{1+5m}\left(6a^5+15a^4bx^{1+5m}+20a^3b^2x^{2+10m}+15a^2b^3x^{3+15m}+6ab^4x^{4+20m}+b^5x^{5+25m}\right)}{6+30m}$$

```
[In] Integrate[(a*x^m + b*x^(1 + 6*m))^5,x]
```

```
[Out] (x^(1 + 5*m)*(6*a^5 + 15*a^4*b*x^(1 + 5*m) + 20*a^3*b^2*x^(2 + 10*m) + 15*a
^2*b^3*x^(3 + 15*m) + 6*a*b^4*x^(4 + 20*m) + b^5*x^(5 + 25*m)))/(6 + 30*m)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(25) = 50$.

Time = 2.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.44

method	result	size
parallelrisch	$\frac{6x\,x^{5m}a^5+15x\,x^{4m}x^{1+6m}a^4b+20x\,x^{3m}x^{2+12m}a^3b^2+15x\,x^{2m}x^{3+18m}a^2b^3+6x\,x^m x^{4+24m}a\,b^4+x\,x^{5+30m}b^5}{6+30m}$	120
risch	$\frac{b^5x^6x^{30m}}{6+30m}+\frac{a\,b^4x^5x^{25m}}{1+5m}+\frac{5a^2b^3x^4x^{20m}}{2(1+5m)}+\frac{10a^3b^2x^3x^{15m}}{3(1+5m)}+\frac{5a^4b\,x^2x^{10m}}{2(1+5m)}+\frac{a^5x\,x^{5m}}{1+5m}$	126

```
[In] int((x^m*a+b*x^(1+6*m))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(6*x*(x^m)^5*a^5+15*x*(x^m)^4*x^(1+6*m)*a^4*b+20*x*(x^m)^3*(x^(1+6*m))^
2*a^3*b^2+15*x*(x^m)^2*(x^(1+6*m))^3*a^2*b^3+6*x*x^m*(x^(1+6*m))^4*a*b^4+x*
(x^(1+6*m))^5*b^5)/(1+5*m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \left(ax^m+bx^{1+6m}\right)^5\,dx$$
$$=\frac{b^5x^6x^{30\,m}+6\,ab^4x^5x^{25\,m}+15\,a^2b^3x^4x^{20\,m}+20\,a^3b^2x^3x^{15\,m}+15\,a^4bx^2x^{10\,m}+6\,a^5xx^{5\,m}}{6\,(5\,m+1)}$$

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 20
*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(19) = 38$.

Time = 0.63 (sec) , antiderivative size = 197, normalized size of antiderivative = 7.30

$$\int \left(ax^m+bx^{1+6m}\right)^5\,dx$$
$$=\begin{cases}\frac{6a^5xx^{5m}}{30m+6}+\frac{15a^4bxx^{4m}x^{6m+1}}{30m+6}+\frac{20a^3b^2xx^{3m}x^{12m+2}}{30m+6}+\frac{15a^2b^3xx^{2m}x^{18m+3}}{30m+6}+\frac{6ab^4xx^mx^{24m+4}}{30m+6}+\frac{b^5xx^{30m+5}}{30m+6} & \text{for } m\neq-\frac{1}{5}\\ a^5\log{(x)}+5a^4b\log{(x)}+10a^3b^2\log{(x)}+10a^2b^3\log{(x)}+5ab^4\log{(x)}+b^5\log{(x)} & \text{otherwise}\end{cases}$$

```
[In] integrate((a*x**m+b*x**(1+6*m))**5,x)
```

```
[Out] Piecewise((6*a**5*x*x**(5*m)/(30*m + 6) + 15*a**4*b*x*x**(4*m)*x**(6*m + 1)
/(30*m + 6) + 20*a**3*b**2*x*x**(3*m)*x**(12*m + 2)/(30*m + 6) + 15*a**2*b*
*3*x*x**(2*m)*x**(18*m + 3)/(30*m + 6) + 6*a*b**4*x*x**m*x**(24*m + 4)/(30*
m + 6) + b**5*x*x**(30*m + 5)/(30*m + 6), Ne(m, -1/5)), (a**5*log(x) + 5*a*
*4*b*log(x) + 10*a**3*b**2*log(x) + 10*a**2*b**3*log(x) + 5*a*b**4*log(x) +
 b**5*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.48

$$\int \left(ax^m+bx^{1+6m}\right)^5\,dx=\frac{b^5x^{30\,m+6}}{6\,(5\,m+1)}+\frac{ab^4x^{25\,m+5}}{5\,m+1}+\frac{5\,a^2b^3x^{20\,m+4}}{2\,(5\,m+1)}+\frac{10\,a^3b^2x^{15\,m+3}}{3\,(5\,m+1)}+\frac{5\,a^4bx^{10\,m+2}}{2\,(5\,m+1)}+\frac{a^5x^{5\,m+1}}{5\,m+1}$$

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^(30*m + 6)/(5*m + 1) + a*b^4*x^(25*m + 5)/(5*m + 1) + 5/2*a^2*b^3
*x^(20*m + 4)/(5*m + 1) + 10/3*a^3*b^2*x^(15*m + 3)/(5*m + 1) + 5/2*a^4*b*x
^(10*m + 2)/(5*m + 1) + a^5*x^(5*m + 1)/(5*m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(25) = 50$.

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \left(ax^m + bx^{1+6m}\right)^5 \, dx = \frac{b^5x^6x^{30\,m} + 6\,ab^4x^5x^{25\,m} + 15\,a^2b^3x^4x^{20\,m} + 20\,a^3b^2x^3x^{15\,m} + 15\,a^4bx^2x^{10\,m} + 6\,a^5xx^{5\,m}}{6\,(5\,m+1)}$$

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="giac")
```

```
[Out] 1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 20
*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.59

$$\int \left(ax^m + bx^{1+6m}\right)^5 \, dx = \frac{b^5\,x^{30\,m}\,x^6}{30\,m+6} + \frac{a^5\,x\,x^{5\,m}}{5\,m+1} + \frac{5\,a^4\,b\,x^{10\,m}\,x^2}{10\,m+2} + \frac{a\,b^4\,x^{25\,m}\,x^5}{5\,m+1} + \frac{5\,a^2\,b^3\,x^{20\,m}\,x^4}{10\,m+2} + \frac{10\,a^3\,b^2\,x^{15\,m}\,x^3}{15\,m+3}$$

```
[In] int((a*x^m + b*x^(6*m + 1))^5,x)
```

```
[Out] (b^5*x^(30*m)*x^6)/(30*m + 6) + (a^5*x*x^(5*m))/(5*m + 1) + (5*a^4*b*x^(10*
m)*x^2)/(10*m + 2) + (a*b^4*x^(25*m)*x^5)/(5*m + 1) + (5*a^2*b^3*x^(20*m)*x
^4)/(10*m + 2) + (10*a^3*b^2*x^(15*m)*x^3)/(15*m + 3)
```

3.337 $\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx=-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

```
[Out] -1/2/b/(1-3*m)/(a+b*x^(1-3*m))^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1607, 267}

$$\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx=-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

```
[In] Int[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

```
[Out] -1/2*1/(b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-3m}}{\left(a+bx^{1-3m}\right)^3}\,dx\\ &= -\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx = -\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

```
[In] Integrate[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
[Out] -1/2*1/(b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

method	result	size
risch	$-\frac{x\left(2a\,x^{3m}+bx\right)}{2(3m-1)a^2\left(a\,x^{3m}+bx\right)^2}$	39

```
[In] int(1/(b*x^(1-2*m)+x^m*a)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*x*(2*a*(x^m)^3+b*x)/(3*m-1)/a^2/(a*(x^m)^3+b*x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(25) = 50.

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\begin{aligned}&\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx\\ &= -\frac{2\,axx^{3\,m}+bx^2}{2\left(2\left(3\,a^3bm-a^3b\right)xx^{3\,m}+\left(3\,a^2b^2m-a^2b^2\right)x^2+\left(3\,a^4m-a^4\right)x^{6\,m}\right)}\end{aligned}$$

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="fricas")
[Out] -1/2*(2*a*x*x^(3*m) + b*x^2)/(2*(3*a^3*b*m - a^3*b)*x*x^(3*m) + (3*a^2*b^2*
m - a^2*b^2)*x^2 + (3*a^4*m - a^4)*x^(6*m))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

```
[In] integrate(1/(b*x**(1-2*m)+a*x**m)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = -\frac{2\,axx^{3\,m}+bx^2}{2\,(2\,a^3b(3\,m-1)xx^{3\,m}+a^2b^2(3\,m-1)x^2+a^4(3\,m-1)x^{6\,m})}$$

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*x*x^(3*m) + b*x^2)/(2*a^3*b*(3*m - 1)*x*x^(3*m) + a^2*b^2*(3*m -
1)*x^2 + a^4*(3*m - 1)*x^(6*m))
```

Giac [F]

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = \int \frac{1}{(ax^m+bx^{-2\,m+1})^3}\,dx$$

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*x^(-2*m + 1))^(-3), x)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{1}{(bx^{1-2m}+ax^m)^3}\,dx = -\frac{x\,(b\,x+2\,a\,x^{3\,m})}{2\,a^2\,(3\,m-1)\,(b\,x+a\,x^{3\,m})^2}$$

```
[In] int(1/(a*x^m + b*x^(1 - 2*m))^3,x)
```

```
[Out] -(x*(b*x + 2*a*x^(3*m)))/(2*a^2*(3*m - 1)*(b*x + a*x^(3*m))^2)
```

3.338 $\int \frac{1}{\frac{b}{x}+ax}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{1}{\frac{b}{x}+ax}\,dx = \frac{\log(b+ax^2)}{2a}$$

```
[Out] 1/2*ln(a*x^2+b)/a
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 266}

$$\int \frac{1}{\frac{b}{x}+ax}\,dx = \frac{\log(ax^2+b)}{2a}$$

```
[In] Int[(b/x + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^2]/(2*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x}{b+ax^2}\,dx\\ &= \frac{\log\left(b+ax^2\right)}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{b}{x}+ax}\,dx = \frac{\log\left(b+ax^2\right)}{2a}$$

```
[In] Integrate[(b/x + a*x)^(-1),x]
[Out] Log[b + a*x^2]/(2*a)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln\left(a\,x^{2}+b\right)}{2a}$	14
norman	$\frac{\ln\left(a\,x^{2}+b\right)}{2a}$	14
risch	$\frac{\ln\left(a\,x^{2}+b\right)}{2a}$	14
parallelrisch	$\frac{\ln\left(a\,x^{2}+b\right)}{2a}$	14

```
[In] int(1/(b/x+a*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(a*x^2+b)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log\left(ax^{2}+b\right)}{2\,a}$$

```
[In] integrate(1/(b/x+a*x),x, algorithm="fricas")
[Out] 1/2*log(a*x^2 + b)/a
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log\left(ax^{2}+b\right)}{2a}$$

```
[In] integrate(1/(b/x+a*x),x)
[Out] log(a*x**2 + b)/(2*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log\left(ax^{2}+b\right)}{2\,a}$$

```
[In] integrate(1/(b/x+a*x),x, algorithm="maxima")
[Out] 1/2*log(a*x^2 + b)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\log\left(|ax^{2}+b|\right)}{2\,a}$$

```
[In] integrate(1/(b/x+a*x),x, algorithm="giac")
[Out] 1/2*log(abs(a*x^2 + b))/a
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x}+ax}\,dx=\frac{\ln\left(a\,x^2+b\right)}{2\,a}$$

```
[In] int(1/(a*x + b/x),x)
```

```
[Out] log(b + a*x^2)/(2*a)
```

3.339 $\int \frac{1}{\frac{b}{x^2}+ax} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{1}{\frac{b}{x^2}+ax} \, dx = \frac{\log(b+ax^3)}{3a}$$

```
[Out] 1/3*ln(a*x^3+b)/a
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 266}

$$\int \frac{1}{\frac{b}{x^2}+ax} \, dx = \frac{\log(ax^3+b)}{3a}$$

```
[In] Int[(b/x^2 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^3]/(3*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{x^2}{b+ax^3}\,dx \\ &= \frac{\log\left(b+ax^3\right)}{3a} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx = \frac{\log\left(b+ax^3\right)}{3a}$$

```
[In] Integrate[(b/x^2 + a*x)^(-1),x]
[Out] Log[b + a*x^3]/(3*a)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln\left(a\,x^{3}+b\right)}{3a}$	14
norman	$\frac{\ln\left(a\,x^{3}+b\right)}{3a}$	14
risch	$\frac{\ln\left(a\,x^{3}+b\right)}{3a}$	14
parallelrisch	$\frac{\ln\left(a\,x^{3}+b\right)}{3a}$	14

```
[In] int(1/(b/x^2+a*x),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(a*x^3+b)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\log\left(ax^3+b\right)}{3\,a}$$

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="fricas")
[Out] 1/3*log(a*x^3 + b)/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\log\left(ax^3+b\right)}{3a}$$

```
[In] integrate(1/(b/x**2+a*x),x)
[Out] log(a*x**3 + b)/(3*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\log\left(ax^3+b\right)}{3\,a}$$

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="maxima")
[Out] 1/3*log(a*x^3 + b)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\log\left(|ax^3+b|\right)}{3\,a}$$

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="giac")
[Out] 1/3*log(abs(a*x^3 + b))/a
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^2}+ax}\,dx=\frac{\ln\left(a\,x^3+b\right)}{3\,a}$$

```
[In] int(1/(a*x + b/x^2),x)
```

```
[Out] log(b + a*x^3)/(3*a)
```

3.340 $\int \frac{1}{\frac{b}{x^3}+ax}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log\left(b+ax^4\right)}{4a}$$

```
[Out] 1/4*ln(a*x^4+b)/a
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 266}

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log\left(ax^4+b\right)}{4a}$$

```
[In] Int[(b/x^3 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^4]/(4*a)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^3}{b+ax^4}\,dx\\ &= \frac{\log\left(b+ax^4\right)}{4a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx = \frac{\log\left(b+ax^4\right)}{4a}$$

```
[In] Integrate[(b/x^3 + a*x)^(-1),x]
[Out] Log[b + a*x^4]/(4*a)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln\left(a\,x^{4}+b\right)}{4a}$	14
norman	$\frac{\ln\left(a\,x^{4}+b\right)}{4a}$	14
risch	$\frac{\ln\left(a\,x^{4}+b\right)}{4a}$	14
parallelrisch	$\frac{\ln\left(a\,x^{4}+b\right)}{4a}$	14

```
[In] int(1/(b/x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(a*x^4+b)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log\left(ax^4+b\right)}{4\,a}$$

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="fricas")
[Out] 1/4*log(a*x^4 + b)/a
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log\left(ax^4+b\right)}{4a}$$

```
[In] integrate(1/(b/x**3+a*x),x)
[Out] log(a*x**4 + b)/(4*a)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log\left(ax^4+b\right)}{4\,a}$$

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="maxima")
[Out] 1/4*log(a*x^4 + b)/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\log\left(|ax^4+b|\right)}{4\,a}$$

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="giac")
[Out] 1/4*log(abs(a*x^4 + b))/a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\frac{b}{x^3}+ax}\,dx=\frac{\ln\left(a\,x^4+b\right)}{4\,a}$$

```
[In] int(1/(a*x + b/x^3),x)
[Out] log(b + a*x^4)/(4*a)
```

3.341 $\int \frac{1}{\left(\frac{b}{x}+ax\right)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3} \, dx=\frac{x^4}{4b\left(b+ax^2\right)^2}$$

```
[Out] 1/4*x^4/b/(a*x^2+b)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 270}

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3} \, dx=\frac{x^4}{4b\left(ax^2+b\right)^2}$$

```
[In] Int[(b/x + a*x)^(-3),x]
```

```
[Out] x^4/(4*b*(b + a*x^2)^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

`PosQ[q - p]`

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^3}{(b+ax^2)^3}\,dx\\ &= \frac{x^4}{4b\left(b+ax^2\right)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = -\frac{b+2ax^2}{4a^2\left(b+ax^2\right)^2}$$

```
[In] Integrate[(b/x + a*x)^(-3),x]
[Out] -1/4*(b + 2*a*x^2)/(a^2*(b + a*x^2)^2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2a\,x^2+b}{4\left(a\,x^2+b\right)^2 a^2}$	23
parallelrisch	$\frac{-2a\,x^2-b}{4a^2\left(a\,x^2+b\right)^2}$	25
norman	$\frac{-\frac{x^2}{2a}-\frac{b}{4a^2}}{\left(a\,x^2+b\right)^2}$	26
risch	$\frac{-\frac{x^2}{2a}-\frac{b}{4a^2}}{\left(a\,x^2+b\right)^2}$	26
default	$-\frac{1}{2a^2\left(a\,x^2+b\right)}+\frac{b}{4a^2\left(a\,x^2+b\right)^2}$	31

```
[In] int(1/(b/x+a*x)^3,x,method=_RETURNVERBOSE)
[Out] -1/4*(2*a*x^2+b)/(a*x^2+b)^2/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = -\frac{2\,ax^2+b}{4\left(a^4x^4+2\,a^3bx^2+a^2b^2\right)}$$

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="fricas")
[Out] -1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = \frac{-2ax^2-b}{4a^4x^4+8a^3bx^2+4a^2b^2}$$

```
[In] integrate(1/(b/x+a*x)**3,x)
[Out] (-2*a*x**2 - b)/(4*a**4*x**4 + 8*a**3*b*x**2 + 4*a**2*b**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = -\frac{2\,ax^2+b}{4\left(a^4x^4+2\,a^3bx^2+a^2b^2\right)}$$

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="maxima")
[Out] -1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = -\frac{2\,ax^2+b}{4\,(ax^2+b)^2a^2}$$

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="giac")
[Out] -1/4*(2*a*x^2 + b)/((a*x^2 + b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx = -\frac{\frac{b}{4\,a^2}+\frac{x^2}{2\,a}}{a^2\,x^4+2\,a\,b\,x^2+b^2}$$

```
[In] int(1/(a*x + b/x)^3,x)
[Out] -(b/(4*a^2) + x^2/(2*a))/(b^2 + a^2*x^4 + 2*a*b*x^2)
```

3.342 $\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx=\frac{x^{10}}{10b\left(b+ax^5\right)^2}$$

```
[Out] 1/10*x^10/b/(a*x^5+b)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1607, 270}

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx=\frac{x^{10}}{10b\left(ax^5+b\right)^2}$$

```
[In] Int[(b/x^3 + a*x^2)^(-3),x]
```

```
[Out] x^10/(10*b*(b + a*x^5)^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^9}{\left(b+ax^5\right)^3}\,dx\\ &= \frac{x^{10}}{10b\left(b+ax^5\right)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{b+2ax^5}{10a^2\left(b+ax^5\right)^2}$$

```
[In] Integrate[(b/x^3 + a*x^2)^(-3),x]
[Out] -1/10*(b + 2*a*x^5)/(a^2*(b + a*x^5)^2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2x^5a+b}{10\left(x^5a+b\right)^2a^2}$	23
parallelrisch	$\frac{-2x^5a-b}{10a^2\left(x^5a+b\right)^2}$	25
norman	$\frac{-\frac{x^5}{5a}-\frac{b}{10a^2}}{\left(x^5a+b\right)^2}$	26
risch	$\frac{-\frac{x^5}{5a}-\frac{b}{10a^2}}{\left(x^5a+b\right)^2}$	26
default	$-\frac{1}{5a^2\left(x^5a+b\right)}+\frac{b}{10a^2\left(x^5a+b\right)^2}$	31

```
[In] int(1/(b/x^3+a*x^2)^3,x,method=_RETURNVERBOSE)
[Out] -1/10*(2*a*x^5+b)/(a*x^5+b)^2/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{2\,ax^5+b}{10\,\left(a^4x^{10}+2\,a^3bx^5+a^2b^2\right)}$$

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="fricas")
[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = \frac{-2ax^5-b}{10a^4x^{10}+20a^3bx^5+10a^2b^2}$$

```
[In] integrate(1/(b/x**3+a*x**2)**3,x)
[Out] (-2*a*x**5 - b)/(10*a**4*x**10 + 20*a**3*b*x**5 + 10*a**2*b**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{2\,ax^5+b}{10\,\left(a^4x^{10}+2\,a^3bx^5+a^2b^2\right)}$$

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="maxima")
[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{2\,ax^5+b}{10\,(ax^5+b)^2a^2}$$

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/10*(2*a*x^5 + b)/((a*x^5 + b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx = -\frac{\frac{b}{10\,a^2}+\frac{x^5}{5\,a}}{a^2\,x^{10}+2\,a\,b\,x^5+b^2}$$

```
[In] int(1/(a*x^2 + b/x^3)^3,x)
```

```
[Out] -(b/(10*a^2) + x^5/(5*a))/(b^2 + a^2*x^10 + 2*a*b*x^5)
```

3.343 $\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\, dx = \frac{x^{16}}{16b\left(b+ax^8\right)^2}$$

```
[Out] 1/16*x^16/b/(a*x^8+b)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1607, 270}

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\, dx = \frac{x^{16}}{16b\left(ax^8+b\right)^2}$$

```
[In] Int[(b/x^5 + a*x^3)^(-3),x]
```

```
[Out] x^16/(16*b*(b + a*x^8)^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^{15}}{(b+ax^8)^3}\,dx \\
&= \frac{x^{16}}{16b\,(b+ax^8)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = -\frac{b+2ax^8}{16a^2\,(b+ax^8)^2}$$

```
[In] Integrate[(b/x^5 + a*x^3)^(-3),x]
[Out] -1/16*(b + 2*a*x^8)/(a^2*(b + a*x^8)^2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2a\,x^8+b}{16(a\,x^8+b)^2a^2}$	23
parallelrisch	$\frac{-2a\,x^8-b}{16a^2(a\,x^8+b)^2}$	25
norman	$\frac{-\frac{b}{16a^2}-\frac{x^8}{8a}}{(a\,x^8+b)^2}$	26
risch	$\frac{-\frac{b}{16a^2}-\frac{x^8}{8a}}{(a\,x^8+b)^2}$	26
default	$-\frac{1}{8a^2(a\,x^8+b)}+\frac{b}{16a^2(a\,x^8+b)^2}$	31

```
[In] int(1/(b/x^5+a*x^3)^3,x,method=_RETURNVERBOSE)
[Out] -1/16*(2*a*x^8+b)/(a*x^8+b)^2/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = -\frac{2\,ax^8+b}{16\,\left(a^4x^{16}+2\,a^3bx^8+a^2b^2\right)}$$

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="fricas")
[Out] -1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = \frac{-2ax^8-b}{16a^4x^{16}+32a^3bx^8+16a^2b^2}$$

```
[In] integrate(1/(b/x**5+a*x**3)**3,x)
[Out] (-2*a*x**8 - b)/(16*a**4*x**16 + 32*a**3*b*x**8 + 16*a**2*b**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx = -\frac{2\,ax^8+b}{16\,\left(a^4x^{16}+2\,a^3bx^8+a^2b^2\right)}$$

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="maxima")
[Out] -1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx=-\frac{2\,ax^8+b}{16\,(ax^8+b)^2a^2}$$

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="giac")
[Out] -1/16*(2*a*x^8 + b)/((a*x^8 + b)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx=-\frac{\frac{b}{16\,a^2}+\frac{x^8}{8\,a}}{a^2\,x^{16}+2\,a\,b\,x^8+b^2}$$

```
[In] int(1/(a*x^3 + b/x^5)^3,x)
[Out] -(b/(16*a^2) + x^8/(8*a))/(b^2 + a^2*x^16 + 2*a*b*x^8)
```

3.344 $\int \left(\frac{a}{x}+bx\right)^2 dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \left(\frac{a}{x}+bx\right)^2 dx=-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

```
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 276}

$$\int \left(\frac{a}{x}+bx\right)^2 dx=-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

```
[In] Int[(a/x + b*x)^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(a+bx^2)^2}{x^2}\,dx\\ &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \left(\frac{a}{x}+bx\right)^2 dx = -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

```
[In] Integrate[(a/x + b*x)^2,x]
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
risch	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
norman	$\frac{\frac{1}{3}b^2x^4+2ab\,x^2-a^2}{x}$	26
parallelrisch	$\frac{b^2x^4+6ab\,x^2-3a^2}{3x}$	26
gosper	$-\frac{-b^2x^4-6ab\,x^2+3a^2}{3x}$	27

```
[In] int((a/x+b*x)^2,x,method=_RETURNVERBOSE)
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \left(\frac{a}{x}+bx\right)^2\,dx=\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

```
[In] integrate((a/x+b*x)^2,x, algorithm="fricas")
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \left(\frac{a}{x}+bx\right)^2\,dx=-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

```
[In] integrate((a/x+b*x)**2,x)
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^2\,dx=\frac{1}{3}\,b^2x^3+2\,abx-\frac{a^2}{x}$$

```
[In] integrate((a/x+b*x)^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^2\,dx=\frac{1}{3}\,b^2x^3+2\,abx-\frac{a^2}{x}$$

```
[In] integrate((a/x+b*x)^2,x, algorithm="giac")
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^2\,dx=\frac{b^2\,x^3}{3}-\frac{a^2}{x}+2\,a\,b\,x$$

```
[In] int((b*x + a/x)^2,x)
```

```
[Out] (b^2*x^3)/3 - a^2/x + 2*a*b*x
```

3.345 $\int \left(\frac{a}{x}+bx\right)^3 dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \left(\frac{a}{x}+bx\right)^3 dx = -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)$$

```
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1607, 272, 45}

$$\int \left(\frac{a}{x}+bx\right)^3 dx = -\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

```
[In] Int[(a/x + b*x)^3,x]
```

```
[Out] -1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{(a+bx^2)^3}{x^3}\,dx \\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^3}{x^2}\,dx, x, x^2\right) \\ &= \frac{1}{2}\text{Subst}\left(\int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right)\,dx, x, x^2\right) \\ &= -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \left(\frac{a}{x}+bx\right)^3\,dx = -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)$$

```
[In] Integrate[(a/x + b*x)^3,x]
[Out] -1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^3}{2x^2}+\frac{3a\,b^2x^2}{2}+\frac{b^3x^4}{4}+3a^2b\ln\left(x\right)$	35
norman	$\frac{-\frac{1}{2}a^3+\frac{1}{4}b^3x^6+\frac{3}{2}a\,b^2x^4}{x^2}+3a^2b\ln\left(x\right)$	37
parallelrisch	$\frac{b^3x^6+6a\,b^2x^4+12a^2b\ln(x)x^2-2a^3}{4x^2}$	39
risch	$\frac{b^3x^4}{4}+\frac{3a\,b^2x^2}{2}+\frac{9a^2b}{4}-\frac{a^3}{2x^2}+3a^2b\ln\left(x\right)$	41

```
[In] int((a/x+b*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \left(\frac{a}{x}+bx\right)^3\,dx=\frac{b^3x^6+6\,ab^2x^4+12\,a^2bx^2\log\left(x\right)-2\,a^3}{4\,x^2}$$

```
[In] integrate((a/x+b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \left(\frac{a}{x}+bx\right)^3\,dx=-\frac{a^3}{2x^2}+3a^2b\log\left(x\right)+\frac{3ab^2x^2}{2}+\frac{b^3x^4}{4}$$

```
[In] integrate((a/x+b*x)**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \left(\frac{a}{x}+bx\right)^3\,dx=\frac{1}{4}\,b^3x^4+\frac{3}{2}\,ab^2x^2+3\,a^2b\log\left(x\right)-\frac{a^3}{2\,x^2}$$

```
[In] integrate((a/x+b*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*log(x) - 1/2*a^3/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \left(\frac{a}{x}+bx\right)^3\,dx=\frac{1}{4}\,b^3x^4+\frac{3}{2}\,ab^2x^2+\frac{3}{2}\,a^2b\log\left(x^2\right)-\frac{3\,a^2bx^2+a^3}{2\,x^2}$$

```
[In] integrate((a/x+b*x)^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/
x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \left(\frac{a}{x}+bx\right)^3\,dx=\frac{b^3\,x^4}{4}-\frac{a^3}{2\,x^2}+\frac{3\,a\,b^2\,x^2}{2}+3\,a^2\,b\,\ln\left(x\right)$$

```
[In] int((b*x + a/x)^3,x)
```

```
[Out] (b^3*x^4)/4 - a^3/(2*x^2) + (3*a*b^2*x^2)/2 + 3*a^2*b*log(x)
```

3.346 $\int \left(\frac{a}{x}+bx\right)^4 dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \left(\frac{a}{x}+bx\right)^4 dx=-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

```
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 276}

$$\int \left(\frac{a}{x}+bx\right)^4 dx=-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

```
[In] Int[(a/x + b*x)^4,x]
```

```
[Out] -1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(a+bx^2\right)^4}{x^4}\,dx\\
&= \int \left(6a^2b^2+\frac{a^4}{x^4}+\frac{4a^3b}{x^2}+4ab^3x^2+b^4x^4\right)dx\\
&= -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(\frac{a}{x}+bx\right)^4 dx = -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

```
[In] Integrate[(a/x + b*x)^4,x]
[Out] -1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4a\,b^3x^3}{3}+\frac{b^4x^5}{5}$	45
risch	$\frac{b^4x^5}{5}+\frac{4a\,b^3x^3}{3}+6a^2b^2x+\frac{-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	47
norman	$\frac{\frac{1}{5}x^8b^4+\frac{4}{3}a\,b^3x^6+6a^2x^4b^2-4a^3b\,x^2-\frac{1}{3}a^4}{x^3}$	48
gosper	$-\frac{-3x^8b^4-20a\,b^3x^6-90a^2x^4b^2+60a^3b\,x^2+5a^4}{15x^3}$	49
parallelrisch	$\frac{3x^8b^4+20a\,b^3x^6+90a^2x^4b^2-60a^3b\,x^2-5a^4}{15x^3}$	49

```
[In] int((a/x+b*x)^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(\frac{a}{x}+bx\right)^4 dx = \frac{3\,b^4x^8+20\,ab^3x^6+90\,a^2b^2x^4-60\,a^3bx^2-5\,a^4}{15\,x^3}$$

```
[In] integrate((a/x+b*x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \left(\frac{a}{x}+bx\right)^4 dx = 6a^2b^2x+\frac{4ab^3x^3}{3}+\frac{b^4x^5}{5}+\frac{-a^4-12a^3bx^2}{3x^3}$$

```
[In] integrate((a/x+b*x)**4,x)
```

```
[Out] 6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 + (-a**4 - 12*a**3*b*x**2)/(3
*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \left(\frac{a}{x}+bx\right)^4 dx = \frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{4\,a^3b}{x}-\frac{a^4}{3\,x^3}$$

```
[In] integrate((a/x+b*x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 4*a^3*b/x - 1/3*a^4/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \left(\frac{a}{x}+bx\right)^4 dx = \frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{12\,a^3bx^2+a^4}{3\,x^3}$$

```
[In] integrate((a/x+b*x)^4,x, algorithm="giac")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \left(\frac{a}{x}+bx\right)^4\,dx=\frac{b^4\,x^5}{5}-\frac{\frac{a^4}{3}+4\,b\,a^3\,x^2}{x^3}+6\,a^2\,b^2\,x+\frac{4\,a\,b^3\,x^3}{3}$$

```
[In] int((b*x + a/x)^4,x)
```

```
[Out] (b^4*x^5)/5 - (a^4/3 + 4*a^3*b*x^2)/x^3 + 6*a^2*b^2*x + (4*a*b^3*x^3)/3
```

3.347 $\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 185

$$\begin{aligned}\int \frac{1}{\frac{1}{x^2}+x^3}\,dx=&-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}+2\sqrt{\frac{2}{5+\sqrt{5}}}x\right)\\&-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}x\right)\\&+\frac{1}{5}\log(1+x)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2\right)\\&-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2\right)\end{aligned}$$

```
[Out] 1/5*ln(1+x)-1/20*ln(1+x^2-1/2*x*(5^(1/2)+1))*(-5^(1/2)+1)-1/20*ln(1+x^2-1/2
*x*(-5^(1/2)+1))*(5^(1/2)+1)-1/10*arctan(1/5*(25-10*5^(1/2))^(1/2)+2*x*2^(1
/2)/(5+5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/10*arctan(1/5*x*(50+10*5^(1/2
))^(1/2)-1/5*(25+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used

$= \{1607, 299, 648, 632, 210, 642, 31\}$

$$\begin{aligned}\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = &-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(2\sqrt{\frac{2}{5+\sqrt{5}}}x+\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}\right)\\&-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}x\right)\\&-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)\\&-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)\end{aligned}$$

```
[In] Int[(x^(-2) + x^3)^(-1),x]
```

```
[Out] -1/5*(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 +
Sqrt[5])]*x]) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqr
t[(2*(5 + Sqrt[5]))/5]*x])/5 + Log[1 + x]/5 - ((1 + Sqrt[5])*Log[1 - ((1 -
Sqrt[5])*x)/2 + x^2])/20 - ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2
])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 299

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x]; (-(-r)^(m + 1)/(a*n*s^m))*Int[1/(r + s*x), x]
 + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Free
Q[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a
/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{1+x^5}\,dx$$

$$= \frac{2}{5}\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx + \frac{2}{5}\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx + \frac{1}{5}\int \frac{1}{1+x}\,dx$$

$$= \frac{1}{5}\log(1+x) + \frac{\int \frac{1}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}} - \frac{\int \frac{1}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}}$$
$$+ \frac{1}{20}\left(-1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1+\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx$$
$$+ \frac{1}{20}\left(-1+\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx$$

$$= \frac{1}{5}\log(1+x) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x-\sqrt{5}x+2x^2\right)$$
$$- \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x+\sqrt{5}x+2x^2\right)$$
$$+ \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{2}\left(-5-\sqrt{5}\right)-x^2}\,dx, x, \frac{1}{2}\left(-1+\sqrt{5}\right)+2x\right)}{\sqrt{5}}$$
$$- \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{2}\left(-5+\sqrt{5}\right)-x^2}\,dx, x, \frac{1}{2}\left(-1-\sqrt{5}\right)+2x\right)}{\sqrt{5}}$$

$$
\begin{aligned}
&= \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) \\
&\quad -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}-4x\right)\right)+\frac{1}{5}\log(1+x) \\
&\quad -\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x-\sqrt{5}x+2x^2\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x+\sqrt{5}x+2x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.78

$$
\begin{aligned}
\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = \frac{1}{20}\Bigg(&-2\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1+\sqrt{5}-4x}{\sqrt{10-2\sqrt{5}}}\right) \\
&-2\sqrt{10-2\sqrt{5}}\arctan\left(\frac{-1+\sqrt{5}+4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)+4\log(1+x) \\
&-\left(1+\sqrt{5}\right)\log\left(1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2\right) \\
&+\left(-1+\sqrt{5}\right)\log\left(1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[(x^(-2) + x^3)^(-1),x]
```

```
[Out] (-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]]
- 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]]
 + 4*Log[1 + x] - (1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] + (-1 +
 Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.18

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^3+\_Z^2+\_Z+1\right)}\_R\ln\left(\_R^2+x\right)\right)}{5}+\frac{\ln(1+x)}{5}$
default	$\frac{\ln(1+x)}{5}-\frac{\left(-\sqrt{5}+1\right)\ln\left(-x\sqrt{5}+2x^2-x+2\right)}{20}-\frac{2\left(-\sqrt{5}+1-\frac{\left(-\sqrt{5}+1\right)\left(-\sqrt{5}-1\right)}{4}\right)\arctan\left(\frac{-\sqrt{5}+4x-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}+\frac{\left(-\sqrt{5}-1\right)\ln\left(x\sqrt{5}+\right.}{20}$

```
[In] int(1/(1/x^2+x^3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*sum(_R*ln(_R^2+x),_R=RootOf(_Z^4+_Z^3+_Z^2+_Z+1))+1/5*ln(1+x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(122) = 244$.

Time = 0.90 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.44

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$$

$$= -\frac{1}{20}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(\frac{1}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+x\right)$$

$$+\frac{1}{20}\Bigg(\sqrt{5}+2\sqrt{-\frac{3}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{1}{8}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\Bigg(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-}$$

$$-\frac{1}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2$$

$$+\frac{1}{2}\sqrt{-\frac{3}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{1}{8}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)}$$

$$+2\,x-1\Bigg)$$

$$+\frac{1}{20}\Bigg(\sqrt{5}-2\sqrt{-\frac{3}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{1}{8}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\Bigg(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-}$$

$$-\frac{1}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2$$

$$-\frac{1}{2}\sqrt{-\frac{3}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+\frac{1}{8}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)}$$

$$+2\,x-1\Bigg)$$

$$+\frac{1}{20}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\log\left(\frac{1}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2+x\right)$$

$$+\frac{1}{5}\log\left(x+1\right)$$

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(1/16*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + x) + 1/20*(sqrt(5) + 2*sqrt(-3/16*(2*sq
rt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5)
```

```
- 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2
*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5)
 + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(
5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 1/2*sqrt
(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) -
1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(
sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x - 1) + 1/20*(sqrt(5)
- 2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqr
t(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sq
rt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/
2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(
sqrt(5) - 5) + sqrt(5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(
5) - 1)^2 - 1/2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2
+ 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)
^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x -
 1) + 1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(1/16*(2*sqrt(1
/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + x) + 1/5*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.19

$$\begin{aligned}\int \frac{1}{\frac{1}{x^2}+x^3}\,dx\\ =\frac{\log(x+1)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(25t^2+x\right)\right)\right)\end{aligned}$$

```
[In] integrate(1/(1/x**2+x**3),x)
```

```
[Out] log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(
_t, _t*log(25*_t**2 + x)))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.67

$$\begin{aligned}\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = {}&-\frac{2\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}\\ &+\frac{\log\left(2x^2-x(\sqrt{5}+1)+2\right)}{5\left(\sqrt{5}+1\right)}-\frac{\log\left(2x^2+x(\sqrt{5}-1)+2\right)}{5\left(\sqrt{5}-1\right)}+\frac{1}{5}\log(x+1)\end{aligned}$$

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*sqrt(5
) + 10) + 2/5*sqrt(5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))/sqr
t(-2*sqrt(5) + 10) + 1/5*log(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) - 1
/5*log(2*x^2 + x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) + 1/5*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.61

$$
\begin{aligned}
\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = {} & \frac{1}{20}\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}\,x\left(\sqrt{5}+1\right)+1\right) \\
& -\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(x^2+\frac{1}{2}\,x\left(\sqrt{5}-1\right)+1\right) \\
& -\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4\,x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right) \\
& +\frac{1}{10}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{4\,x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{1}{5}\log\left(|x+1|\right)
\end{aligned}
$$

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*(sqrt(5) + 1)*
log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*x
 + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan((4
*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) + 1/5*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06

$$
\begin{aligned}
\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = {} & \frac{\ln\left(x+1\right)}{5} \\
& -\ln\left(1-\frac{x\left(\sqrt{2}\,\sqrt{-\sqrt{5}-5}-\sqrt{5}+1\right)^3}{64}\right)\left(\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right) \\
& +\ln\left(\frac{x\left(\sqrt{2}\,\sqrt{-\sqrt{5}-5}+\sqrt{5}-1\right)^3}{64}+1\right)\left(\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{20}+\frac{\sqrt{5}}{20}-\frac{1}{20}\right) \\
& -\ln\left(1-\frac{x\left(\sqrt{5}+\sqrt{2}\,\sqrt{\sqrt{5}-5}+1\right)^3}{64}\right)\left(\frac{\sqrt{5}}{20}+\frac{\sqrt{2}\,\sqrt{\sqrt{5}-5}}{20}+\frac{1}{20}\right) \\
& -\ln\left(1-\frac{x\left(\sqrt{5}-\sqrt{2}\,\sqrt{\sqrt{5}-5}+1\right)^3}{64}\right)\left(\frac{\sqrt{5}}{20}-\frac{\sqrt{2}\,\sqrt{\sqrt{5}-5}}{20}+\frac{1}{20}\right)
\end{aligned}
$$

```
[In] int(1/(1/x^2 + x^3),x)
```

```
[Out] log(x + 1)/5 - log(1 - (x*(2^(1/2)*(- 5^(1/2) - 5)^(1/2) - 5^(1/2) + 1)^3)/
64)*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) + log((x*(2^(1
/2)*(- 5^(1/2) - 5)^(1/2) + 5^(1/2) - 1)^3)/64 + 1)*((2^(1/2)*(- 5^(1/2) -
5)^(1/2))/20 + 5^(1/2)/20 - 1/20) - log(1 - (x*(5^(1/2) + 2^(1/2)*(5^(1/2)
- 5)^(1/2) + 1)^3)/64)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/2
0) - log(1 - (x*(5^(1/2) - 2^(1/2)*(5^(1/2) - 5)^(1/2) + 1)^3)/64)*(5^(1/2)
/20 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20)
```

3.348 $\int x^p(ax^n+bx^{1+13n+p})^{12}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 29

$$\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx=\frac{\left(a+bx^{1+12n+p}\right)^{13}}{13b(1+12n+p)}$$

```
[Out] 1/13*(a+b*x^(1+12*n+p))^13/b/(1+12*n+p)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1598, 267}

$$\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx=\frac{\left(a+bx^{12n+p+1}\right)^{13}}{13b(12n+p+1)}$$

```
[In] Int[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

```
[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int x^{12n+p}\left(a+bx^{1+12n+p}\right)^{12}\,dx \\ &= \frac{\left(a+bx^{1+12n+p}\right)^{13}}{13b(1+12n+p)} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 232 vs. $2(29) = 58$.

Time = 0.19 (sec) , antiderivative size = 232, normalized size of antiderivative = 8.00

$$\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx$$

$$= \frac{x^{1+12n+p}(13a^{12}+78a^{11}bx^{1+12n+p}+286a^{10}b^2x^{2+24n+2p}+715a^9b^3x^{3+36n+3p}+1287a^8b^4x^{4+48n+4p}+1716a^7b}{}$$

```
[In] Integrate[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

```
[Out] (x^(1 + 12*n + p)*(13*a^12 + 78*a^11*b*x^(1 + 12*n + p) + 286*a^10*b^2*x^(2
 + 24*n + 2*p) + 715*a^9*b^3*x^(3 + 36*n + 3*p) + 1287*a^8*b^4*x^(4 + 48*n
+ 4*p) + 1716*a^7*b^5*x^(5 + 60*n + 5*p) + 1716*a^6*b^6*x^(6 + 72*n + 6*p)
+ 1287*a^5*b^7*x^(7 + 84*n + 7*p) + 715*a^4*b^8*x^(8 + 96*n + 8*p) + 286*a^
3*b^9*x^(9 + 108*n + 9*p) + 78*a^2*b^10*x^(10 + 120*n + 10*p) + 13*a*b^11*x
^(11 + 132*n + 11*p) + b^12*x^(12 + 144*n + 12*p)))/(13*(1 + 12*n + p))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 363, normalized size of antiderivative = 12.52

$$\frac{b^{12}x^{13}x^{156n}x^{13p}}{13+156n+13p}+\frac{a\,b^{11}x^{12}x^{144n}x^{12p}}{1+12n+p}+\frac{6a^2b^{10}x^{11}x^{132n}x^{11p}}{1+12n+p}+\frac{22a^3b^9x^{10}x^{120n}x^{10p}}{1+12n+p}+\frac{55a^4b^8x^9x^{108n}x^{9p}}{1+12n+p}+\frac{99a^5b^7x}{1+12}$$

```
[In] int(x^p*(a*x^n+b*x^(1+13*n+p))^12,x)
```

```
[Out] 1/13*b^12*x^13*(x^n)^156/(1+12*n+p)*(x^p)^13+a*b^11*x^12*(x^n)^144/(1+12*n+
p)*(x^p)^12+6*a^2*b^10*x^11*(x^n)^132/(1+12*n+p)*(x^p)^11+22*a^3*b^9*x^10*(
x^n)^120/(1+12*n+p)*(x^p)^10+55*a^4*b^8*x^9*(x^n)^108/(1+12*n+p)*(x^p)^9+99
*a^5*b^7*x^8*(x^n)^96/(1+12*n+p)*(x^p)^8+132*a^6*b^6*x^7*(x^n)^84/(1+12*n+p
)*(x^p)^7+132*a^7*b^5*x^6*(x^n)^72/(1+12*n+p)*(x^p)^6+99*a^8*b^4*x^5*(x^n)^
60/(1+12*n+p)*(x^p)^5+55*a^9*b^3*x^4*(x^n)^48/(1+12*n+p)*(x^p)^4+22*a^10*b^
2*x^3*(x^n)^36/(1+12*n+p)*(x^p)^3+6*b*a^11*x^2*(x^n)^24/(1+12*n+p)*(x^p)^2+
a^12/(1+12*n+p)*x*(x^n)^12*x^p
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 297, normalized size of antiderivative = 10.24

$$\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} \, dx$$
$$= \frac{78\, a^2 b^{10} x^{2\,n} x^{143\,n+11\,p+11} + 286\, a^3 b^9 x^{3\,n} x^{130\,n+10\,p+10} + 715\, a^4 b^8 x^{4\,n} x^{117\,n+9\,p+9} + 1287\, a^5 b^7 x^{5\,n} x^{104\,n+8\,p+8} +}{}$$

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(78*a^2*b^10*x^(2*n)*x^(143*n + 11*p + 11) + 286*a^3*b^9*x^(3*n)*x^(13
0*n + 10*p + 10) + 715*a^4*b^8*x^(4*n)*x^(117*n + 9*p + 9) + 1287*a^5*b^7*x
^(5*n)*x^(104*n + 8*p + 8) + 1716*a^6*b^6*x^(6*n)*x^(91*n + 7*p + 7) + 1716
*a^7*b^5*x^(7*n)*x^(78*n + 6*p + 6) + 1287*a^8*b^4*x^(8*n)*x^(65*n + 5*p +
5) + 715*a^9*b^3*x^(9*n)*x^(52*n + 4*p + 4) + 286*a^10*b^2*x^(10*n)*x^(39*n
 + 3*p + 3) + 78*a^11*b*x^(11*n)*x^(26*n + 2*p + 2) + 13*a^12*x^(12*n)*x^(1
3*n + p + 1) + 13*a*b^11*x^(156*n + 12*p + 12)*x^n + b^12*x^(169*n + 13*p +
 13))/((12*n + p + 1)*x^(13*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(22) = 44.

Time = 149.34 (sec) , antiderivative size = 690, normalized size of antiderivative = 23.79

$$\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} \, dx = \text{Too large to display}$$

```
[In] integrate(x**p*(a*x**n+b*x**(1+13*n+p))**12,x)
```

```
[Out] Piecewise((13*a**12*x*x**(12*n)*x**p/(156*n + 13*p + 13) + 78*a**11*b*x*x**
(11*n)*x**p*x**(13*n + p + 1)/(156*n + 13*p + 13) + 286*a**10*b**2*x*x**(10
*n)*x**p*x**(26*n + 2*p + 2)/(156*n + 13*p + 13) + 715*a**9*b**3*x*x**(9*n)
*x**p*x**(39*n + 3*p + 3)/(156*n + 13*p + 13) + 1287*a**8*b**4*x*x**(8*n)*x
**p*x**(52*n + 4*p + 4)/(156*n + 13*p + 13) + 1716*a**7*b**5*x*x**(7*n)*x**
p*x**(65*n + 5*p + 5)/(156*n + 13*p + 13) + 1716*a**6*b**6*x*x**(6*n)*x**p*
x**(78*n + 6*p + 6)/(156*n + 13*p + 13) + 1287*a**5*b**7*x*x**(5*n)*x**p*x*
*(91*n + 7*p + 7)/(156*n + 13*p + 13) + 715*a**4*b**8*x*x**(4*n)*x**p*x**(1
04*n + 8*p + 8)/(156*n + 13*p + 13) + 286*a**3*b**9*x*x**(3*n)*x**p*x**(117
*n + 9*p + 9)/(156*n + 13*p + 13) + 78*a**2*b**10*x*x**(2*n)*x**p*x**(130*n
 + 10*p + 10)/(156*n + 13*p + 13) + 13*a*b**11*x*x**n*x**p*x**(143*n + 11*p
 + 11)/(156*n + 13*p + 13) + b**12*x*x**p*x**(156*n + 12*p + 12)/(156*n + 1
3*p + 13), Ne(n, -p/12 - 1/12)), (a**12*Piecewise((log(x), Eq(p, 0)), (log(
```

```
x**p)/p, True)) + 12*a**11*b*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, Tr
ue)) + 66*a**10*b**2*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) + 2
20*a**9*b**3*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) + 495*a**8*
b**4*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) + 792*a**7*b**5*Pie
cewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) + 924*a**6*b**6*Piecewise((
log(x), Eq(p, 0)), (log(x**p)/p, True)) + 792*a**5*b**7*Piecewise((log(x),
Eq(p, 0)), (log(x**p)/p, True)) + 495*a**4*b**8*Piecewise((log(x), Eq(p, 0)
), (log(x**p)/p, True)) + 220*a**3*b**9*Piecewise((log(x), Eq(p, 0)), (log(
x**p)/p, True)) + 66*a**2*b**10*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p,
 True)) + 12*a*b**11*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)) + b
**12*Piecewise((log(x), Eq(p, 0)), (log(x**p)/p, True)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(27) = 54$.

Time = 0.22 (sec) , antiderivative size = 325, normalized size of antiderivative = 11.21

$$\begin{aligned}\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx=&\ \frac{b^{12}x^{156\,n+13\,p+13}}{13\,(12\,n+p+1)}+\frac{ab^{11}x^{144\,n+12\,p+12}}{12\,n+p+1}+\frac{6\,a^2b^{10}x^{132\,n+11\,p+11}}{12\,n+p+1}\\&+\frac{22\,a^3b^9x^{120\,n+10\,p+10}}{12\,n+p+1}+\frac{55\,a^4b^8x^{108\,n+9\,p+9}}{12\,n+p+1}\\&+\frac{99\,a^5b^7x^{96\,n+8\,p+8}}{12\,n+p+1}+\frac{132\,a^6b^6x^{84\,n+7\,p+7}}{12\,n+p+1}\\&+\frac{132\,a^7b^5x^{72\,n+6\,p+6}}{12\,n+p+1}+\frac{99\,a^8b^4x^{60\,n+5\,p+5}}{12\,n+p+1}+\frac{55\,a^9b^3x^{48\,n+4\,p+4}}{12\,n+p+1}\\&+\frac{22\,a^{10}b^2x^{36\,n+3\,p+3}}{12\,n+p+1}+\frac{6\,a^{11}bx^{24\,n+2\,p+2}}{12\,n+p+1}+\frac{a^{12}x^{12\,n+p+1}}{12\,n+p+1}\end{aligned}$$

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*n + 13*p + 13)/(12*n + p + 1) + a*b^11*x^(144*n + 12*p + 1
2)/(12*n + p + 1) + 6*a^2*b^10*x^(132*n + 11*p + 11)/(12*n + p + 1) + 22*a^
3*b^9*x^(120*n + 10*p + 10)/(12*n + p + 1) + 55*a^4*b^8*x^(108*n + 9*p + 9)
/(12*n + p + 1) + 99*a^5*b^7*x^(96*n + 8*p + 8)/(12*n + p + 1) + 132*a^6*b^
6*x^(84*n + 7*p + 7)/(12*n + p + 1) + 132*a^7*b^5*x^(72*n + 6*p + 6)/(12*n
+ p + 1) + 99*a^8*b^4*x^(60*n + 5*p + 5)/(12*n + p + 1) + 55*a^9*b^3*x^(48*
n + 4*p + 4)/(12*n + p + 1) + 22*a^10*b^2*x^(36*n + 3*p + 3)/(12*n + p + 1)
 + 6*a^11*b*x^(24*n + 2*p + 2)/(12*n + p + 1) + a^12*x^(12*n + p + 1)/(12*n
 + p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50971 vs. 2(27) = 54.

Time = 1.46 (sec) , antiderivative size = 50971, normalized size of antiderivative = 1757.62

$$\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx=\text{Too large to display}$$

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="giac")
```

```
[Out] (31408819200*a^2*b^10*n^10*p*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) +
35*log(x)) + 331405966080*a^2*b^10*n^9*p^2*x*x^(2*n)*x^p*e^(455*n*log(x) +
35*p*log(x) + 35*log(x)) + 1230778965888*a^2*b^10*n^8*p^3*x*x^(2*n)*x^p*e^(
455*n*log(x) + 35*p*log(x) + 35*log(x)) + 2139674600448*a^2*b^10*n^7*p^4*x*
x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 1890383812992*a^2*
b^10*n^6*p^5*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 874
552702464*a^2*b^10*n^5*p^6*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35
*log(x)) + 222844093056*a^2*b^10*n^4*p^7*x*x^(2*n)*x^p*e^(455*n*log(x) + 35
*p*log(x) + 35*log(x)) + 32330382336*a^2*b^10*n^3*p^8*x*x^(2*n)*x^p*e^(455*
n*log(x) + 35*p*log(x) + 35*log(x)) + 2661766272*a^2*b^10*n^2*p^9*x*x^(2*n)
*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 115879680*a^2*b^10*n*p^10
*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 2073600*a^2*b^1
0*p^11*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 315658632
9600*a^5*b^7*n^10*p*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)
) + 3330201657024 0*a^5*b^7*n^9*p^2*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log
(x) + 34*log(x)) + 123648483714624*a^5*b^7*n^8*p^3*x*x^(5*n)*x^p*e^(442*n*l
og(x) + 34*p*log(x) + 34*log(x)) + 214873536791232*a^5*b^7*n^7*p^4*x*x^(5*n
)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 189706686719616*a^5*b^7*
n^6*p^5*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 87659938
485504*a^5*b^7*n^5*p^6*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log
(x)) + 22297720357440*a^5*b^7*n^4*p^7*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*
log(x) + 34*log(x)) + 3227734775232*a^5*b^7*n^3*p^8*x*x^(5*n)*x^p*e^(442*n*
log(x) + 34*p*log(x) + 34*log(x)) + 265050517248*a^5*b^7*n^2*p^9*x*x^(5*n)*
x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 11506302720*a^5*b^7*n*p^10
*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 205286400*a^5*b
^7*p^11*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 42087817
72800*a^7*b^5*n^10*p*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x
)) + 44402688760320*a^7*b^5*n^9*p^2*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*lo
g(x) + 33*log(x)) + 164864644952832*a^7*b^5*n^8*p^3*x*x^(7*n)*x^p*e^(429*n*
log(x) + 33*p*log(x) + 33*log(x)) + 286498049054976*a^7*b^5*n^7*p^4*x*x^(7*
n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 252942248959488*a^7*b^5
*n^6*p^5*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1168799
17980672*a^7*b^5*n^5*p^6*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*l
og(x)) + 29730293809920*a^7*b^5*n^4*p^7*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*
p*log(x) + 33*log(x)) + 430364636976*a^7*b^5*n^3*p^8*x*x^(7*n)*x^p*e^(429*
```

```
n*log(x) + 33*p*log(x) + 33*log(x)) + 353400689664*a^7*b^5*n^2*p^9*x*x^(7*n
)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 15341736960*a^7*b^5*n*p^
10*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 273715200*a^7
*b^5*p^11*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 360084
6627840*a^6*b^6*n^10*p*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log
(x)) + 37996241488128*a^6*b^6*n^9*p^2*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*
log(x) + 33*log(x)) + 141126961675392*a^6*b^6*n^8*p^3*x*x^(6*n)*x^p*e^(429*
n*log(x) + 33*p*log(x) + 33*log(x)) + 245393184427776*a^6*b^6*n^7*p^4*x*x^(
6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 216876600420480*a^6*b
^6*n^6*p^5*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 10039
2629154048*a^6*b^6*n^5*p^6*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33
*log(x)) + 25602789066624*a^6*b^6*n^4*p^7*x*x^(6*n)*x^p*e^(429*n*log(x) + 3
3*p*log(x) + 33*log(x)) + 3718561271040*a^6*b^6*n^3*p^8*x*x^(6*n)*x^p*e^(42
9*n*log(x) + 33*p*log(x) + 33*log(x)) + 306543194496*a^6*b^6*n^2*p^9*x*x^(6
*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 13364144640*a^6*b^6*n*
p^10*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 239500800*a
^6*b^6*p^11*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 3600
846627840*a^6*b^6*n^10*p*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*l
og(x)) + 37996241488128*a^6*b^6*n^9*p^2*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*
p*log(x) + 33*log(x)) + 141126961675392*a^6*b^6*n^8*p^3*x*x^(5*n)*x^p*e^(42
9*n*log(x) + 33*p*log(x) + 33*log(x)) + 245393184427776*a^6*b^6*n^7*p^4*x*x
^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 216876600420480*a^6
*b^6*n^6*p^5*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 100
392629154048*a^6*b^6*n^5*p^6*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) +
33*log(x)) + 25602789066624*a^6*b^6*n^4*p^7*x*x^(5*n)*x^p*e^(429*n*log(x) +
 33*p*log(x) + 33*log(x)) + 3718561271040*a^6*b^6*n^3*p^8*x*x^(5*n)*x^p*e^(
429*n*log(x) + 33*p*log(x) + 33*log(x)) + 306543194496*a^6*b^6*n^2*p^9*x*x^
(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 13364144640*a^6*b^6*
n*p^10*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 239500800
*a^6*b^6*p^11*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 21
04390886400*a*b^11*n^10*p*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*
log(x)) + 11870698210560*a*b^11*n^9*p^2*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*
p*log(x) + 32*log(x)) + 25097135458176*a*b^11*n^8*p^3*x*x^n*x^(6*p)*e^(416*
n*log(x) + 32*p*log(x) + 32*log(x)) + 25257194734464*a*b^11*n^7*p^4*x*x^n*x
^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 13207789400832*a*b^11*n
^6*p^5*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 389838693
8880*a*b^11*n^5*p^6*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)
) + 685935755136*a*b^11*n^4*p^7*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x)
 + 32*log(x)) + 73521399168*a*b^11*n^3*p^8*x*x^n*x^(6*p)*e^(416*n*log(x) +
32*p*log(x) + 32*log(x)) + 4713638400*a*b^11*n^2*p^9*x*x^n*x^(6*p)*e^(416*n
*log(x) + 32*p*log(x) + 32*log(x)) + 166344192*a*b^11*n*p^10*x*x^n*x^(6*p)*
e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2488320*a*b^11*p^11*x*x^n*x^(6
*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 4734879494400*a^2*b^10*n^1
0*p*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 40913709
456960*a^2*b^10*n^9*p^2*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 3
```

```
2*log(x)) + 108186490735776*a^2*b^10*n^8*p^3*x*x^(2*n)*x^(5*p)*e^(416*n*log
(x) + 32*p*log(x) + 32*log(x)) + 122798480585472*a^2*b^10*n^7*p^4*x*x^(2*n)
*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 68264005743648*a^2*b^
10*n^6*p^5*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2
0830989884544*a^2*b^10*n^5*p^6*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log
(x) + 32*log(x)) + 3738228742368*a^2*b^10*n^4*p^7*x*x^(2*n)*x^(5*p)*e^(416*
n*log(x) + 32*p*log(x) + 32*log(x)) + 405742908672*a^2*b^10*n^3*p^8*x*x^(2*
n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 26235609696*a^2*b^1
0*n^2*p^9*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 93
1487040*a^2*b^10*n*p^10*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 3
2*log(x)) + 13996800*a^2*b^10*p^11*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p
*log(x) + 32*log(x)) + 2553121296000*a^8*b^4*n^10*p*x*x^(8*n)*x^p*e^(416*n*
log(x) + 32*p*log(x) + 32*log(x)) + 26942281106400*a^8*b^4*n^9*p^2*x*x^(8*n
)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 100081220595840*a^8*b^4*
n^8*p^3*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 17405587
6742880*a^8*b^4*n^7*p^4*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*lo
g(x)) + 153880885192320*a^8*b^4*n^6*p^5*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*
p*log(x) + 32*log(x)) + 71272716752160*a^8*b^4*n^5*p^6*x*x^(8*n)*x^p*e^(416
*n*log(x) + 32*p*log(x) + 32*log(x)) + 18191735464320*a^8*b^4*n^4*p^7*x*x^(
8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2645052965280*a^8*b^4
*n^3*p^8*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2183255
07840*a^8*b^4*n^2*p^9*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(
x)) + 9531561600*a^8*b^4*n*p^10*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x)
 + 32*log(x)) + 171072000*a^8*b^4*p^11*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p
*log(x) + 32*log(x)) + 2553121296000*a^8*b^4*n^10*p*x*x^(7*n)*x^p*e^(416*n*
log(x) + 32*p*log(x) + 32*log(x)) + 26942281106400*a^8*b^4*n^9*p^2*x*x^(7*n
)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 100081220595840*a^8*b^4*
n^8*p^3*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 17405587
6742880*a^8*b^4*n^7*p^4*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*lo
g(x)) + 153880885192320*a^8*b^4*n^6*p^5*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*
p*log(x) + 32*log(x)) + 71272716752160*a^8*b^4*n^5*p^6*x*x^(7*n)*x^p*e^(416
*n*log(x) + 32*p*log(x) + 32*log(x)) + 18191735464320*a^8*b^4*n^4*p^7*x*x^(
7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2645052965280*a^8*b^4
*n^3*p^8*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2183255
07840*a^8*b^4*n^2*p^9*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(
x)) + 9531561600*a^8*b^4*n*p^10*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x)
 + 32*log(x)) + 171072000*a^8*b^4*p^11*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p
*log(x) + 32*log(x)) + 87682953600*a^11*b*n^10*p*x*x^(11*n)*x^p*e^(403*n*lo
g(x) + 31*p*log(x) + 31*log(x)) + 925720280640*a^11*b*n^9*p^2*x*x^(11*n)*x^
p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 3441632747904*a^11*b*n^8*p^3
*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 5994150272064*
a^11*b*n^7*p^4*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) +
5312727605376*a^11*b*n^6*p^5*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) +
 31*log(x)) + 2471328280512*a^11*b*n^5*p^6*x*x^(11*n)*x^p*e^(403*n*log(x) +
 31*p*log(x) + 31*log(x)) + 634800578688*a^11*b*n^4*p^7*x*x^(11*n)*x^p*e^(4
```

```
03*n*log(x) + 31*p*log(x) + 31*log(x)) + 9306625728*a^11*b*n^3*p^8*x*x^(11
*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 7757807616*a^11*b*n^2*
p^9*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 342455040*a
^11*b*n*p^10*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 62
20800*a^11*b*p^11*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x))
 + 12860166528000*a^3*b^9*n^10*p*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*l
og(x) + 30*log(x)) + 115410377491200*a^3*b^9*n^9*p^2*x*x^(3*n)*x^(6*p)*e^(3
90*n*log(x) + 30*p*log(x) + 30*log(x)) + 323736532859520*a^3*b^9*n^8*p^3*x*
x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 38164555205568
0*a^3*b^9*n^7*p^4*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(
x)) + 216385965354240*a^3*b^9*n^6*p^5*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 66751542950400*a^3*b^9*n^5*p^6*x*x^(3*n)*x^(6*p)*
e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 12056636718720*a^3*b^9*n^4*p^7
*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 13140509817
60*a^3*b^9*n^3*p^8*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log
(x)) + 85207795200*a^3*b^9*n^2*p^9*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p
*log(x) + 30*log(x)) + 3031395840*a^3*b^9*n*p^10*x*x^(3*n)*x^(6*p)*e^(390*n
*log(x) + 30*p*log(x) + 30*log(x)) + 45619200*a^3*b^9*p^11*x*x^(3*n)*x^(6*p
)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 21701531016000*a^4*b^8*n^10*
p*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2092226993
60400*a^4*b^8*n^9*p^2*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*
log(x)) + 661674386534040*a^4*b^8*n^8*p^3*x*x^(4*n)*x^(5*p)*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 892861481227320*a^4*b^8*n^7*p^4*x*x^(4*n)*x^(
5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 545667950464560*a^4*b^8*n
^6*p^5*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 17556
0805823040*a^4*b^8*n^5*p^6*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x)
+ 30*log(x)) + 32523483187800*a^4*b^8*n^4*p^7*x*x^(4*n)*x^(5*p)*e^(390*n*lo
g(x) + 30*p*log(x) + 30*log(x)) + 3603268615320*a^4*b^8*n^3*p^8*x*x^(4*n)*x
^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 236287925280*a^4*b^8*n^
2*p^9*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 847447
9200*a^4*b^8*n*p^10*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*lo
g(x)) + 128304000*a^4*b^8*p^11*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log
(x) + 30*log(x)) + 4286722176000*a^9*b^3*n^10*p*x*x^(9*n)*x^(4*p)*e^(390*n*
log(x) + 30*p*log(x) + 30*log(x)) + 44185755398400*a^9*b^3*n^9*p^2*x*x^(9*n
)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 157300468871040*a^9*
b^3*n^8*p^3*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
254635323761280*a^9*b^3*n^7*p^4*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*lo
g(x) + 30*log(x)) + 199275520561920*a^9*b^3*n^6*p^5*x*x^(9*n)*x^(4*p)*e^(39
0*n*log(x) + 30*p*log(x) + 30*log(x)) + 76039765992960*a^9*b^3*n^5*p^6*x*x^
(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 15756480769920*a
^9*b^3*n^4*p^7*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
 + 1883988247680*a^9*b^3*n^3*p^8*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*l
og(x) + 30*log(x)) + 130434839040*a^9*b^3*n^2*p^9*x*x^(9*n)*x^(4*p)*e^(390*
n*log(x) + 30*p*log(x) + 30*log(x)) + 4869849600*a^9*b^3*n*p^10*x*x^(9*n)*x
^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 76032000*a^9*b^3*p^11*x
```

```
*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1157414987520
*a^10*b^2*n^10*p*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(
x)) + 12084475955904*a^10*b^2*n^9*p^2*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) +
30*p*log(x) + 30*log(x)) + 44030949389856*a^10*b^2*n^8*p^3*x*x^(10*n)*x^(3*
p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 74102389735968*a^10*b^2*n^7
*p^4*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 619010
91743040*a^10*b^2*n^6*p^5*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x)
+ 30*log(x)) + 26085315320064*a^10*b^2*n^5*p^6*x*x^(10*n)*x^(3*p)*e^(390*n*
log(x) + 30*p*log(x) + 30*log(x)) + 5880765683232*a^10*b^2*n^4*p^7*x*x^(10*
n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 750606959520*a^10*b
^2*n^3*p^8*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
54654956928*a^10*b^2*n^2*p^9*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(
x) + 30*log(x)) + 2123003520*a^10*b^2*n*p^10*x*x^(10*n)*x^(3*p)*e^(390*n*lo
g(x) + 30*p*log(x) + 30*log(x)) + 34214400*a^10*b^2*p^11*x*x^(10*n)*x^(3*p)
*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 14613825600*a^12*n^10*p*x*x^(
12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 154286713440*a^12*n^
9*p^2*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 573605457
984*a^12*n^8*p^3*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
+ 999025045344*a^12*n^7*p^4*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) +
30*log(x)) + 885454600896*a^12*n^6*p^5*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 411888046752*a^12*n^5*p^6*x*x^(12*n)*x^p*e^(390*n*l
og(x) + 30*p*log(x) + 30*log(x)) + 105800096448*a^12*n^4*p^7*x*x^(12*n)*x^p
*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 15511104288*a^12*n^3*p^8*x*x^
(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1292967936*a^12*n^2
*p^9*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 57075840*a
^12*n*p^10*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1036
800*a^12*p^11*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 3
1408819200*a^2*b^10*n^10*p*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25
*log(x)) + 331405966080*a^2*b^10*n^9*p^2*x*x^(2*n)*x^p*e^(325*n*log(x) + 25
*p*log(x) + 25*log(x)) + 1230778965888*a^2*b^10*n^8*p^3*x*x^(2*n)*x^p*e^(32
5*n*log(x) + 25*p*log(x) + 25*log(x)) + 2139674600448*a^2*b^10*n^7*p^4*x*x^
(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 1890383812992*a^2*b^
10*n^6*p^5*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 87455
2702464*a^2*b^10*n^5*p^6*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*l
og(x)) + 222844093056*a^2*b^10*n^4*p^7*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p
*log(x) + 25*log(x)) + 32330382336*a^2*b^10*n^3*p^8*x*x^(2*n)*x^p*e^(325*n*
log(x) + 25*p*log(x) + 25*log(x)) + 2661766272*a^2*b^10*n^2*p^9*x*x^(2*n)*x
^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 115879680*a^2*b^10*n*p^10*x
*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 2073600*a^2*b^10*
p^11*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 5234803200*
a^2*b^10*n^10*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 13
8336128640*a^2*b^10*n^9*p*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*
log(x)) + 865451070720*a^2*b^10*n^8*p^2*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*
p*log(x) + 35*log(x)) + 2214049278240*a^2*b^10*n^7*p^3*x*x^(2*n)*x^p*e^(455
*n*log(x) + 35*p*log(x) + 35*log(x)) + 2727644406624*a^2*b^10*n^6*p^4*x*x^(
```

```
2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 172244634368*a^2*b^1
0*n^5*p^5*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 582048
783936*a^2*b^10*n^4*p^6*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*lo
g(x)) + 108313627680*a^2*b^10*n^3*p^7*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*
log(x) + 35*log(x)) + 11065905504*a^2*b^10*n^2*p^8*x*x^(2*n)*x^p*e^(455*n*l
og(x) + 35*p*log(x) + 35*log(x)) + 580801536*a^2*b^10*n*p^9*x*x^(2*n)*x^p*e
^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 12234240*a^2*b^10*p^10*x*x^(2*n
)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 526097721600*a^5*b^7*n^1
0*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 13897784070720
*a^5*b^7*n^9*p*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 8
6914620492480*a^5*b^7*n^8*p^2*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) +
 34*log(x)) + 222235922410128*a^5*b^7*n^7*p^3*x*x^(5*n)*x^p*e^(442*n*log(x)
 + 34*p*log(x) + 34*log(x)) + 273574554210144*a^5*b^7*n^6*p^4*x*x^(5*n)*x^p
*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 172551644182368*a^5*b^7*n^5*p
^5*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 5821329659270
4*a^5*b^7*n^4*p^6*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x))
+ 10810372115664*a^5*b^7*n^3*p^7*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x
) + 34*log(x)) + 1101728057760*a^5*b^7*n^2*p^8*x*x^(5*n)*x^p*e^(442*n*log(x
) + 34*p*log(x) + 34*log(x)) + 57667002624*a^5*b^7*n*p^9*x*x^(5*n)*x^p*e^(4
42*n*log(x) + 34*p*log(x) + 34*log(x)) + 1211189760*a^5*b^7*p^10*x*x^(5*n)*
x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 701463628800*a^7*b^5*n^10*
x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 18530378760960*a
^7*b^5*n^9*p*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 115
886160656640*a^7*b^5*n^8*p^2*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) +
33*log(x)) + 296314563213504*a^7*b^5*n^7*p^3*x*x^(7*n)*x^p*e^(429*n*log(x)
+ 33*p*log(x) + 33*log(x)) + 364766072280192*a^7*b^5*n^6*p^4*x*x^(7*n)*x^p*
e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 230068858909824*a^7*b^5*n^5*p^
5*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 77617728790272
*a^7*b^5*n^4*p^6*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) +
 14413829487552*a^7*b^5*n^3*p^7*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x)
 + 33*log(x)) + 1468970743680*a^7*b^5*n^2*p^8*x*x^(7*n)*x^p*e^(429*n*log(x)
 + 33*p*log(x) + 33*log(x)) + 76889336832*a^7*b^5*n*p^9*x*x^(7*n)*x^p*e^(42
9*n*log(x) + 33*p*log(x) + 33*log(x)) + 1614919680*a^7*b^5*p^10*x*x^(7*n)*x
^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 600141104640*a^6*b^6*n^10*x
*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 15862255339392*a^
6*b^6*n^9*p*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 9925
4420762112*a^6*b^6*n^8*p^2*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33
*log(x)) + 253982510524512*a^6*b^6*n^7*p^3*x*x^(6*n)*x^p*e^(429*n*log(x) +
33*p*log(x) + 33*log(x)) + 313018558798176*a^6*b^6*n^6*p^4*x*x^(6*n)*x^p*e^
(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 197778879010944*a^6*b^6*n^5*p^5*
x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 66887049886272*a
^6*b^6*n^4*p^6*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1
2459803523552*a^6*b^6*n^3*p^7*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) +
 33*log(x)) + 1274512887648*a^6*b^6*n^2*p^8*x*x^(6*n)*x^p*e^(429*n*log(x) +
 33*p*log(x) + 33*log(x)) + 66984781248*a^6*b^6*n*p^9*x*x^(6*n)*x^p*e^(429*
```

```
n*log(x) + 33*p*log(x) + 33*log(x)) + 1413054720*a^6*b^6*p^10*x*x^(6*n)*x^p
*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 600141104640*a^6*b^6*n^10*x*x
^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 15862255339392*a^6*
b^6*n^9*p*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 992544
20762112*a^6*b^6*n^8*p^2*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*l
og(x)) + 253982510524512*a^6*b^6*n^7*p^3*x*x^(5*n)*x^p*e^(429*n*log(x) + 33
*p*log(x) + 33*log(x)) + 313018558798176*a^6*b^6*n^6*p^4*x*x^(5*n)*x^p*e^(4
29*n*log(x) + 33*p*log(x) + 33*log(x)) + 197778879010944*a^6*b^6*n^5*p^5*x*
x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 66887049886272*a^6
*b^6*n^4*p^6*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 124
59803523552*a^6*b^6*n^3*p^7*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 3
3*log(x)) + 1274512887648*a^6*b^6*n^2*p^8*x*x^(5*n)*x^p*e^(429*n*log(x) + 3
3*p*log(x) + 33*log(x)) + 66984781248*a^6*b^6*n*p^9*x*x^(5*n)*x^p*e^(429*n*
log(x) + 33*p*log(x) + 33*log(x)) + 1413054720*a^6*b^6*p^10*x*x^(5*n)*x^p*e
^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 350731814400*a*b^11*n^10*x*x^n*
x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 5630332394880*a*b^11*n
^9*p*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 20781314121
600*a*b^11*n^8*p^2*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x))
 + 31914975795552*a*b^11*n^7*p^3*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x
) + 32*log(x)) + 24042444803712*a*b^11*n^6*p^4*x*x^n*x^(6*p)*e^(416*n*log(x
) + 32*p*log(x) + 32*log(x)) + 9692067000960*a*b^11*n^5*p^5*x*x^n*x^(6*p)*e
^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2220541401024*a*b^11*n^4*p^6*x*
x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 297727556256*a*b^1
1*n^3*p^7*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 231205
48416*a*b^11*n^2*p^8*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x
)) + 963353088*a*b^11*n*p^9*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 3
2*log(x)) + 16671744*a*b^11*p^10*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x
) + 32*log(x)) + 789146582400*a^2*b^10*n^10*x*x^(2*n)*x^(5*p)*e^(416*n*log(
x) + 32*p*log(x) + 32*log(x)) + 17403127382880*a^2*b^10*n^9*p*x*x^(2*n)*x^(
5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 81545157695520*a^2*b^10*n
^8*p^2*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 14488
3473429960*a^2*b^10*n^7*p^3*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x)
 + 32*log(x)) + 118801479508200*a^2*b^10*n^6*p^4*x*x^(2*n)*x^(5*p)*e^(416*n
*log(x) + 32*p*log(x) + 32*log(x)) + 50267219057568*a^2*b^10*n^5*p^5*x*x^(2
*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 11853409468176*a^2
*b^10*n^4*p^6*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x))
+ 1618399904616*a^2*b^10*n^3*p^7*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*l
og(x) + 32*log(x)) + 127208126376*a^2*b^10*n^2*p^8*x*x^(2*n)*x^(5*p)*e^(416
*n*log(x) + 32*p*log(x) + 32*log(x)) + 5345342928*a^2*b^10*n*p^9*x*x^(2*n)*
x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 93078720*a^2*b^10*p^10
*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 42552021600
0*a^8*b^4*n^10*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 1
1248819045200*a^8*b^4*n^9*p*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 3
2*log(x)) + 70399360726200*a^8*b^4*n^8*p^2*x*x^(8*n)*x^p*e^(416*n*log(x) +
32*p*log(x) + 32*log(x)) + 180189820083240*a^8*b^4*n^7*p^3*x*x^(8*n)*x^p*e^
```

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(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 222156964656000*a^8*b^4*n^6*p^4*
x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 140448816699840*
a^8*b^4*n^5*p^5*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) +
47536176841560*a^8*b^4*n^4*p^6*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x)
+ 32*log(x)) + 8864089371720*a^8*b^4*n^3*p^7*x*x^(8*n)*x^p*e^(416*n*log(x)
+ 32*p*log(x) + 32*log(x)) + 907804758960*a^8*b^4*n^2*p^8*x*x^(8*n)*x^p*e^(
416*n*log(x) + 32*p*log(x) + 32*log(x)) + 47776417920*a^8*b^4*n*p^9*x*x^(8*
n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 1009324800*a^8*b^4*p^10
*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 425520216000*a^
8*b^4*n^10*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 11248
819045200*a^8*b^4*n^9*p*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*lo
g(x)) + 70399360726200*a^8*b^4*n^8*p^2*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p
*log(x) + 32*log(x)) + 180189820083240*a^8*b^4*n^7*p^3*x*x^(7*n)*x^p*e^(416
*n*log(x) + 32*p*log(x) + 32*log(x)) + 222156964656000*a^8*b^4*n^6*p^4*x*x^
(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 140448816699840*a^8*
b^4*n^5*p^5*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 4753
6176841560*a^8*b^4*n^4*p^6*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32
*log(x)) + 8864089371720*a^8*b^4*n^3*p^7*x*x^(7*n)*x^p*e^(416*n*log(x) + 32
*p*log(x) + 32*log(x)) + 907804758960*a^8*b^4*n^2*p^8*x*x^(7*n)*x^p*e^(416*
n*log(x) + 32*p*log(x) + 32*log(x)) + 47776417920*a^8*b^4*n*p^9*x*x^(7*n)*x
^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 1009324800*a^8*b^4*p^10*x*x
^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 14613825600*a^11*b*
n^10*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 3868245331
20*a^11*b*n^9*p*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) +
 2424107694960*a^11*b*n^8*p^2*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x)
+ 31*log(x)) + 6216133374000*a^11*b*n^7*p^3*x*x^(11*n)*x^p*e^(403*n*log(x)
+ 31*p*log(x) + 31*log(x)) + 7685581300992*a^11*b*n^6*p^4*x*x^(11*n)*x^p*e^
(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 4879810941984*a^11*b*n^5*p^5*x*x
^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 1661541037488*a^11
*b*n^4*p^6*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 3122
36961840*a^11*b*n^3*p^7*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*l
og(x)) + 32278023072*a^11*b*n^2*p^8*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*l
og(x) + 31*log(x)) + 1717003008*a^11*b*n*p^9*x*x^(11*n)*x^p*e^(403*n*log(x)
 + 31*p*log(x) + 31*log(x)) + 36702720*a^11*b*p^10*x*x^(11*n)*x^p*e^(403*n*
log(x) + 31*p*log(x) + 31*log(x)) + 29227651200*b^12*n^11*x*x^(7*p)*e^(390*
n*log(x) + 30*p*log(x) + 30*log(x)) + 311009064480*b^12*n^10*p*x*x^(7*p)*e^
(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1172925368208*b^12*n^9*p^2*x*x^(
7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2093651000352*b^12*n^8*p^
3*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1937413376016*b^12
*n^7*p^4*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 97135186032
0*b^12*n^6*p^5*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 28024
8200688*b^12*n^5*p^6*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
 48655557984*b^12*n^4*p^7*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(
x)) + 5171119920*b^12*n^3*p^8*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*
log(x)) + 329646336*b^12*n^2*p^9*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) +
```

```
30*log(x)) + 11586240*b^12*n*p^10*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) +
 30*log(x)) + 172800*b^12*p^11*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30
*log(x)) + 2143361088000*a^3*b^9*n^10*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 50125568169600*a^3*b^9*n^9*p*x*x^(3*n)*x^(6*p)*e^
(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 249565270204800*a^3*b^9*n^8*p^2*
x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 459537650082
720*a^3*b^9*n^7*p^3*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*lo
g(x)) + 383344123440960*a^3*b^9*n^6*p^4*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) +
 30*p*log(x) + 30*log(x)) + 163538297271360*a^3*b^9*n^5*p^5*x*x^(3*n)*x^(6*
p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 38725599559680*a^3*b^9*n^4*
p^6*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 52996070
95200*a^3*b^9*n^3*p^7*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*
log(x)) + 417125079360*a^3*b^9*n^2*p^8*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) +
30*p*log(x) + 30*log(x)) + 17542863360*a^3*b^9*n*p^9*x*x^(3*n)*x^(6*p)*e^(3
90*n*log(x) + 30*p*log(x) + 30*log(x)) + 305648640*a^3*b^9*p^10*x*x^(3*n)*x
^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 3616921836000*a^4*b^8*n
^10*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 88806638
428200*a^4*b^8*n^9*p*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*l
og(x)) + 485925578018100*a^4*b^8*n^8*p^2*x*x^(4*n)*x^(5*p)*e^(390*n*log(x)
+ 30*p*log(x) + 30*log(x)) + 1008129732047460*a^4*b^8*n^7*p^3*x*x^(4*n)*x^(
5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 922673369841300*a^4*b^8*n
^6*p^4*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 41675
2971877440*a^4*b^8*n^5*p^5*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x)
+ 30*log(x)) + 102205479909060*a^4*b^8*n^4*p^6*x*x^(4*n)*x^(5*p)*e^(390*n*l
og(x) + 30*p*log(x) + 30*log(x)) + 14304343183020*a^4*b^8*n^3*p^7*x*x^(4*n)
*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1143036867060*a^4*b^8
*n^2*p^8*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 485
88404040*a^4*b^8*n*p^9*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30
*log(x)) + 853221600*a^4*b^8*p^10*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*
log(x) + 30*log(x)) + 714453696000*a^9*b^3*n^10*x*x^(9*n)*x^(4*p)*e^(390*n*
log(x) + 30*p*log(x) + 30*log(x)) + 18613732579200*a^9*b^3*n^9*p*x*x^(9*n)*
x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 112745230646400*a^9*b^
3*n^8*p^2*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 27
2737741480160*a^9*b^3*n^7*p^3*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(
x) + 30*log(x)) + 306279736065600*a^9*b^3*n^6*p^4*x*x^(9*n)*x^(4*p)*e^(390*
n*log(x) + 30*p*log(x) + 30*log(x)) + 167289638401600*a^9*b^3*n^5*p^5*x*x^(
9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 46967905198720*a^
9*b^3*n^4*p^6*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
+ 7213695492960*a^9*b^3*n^3*p^7*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*lo
g(x) + 30*log(x)) + 615376329920*a^9*b^3*n^2*p^8*x*x^(9*n)*x^(4*p)*e^(390*n
*log(x) + 30*p*log(x) + 30*log(x)) + 27433105920*a^9*b^3*n*p^9*x*x^(9*n)*x^
(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 499276800*a^9*b^3*p^10*x
*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 192902497920*
a^10*b^2*n^10*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
 + 5064288295968*a^10*b^2*n^9*p*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*l
```

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og(x) + 30*log(x)) + 31220715613824*a^10*b^2*n^8*p^2*x*x^(10*n)*x^(3*p)*e^(
390*n*log(x) + 30*p*log(x) + 30*log(x)) + 77858036962200*a^10*b^2*n^7*p^3*x
*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 918991388177
28*a^10*b^2*n^6*p^4*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*l
og(x)) + 54141707293344*a^10*b^2*n^5*p^5*x*x^(10*n)*x^(3*p)*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 16581721848912*a^10*b^2*n^4*p^6*x*x^(10*n)*x^
(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2743082735976*a^10*b^2*n
^3*p^7*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2482
26607728*a^10*b^2*n^2*p^8*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x)
+ 30*log(x)) + 11593036224*a^10*b^2*n*p^9*x*x^(10*n)*x^(3*p)*e^(390*n*log(x
) + 30*p*log(x) + 30*log(x)) + 218972160*a^10*b^2*p^10*x*x^(10*n)*x^(3*p)*e
^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2435637600*a^12*n^10*x*x^(12*n)
*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 64470755520*a^12*n^9*p*x*
x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 404017949160*a^12
*n^8*p^2*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 103602
2229000*a^12*n^7*p^3*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(
x)) + 1280930216832*a^12*n^6*p^4*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(
x) + 30*log(x)) + 813301823664*a^12*n^5*p^5*x*x^(12*n)*x^p*e^(390*n*log(x)
+ 30*p*log(x) + 30*log(x)) + 276923506248*a^12*n^4*p^6*x*x^(12*n)*x^p*e^(39
0*n*log(x) + 30*p*log(x) + 30*log(x)) + 52039493640*a^12*n^3*p^7*x*x^(12*n)
*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 5379670512*a^12*n^2*p^8*x
*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 286167168*a^12*n
*p^9*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 6117120*a^
12*p^10*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 5234803
200*a^2*b^10*n^10*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x))
+ 138336128640*a^2*b^10*n^9*p*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) +
 25*log(x)) + 865451070720*a^2*b^10*n^8*p^2*x*x^(2*n)*x^p*e^(325*n*log(x) +
 25*p*log(x) + 25*log(x)) + 2214049278240*a^2*b^10*n^7*p^3*x*x^(2*n)*x^p*e^
(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 2727644406624*a^2*b^10*n^6*p^4*x
*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 1722446434368*a^2
*b^10*n^5*p^5*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 58
2048783936*a^2*b^10*n^4*p^6*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 2
5*log(x)) + 108313627680*a^2*b^10*n^3*p^7*x*x^(2*n)*x^p*e^(325*n*log(x) + 2
5*p*log(x) + 25*log(x)) + 11065905504*a^2*b^10*n^2*p^8*x*x^(2*n)*x^p*e^(325
*n*log(x) + 25*p*log(x) + 25*log(x)) + 580801536*a^2*b^10*n*p^9*x*x^(2*n)*x
^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 12234240*a^2*b^10*p^10*x*x^
(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 13850300160*a^2*b^10
*n^9*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 19679627721
6*a^2*b^10*n^8*p*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) +
 832771425456*a^2*b^10*n^7*p^2*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x)
+ 35*log(x)) + 1511260255824*a^2*b^10*n^6*p^3*x*x^(2*n)*x^p*e^(455*n*log(x)
 + 35*p*log(x) + 35*log(x)) + 1335169607184*a^2*b^10*n^5*p^4*x*x^(2*n)*x^p*
e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 610000650336*a^2*b^10*n^4*p^5*
x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 148575844848*a^2
*b^10*n^3*p^6*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 19
```

```
239408432*a^2*b^10*n^2*p^7*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35
*log(x)) + 1241739600*a^2*b^10*n*p^8*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*l
og(x) + 35*log(x)) + 31328064*a^2*b^10*p^9*x*x^(2*n)*x^p*e^(455*n*log(x) +
35*p*log(x) + 35*log(x)) + 1391241329280*a^5*b^7*n^9*x*x^(5*n)*x^p*e^(442*n
*log(x) + 34*p*log(x) + 34*log(x)) + 19757792319408*a^5*b^7*n^8*p*x*x^(5*n)
*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 83556262315944*a^5*b^7*n^
7*p^2*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 1515037525
23024*a^5*b^7*n^6*p^3*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(
x)) + 133691205053736*a^5*b^7*n^5*p^4*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*
log(x) + 34*log(x)) + 60983350711272*a^5*b^7*n^4*p^5*x*x^(5*n)*x^p*e^(442*n
*log(x) + 34*p*log(x) + 34*log(x)) + 14824258469856*a^5*b^7*n^3*p^6*x*x^(5*
n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 1915144837848*a^5*b^7*n
^2*p^7*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 123281482
896*a^5*b^7*n*p^8*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x))
+ 3101478336*a^5*b^7*p^9*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*l
og(x)) + 1854988439040*a^7*b^5*n^9*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log
(x) + 33*log(x)) + 26343723092544*a^7*b^5*n^8*p*x*x^(7*n)*x^p*e^(429*n*log(
x) + 33*p*log(x) + 33*log(x)) + 111408349754592*a^7*b^5*n^7*p^2*x*x^(7*n)*x
^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 202005003364032*a^7*b^5*n^6
*p^3*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 17825494007
1648*a^7*b^5*n^5*p^4*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x
)) + 81311134281696*a^7*b^5*n^4*p^5*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*lo
g(x) + 33*log(x)) + 19765677959808*a^7*b^5*n^3*p^6*x*x^(7*n)*x^p*e^(429*n*l
og(x) + 33*p*log(x) + 33*log(x)) + 2553526450464*a^7*b^5*n^2*p^7*x*x^(7*n)*
x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 164375310528*a^7*b^5*n*p^8
*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 4135304448*a^7*
b^5*p^9*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 15882580
70784*a^6*b^6*n^9*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x))
+ 22572890458272*a^6*b^6*n^8*p*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x)
+ 33*log(x)) + 95549262528720*a^6*b^6*n^7*p^2*x*x^(6*n)*x^p*e^(429*n*log(x)
 + 33*p*log(x) + 33*log(x)) + 173468794059792*a^6*b^6*n^6*p^3*x*x^(6*n)*x^p
*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 153345878801808*a^6*b^6*n^5*p
^4*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 7011362077843
2*a^6*b^6*n^4*p^5*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x))
+ 17093931989904*a^6*b^6*n^3*p^6*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x
) + 33*log(x)) + 2216091163440*a^6*b^6*n^2*p^7*x*x^(6*n)*x^p*e^(429*n*log(x
) + 33*p*log(x) + 33*log(x)) + 143217187344*a^6*b^6*n*p^8*x*x^(6*n)*x^p*e^(
429*n*log(x) + 33*p*log(x) + 33*log(x)) + 3618391392*a^6*b^6*p^9*x*x^(6*n)*
x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1588258070784*a^6*b^6*n^9*
x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 22572890458272*a
^6*b^6*n^8*p*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 955
49262528720*a^6*b^6*n^7*p^2*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 3
3*log(x)) + 173468794059792*a^6*b^6*n^6*p^3*x*x^(5*n)*x^p*e^(429*n*log(x) +
 33*p*log(x) + 33*log(x)) + 153345878801808*a^6*b^6*n^5*p^4*x*x^(5*n)*x^p*e
^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 70113620778432*a^6*b^6*n^4*p^5*
```

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x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 17093931989904*a
^6*b^6*n^3*p^6*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 2
216091163440*a^6*b^6*n^2*p^7*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) +
33*log(x)) + 143217187344*a^6*b^6*n*p^8*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*
p*log(x) + 33*log(x)) + 3618391392*a^6*b^6*p^9*x*x^(5*n)*x^p*e^(429*n*log(x
) + 33*p*log(x) + 33*log(x)) + 608647115520*a*b^11*n^9*x*x^n*x^(6*p)*e^(416
*n*log(x) + 32*p*log(x) + 32*log(x)) + 5424901173792*a*b^11*n^8*p*x*x^n*x^(
6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 14263349602608*a*b^11*n^7
*p^2*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 16303637553
792*a*b^11*n^6*p^3*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x))
 + 9329159083344*a*b^11*n^5*p^4*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x)
 + 32*log(x)) + 2877877404144*a*b^11*n^4*p^5*x*x^n*x^(6*p)*e^(416*n*log(x)
+ 32*p*log(x) + 32*log(x)) + 496949788704*a*b^11*n^3*p^6*x*x^n*x^(6*p)*e^(4
16*n*log(x) + 32*p*log(x) + 32*log(x)) + 47900700336*a*b^11*n^2*p^7*x*x^n*x
^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2404386720*a*b^11*n*p^8
*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 48940416*a*b^11
*p^9*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 17640293011
20*a^2*b^10*n^9*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)
) + 19491933285432*a^2*b^10*n^8*p*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*
log(x) + 32*log(x)) + 60124186166004*a^2*b^10*n^7*p^2*x*x^(2*n)*x^(5*p)*e^(
416*n*log(x) + 32*p*log(x) + 32*log(x)) + 76264454018376*a^2*b^10*n^6*p^3*x
*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 4657238343518
4*a^2*b^10*n^5*p^4*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log
(x)) + 14957038199952*a^2*b^10*n^4*p^5*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) +
32*p*log(x) + 32*log(x)) + 2649694839516*a^2*b^10*n^3*p^6*x*x^(2*n)*x^(5*p)
*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 259759068912*a^2*b^10*n^2*p^7
*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 13191781824
*a^2*b^10*n*p^8*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)
) + 270775872*a^2*b^10*p^9*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x)
+ 32*log(x)) + 1126406476800*a^8*b^4*n^9*x*x^(8*n)*x^p*e^(416*n*log(x) + 32
*p*log(x) + 32*log(x)) + 16012828697580*a^8*b^4*n^8*p*x*x^(8*n)*x^p*e^(416*
n*log(x) + 32*p*log(x) + 32*log(x)) + 67801225807260*a^8*b^4*n^7*p^2*x*x^(8
*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 123142956427260*a^8*b^
4*n^6*p^3*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 108920
710075320*a^8*b^4*n^5*p^4*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*
log(x)) + 49839431980140*a^8*b^4*n^4*p^5*x*x^(8*n)*x^p*e^(416*n*log(x) + 32
*p*log(x) + 32*log(x)) + 12162702932820*a^8*b^4*n^3*p^6*x*x^(8*n)*x^p*e^(41
6*n*log(x) + 32*p*log(x) + 32*log(x)) + 1578608311500*a^8*b^4*n^2*p^7*x*x^(
8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 102152464920*a^8*b^4*
n*p^8*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2584565280
*a^8*b^4*p^9*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 112
6406476800*a^8*b^4*n^9*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log
(x)) + 16012828697580*a^8*b^4*n^8*p*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*lo
g(x) + 32*log(x)) + 67801225807260*a^8*b^4*n^7*p^2*x*x^(7*n)*x^p*e^(416*n*l
og(x) + 32*p*log(x) + 32*log(x)) + 123142956427260*a^8*b^4*n^6*p^3*x*x^(7*n
```

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)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 108920710075320*a^8*b^4*
n^5*p^4*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 49839431
980140*a^8*b^4*n^4*p^5*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log
(x)) + 12162702932820*a^8*b^4*n^3*p^6*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*
log(x) + 32*log(x)) + 1578608311500*a^8*b^4*n^2*p^7*x*x^(7*n)*x^p*e^(416*n*
log(x) + 32*p*log(x) + 32*log(x)) + 102152464920*a^8*b^4*n*p^8*x*x^(7*n)*x^
p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2584565280*a^8*b^4*p^9*x*x^(
7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 38756303280*a^11*b*n^
9*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 551971144728*
a^11*b*n^8*p*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 23
42375496408*a^11*b*n^7*p^2*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 3
1*log(x)) + 4267430984232*a^11*b*n^6*p^3*x*x^(11*n)*x^p*e^(403*n*log(x) + 3
1*p*log(x) + 31*log(x)) + 3790990295232*a^11*b*n^5*p^4*x*x^(11*n)*x^p*e^(40
3*n*log(x) + 31*p*log(x) + 31*log(x)) + 1744755722808*a^11*b*n^4*p^5*x*x^(1
1*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 428928528744*a^11*b*n
^3*p^6*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 56168592
456*a^11*b*n^2*p^7*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)
) + 3672300240*a^11*b*n*p^8*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) +
31*log(x)) + 93984192*a^11*b*p^9*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(
x) + 31*log(x)) + 79948244160*b^12*n^10*x*x^(7*p)*e^(390*n*log(x) + 30*p*lo
g(x) + 30*log(x)) + 649007915112*b^12*n^9*p*x*x^(7*p)*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 1886039859132*b^12*n^8*p^2*x*x^(7*p)*e^(390*n*log(x
) + 30*p*log(x) + 30*log(x)) + 2578132750052*b^12*n^7*p^3*x*x^(7*p)*e^(390*
n*log(x) + 30*p*log(x) + 30*log(x)) + 1825775806896*b^12*n^6*p^4*x*x^(7*p)*
e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 711337290784*b^12*n^5*p^5*x*x^
(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 159756994732*b^12*n^4*p^
6*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 21155920692*b^12*n
^3*p^7*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1629499688*b^
12*n^2*p^8*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 67510368*
b^12*n*p^9*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1163520*b
^12*p^10*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 51484175424
00*a^3*b^9*n^9*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
 + 60868092822240*a^3*b^9*n^8*p*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*lo
g(x) + 30*log(x)) + 195287458297360*a^3*b^9*n^7*p^2*x*x^(3*n)*x^(6*p)*e^(39
0*n*log(x) + 30*p*log(x) + 30*log(x)) + 251835561135840*a^3*b^9*n^6*p^3*x*x
^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 154596972635600
*a^3*b^9*n^5*p^4*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x
)) + 49699578761360*a^3*b^9*n^4*p^5*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 8800869052800*a^3*b^9*n^3*p^6*x*x^(3*n)*x^(6*p)*e^(
390*n*log(x) + 30*p*log(x) + 30*log(x)) + 862098769840*a^3*b^9*n^2*p^7*x*x^
(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 43745248800*a^3*
b^9*n*p^8*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 89
7240960*a^3*b^9*p^9*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*lo
g(x)) + 8989364755800*a^4*b^8*n^9*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*
log(x) + 30*log(x)) + 114199438708080*a^4*b^8*n^8*p*x*x^(4*n)*x^(5*p)*e^(39
```

```
0*n*log(x) + 30*p*log(x) + 30*log(x)) + 405438728216940*a^4*b^8*n^7*p^2*x*x
^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 575237252820780
*a^4*b^8*n^6*p^3*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x
)) + 378895381171560*a^4*b^8*n^5*p^4*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30
*p*log(x) + 30*log(x)) + 127578692771640*a^4*b^8*n^4*p^5*x*x^(4*n)*x^(5*p)*
e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 23284618796340*a^4*b^8*n^3*p^6
*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 23275602840
60*a^4*b^8*n^2*p^7*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log
(x)) + 119786253840*a^4*b^8*n*p^8*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*
log(x) + 30*log(x)) + 2482112160*a^4*b^8*p^9*x*x^(4*n)*x^(5*p)*e^(390*n*log
(x) + 30*p*log(x) + 30*log(x)) + 1874906668800*a^9*b^3*n^9*x*x^(9*n)*x^(4*p
)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 26025942418080*a^9*b^3*n^8*p
*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 10543833903
4160*a^9*b^3*n^7*p^2*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*l
og(x)) + 178047651821600*a^9*b^3*n^6*p^3*x*x^(9*n)*x^(4*p)*e^(390*n*log(x)
+ 30*p*log(x) + 30*log(x)) + 140654461721840*a^9*b^3*n^5*p^4*x*x^(9*n)*x^(4
*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 55152097018480*a^9*b^3*n^4
*p^5*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1124465
6030080*a^9*b^3*n^3*p^6*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 3
0*log(x)) + 1215842787280*a^9*b^3*n^2*p^7*x*x^(9*n)*x^(4*p)*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 66216664800*a^9*b^3*n*p^8*x*x^(9*n)*x^(4*p)*e
^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1430858880*a^9*b^3*p^9*x*x^(9*n
)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 508368161664*a^10*b^
2*n^9*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 71460
55568520*a^10*b^2*n^8*p*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) +
30*log(x)) + 29641760601156*a^10*b^2*n^7*p^2*x*x^(10*n)*x^(3*p)*e^(390*n*lo
g(x) + 30*p*log(x) + 30*log(x)) + 52030292932860*a^10*b^2*n^6*p^3*x*x^(10*n
)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 43593412942464*a^10*
b^2*n^5*p^4*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
 18435803584176*a^10*b^2*n^4*p^5*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*
log(x) + 30*log(x)) + 4063450114020*a^10*b^2*n^3*p^6*x*x^(10*n)*x^(3*p)*e^(
390*n*log(x) + 30*p*log(x) + 30*log(x)) + 469709163660*a^10*b^2*n^2*p^7*x*x
^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 27001146744*a^
10*b^2*n*p^8*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
+ 609291936*a^10*b^2*p^9*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) +
 30*log(x)) + 6459383880*a^12*n^9*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log
(x) + 30*log(x)) + 91995190788*a^12*n^8*p*x*x^(12*n)*x^p*e^(390*n*log(x) +
30*p*log(x) + 30*log(x)) + 390395916068*a^12*n^7*p^2*x*x^(12*n)*x^p*e^(390*
n*log(x) + 30*p*log(x) + 30*log(x)) + 711238497372*a^12*n^6*p^3*x*x^(12*n)*
x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 631831715872*a^12*n^5*p^4*
x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 290792620468*a^
12*n^4*p^5*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 7148
8088124*a^12*n^3*p^6*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(
x)) + 9361432076*a^12*n^2*p^7*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x)
+ 30*log(x)) + 612050040*a^12*n*p^8*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*l
```

```
og(x) + 30*log(x)) + 15664032*a^12*p^9*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 13850300160*a^2*b^10*n^9*x*x^(2*n)*x^p*e^(325*n*log
(x) + 25*p*log(x) + 25*log(x)) + 196796277216*a^2*b^10*n^8*p*x*x^(2*n)*x^p*
e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 832771425456*a^2*b^10*n^7*p^2*
x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 1511260255824*a^
2*b^10*n^6*p^3*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 1
335169607184*a^2*b^10*n^5*p^4*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) +
 25*log(x)) + 610000650336*a^2*b^10*n^4*p^5*x*x^(2*n)*x^p*e^(325*n*log(x) +
 25*p*log(x) + 25*log(x)) + 148575844848*a^2*b^10*n^3*p^6*x*x^(2*n)*x^p*e^(
325*n*log(x) + 25*p*log(x) + 25*log(x)) + 19239408432*a^2*b^10*n^2*p^7*x*x^
(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 1241739600*a^2*b^10*
n*p^8*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 31328064*a
^2*b^10*p^9*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 1445
7122784*a^2*b^10*n^8*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x
)) + 135174661344*a^2*b^10*n^7*p*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x
) + 35*log(x)) + 404124332064*a^2*b^10*n^6*p^2*x*x^(2*n)*x^p*e^(455*n*log(x
) + 35*p*log(x) + 35*log(x)) + 526747158528*a^2*b^10*n^5*p^3*x*x^(2*n)*x^p*
e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 335371910208*a^2*b^10*n^4*p^4*
x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 109417817568*a^2
*b^10*n^3*p^5*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 18
349693248*a^2*b^10*n^2*p^6*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35
*log(x)) + 1486943616*a^2*b^10*n*p^7*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*l
og(x) + 35*log(x)) + 45790272*a^2*b^10*p^8*x*x^(2*n)*x^p*e^(455*n*log(x) +
35*p*log(x) + 35*log(x)) + 1451117056752*a^5*b^7*n^8*x*x^(5*n)*x^p*e^(442*n
*log(x) + 34*p*log(x) + 34*log(x)) + 13558251394512*a^5*b^7*n^7*p*x*x^(5*n)
*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 40497417998136*a^5*b^7*n^
6*p^2*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 5272189885
7496*a^5*b^7*n^5*p^3*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x
)) + 33515482057200*a^5*b^7*n^4*p^4*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*lo
g(x) + 34*log(x)) + 10914040731528*a^5*b^7*n^3*p^5*x*x^(5*n)*x^p*e^(442*n*l
og(x) + 34*p*log(x) + 34*log(x)) + 1826241751224*a^5*b^7*n^2*p^6*x*x^(5*n)*
x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 147613694976*a^5*b^7*n*p^7
*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 4533236928*a^5*
b^7*p^8*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 19348227
42336*a^7*b^5*n^8*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x))
+ 18077668526016*a^7*b^5*n^7*p*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x)
+ 33*log(x)) + 53996557330848*a^7*b^5*n^6*p^2*x*x^(7*n)*x^p*e^(429*n*log(x)
 + 33*p*log(x) + 33*log(x)) + 70295865143328*a^7*b^5*n^5*p^3*x*x^(7*n)*x^p*
e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 44687309409600*a^7*b^5*n^4*p^4
*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 14552054308704*
a^7*b^5*n^3*p^5*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) +
2434989001632*a^7*b^5*n^2*p^6*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) +
 33*log(x)) + 196818259968*a^7*b^5*n*p^7*x*x^(7*n)*x^p*e^(429*n*log(x) + 33
*p*log(x) + 33*log(x)) + 6044315904*a^7*b^5*p^8*x*x^(7*n)*x^p*e^(429*n*log(
x) + 33*p*log(x) + 33*log(x)) + 1658446729632*a^6*b^6*n^8*x*x^(6*n)*x^p*e^(
```

```
429*n*log(x) + 33*p*log(x) + 33*log(x)) + 15511995553824*a^6*b^6*n^7*p*x*x^
(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 46396077530880*a^6*b
^6*n^6*p^2*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 60509
706313056*a^6*b^6*n^5*p^3*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*
log(x)) + 38554819779168*a^6*b^6*n^4*p^4*x*x^(6*n)*x^p*e^(429*n*log(x) + 33
*p*log(x) + 33*log(x)) + 12590552923872*a^6*b^6*n^3*p^5*x*x^(6*n)*x^p*e^(42
9*n*log(x) + 33*p*log(x) + 33*log(x)) + 2113804441056*a^6*b^6*n^2*p^6*x*x^(
6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 171504992736*a^6*b^6*
n*p^7*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 5288776416
*a^6*b^6*p^8*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 165
8446729632*a^6*b^6*n^8*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log
(x)) + 15511995553824*a^6*b^6*n^7*p*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*lo
g(x) + 33*log(x)) + 46396077530880*a^6*b^6*n^6*p^2*x*x^(5*n)*x^p*e^(429*n*l
og(x) + 33*p*log(x) + 33*log(x)) + 60509706313056*a^6*b^6*n^5*p^3*x*x^(5*n)
*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 38554819779168*a^6*b^6*n^
4*p^4*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1259055292
3872*a^6*b^6*n^3*p^5*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x
)) + 2113804441056*a^6*b^6*n^2*p^6*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log
(x) + 33*log(x)) + 171504992736*a^6*b^6*n*p^7*x*x^(5*n)*x^p*e^(429*n*log(x)
 + 33*p*log(x) + 33*log(x)) + 5288776416*a^6*b^6*p^8*x*x^(5*n)*x^p*e^(429*n
*log(x) + 33*p*log(x) + 33*log(x)) + 443081911968*a*b^11*n^8*x*x^n*x^(6*p)*
e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2683911976416*a*b^11*n^7*p*x*x
^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 5201097284400*a*b^1
1*n^6*p^2*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 447524
3172912*a*b^11*n^5*p^3*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log
(x)) + 1936043630304*a*b^11*n^4*p^4*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*lo
g(x) + 32*log(x)) + 445133584656*a*b^11*n^3*p^5*x*x^n*x^(6*p)*e^(416*n*log(
x) + 32*p*log(x) + 32*log(x)) + 54773710128*a*b^11*n^2*p^6*x*x^n*x^(6*p)*e^
(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 3390581376*a*b^11*n*p^7*x*x^n*x^
(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 82753920*a*b^11*p^8*x*x^
n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 1484375661288*a^2*b^
10*n^8*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 10514
783132316*a^2*b^10*n^7*p*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) +
32*log(x)) + 22897355882814*a^2*b^10*n^6*p^2*x*x^(2*n)*x^(5*p)*e^(416*n*log
(x) + 32*p*log(x) + 32*log(x)) + 21337887309768*a^2*b^10*n^5*p^3*x*x^(2*n)*
x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 9731225463840*a^2*b^10
*n^4*p^4*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 231
6295538064*a^2*b^10*n^3*p^5*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x)
 + 32*log(x)) + 291692203866*a^2*b^10*n^2*p^6*x*x^(2*n)*x^(5*p)*e^(416*n*lo
g(x) + 32*p*log(x) + 32*log(x)) + 18345680604*a^2*b^10*n*p^7*x*x^(2*n)*x^(5
*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 452854800*a^2*b^10*p^8*x*x
^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 117605969220*a
^8*b^4*n^8*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 11008
996552620*a^8*b^4*n^7*p*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*lo
g(x)) + 32942194474860*a^8*b^4*n^6*p^2*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p
```

```
*log(x) + 32*log(x)) + 42988226240520*a^8*b^4*n^5*p^3*x*x^(8*n)*x^p*e^(416*
n*log(x) + 32*p*log(x) + 32*log(x)) + 27411226688340*a^8*b^4*n^4*p^4*x*x^(8
*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 8959741942140*a^8*b^4*
n^3*p^5*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 15058841
62860*a^8*b^4*n^2*p^6*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(
x)) + 122334284160*a^8*b^4*n*p^7*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x
) + 32*log(x)) + 3777697440*a^8*b^4*p^8*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*
p*log(x) + 32*log(x)) + 1176605969220*a^8*b^4*n^8*x*x^(7*n)*x^p*e^(416*n*lo
g(x) + 32*p*log(x) + 32*log(x)) + 11008996552620*a^8*b^4*n^7*p*x*x^(7*n)*x^
p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 32942194474860*a^8*b^4*n^6*p
^2*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 4298822624052
0*a^8*b^4*n^5*p^3*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x))
 + 27411226688340*a^8*b^4*n^4*p^4*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x
) + 32*log(x)) + 8959741942140*a^8*b^4*n^3*p^5*x*x^(7*n)*x^p*e^(416*n*log(x
) + 32*p*log(x) + 32*log(x)) + 1505884162860*a^8*b^4*n^2*p^6*x*x^(7*n)*x^p*
e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 122334284160*a^8*b^4*n*p^7*x*x
^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 3777697440*a^8*b^4*
p^8*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 40592350872*
a^11*b*n^8*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 3807
95654952*a^11*b*n^7*p*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log
(x)) + 1143236019432*a^11*b*n^6*p^2*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*l
og(x) + 31*log(x)) + 1498459714224*a^11*b*n^5*p^3*x*x^(11*n)*x^p*e^(403*n*l
og(x) + 31*p*log(x) + 31*log(x)) + 960938273304*a^11*b*n^4*p^4*x*x^(11*n)*x
^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 316326714504*a^11*b*n^3*p^5
*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 53620038984*a^
11*b*n^2*p^6*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 43
99273728*a^11*b*n*p^7*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log
(x)) + 137370816*a^11*b*p^8*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) +
31*log(x)) + 87644085624*b^12*n^9*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) +
 30*log(x)) + 549120712272*b^12*n^8*p*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(
x) + 30*log(x)) + 1230524560580*b^12*n^7*p^2*x*x^(7*p)*e^(390*n*log(x) + 30
*p*log(x) + 30*log(x)) + 1291395582962*b^12*n^6*p^3*x*x^(7*p)*e^(390*n*log(
x) + 30*p*log(x) + 30*log(x)) + 703884615728*b^12*n^5*p^4*x*x^(7*p)*e^(390*
n*log(x) + 30*p*log(x) + 30*log(x)) + 210969364912*b^12*n^4*p^5*x*x^(7*p)*e
^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 35784807664*b^12*n^3*p^6*x*x^(7
*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 3409367230*b^12*n^2*p^7*x*
x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 169779204*b^12*n*p^8*x
*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 3436272*b^12*p^9*x*x^
(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 4711093764960*a^3*b^9*n^
8*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 3495128490
1280*a^3*b^9*n^7*p*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log
(x)) + 77609858209840*a^3*b^9*n^6*p^2*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 72609515180400*a^3*b^9*n^5*p^3*x*x^(3*n)*x^(6*p)*
e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 33038264020960*a^3*b^9*n^4*p^4
*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 78338784576
```

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80*a^3*b^9*n^3*p^5*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log
(x)) + 982860465840*a^3*b^9*n^2*p^6*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 61620831360*a^3*b^9*n*p^7*x*x^(3*n)*x^(6*p)*e^(390*
n*log(x) + 30*p*log(x) + 30*log(x)) + 1517155200*a^3*b^9*p^8*x*x^(3*n)*x^(6
*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 8598614407020*a^4*b^8*n^8*
x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 693200726133
30*a^4*b^8*n^7*p*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x
)) + 168436565138655*a^4*b^8*n^6*p^2*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30
*p*log(x) + 30*log(x)) + 170217907877160*a^4*b^8*n^5*p^3*x*x^(4*n)*x^(5*p)*
e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 81985375917720*a^4*b^8*n^4*p^4
*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 20215751398
920*a^4*b^8*n^3*p^5*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*lo
g(x)) + 2604942942525*a^4*b^8*n^2*p^6*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 166377757590*a^4*b^8*n*p^7*x*x^(4*n)*x^(5*p)*e^(3
90*n*log(x) + 30*p*log(x) + 30*log(x)) + 4151169000*a^4*b^8*p^8*x*x^(4*n)*x
^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1934088018720*a^9*b^3*n
^8*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 174943566
52960*a^9*b^3*n^7*p*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*lo
g(x)) + 49439116493360*a^9*b^3*n^6*p^2*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) +
30*p*log(x) + 30*log(x)) + 59038710713840*a^9*b^3*n^5*p^3*x*x^(9*n)*x^(4*p)
*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 33220971859680*a^9*b^3*n^4*p^
4*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 9286391293
520*a^9*b^3*n^3*p^5*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*lo
g(x)) + 1312779203120*a^9*b^3*n^2*p^6*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 89675910720*a^9*b^3*n*p^7*x*x^(9*n)*x^(4*p)*e^(39
0*n*log(x) + 30*p*log(x) + 30*log(x)) + 2350719360*a^9*b^3*p^8*x*x^(9*n)*x^
(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 527614148952*a^10*b^2*n^
8*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 485819410
3884*a^10*b^2*n^7*p*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*l
og(x)) + 14150388842106*a^10*b^2*n^6*p^2*x*x^(10*n)*x^(3*p)*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 17698294870080*a^10*b^2*n^5*p^3*x*x^(10*n)*x^
(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 10603779949752*a^10*b^2*
n^4*p^4*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 318
7439678808*a^10*b^2*n^3*p^5*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x
) + 30*log(x)) + 483732334998*a^10*b^2*n^2*p^6*x*x^(10*n)*x^(3*p)*e^(390*n*
log(x) + 30*p*log(x) + 30*log(x)) + 35120524428*a^10*b^2*n*p^7*x*x^(10*n)*x
^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 967806576*a^10*b^2*p^8*
x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 6765391812*
a^12*n^8*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 634659
42492*a^12*n^7*p*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
+ 190539336572*a^12*n^6*p^2*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) +
30*log(x)) + 249743285704*a^12*n^5*p^3*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 160156378884*a^12*n^4*p^4*x*x^(12*n)*x^p*e^(390*n*l
og(x) + 30*p*log(x) + 30*log(x)) + 52721119084*a^12*n^3*p^5*x*x^(12*n)*x^p*
e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 8936673164*a^12*n^2*p^6*x*x^(1
```

```
2*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 733212288*a^12*n*p^7*
x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 22895136*a^12*p
^8*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 14457122784*
a^2*b^10*n^8*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 135
174661344*a^2*b^10*n^7*p*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*l
og(x)) + 404124332064*a^2*b^10*n^6*p^2*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p
*log(x) + 25*log(x)) + 526747158528*a^2*b^10*n^5*p^3*x*x^(2*n)*x^p*e^(325*n
*log(x) + 25*p*log(x) + 25*log(x)) + 335371910208*a^2*b^10*n^4*p^4*x*x^(2*n
)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 109417817568*a^2*b^10*n^
3*p^5*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 1834969324
8*a^2*b^10*n^2*p^6*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x))
 + 1486943616*a^2*b^10*n*p^7*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) +
25*log(x)) + 45790272*a^2*b^10*p^8*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log
(x) + 25*log(x)) + 7995815280*a^2*b^10*n^7*x*x^(2*n)*x^p*e^(455*n*log(x) +
35*p*log(x) + 35*log(x)) + 52348311192*a^2*b^10*n^6*p*x*x^(2*n)*x^p*e^(455*
n*log(x) + 35*p*log(x) + 35*log(x)) + 112336940436*a^2*b^10*n^5*p^2*x*x^(2*
n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 105319914360*a^2*b^10*n
^4*p^3*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 476065887
84*a^2*b^10*n^3*p^4*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)
) + 10605764832*a^2*b^10*n^2*p^5*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x
) + 35*log(x)) + 1103367084*a^2*b^10*n*p^6*x*x^(2*n)*x^p*e^(455*n*log(x) +
35*p*log(x) + 35*log(x)) + 42331248*a^2*b^10*p^7*x*x^(2*n)*x^p*e^(455*n*log
(x) + 35*p*log(x) + 35*log(x)) + 801773808264*a^5*b^7*n^7*x*x^(5*n)*x^p*e^(
442*n*log(x) + 34*p*log(x) + 34*log(x)) + 5244082727796*a^5*b^7*n^6*p*x*x^(
5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 11239798055310*a^5*b^
7*n^5*p^2*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 105217
08564510*a^5*b^7*n^4*p^3*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*l
og(x)) + 4747307628294*a^5*b^7*n^3*p^4*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p
*log(x) + 34*log(x)) + 1055340080982*a^5*b^7*n^2*p^5*x*x^(5*n)*x^p*e^(442*n
*log(x) + 34*p*log(x) + 34*log(x)) + 109525529916*a^5*b^7*n*p^6*x*x^(5*n)*x
^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 4190793552*a^5*b^7*p^7*x*x^
(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 1069031744352*a^7*b^
5*n^7*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 6992110303
728*a^7*b^5*n^6*p*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x))
 + 14986397407080*a^7*b^5*n^5*p^2*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x
) + 33*log(x)) + 14028944752680*a^7*b^5*n^4*p^3*x*x^(7*n)*x^p*e^(429*n*log(
x) + 33*p*log(x) + 33*log(x)) + 6329743504392*a^7*b^5*n^3*p^4*x*x^(7*n)*x^p
*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1407120107976*a^7*b^5*n^2*p^5
*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 146034039888*a^
7*b^5*n*p^6*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 5587
724736*a^7*b^5*p^7*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x))
 + 917684774160*a^6*b^6*n^7*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 3
3*log(x)) + 6010910585064*a^6*b^6*n^6*p*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*
p*log(x) + 33*log(x)) + 12906859653996*a^6*b^6*n^5*p^2*x*x^(6*n)*x^p*e^(429
*n*log(x) + 33*p*log(x) + 33*log(x)) + 12109689511920*a^6*b^6*n^4*p^3*x*x^(
```

```
6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 5478755051616*a^6*b^6
*n^3*p^4*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1221848
546688*a^6*b^6*n^2*p^5*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log
(x)) + 127268454852*a^6*b^6*n*p^6*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(
x) + 33*log(x)) + 4889259144*a^6*b^6*p^7*x*x^(6*n)*x^p*e^(429*n*log(x) + 33
*p*log(x) + 33*log(x)) + 917684774160*a^6*b^6*n^7*x*x^(5*n)*x^p*e^(429*n*lo
g(x) + 33*p*log(x) + 33*log(x)) + 6010910585064*a^6*b^6*n^6*p*x*x^(5*n)*x^p
*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 12906859653996*a^6*b^6*n^5*p^
2*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 12109689511920
*a^6*b^6*n^4*p^3*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) +
 5478755051616*a^6*b^6*n^3*p^4*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x)
+ 33*log(x)) + 1221848546688*a^6*b^6*n^2*p^5*x*x^(5*n)*x^p*e^(429*n*log(x)
+ 33*p*log(x) + 33*log(x)) + 127268454852*a^6*b^6*n*p^6*x*x^(5*n)*x^p*e^(42
9*n*log(x) + 33*p*log(x) + 33*log(x)) + 4889259144*a^6*b^6*p^7*x*x^(5*n)*x^
p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 179380448496*a*b^11*n^7*x*x^
n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 785545490712*a*b^11*
n^6*p*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 1136586130
788*a*b^11*n^5*p^2*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x))
 + 732765029052*a*b^11*n^4*p^3*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x)
+ 32*log(x)) + 233923620372*a*b^11*n^3*p^4*x*x^n*x^(6*p)*e^(416*n*log(x) +
32*p*log(x) + 32*log(x)) + 37980995340*a*b^11*n^2*p^5*x*x^n*x^(6*p)*e^(416*
n*log(x) + 32*p*log(x) + 32*log(x)) + 2979656280*a*b^11*n*p^6*x*x^n*x^(6*p)
*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 89195040*a*b^11*p^7*x*x^n*x^(
6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 658352480400*a^2*b^10*n^7
*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 32502128196
60*a^2*b^10*n^6*p*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(
x)) + 5148070873956*a^2*b^10*n^5*p^2*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32
*p*log(x) + 32*log(x)) + 3539527022988*a^2*b^10*n^4*p^3*x*x^(2*n)*x^(5*p)*e
^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 1181366112960*a^2*b^10*n^3*p^4*
x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 197768539224
*a^2*b^10*n^2*p^5*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(
x)) + 15848091612*a^2*b^10*n*p^6*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*l
og(x) + 32*log(x)) + 481606560*a^2*b^10*p^7*x*x^(2*n)*x^(5*p)*e^(416*n*log(
x) + 32*p*log(x) + 32*log(x)) + 651375131220*a^8*b^4*n^7*x*x^(8*n)*x^p*e^(4
16*n*log(x) + 32*p*log(x) + 32*log(x)) + 4268565381465*a^8*b^4*n^6*p*x*x^(8
*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 9171056434320*a^8*b^4*
n^5*p^2*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 86109817
58280*a^8*b^4*n^4*p^3*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(
x)) + 3899332676250*a^8*b^4*n^3*p^4*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*lo
g(x) + 32*log(x)) + 870529390335*a^8*b^4*n^2*p^5*x*x^(8*n)*x^p*e^(416*n*log
(x) + 32*p*log(x) + 32*log(x)) + 90784293930*a^8*b^4*n*p^6*x*x^(8*n)*x^p*e^
(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 3492327960*a^8*b^4*p^7*x*x^(8*n)
*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 651375131220*a^8*b^4*n^7*
x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 4268565381465*a^
8*b^4*n^6*p*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 9171
```

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056434320*a^8*b^4*n^5*p^2*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*
log(x)) + 8610981758280*a^8*b^4*n^4*p^3*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*
p*log(x) + 32*log(x)) + 3899332676250*a^8*b^4*n^3*p^4*x*x^(7*n)*x^p*e^(416*
n*log(x) + 32*p*log(x) + 32*log(x)) + 870529390335*a^8*b^4*n^2*p^5*x*x^(7*n
)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 90784293930*a^8*b^4*n*p^
6*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 3492327960*a^8
*b^4*p^7*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2255323
5780*a^11*b*n^7*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) +
 148319247006*a^11*b*n^6*p*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 3
1*log(x)) + 320095847328*a^11*b*n^5*p^2*x*x^(11*n)*x^p*e^(403*n*log(x) + 31
*p*log(x) + 31*log(x)) + 302242822200*a^11*b*n^4*p^3*x*x^(11*n)*x^p*e^(403*
n*log(x) + 31*p*log(x) + 31*log(x)) + 137809591872*a^11*b*n^3*p^4*x*x^(11*n
)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 31019169546*a^11*b*n^2*p
^5*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 3265830252*a
^11*b*n*p^6*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 126
993744*a^11*b*p^7*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x))
 + 51871863372*b^12*n^8*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)
) + 251634889474*b^12*n^7*p*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*lo
g(x)) + 434550279905*b^12*n^6*p^2*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) +
 30*log(x)) + 349927822076*b^12*n^5*p^3*x*x^(7*p)*e^(390*n*log(x) + 30*p*lo
g(x) + 30*log(x)) + 145438843908*b^12*n^4*p^4*x*x^(7*p)*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 32610746252*b^12*n^3*p^5*x*x^(7*p)*e^(390*n*log(x
) + 30*p*log(x) + 30*log(x)) + 3947880911*b^12*n^2*p^6*x*x^(7*p)*e^(390*n*l
og(x) + 30*p*log(x) + 30*log(x)) + 241704126*b^12*n*p^7*x*x^(7*p)*e^(390*n*
log(x) + 30*p*log(x) + 30*log(x)) + 5853816*b^12*p^8*x*x^(7*p)*e^(390*n*log
(x) + 30*p*log(x) + 30*log(x)) + 2233572058960*a^3*b^9*n^7*x*x^(3*n)*x^(6*p
)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 11310260937800*a^3*b^9*n^6*p
*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 18012678787
180*a^3*b^9*n^5*p^2*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*lo
g(x)) + 12331434413980*a^3*b^9*n^4*p^3*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) +
30*p*log(x) + 30*log(x)) + 4085808167100*a^3*b^9*n^3*p^4*x*x^(3*n)*x^(6*p)*
e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 679059958060*a^3*b^9*n^2*p^5*x
*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 54080900280*a
^3*b^9*n*p^6*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
 1635242400*a^3*b^9*p^7*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 3
0*log(x)) + 4269489490530*a^4*b^8*n^7*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 23428576861470*a^4*b^8*n^6*p*x*x^(4*n)*x^(5*p)*e^
(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 40324331437830*a^4*b^8*n^5*p^2*x
*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2943399854439
0*a^4*b^8*n^4*p^3*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(
x)) + 10230641559270*a^4*b^8*n^3*p^4*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30
*p*log(x) + 30*log(x)) + 1759226372640*a^4*b^8*n^2*p^5*x*x^(4*n)*x^(5*p)*e^
(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 143514555030*a^4*b^8*n*p^6*x*x^(
4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 4414726800*a^4*b^
8*p^7*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 105308
```

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0213680*a^9*b^3*n^7*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*lo
g(x)) + 6599592301720*a^9*b^3*n^6*p*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 13217922873540*a^9*b^3*n^5*p^2*x*x^(9*n)*x^(4*p)*e^
(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 11219414780420*a^9*b^3*n^4*p^3*x
*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 4452627154580
*a^9*b^3*n^3*p^4*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x
)) + 850647378900*a^9*b^3*n^2*p^5*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*
log(x) + 30*log(x)) + 75075735240*a^9*b^3*n*p^6*x*x^(9*n)*x^(4*p)*e^(390*n*
log(x) + 30*p*log(x) + 30*log(x)) + 2447549280*a^9*b^3*p^7*x*x^(9*n)*x^(4*p
)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 289592883360*a^10*b^2*n^7*x*
x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1857759853224
*a^10*b^2*n^6*p*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x
)) + 3859385122956*a^10*b^2*n^5*p^2*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30
*p*log(x) + 30*log(x)) + 3448309859148*a^10*b^2*n^4*p^3*x*x^(10*n)*x^(3*p)*
e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1458545850960*a^10*b^2*n^3*p^4
*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2985498369
48*a^10*b^2*n^2*p^5*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*l
og(x)) + 28128887820*a^10*b^2*n*p^6*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30
*p*log(x) + 30*log(x)) + 970101792*a^10*b^2*p^7*x*x^(10*n)*x^(3*p)*e^(390*n
*log(x) + 30*p*log(x) + 30*log(x)) + 3758872630*a^12*n^7*x*x^(12*n)*x^p*e^(
390*n*log(x) + 30*p*log(x) + 30*log(x)) + 24719874501*a^12*n^6*p*x*x^(12*n)
*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 53349307888*a^12*n^5*p^2*
x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 50373803700*a^1
2*n^4*p^3*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 22968
265312*a^12*n^3*p^4*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x
)) + 5169861591*a^12*n^2*p^5*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) +
 30*log(x)) + 544305042*a^12*n*p^6*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*lo
g(x) + 30*log(x)) + 21165624*a^12*p^7*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p
*log(x) + 30*log(x)) + 7995815280*a^2*b^10*n^7*x*x^(2*n)*x^p*e^(325*n*log(x
) + 25*p*log(x) + 25*log(x)) + 52348311192*a^2*b^10*n^6*p*x*x^(2*n)*x^p*e^(
325*n*log(x) + 25*p*log(x) + 25*log(x)) + 112336940436*a^2*b^10*n^5*p^2*x*x
^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 105319914360*a^2*b^
10*n^4*p^3*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 47606
588784*a^2*b^10*n^3*p^4*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*lo
g(x)) + 10605764832*a^2*b^10*n^2*p^5*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*l
og(x) + 25*log(x)) + 1103367084*a^2*b^10*n*p^6*x*x^(2*n)*x^p*e^(325*n*log(x
) + 25*p*log(x) + 25*log(x)) + 42331248*a^2*b^10*p^7*x*x^(2*n)*x^p*e^(325*n
*log(x) + 25*p*log(x) + 25*log(x)) + 2634058920*a^2*b^10*n^6*x*x^(2*n)*x^p*
e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 12340400088*a^2*b^10*n^5*p*x*x
^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 19018570608*a^2*b^1
0*n^4*p^2*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 126122
81208*a^2*b^10*n^3*p^3*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log
(x)) + 3869339208*a^2*b^10*n^2*p^4*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log
(x) + 35*log(x)) + 530872128*a^2*b^10*n*p^5*x*x^(2*n)*x^p*e^(455*n*log(x) +
 35*p*log(x) + 35*log(x)) + 25957872*a^2*b^10*p^6*x*x^(2*n)*x^p*e^(455*n*lo
```

```
g(x) + 35*p*log(x) + 35*log(x)) + 263796730956*a^5*b^7*n^6*x*x^(5*n)*x^p*e^
(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 1234335225420*a^5*b^7*n^5*p*x*x^
(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 1899450294390*a^5*b^
7*n^4*p^2*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 125738
4366216*a^5*b^7*n^3*p^3*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*lo
g(x)) + 384957329922*a^5*b^7*n^2*p^4*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*l
og(x) + 34*log(x)) + 52692456960*a^5*b^7*n*p^5*x*x^(5*n)*x^p*e^(442*n*log(x
) + 34*p*log(x) + 34*log(x)) + 2569829328*a^5*b^7*p^6*x*x^(5*n)*x^p*e^(442*
n*log(x) + 34*p*log(x) + 34*log(x)) + 351728974608*a^7*b^5*n^6*x*x^(7*n)*x^
p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1645780300560*a^7*b^5*n^5*p*
x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 2532600392520*a^
7*b^5*n^4*p^2*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 16
76512488288*a^7*b^5*n^3*p^3*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 3
3*log(x)) + 513276439896*a^7*b^5*n^2*p^4*x*x^(7*n)*x^p*e^(429*n*log(x) + 33
*p*log(x) + 33*log(x)) + 70256609280*a^7*b^5*n*p^5*x*x^(7*n)*x^p*e^(429*n*l
og(x) + 33*p*log(x) + 33*log(x)) + 3426439104*a^7*b^5*p^6*x*x^(7*n)*x^p*e^(
429*n*log(x) + 33*p*log(x) + 33*log(x)) + 302498611800*a^6*b^6*n^6*x*x^(6*n
)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1418052050184*a^6*b^6*n^
5*p*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 218706867271
2*a^6*b^6*n^4*p^2*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x))
+ 1451646801528*a^6*b^6*n^3*p^3*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x)
 + 33*log(x)) + 445809588840*a^6*b^6*n^2*p^4*x*x^(6*n)*x^p*e^(429*n*log(x)
+ 33*p*log(x) + 33*log(x)) + 61236329616*a^6*b^6*n*p^5*x*x^(6*n)*x^p*e^(429
*n*log(x) + 33*p*log(x) + 33*log(x)) + 2998134216*a^6*b^6*p^6*x*x^(6*n)*x^p
*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 302498611800*a^6*b^6*n^6*x*x^
(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1418052050184*a^6*b^
6*n^5*p*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 21870686
72712*a^6*b^6*n^4*p^2*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(
x)) + 1451646801528*a^6*b^6*n^3*p^3*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*lo
g(x) + 33*log(x)) + 445809588840*a^6*b^6*n^2*p^4*x*x^(5*n)*x^p*e^(429*n*log
(x) + 33*p*log(x) + 33*log(x)) + 61236329616*a^6*b^6*n*p^5*x*x^(5*n)*x^p*e^
(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 2998134216*a^6*b^6*p^6*x*x^(5*n)
*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 45076390824*a*b^11*n^6*x*
x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 145773581736*a*b^1
1*n^5*p*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 15685423
4412*a*b^11*n^4*p^2*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)
) + 74017549632*a*b^11*n^3*p^3*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x)
+ 32*log(x)) + 16547578356*a*b^11*n^2*p^4*x*x^n*x^(6*p)*e^(416*n*log(x) + 3
2*p*log(x) + 32*log(x)) + 1700169696*a*b^11*n*p^5*x*x^n*x^(6*p)*e^(416*n*lo
g(x) + 32*p*log(x) + 32*log(x)) + 64107936*a*b^11*p^6*x*x^n*x^(6*p)*e^(416*
n*log(x) + 32*p*log(x) + 32*log(x)) + 175851648270*a^2*b^10*n^6*x*x^(2*n)*x
^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 625767603408*a^2*b^10*n
^5*p*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 7246229
72436*a^2*b^10*n^4*p^2*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32
*log(x)) + 360861482808*a^2*b^10*n^3*p^3*x*x^(2*n)*x^(5*p)*e^(416*n*log(x)
```

```
+ 32*p*log(x) + 32*log(x)) + 83851608402*a^2*b^10*n^2*p^4*x*x^(2*n)*x^(5*p)
*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 8855391420*a^2*b^10*n*p^5*x*x
^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 340549920*a^2*b
^10*p^6*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2148
44683185*a^8*b^4*n^6*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x
)) + 1007755504470*a^8*b^4*n^5*p*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x
) + 32*log(x)) + 1555405308270*a^8*b^4*n^4*p^2*x*x^(8*n)*x^p*e^(416*n*log(x
) + 32*p*log(x) + 32*log(x)) + 1033288412310*a^8*b^4*n^3*p^3*x*x^(8*n)*x^p*
e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 317652743925*a^8*b^4*n^2*p^4*x
*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 43683520320*a^8*b
^4*n*p^5*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2141524
440*a^8*b^4*p^6*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) +
214844683185*a^8*b^4*n^6*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*l
og(x)) + 1007755504470*a^8*b^4*n^5*p*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*l
og(x) + 32*log(x)) + 1555405308270*a^8*b^4*n^4*p^2*x*x^(7*n)*x^p*e^(416*n*l
og(x) + 32*p*log(x) + 32*log(x)) + 1033288412310*a^8*b^4*n^3*p^3*x*x^(7*n)*
x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 317652743925*a^8*b^4*n^2*p
^4*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 43683520320*a
^8*b^4*n*p^5*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 214
1524440*a^8*b^4*p^6*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)
) + 7472931990*a^11*b*n^6*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31
*log(x)) + 35213194524*a^11*b*n^5*p*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*l
og(x) + 31*log(x)) + 54653757924*a^11*b*n^4*p^2*x*x^(11*n)*x^p*e^(403*n*log
(x) + 31*p*log(x) + 31*log(x)) + 36552524604*a^11*b*n^3*p^3*x*x^(11*n)*x^p*
e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 11326446414*a^11*b*n^2*p^4*x*x
^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 1571992704*a^11*b*
n*p^5*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 77873616*
a^11*b*p^6*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 1870
4736610*b^12*n^7*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 701
50104304*b^12*n^6*p*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
92941735986*b^12*n^5*p^2*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x
)) + 56785087070*b^12*n^4*p^3*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*
log(x)) + 17519496206*b^12*n^3*p^4*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x)
+ 30*log(x)) + 2781719058*b^12*n^2*p^5*x*x^(7*p)*e^(390*n*log(x) + 30*p*log
(x) + 30*log(x)) + 214966782*b^12*n*p^6*x*x^(7*p)*e^(390*n*log(x) + 30*p*lo
g(x) + 30*log(x)) + 6367824*b^12*p^7*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x
) + 30*log(x)) + 627156665080*a^3*b^9*n^6*x*x^(3*n)*x^(6*p)*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 2253380407960*a^3*b^9*n^5*p*x*x^(3*n)*x^(6*p)
*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2598690286940*a^3*b^9*n^4*p^2
*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 12814776383
20*a^3*b^9*n^3*p^3*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log
(x)) + 294572555860*a^3*b^9*n^2*p^4*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 30808879200*a^3*b^9*n*p^5*x*x^(3*n)*x^(6*p)*e^(390*
n*log(x) + 30*p*log(x) + 30*log(x)) + 1175312160*a^3*b^9*p^6*x*x^(3*n)*x^(6
*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1247338782855*a^4*b^8*n^6*
```

```
x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 482399633826
0*a^4*b^8*n^5*p*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)
) + 5949974876670*a^4*b^8*n^4*p^2*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*
log(x) + 30*log(x)) + 3099267291780*a^4*b^8*n^3*p^3*x*x^(4*n)*x^(5*p)*e^(39
0*n*log(x) + 30*p*log(x) + 30*log(x)) + 742586458185*a^4*b^8*n^2*p^4*x*x^(4
*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 80053472070*a^4*b^
8*n*p^5*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 3121
707600*a^4*b^8*p^6*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log
(x)) + 340217982280*a^9*b^3*n^6*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*lo
g(x) + 30*log(x)) + 1509548804280*a^9*b^3*n^5*p*x*x^(9*n)*x^(4*p)*e^(390*n*
log(x) + 30*p*log(x) + 30*log(x)) + 2147675822380*a^9*b^3*n^4*p^2*x*x^(9*n)
*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1278541835120*a^9*b^3
*n^3*p^3*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 343
184596260*a^9*b^3*n^2*p^4*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) +
 30*log(x)) + 40460911920*a^9*b^3*n*p^5*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) +
 30*p*log(x) + 30*log(x)) + 1688084640*a^9*b^3*p^6*x*x^(9*n)*x^(4*p)*e^(390
*n*log(x) + 30*p*log(x) + 30*log(x)) + 94491973290*a^10*b^2*n^6*x*x^(10*n)*
x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 431511184632*a^10*b^2*
n^5*p*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 64002
8801412*a^10*b^2*n^4*p^2*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) +
 30*log(x)) + 402242816712*a^10*b^2*n^3*p^3*x*x^(10*n)*x^(3*p)*e^(390*n*log
(x) + 30*p*log(x) + 30*log(x)) + 115004272350*a^10*b^2*n^2*p^4*x*x^(10*n)*x
^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 14474643084*a^10*b^2*n*
p^5*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 6417195
84*a^10*b^2*p^6*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x
)) + 1245488665*a^12*n^6*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*
log(x)) + 5868865754*a^12*n^5*p*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x
) + 30*log(x)) + 9108959654*a^12*n^4*p^2*x*x^(12*n)*x^p*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 6092087434*a^12*n^3*p^3*x*x^(12*n)*x^p*e^(390*n*l
og(x) + 30*p*log(x) + 30*log(x)) + 1887741069*a^12*n^2*p^4*x*x^(12*n)*x^p*e
^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 261998784*a^12*n*p^5*x*x^(12*n)
*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 12978936*a^12*p^6*x*x^(12
*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2634058920*a^2*b^10*n^
6*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 12340400088*a^
2*b^10*n^5*p*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 190
18570608*a^2*b^10*n^4*p^2*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*
log(x)) + 12612281208*a^2*b^10*n^3*p^3*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p
*log(x) + 25*log(x)) + 3869339208*a^2*b^10*n^2*p^4*x*x^(2*n)*x^p*e^(325*n*l
og(x) + 25*p*log(x) + 25*log(x)) + 530872128*a^2*b^10*n*p^5*x*x^(2*n)*x^p*e
^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 25957872*a^2*b^10*p^6*x*x^(2*n)
*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 547446900*a^2*b^10*n^5*x*
x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 1838120472*a^2*b^1
0*n^4*p*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 19985597
04*a^2*b^10*n^3*p^2*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)
) + 895658640*a^2*b^10*n^2*p^3*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x)
```

```
+ 35*log(x)) + 168259788*a^2*b^10*n*p^4*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*
p*log(x) + 35*log(x)) + 10781424*a^2*b^10*p^5*x*x^(2*n)*x^p*e^(455*n*log(x)
 + 35*p*log(x) + 35*log(x)) + 54743422734*a^5*b^7*n^5*x*x^(5*n)*x^p*e^(442*
n*log(x) + 34*p*log(x) + 34*log(x)) + 183532762314*a^5*b^7*n^4*p*x*x^(5*n)*
x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 199203373908*a^5*b^7*n^3*p
^2*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 89094194244*a
^5*b^7*n^2*p^3*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 1
6699496220*a^5*b^7*n*p^4*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*l
og(x)) + 1067360976*a^5*b^7*p^5*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x)
 + 34*log(x)) + 72991230312*a^7*b^5*n^5*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*
p*log(x) + 33*log(x)) + 244710349752*a^7*b^5*n^4*p*x*x^(7*n)*x^p*e^(429*n*l
og(x) + 33*p*log(x) + 33*log(x)) + 265604498544*a^7*b^5*n^3*p^2*x*x^(7*n)*x
^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 118792258992*a^7*b^5*n^2*p^
3*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 22265994960*a^
7*b^5*n*p^4*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1423
147968*a^7*b^5*p^5*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x))
 + 62915985132*a^6*b^6*n^5*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33
*log(x)) + 211403926272*a^6*b^6*n^4*p*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*
log(x) + 33*log(x)) + 230054897688*a^6*b^6*n^3*p^2*x*x^(6*n)*x^p*e^(429*n*l
og(x) + 33*p*log(x) + 33*log(x)) + 103202325072*a^6*b^6*n^2*p^3*x*x^(6*n)*x
^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 19409635476*a^6*b^6*n*p^4*x
*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1245254472*a^6*b^
6*p^5*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 6291598513
2*a^6*b^6*n^5*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 21
1403926272*a^6*b^6*n^4*p*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*l
og(x)) + 230054897688*a^6*b^6*n^3*p^2*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*
log(x) + 33*log(x)) + 103202325072*a^6*b^6*n^2*p^3*x*x^(5*n)*x^p*e^(429*n*l
og(x) + 33*p*log(x) + 33*log(x)) + 19409635476*a^6*b^6*n*p^4*x*x^(5*n)*x^p*
e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1245254472*a^6*b^6*p^5*x*x^(5*
n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 7406904996*a*b^11*n^5*x
*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 17673046836*a*b^1
1*n^4*p*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 13824178
056*a*b^11*n^3*p^2*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x))
 + 4535734440*a*b^11*n^2*p^3*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) +
32*log(x)) + 637142328*a*b^11*n*p^4*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*lo
g(x) + 32*log(x)) + 31265568*a*b^11*p^5*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*
p*log(x) + 32*log(x)) + 30161591460*a^2*b^10*n^5*x*x^(2*n)*x^(5*p)*e^(416*n
*log(x) + 32*p*log(x) + 32*log(x)) + 77875610868*a^2*b^10*n^4*p*x*x^(2*n)*x
^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 64785497004*a^2*b^10*n^
3*p^2*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 222651
56880*a^2*b^10*n^2*p^3*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32
*log(x)) + 3236516028*a^2*b^10*n*p^4*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32
*p*log(x) + 32*log(x)) + 162852768*a^2*b^10*p^5*x*x^(2*n)*x^(5*p)*e^(416*n*
log(x) + 32*p*log(x) + 32*log(x)) + 44717703030*a^8*b^4*n^5*x*x^(8*n)*x^p*e
^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 150365528280*a^8*b^4*n^4*p*x*x^
```

```
(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 163771703370*a^8*b^4
*n^3*p^2*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 7354056
8970*a^8*b^4*n^2*p^3*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x
)) + 13846618170*a^8*b^4*n*p^4*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x)
+ 32*log(x)) + 889467480*a^8*b^4*p^5*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*l
og(x) + 32*log(x)) + 44717703030*a^8*b^4*n^5*x*x^(7*n)*x^p*e^(416*n*log(x)
+ 32*p*log(x) + 32*log(x)) + 150365528280*a^8*b^4*n^4*p*x*x^(7*n)*x^p*e^(41
6*n*log(x) + 32*p*log(x) + 32*log(x)) + 163771703370*a^8*b^4*n^3*p^2*x*x^(7
*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 73540568970*a^8*b^4*n^
2*p^3*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 1384661817
0*a^8*b^4*n*p^4*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) +
889467480*a^8*b^4*p^5*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(
x)) + 1564059660*a^11*b*n^5*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) +
31*log(x)) + 5288610216*a^11*b*n^4*p*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*
log(x) + 31*log(x)) + 5798287932*a^11*b*n^3*p^2*x*x^(11*n)*x^p*e^(403*n*log
(x) + 31*p*log(x) + 31*log(x)) + 2623856220*a^11*b*n^2*p^3*x*x^(11*n)*x^p*e
^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 498449484*a^11*b*n*p^4*x*x^(11*
n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 32344272*a^11*b*p^5*x*x
^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 4373607985*b^12*n^
6*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 12597325584*b^12*n
^5*p*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 12633787582*b^1
2*n^4*p^2*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 5698958228
*b^12*n^3*p^3*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 123664
4835*b^12*n^2*p^4*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 12
4514046*b^12*n*p^5*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 4
629456*b^12*p^6*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1116
79030540*a^3*b^9*n^5*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*l
og(x)) + 287904084820*a^3*b^9*n^4*p*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 236974270280*a^3*b^9*n^3*p^2*x*x^(3*n)*x^(6*p)*e^(3
90*n*log(x) + 30*p*log(x) + 30*log(x)) + 80336957800*a^3*b^9*n^2*p^3*x*x^(3
*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 11523841560*a^3*b^
9*n*p^4*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 5732
02080*a^3*b^9*p^5*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(
x)) + 229399292430*a^4*b^8*n^5*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log
(x) + 30*log(x)) + 632063519010*a^4*b^8*n^4*p*x*x^(4*n)*x^(5*p)*e^(390*n*lo
g(x) + 30*p*log(x) + 30*log(x)) + 551830367880*a^4*b^8*n^3*p^2*x*x^(4*n)*x^
(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 196242732180*a^4*b^8*n^2
*p^3*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2920189
2390*a^4*b^8*n*p^4*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log
(x)) + 1492817040*a^4*b^8*p^5*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(
x) + 30*log(x)) + 69108419380*a^9*b^3*n^5*x*x^(9*n)*x^(4*p)*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 217623750300*a^9*b^3*n^4*p*x*x^(9*n)*x^(4*p)*
e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 216509605400*a^9*b^3*n^3*p^2*x
*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 86587690200*a
^9*b^3*n^2*p^3*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
```

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 + 14209022520*a^9*b^3*n*p^4*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x
) + 30*log(x)) + 784412640*a^9*b^3*p^5*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) +
30*p*log(x) + 30*log(x)) + 19416615372*a^10*b^2*n^5*x*x^(10*n)*x^(3*p)*e^(3
90*n*log(x) + 30*p*log(x) + 30*log(x)) + 63230192916*a^10*b^2*n^4*p*x*x^(10
*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 65825086932*a^10*b
^2*n^3*p^2*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
27831496836*a^10*b^2*n^2*p^3*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(
x) + 30*log(x)) + 4858139484*a^10*b^2*n*p^4*x*x^(10*n)*x^(3*p)*e^(390*n*log
(x) + 30*p*log(x) + 30*log(x)) + 285338592*a^10*b^2*p^5*x*x^(10*n)*x^(3*p)*
e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 260676610*a^12*n^5*x*x^(12*n)*
x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 881435036*a^12*n^4*p*x*x^(
12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 966381322*a^12*n^3*p
^2*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 437309370*a^
12*n^2*p^3*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 8307
4914*a^12*n*p^4*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
 5390712*a^12*p^5*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
 + 547446900*a^2*b^10*n^5*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*
log(x)) + 1838120472*a^2*b^10*n^4*p*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*lo
g(x) + 25*log(x)) + 1998559704*a^2*b^10*n^3*p^2*x*x^(2*n)*x^p*e^(325*n*log(
x) + 25*p*log(x) + 25*log(x)) + 895658640*a^2*b^10*n^2*p^3*x*x^(2*n)*x^p*e^
(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 168259788*a^2*b^10*n*p^4*x*x^(2*
n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 10781424*a^2*b^10*p^5*x
*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 73644672*a^2*b^10
*n^4*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 174053736*a
^2*b^10*n^3*p*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 12
7482864*a^2*b^10*n^2*p^2*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*l
og(x)) + 34831008*a^2*b^10*n*p^3*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x
) + 35*log(x)) + 3039984*a^2*b^10*p^4*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*
log(x) + 35*log(x)) + 7351636842*a^5*b^7*n^4*x*x^(5*n)*x^p*e^(442*n*log(x)
+ 34*p*log(x) + 34*log(x)) + 17345159568*a^5*b^7*n^3*p*x*x^(5*n)*x^p*e^(442
*n*log(x) + 34*p*log(x) + 34*log(x)) + 12679318476*a^5*b^7*n^2*p^2*x*x^(5*n
)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 3456652320*a^5*b^7*n*p^3
*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 300958416*a^5*b
^7*p^4*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 980218245
6*a^7*b^5*n^4*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 23
126879424*a^7*b^5*n^3*p*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*lo
g(x)) + 16905757968*a^7*b^5*n^2*p^2*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*lo
g(x) + 33*log(x)) + 4608869760*a^7*b^5*n*p^3*x*x^(7*n)*x^p*e^(429*n*log(x)
+ 33*p*log(x) + 33*log(x)) + 401277888*a^7*b^5*p^4*x*x^(7*n)*x^p*e^(429*n*l
og(x) + 33*p*log(x) + 33*log(x)) + 8470875336*a^6*b^6*n^4*x*x^(6*n)*x^p*e^(
429*n*log(x) + 33*p*log(x) + 33*log(x)) + 20037322536*a^6*b^6*n^3*p*x*x^(6*
n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 14690280528*a^6*b^6*n^2
*p^2*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 4018091616*
a^6*b^6*n*p^3*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 35
1118152*a^6*b^6*p^4*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)
```

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) + 8470875336*a^6*b^6*n^4*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33
*log(x)) + 20037322536*a^6*b^6*n^3*p*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*l
og(x) + 33*log(x)) + 14690280528*a^6*b^6*n^2*p^2*x*x^(5*n)*x^p*e^(429*n*log
(x) + 33*p*log(x) + 33*log(x)) + 4018091616*a^6*b^6*n*p^3*x*x^(5*n)*x^p*e^(
429*n*log(x) + 33*p*log(x) + 33*log(x)) + 351118152*a^6*b^6*p^4*x*x^(5*n)*x
^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 811969428*a*b^11*n^4*x*x^n*
x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 1399297200*a*b^11*n^3*
p*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 756368472*a*b^
11*n^2*p^2*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 15494
9760*a*b^11*n*p^3*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x))
 + 10332576*a*b^11*p^4*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(
x)) + 3411671724*a^2*b^10*n^4*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(
x) + 32*log(x)) + 6286281264*a^2*b^10*n^3*p*x*x^(2*n)*x^(5*p)*e^(416*n*log(
x) + 32*p*log(x) + 32*log(x)) + 3583012158*a^2*b^10*n^2*p^2*x*x^(2*n)*x^(5*
p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 764582004*a^2*b^10*n*p^3*x*
x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 52581312*a^2*b
^10*p^4*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 6025
748850*a^8*b^4*n^4*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x))
 + 14265560430*a^8*b^4*n^3*p*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) +
32*log(x)) + 10468847070*a^8*b^4*n^2*p^2*x*x^(8*n)*x^p*e^(416*n*log(x) + 32
*p*log(x) + 32*log(x)) + 2866572720*a^8*b^4*n*p^3*x*x^(8*n)*x^p*e^(416*n*lo
g(x) + 32*p*log(x) + 32*log(x)) + 250798680*a^8*b^4*p^4*x*x^(8*n)*x^p*e^(41
6*n*log(x) + 32*p*log(x) + 32*log(x)) + 6025748850*a^8*b^4*n^4*x*x^(7*n)*x^
p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 14265560430*a^8*b^4*n^3*p*x*
x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 10468847070*a^8*b^
4*n^2*p^2*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 286657
2720*a^8*b^4*n*p^3*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x))
 + 250798680*a^8*b^4*p^4*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*l
og(x)) + 212114796*a^11*b*n^4*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p*log(x)
+ 31*log(x)) + 505447548*a^11*b*n^3*p*x*x^(11*n)*x^p*e^(403*n*log(x) + 31*p
*log(x) + 31*log(x)) + 373731732*a^11*b*n^2*p^2*x*x^(11*n)*x^p*e^(403*n*log
(x) + 31*p*log(x) + 31*log(x)) + 103222944*a^11*b*n*p^3*x*x^(11*n)*x^p*e^(4
03*n*log(x) + 31*p*log(x) + 31*log(x)) + 9119952*a^11*b*p^4*x*x^(11*n)*x^p*
e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 684906202*b^12*n^5*x*x^(7*p)*e
^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1492197162*b^12*n^4*p*x*x^(7*p)
*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1101203828*b^12*n^3*p^2*x*x^(
7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 347550762*b^12*n^2*p^3*x*
x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 47541366*b^12*n*p^4*x*
x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2290032*b^12*p^5*x*x^(
7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 13000111300*a^3*b^9*n^4*x
*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 23728510080*a
^3*b^9*n^3*p*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
 13320703000*a^3*b^9*n^2*p^2*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x
) + 30*log(x)) + 2796393600*a^3*b^9*n*p^3*x*x^(3*n)*x^(6*p)*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 189430560*a^3*b^9*p^4*x*x^(3*n)*x^(6*p)*e^(39
```

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0*n*log(x) + 30*p*log(x) + 30*log(x)) + 27407319390*a^4*b^8*n^4*x*x^(4*n)*x
^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 53127522360*a^4*b^8*n^3
*p*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 314280336
15*a^4*b^8*n^2*p^2*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log
(x)) + 6884213490*a^4*b^8*n*p^3*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*lo
g(x) + 30*log(x)) + 481995360*a^4*b^8*p^4*x*x^(4*n)*x^(5*p)*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 9063438340*a^9*b^3*n^4*x*x^(9*n)*x^(4*p)*e^(3
90*n*log(x) + 30*p*log(x) + 30*log(x)) + 19919567360*a^9*b^3*n^3*p*x*x^(9*n
)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 13256735800*a^9*b^3*
n^2*p^2*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 3219
084000*a^9*b^3*n*p^3*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*l
og(x)) + 245314080*a^9*b^3*p^4*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log
(x) + 30*log(x)) + 2578719396*a^10*b^2*n^4*x*x^(10*n)*x^(3*p)*e^(390*n*log(
x) + 30*p*log(x) + 30*log(x)) + 5883941976*a^10*b^2*n^3*p*x*x^(10*n)*x^(3*p
)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 4108070802*a^10*b^2*n^2*p^2*
x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1055125764*
a^10*b^2*n*p^3*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)
) + 85383936*a^10*b^2*p^4*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x)
+ 30*log(x)) + 35352466*a^12*n^4*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(
x) + 30*log(x)) + 84241258*a^12*n^3*p*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p
*log(x) + 30*log(x)) + 62288622*a^12*n^2*p^2*x*x^(12*n)*x^p*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 17203824*a^12*n*p^3*x*x^(12*n)*x^p*e^(390*n*l
og(x) + 30*p*log(x) + 30*log(x)) + 1519992*a^12*p^4*x*x^(12*n)*x^p*e^(390*n
*log(x) + 30*p*log(x) + 30*log(x)) + 73644672*a^2*b^10*n^4*x*x^(2*n)*x^p*e^
(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 174053736*a^2*b^10*n^3*p*x*x^(2*
n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 127482864*a^2*b^10*n^2*
p^2*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 34831008*a^2
*b^10*n*p^3*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 3039
984*a^2*b^10*p^4*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) +
 6404520*a^2*b^10*n^3*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(
x)) + 10170528*a^2*b^10*n^2*p*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) +
 35*log(x)) + 4526148*a^2*b^10*n*p^2*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*l
og(x) + 35*log(x)) + 572112*a^2*b^10*p^3*x*x^(2*n)*x^p*e^(455*n*log(x) + 35
*p*log(x) + 35*log(x)) + 638125686*a^5*b^7*n^3*x*x^(5*n)*x^p*e^(442*n*log(x
) + 34*p*log(x) + 34*log(x)) + 1011418254*a^5*b^7*n^2*p*x*x^(5*n)*x^p*e^(44
2*n*log(x) + 34*p*log(x) + 34*log(x)) + 449145972*a^5*b^7*n*p^2*x*x^(5*n)*x
^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 56639088*a^5*b^7*p^3*x*x^(5
*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 850834248*a^7*b^5*n^3*
x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 1348557672*a^7*b
^5*n^2*p*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 5988612
96*a^7*b^5*n*p^2*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) +
 75518784*a^7*b^5*p^3*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(
x)) + 737361240*a^6*b^6*n^3*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 3
3*log(x)) + 1172060736*a^6*b^6*n^2*p*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*l
og(x) + 33*log(x)) + 522153324*a^6*b^6*n*p^2*x*x^(6*n)*x^p*e^(429*n*log(x)
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+ 33*p*log(x) + 33*log(x)) + 66078936*a^6*b^6*p^3*x*x^(6*n)*x^p*e^(429*n*lo
g(x) + 33*p*log(x) + 33*log(x)) + 737361240*a^6*b^6*n^3*x*x^(5*n)*x^p*e^(42
9*n*log(x) + 33*p*log(x) + 33*log(x)) + 1172060736*a^6*b^6*n^2*p*x*x^(5*n)*
x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 522153324*a^6*b^6*n*p^2*x*
x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 66078936*a^6*b^6*p
^3*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 59001444*a*b^
11*n^3*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 69870492*
a*b^11*n^2*p*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 234
51336*a*b^11*n*p^2*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x))
 + 2268000*a*b^11*p^3*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(
x)) + 253848600*a^2*b^10*n^3*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x
) + 32*log(x)) + 318555720*a^2*b^10*n^2*p*x*x^(2*n)*x^(5*p)*e^(416*n*log(x)
 + 32*p*log(x) + 32*log(x)) + 112020084*a^2*b^10*n*p^2*x*x^(2*n)*x^(5*p)*e^
(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 11236320*a^2*b^10*p^3*x*x^(2*n)*
x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 525008880*a^8*b^4*n^3*
x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 835309035*a^8*b^
4*n^2*p*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 37252611
0*a^8*b^4*n*p^2*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) +
47199240*a^8*b^4*p^3*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(x
)) + 525008880*a^8*b^4*n^3*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32
*log(x)) + 835309035*a^8*b^4*n^2*p*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*log
(x) + 32*log(x)) + 372526110*a^8*b^4*n*p^2*x*x^(7*n)*x^p*e^(416*n*log(x) +
32*p*log(x) + 32*log(x)) + 47199240*a^8*b^4*p^3*x*x^(7*n)*x^p*e^(416*n*log(
x) + 32*p*log(x) + 32*log(x)) + 18614220*a^11*b*n^3*x*x^(11*n)*x^p*e^(403*n
*log(x) + 31*p*log(x) + 31*log(x)) + 29835474*a^11*b*n^2*p*x*x^(11*n)*x^p*e
^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 13418244*a^11*b*n*p^2*x*x^(11*n
)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 1716336*a^11*b*p^3*x*x^(
11*n)*x^p*e^(403*n*log(x) + 31*p*log(x) + 31*log(x)) + 72580906*b^12*n^4*x*
x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 116147036*b^12*n^3*p*x
*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 59748853*b^12*n^2*p^2
*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 11829546*b^12*n*p^3
*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 770112*b^12*p^4*x*x
^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 989229340*a^3*b^9*n^3*x
*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1222849980*a^
3*b^9*n^2*p*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
422179560*a^3*b^9*n*p^2*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 3
0*log(x)) + 41580000*a^3*b^9*p^3*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30*p*l
og(x) + 30*log(x)) + 2130057270*a^4*b^8*n^3*x*x^(4*n)*x^(5*p)*e^(390*n*log(
x) + 30*p*log(x) + 30*log(x)) + 2781115920*a^4*b^8*n^2*p*x*x^(4*n)*x^(5*p)*
e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1006464690*a^4*b^8*n*p^2*x*x^(
4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 102999600*a^4*b^8
*p^3*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 7671512
20*a^9*b^3*n^3*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
 + 1124687300*a^9*b^3*n^2*p*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(x)
 + 30*log(x)) + 452299320*a^9*b^3*n*p^2*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) +
```

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 30*p*log(x) + 30*log(x)) + 50672160*a^9*b^3*p^3*x*x^(9*n)*x^(4*p)*e^(390*n
*log(x) + 30*p*log(x) + 30*log(x)) + 221162040*a^10*b^2*n^3*x*x^(10*n)*x^(3
*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 337669596*a^10*b^2*n^2*p*x
*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 142703748*a^
10*b^2*n*p^2*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
+ 16907616*a^10*b^2*p^3*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) +
30*log(x)) + 3102370*a^12*n^3*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x)
+ 30*log(x)) + 4972579*a^12*n^2*p*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log
(x) + 30*log(x)) + 2236374*a^12*n*p^2*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p
*log(x) + 30*log(x)) + 286056*a^12*p^3*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 6404520*a^2*b^10*n^3*x*x^(2*n)*x^p*e^(325*n*log(x)
+ 25*p*log(x) + 25*log(x)) + 10170528*a^2*b^10*n^2*p*x*x^(2*n)*x^p*e^(325*n
*log(x) + 25*p*log(x) + 25*log(x)) + 4526148*a^2*b^10*n*p^2*x*x^(2*n)*x^p*e
^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 572112*a^2*b^10*p^3*x*x^(2*n)*x
^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 347880*a^2*b^10*n^2*x*x^(2*
n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 334752*a^2*b^10*n*p*x*x
^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 68688*a^2*b^10*p^2*
x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 34591194*a^5*b^7
*n^2*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 33216480*a^
5*b^7*n*p*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 680011
2*a^5*b^7*p^2*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 46
121592*a^7*b^5*n^2*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x))
 + 44288640*a^7*b^5*n*p*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*lo
g(x)) + 9066816*a^7*b^5*p^2*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 3
3*log(x)) + 40092360*a^6*b^6*n^2*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x
) + 33*log(x)) + 38619504*a^6*b^6*n*p*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*
log(x) + 33*log(x)) + 7933464*a^6*b^6*p^2*x*x^(6*n)*x^p*e^(429*n*log(x) + 3
3*p*log(x) + 33*log(x)) + 40092360*a^6*b^6*n^2*x*x^(5*n)*x^p*e^(429*n*log(x
) + 33*p*log(x) + 33*log(x)) + 38619504*a^6*b^6*n*p*x*x^(5*n)*x^p*e^(429*n*
log(x) + 33*p*log(x) + 33*log(x)) + 7933464*a^6*b^6*p^2*x*x^(5*n)*x^p*e^(42
9*n*log(x) + 33*p*log(x) + 33*log(x)) + 2733636*a*b^11*n^2*x*x^n*x^(6*p)*e^
(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 2000160*a*b^11*n*p*x*x^n*x^(6*p)
*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 315360*a*b^11*p^2*x*x^n*x^(6*
p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 11979630*a^2*b^10*n^2*x*x^(
2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 9225252*a^2*b^10*
n*p*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 1516320*
a^2*b^10*p^2*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) +
 28574865*a^8*b^4*n^2*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32*log(
x)) + 27553680*a^8*b^4*n*p*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x) + 32
*log(x)) + 5666760*a^8*b^4*p^2*x*x^(8*n)*x^p*e^(416*n*log(x) + 32*p*log(x)
+ 32*log(x)) + 28574865*a^8*b^4*n^2*x*x^(7*n)*x^p*e^(416*n*log(x) + 32*p*lo
g(x) + 32*log(x)) + 27553680*a^8*b^4*n*p*x*x^(7*n)*x^p*e^(416*n*log(x) + 32
*p*log(x) + 32*log(x)) + 5666760*a^8*b^4*p^2*x*x^(7*n)*x^p*e^(416*n*log(x)
+ 32*p*log(x) + 32*log(x)) + 1021110*a^11*b*n^2*x*x^(11*n)*x^p*e^(403*n*log
(x) + 31*p*log(x) + 31*log(x)) + 992736*a^11*b*n*p*x*x^(11*n)*x^p*e^(403*n*
```

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log(x) + 31*p*log(x) + 31*log(x)) + 206064*a^11*b*p^2*x*x^(11*n)*x^p*e^(403
*n*log(x) + 31*p*log(x) + 31*log(x)) + 5144590*b^12*n^3*x*x^(7*p)*e^(390*n*
log(x) + 30*p*log(x) + 30*log(x)) + 5724806*b^12*n^2*p*x*x^(7*p)*e^(390*n*l
og(x) + 30*p*log(x) + 30*log(x)) + 1840554*b^12*n*p^2*x*x^(7*p)*e^(390*n*lo
g(x) + 30*p*log(x) + 30*log(x)) + 172656*b^12*p^3*x*x^(7*p)*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 47526820*a^3*b^9*n^2*x*x^(3*n)*x^(6*p)*e^(390
*n*log(x) + 30*p*log(x) + 30*log(x)) + 35909280*a^3*b^9*n*p*x*x^(3*n)*x^(6*
p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 5781600*a^3*b^9*p^2*x*x^(3*
n)*x^(6*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 104128695*a^4*b^8*n
^2*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 82711530*
a^4*b^8*n*p*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
13899600*a^4*b^8*p^2*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) + 30*p*log(x) + 30*l
og(x)) + 40517620*a^9*b^3*n^2*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) + 30*p*log(
x) + 30*log(x)) + 35774640*a^9*b^3*n*p*x*x^(9*n)*x^(4*p)*e^(390*n*log(x) +
30*p*log(x) + 30*log(x)) + 6605280*a^9*b^3*p^2*x*x^(9*n)*x^(4*p)*e^(390*n*l
og(x) + 30*p*log(x) + 30*log(x)) + 11838090*a^10*b^2*n^2*x*x^(10*n)*x^(3*p)
*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 10911780*a^10*b^2*n*p*x*x^(10
*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2119392*a^10*b^2*p
^2*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 170185*a
^12*n^2*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 165456*
a^12*n*p*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 34344*
a^12*p^2*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 347880
*a^2*b^10*n^2*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 33
4752*a^2*b^10*n*p*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x))
+ 68688*a^2*b^10*p^2*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x
)) + 10740*a^2*b^10*n*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(
x)) + 4752*a^2*b^10*p*x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(
x)) + 1065636*a^5*b^7*n*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*lo
g(x)) + 470448*a^5*b^7*p*x*x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*l
og(x)) + 1420848*a^7*b^5*n*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33
*log(x)) + 627264*a^7*b^5*p*x*x^(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 3
3*log(x)) + 1239084*a^6*b^6*n*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x) +
 33*log(x)) + 548856*a^6*b^6*p*x*x^(6*n)*x^p*e^(429*n*log(x) + 33*p*log(x)
+ 33*log(x)) + 1239084*a^6*b^6*n*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(x
) + 33*log(x)) + 548856*a^6*b^6*p*x*x^(5*n)*x^p*e^(429*n*log(x) + 33*p*log(
x) + 33*log(x)) + 73224*a*b^11*n*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x
) + 32*log(x)) + 25056*a*b^11*p*x*x^n*x^(6*p)*e^(416*n*log(x) + 32*p*log(x)
 + 32*log(x)) + 325620*a^2*b^10*n*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) + 32*p*
log(x) + 32*log(x)) + 116640*a^2*b^10*p*x*x^(2*n)*x^(5*p)*e^(416*n*log(x) +
 32*p*log(x) + 32*log(x)) + 884070*a^8*b^4*n*x*x^(8*n)*x^p*e^(416*n*log(x)
+ 32*p*log(x) + 32*log(x)) + 392040*a^8*b^4*p*x*x^(8*n)*x^p*e^(416*n*log(x)
 + 32*p*log(x) + 32*log(x)) + 884070*a^8*b^4*n*x*x^(7*n)*x^p*e^(416*n*log(x
) + 32*p*log(x) + 32*log(x)) + 392040*a^8*b^4*p*x*x^(7*n)*x^p*e^(416*n*log(
x) + 32*p*log(x) + 32*log(x)) + 31860*a^11*b*n*x*x^(11*n)*x^p*e^(403*n*log(
x) + 31*p*log(x) + 31*log(x)) + 14256*a^11*b*p*x*x^(11*n)*x^p*e^(403*n*log(
```

```
x) + 31*p*log(x) + 31*log(x)) + 233905*b^12*n^2*x*x^(7*p)*e^(390*n*log(x) +
 30*p*log(x) + 30*log(x)) + 162234*b^12*n*p*x*x^(7*p)*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 24624*b^12*p^2*x*x^(7*p)*e^(390*n*log(x) + 30*p*log
(x) + 30*log(x)) + 1310760*a^3*b^9*n*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 30
*p*log(x) + 30*log(x)) + 459360*a^3*b^9*p*x*x^(3*n)*x^(6*p)*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 2913570*a^4*b^8*n*x*x^(4*n)*x^(5*p)*e^(390*n*
log(x) + 30*p*log(x) + 30*log(x)) + 1069200*a^4*b^8*p*x*x^(4*n)*x^(5*p)*e^(
390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1215720*a^9*b^3*n*x*x^(9*n)*x^(4*
p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 491040*a^9*b^3*p*x*x^(9*n)*
x^(4*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 359964*a^10*b^2*n*x*x^
(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 152064*a^10*b^2
*p*x*x^(10*n)*x^(3*p)*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 5310*a^1
2*n*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2376*a^12*p
*x*x^(12*n)*x^p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 10740*a^2*b^10
*n*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 4752*a^2*b^10
*p*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p*log(x) + 25*log(x)) + 144*a^2*b^10*
x*x^(2*n)*x^p*e^(455*n*log(x) + 35*p*log(x) + 35*log(x)) + 14256*a^5*b^7*x*
x^(5*n)*x^p*e^(442*n*log(x) + 34*p*log(x) + 34*log(x)) + 19008*a^7*b^5*x*x^
(7*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 16632*a^6*b^6*x*x^(6
*n)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 16632*a^6*b^6*x*x^(5*n
)*x^p*e^(429*n*log(x) + 33*p*log(x) + 33*log(x)) + 864*a*b^11*x*x^n*x^(6*p)
*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 3888*a^2*b^10*x*x^(2*n)*x^(5*
p)*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 11880*a^8*b^4*x*x^(8*n)*x^p
*e^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 11880*a^8*b^4*x*x^(7*n)*x^p*e
^(416*n*log(x) + 32*p*log(x) + 32*log(x)) + 432*a^11*b*x*x^(11*n)*x^p*e^(40
3*n*log(x) + 31*p*log(x) + 31*log(x)) + 6174*b^12*n*x*x^(7*p)*e^(390*n*log(
x) + 30*p*log(x) + 30*log(x)) + 2016*b^12*p*x*x^(7*p)*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 15840*a^3*b^9*x*x^(3*n)*x^(6*p)*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 35640*a^4*b^8*x*x^(4*n)*x^(5*p)*e^(390*n*log(x) +
 30*p*log(x) + 30*log(x)) + 15840*a^9*b^3*x*x^(9*n)*x^(4*p)*e^(390*n*log(x)
 + 30*p*log(x) + 30*log(x)) + 4752*a^10*b^2*x*x^(10*n)*x^(3*p)*e^(390*n*log
(x) + 30*p*log(x) + 30*log(x)) + 72*a^12*x*x^(12*n)*x^p*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 144*a^2*b^10*x*x^(2*n)*x^p*e^(325*n*log(x) + 25*p
*log(x) + 25*log(x)) + 72*b^12*x*x^(7*p)*e^(390*n*log(x) + 30*p*log(x) + 30
*log(x)))/(175365907200*n^11*p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
 1866054386880*n^10*p^2*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 703755
2209248*n^9*p^3*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 12561906002112
*n^8*p^4*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 11624480256096*n^7*p^
5*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 5828111161920*n^6*p^6*e^(390
*n*log(x) + 30*p*log(x) + 30*log(x)) + 1681489204128*n^5*p^7*e^(390*n*log(x
) + 30*p*log(x) + 30*log(x)) + 291933347904*n^4*p^8*e^(390*n*log(x) + 30*p*
log(x) + 30*log(x)) + 31026719520*n^3*p^9*e^(390*n*log(x) + 30*p*log(x) + 3
0*log(x)) + 1977878016*n^2*p^10*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
 + 69517440*n*p^11*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1036800*p^12
*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 29227651200*n^11*e^(390*n*log
```

```
(x) + 30*p*log(x) + 30*log(x)) + 790698529440*n^10*p*e^(390*n*log(x) + 30*p
*log(x) + 30*log(x)) + 5066972858880*n^9*p^2*e^(390*n*log(x) + 30*p*log(x)
+ 30*log(x)) + 13409890155144*n^8*p^3*e^(390*n*log(x) + 30*p*log(x) + 30*lo
g(x)) + 17406209876328*n^7*p^4*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
 11926006701696*n^6*p^5*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 454827
1945392*n^5*p^6*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1007197526376*
n^4*p^7*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 132106644072*n^3*p^8*e
^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 10106644464*n^2*p^9*e^(390*n*lo
g(x) + 30*p*log(x) + 30*log(x)) + 416648448*n*p^10*e^(390*n*log(x) + 30*p*l
og(x) + 30*log(x)) + 7153920*p^11*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)
) + 79948244160*n^10*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 117487242
8856*n^9*p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 5180764132764*n^8*p
^2*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 9961280113532*n^7*p^3*e^(39
0*n*log(x) + 30*p*log(x) + 30*log(x)) + 9574149304668*n^6*p^4*e^(390*n*log(
x) + 30*p*log(x) + 30*log(x)) + 4934644985152*n^5*p^5*e^(390*n*log(x) + 30*
p*log(x) + 30*log(x)) + 1425573184204*n^4*p^6*e^(390*n*log(x) + 30*p*log(x)
 + 30*log(x)) + 235864766676*n^3*p^7*e^(390*n*log(x) + 30*p*log(x) + 30*log
(x)) + 22085703068*n^2*p^8*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 108
6185592*n*p^9*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 21781152*p^10*e^
(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 87644085624*n^9*e^(390*n*log(x)
+ 30*p*log(x) + 30*log(x)) + 860351892504*n^8*p*e^(390*n*log(x) + 30*p*log(
x) + 30*log(x)) + 2740333897424*n^7*p^2*e^(390*n*log(x) + 30*p*log(x) + 30*
log(x)) + 3898697262392*n^6*p^3*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
+ 2803451548184*n^5*p^4*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 108360
2428360*n^4*p^5*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 231449285176*n
^3*p^6*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 27096652696*n^2*p^7*e^(
390*n*log(x) + 30*p*log(x) + 30*log(x)) + 1620003960*n*p^8*e^(390*n*log(x)
+ 30*p*log(x) + 30*log(x)) + 38559168*p^9*e^(390*n*log(x) + 30*p*log(x) + 3
0*log(x)) + 51871863372*n^8*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 36
3863309134*n^7*p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 855450905729*
n^6*p^2*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 907578237992*n^5*p^3*e
^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 486149366328*n^4*p^4*e^(390*n*l
og(x) + 30*p*log(x) + 30*log(x)) + 137727723488*n^3*p^5*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 20638195259*n^2*p^6*e^(390*n*log(x) + 30*p*log(x)
 + 30*log(x)) + 1531504818*n*p^7*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
 + 44060760*p^8*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 18704736610*n^
7*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 96391752214*n^6*p*e^(390*n*l
og(x) + 30*p*log(x) + 30*log(x)) + 168525689490*n^5*p^2*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 132587812562*n^4*p^3*e^(390*n*log(x) + 30*p*log(x
) + 30*log(x)) + 51713245574*n^3*p^4*e^(390*n*log(x) + 30*p*log(x) + 30*log
(x)) + 10201588068*n^2*p^5*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 962
051058*n*p^6*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 34144560*p^7*e^(3
90*n*log(x) + 30*p*log(x) + 30*log(x)) + 4373607985*n^6*e^(390*n*log(x) + 3
0*p*log(x) + 30*log(x)) + 16706762796*n^5*p*e^(390*n*log(x) + 30*p*log(x) +
 30*log(x)) + 21586970554*n^4*p^2*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)
```

```
) + 12306181196*n^3*p^3*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 332194
9407*n^2*p^4*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 409762242*n*p^5*e
^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 18369648*p^6*e^(390*n*log(x) +
30*p*log(x) + 30*log(x)) + 684906202*n^5*e^(390*n*log(x) + 30*p*log(x) + 30
*log(x)) + 1927682598*n^4*p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 17
98086044*n^3*p^2*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 706043880*n^2
*p^3*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 118518642*n*p^4*e^(390*n*
log(x) + 30*p*log(x) + 30*log(x)) + 6910704*p^5*e^(390*n*log(x) + 30*p*log(
x) + 30*log(x)) + 72580906*n^4*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) +
 147014576*n^3*p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 94097689*n^2*
p^2*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 22872870*n*p^3*e^(390*n*lo
g(x) + 30*p*log(x) + 30*log(x)) + 1806048*p^4*e^(390*n*log(x) + 30*p*log(x)
 + 30*log(x)) + 5144590*n^3*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 71
28236*n^2*p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 2813958*n*p^2*e^(3
90*n*log(x) + 30*p*log(x) + 30*log(x)) + 320400*p^3*e^(390*n*log(x) + 30*p*
log(x) + 30*log(x)) + 233905*n^2*e^(390*n*log(x) + 30*p*log(x) + 30*log(x))
 + 199278*n*p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 36720*p^2*e^(390
*n*log(x) + 30*p*log(x) + 30*log(x)) + 6174*n*e^(390*n*log(x) + 30*p*log(x)
 + 30*log(x)) + 2448*p*e^(390*n*log(x) + 30*p*log(x) + 30*log(x)) + 72*e^(3
90*n*log(x) + 30*p*log(x) + 30*log(x)))
```

Mupad [B] (verification not implemented)

Time = 10.38 (sec) , antiderivative size = 363, normalized size of antiderivative = 12.52

$$
\begin{aligned}
\int x^p\left(ax^n+bx^{1+13n+p}\right)^{12}\,dx &= \frac{a^{12}\,x\,x^p\,x^{12\,n}}{12\,n+p+1}+\frac{b^{12}\,x^{156\,n}\,x^{13\,p}\,x^{13}}{156\,n+13\,p+13}+\frac{22\,a^{10}\,b^2\,x^{36\,n}\,x^{3\,p}\,x^3}{12\,n+p+1}\\
&\quad+\frac{55\,a^9\,b^3\,x^{48\,n}\,x^{4\,p}\,x^4}{12\,n+p+1}+\frac{99\,a^8\,b^4\,x^{60\,n}\,x^{5\,p}\,x^5}{12\,n+p+1}\\
&\quad+\frac{132\,a^7\,b^5\,x^{72\,n}\,x^{6\,p}\,x^6}{12\,n+p+1}+\frac{132\,a^6\,b^6\,x^{84\,n}\,x^{7\,p}\,x^7}{12\,n+p+1}\\
&\quad+\frac{99\,a^5\,b^7\,x^{96\,n}\,x^{8\,p}\,x^8}{12\,n+p+1}+\frac{55\,a^4\,b^8\,x^{108\,n}\,x^{9\,p}\,x^9}{12\,n+p+1}\\
&\quad+\frac{22\,a^3\,b^9\,x^{120\,n}\,x^{10\,p}\,x^{10}}{12\,n+p+1}+\frac{6\,a^2\,b^{10}\,x^{132\,n}\,x^{11\,p}\,x^{11}}{12\,n+p+1}\\
&\quad+\frac{6\,a^{11}\,b\,x^{24\,n}\,x^{2\,p}\,x^2}{12\,n+p+1}+\frac{a\,b^{11}\,x^{144\,n}\,x^{12\,p}\,x^{12}}{12\,n+p+1}
\end{aligned}
$$

```
[In] int(x^p*(a*x^n + b*x^(13*n + p + 1))^12,x)
```

```
[Out] (a^12*x*x^p*x^(12*n))/(12*n + p + 1) + (b^12*x^(156*n)*x^(13*p)*x^13)/(156*
n + 13*p + 13) + (22*a^10*b^2*x^(36*n)*x^(3*p)*x^3)/(12*n + p + 1) + (55*a^
9*b^3*x^(48*n)*x^(4*p)*x^4)/(12*n + p + 1) + (99*a^8*b^4*x^(60*n)*x^(5*p)*x
^5)/(12*n + p + 1) + (132*a^7*b^5*x^(72*n)*x^(6*p)*x^6)/(12*n + p + 1) + (1
```

```
32*a^6*b^6*x^(84*n)*x^(7*p)*x^7)/(12*n + p + 1) + (99*a^5*b^7*x^(96*n)*x^(8
*p)*x^8)/(12*n + p + 1) + (55*a^4*b^8*x^(108*n)*x^(9*p)*x^9)/(12*n + p + 1)
 + (22*a^3*b^9*x^(120*n)*x^(10*p)*x^10)/(12*n + p + 1) + (6*a^2*b^10*x^(132
*n)*x^(11*p)*x^11)/(12*n + p + 1) + (6*a^11*b*x^(24*n)*x^(2*p)*x^2)/(12*n +
 p + 1) + (a*b^11*x^(144*n)*x^(12*p)*x^12)/(12*n + p + 1)
```

3.349 $\int x^{12}\left(a+bx^{13}\right)^{12}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

```
[Out] 1/169*(b*x^13+a)^13/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {267}

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

```
[In] Int[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. $2(16) = 32$.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=&\ \frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}\\&+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}\\&+\frac{22}{13}a^3b^9x^{130}+\frac{6}{13}a^2b^{10}x^{143}+\frac{1}{13}ab^{11}x^{156}+\frac{b^{12}x^{169}}{169}\end{aligned}$$

```
[In] Integrate[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(b\,x^{13}+a\right)^{13}}{169b}$
gosper	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{2}b^{10}x^{143}+\frac{22}{13}a^{3}b^{9}x^{130}+\frac{55}{13}a^{4}b^{8}x^{117}+\frac{99}{13}a^{5}b^{7}x^{104}+\frac{132}{13}a^{6}b^{6}x^{91}+$
parallelrisch	$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{2}b^{10}x^{143}+\frac{22}{13}a^{3}b^{9}x^{130}+\frac{55}{13}a^{4}b^{8}x^{117}+\frac{99}{13}a^{5}b^{7}x^{104}+\frac{132}{13}a^{6}b^{6}x^{91}+$
risch	$\frac{b^{12}x^{169}}{169}+\frac{a\,b^{11}x^{156}}{13}+\frac{6a^{2}b^{10}x^{143}}{13}+\frac{22a^{3}b^{9}x^{130}}{13}+\frac{55a^{4}b^{8}x^{117}}{13}+\frac{99a^{5}b^{7}x^{104}}{13}+\frac{132a^{6}b^{6}x^{91}}{13}+\frac{132a^{7}b^{5}x^{78}}{13}+$

```
[In] int(x^12*(b*x^13+a)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/169*(b*x^13+a)^13/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{12}\left(a+bx^{13}\right)^{12}\,dx &= \frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^2b^{10}x^{143}+\frac{22}{13}\,a^3b^9x^{130}\\&\quad+\frac{55}{13}\,a^4b^8x^{117}+\frac{99}{13}\,a^5b^7x^{104}+\frac{132}{13}\,a^6b^6x^{91}+\frac{132}{13}\,a^7b^5x^{78}\\&\quad+\frac{99}{13}\,a^8b^4x^{65}+\frac{55}{13}\,a^9b^3x^{52}+\frac{22}{13}\,a^{10}b^2x^{39}+\frac{6}{13}\,a^{11}bx^{26}+\frac{1}{13}\,a^{12}x^{13}\end{aligned}$$

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="fricas")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{12}\left(a+bx^{13}\right)^{12}\,dx &= \frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^2x^{39}}{13}+\frac{55a^9b^3x^{52}}{13}+\frac{99a^8b^4x^{65}}{13}\\&\quad+\frac{132a^7b^5x^{78}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{55a^4b^8x^{117}}{13}\\&\quad+\frac{22a^3b^9x^{130}}{13}+\frac{6a^2b^{10}x^{143}}{13}+\frac{ab^{11}x^{156}}{13}+\frac{b^{12}x^{169}}{169}\end{aligned}$$

```
[In] integrate(x**12*(b*x**13+a)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{\left(bx^{13}+a\right)^{13}}{169\,b}$$

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="maxima")
[Out] 1/169*(b*x^13 + a)^13/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{\left(bx^{13}+a\right)^{13}}{169\,b}$$

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="giac")
[Out] 1/169*(b*x^13 + a)^13/b
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{\left(b\,x^{13}+a\right)^{13}}{169\,b}$$

```
[In] int(x^12*(a + b*x^13)^12,x)
[Out] (a + b*x^13)^13/(169*b)
```

3.350 $\int x^{12}(ax+bx^{26})^{12}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {1598, 267}

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[In] Int[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int x^{24}\left(a+bx^{25}\right)^{12}\,dx \\
&= \frac{\left(a+bx^{25}\right)^{13}}{325b}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$
\begin{aligned}
\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx = {} & \frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125} \\
& +\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225} \\
& +\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}
\end{aligned}
$$

```
[In] Integrate[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 2.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}+$
parallelrisch	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}+$
gosper	$\frac{x^{25}\left(b^{12}x^{300}+13a\,b^{11}x^{275}+78a^2b^{10}x^{250}+286a^3b^9x^{225}+715a^4b^8x^{200}+1287a^5b^7x^{175}+1716a^6b^6x^{150}+1716a^7b^5x^{125}+1287a^8b^4\right.}{325}$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{22a^3b^9x^{250}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{99a^5b^7x^{200}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{132a^7b^5x^{150}}{25}+99$

```
[In] int(x^12*(b*x^26+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 99/25*a^8*b^4*x^125+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+11/5*a^4*b^8*x^
225+1/325*b^12*x^325+99/25*a^5*b^7*x^200+132/25*a^7*b^5*x^150+132/25*a^6*b^
6*x^175+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*b*a^11*x^50+1/25*a*b^11
*x^300+1/25*a^12*x^25
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}+\frac{99}{25}\,a^{8}b^{4}x^{125}+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}$$

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}$$

```
[In] integrate(x**12*(b*x**26+a*x)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx = {} & \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}\\ & +\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}\\ & +\frac{99}{25}\,a^{8}b^{4}x^{125}+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}\end{aligned}$$

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx = {} & \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}\\ & +\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}\\ & +\frac{99}{25}\,a^{8}b^{4}x^{125}+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}\end{aligned}$$

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx=\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a^4\,b^8\,x^{225}}{5}+\frac{22\,a^3\,b^9\,x^{250}}{25}+\frac{6\,a^2\,b^{10}\,x^{275}}{25}+\frac{a\,b^{11}\,x^{300}}{25}+\frac{b^{12}\,x^{325}}{325}$$

```
[In] int(x^12*(a*x + b*x^26)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

3.351 $\int x^{12}(ax^2+bx^{39})^{12}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 16

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1598, 267}

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[In] Int[x^12*(a*x^2 + b*x^39)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x^{36}\left(a+bx^{37}\right)^{12}\,dx$$

$$= \frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx = \frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185} + \frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333} + \frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}$$

```
[In] Integrate[x^12*(a*x^2 + b*x^39)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 2.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
parallelrisch	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
gosper	$\frac{x^{37}(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a^8b}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{132a^7b^5x^{222}}{37}+$

```
[In] int(x^12*(b*x^39+a*x^2)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 6/37*b*a^11*x^74+1/481*b^12*x^481+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+1
32/37*a^7*b^5*x^222+132/37*a^6*b^6*x^259+22/37*a^3*b^9*x^370+1/37*a^12*x^37
+99/37*a^8*b^4*x^185+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/37*a*b^11*x
^444+6/37*a^2*b^10*x^407
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}+\frac{22}{37}\,a^3b^9x^{370}+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}+\frac{132}{37}\,a^6b^6x^{259}+\frac{132}{37}\,a^7b^5x^{222}+\frac{99}{37}\,a^8b^4x^{185}+\frac{55}{37}\,a^9b^3x^{148}+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="fricas")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^2x^{111}}{37}+\frac{55a^9b^3x^{148}}{37}+\frac{99a^8b^4x^{185}}{37}+\frac{132a^7b^5x^{222}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{ab^{11}x^{444}}{37}+\frac{b^{12}x^{481}}{481}$$

```
[In] integrate(x**12*(b*x**39+a*x**2)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}+\frac{22}{37}\,a^3b^9x^{370}+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}+\frac{132}{37}\,a^6b^6x^{259}+\frac{132}{37}\,a^7b^5x^{222}+\frac{99}{37}\,a^8b^4x^{185}+\frac{55}{37}\,a^9b^3x^{148}+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^2b^{10}x^{407}+\frac{22}{37}\,a^3b^9x^{370}+\frac{55}{37}\,a^4b^8x^{333}+\frac{99}{37}\,a^5b^7x^{296}+\frac{132}{37}\,a^6b^6x^{259}+\frac{132}{37}\,a^7b^5x^{222}+\frac{99}{37}\,a^8b^4x^{185}+\frac{55}{37}\,a^9b^3x^{148}+\frac{22}{37}\,a^{10}b^2x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}$$

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{12}\left(ax^2+bx^{39}\right)^{12}\,dx=\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

```
[In] int(x^12*(a*x^2 + b*x^39)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.352 $\int x^{24}(a+bx^{25})^{12}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {267}

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[In] Int[x^24*(a + b*x^25)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. $2(16) = 32$.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=&\ \frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}\\&+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}\\&+\frac{22}{25}a^3b^9x^{250}+\frac{6}{25}a^2b^{10}x^{275}+\frac{1}{25}ab^{11}x^{300}+\frac{b^{12}x^{325}}{325}\end{aligned}$$

```
[In] Integrate[x^24*(a + b*x^25)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(b\,x^{25}+a\right)^{13}}{325b}$
gosper	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}+$
parallelrisch	$\frac{99}{25}a^8b^4x^{125}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{1}{325}b^{12}x^{325}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^7b^5x^{150}+$
risch	$\frac{b^{12}x^{325}}{325}+\frac{a\,b^{11}x^{300}}{25}+\frac{6a^2b^{10}x^{275}}{25}+\frac{22a^3b^9x^{250}}{25}+\frac{11a^4b^8x^{225}}{5}+\frac{99a^5b^7x^{200}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{132a^7b^5x^{150}}{25}+$

```
[In] int(x^24*(b*x^25+a)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/325*(b*x^25+a)^13/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{24}\left(a+bx^{25}\right)^{12}\,dx &= \frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}\\&\quad+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}\\&\quad+\frac{99}{25}\,a^{8}b^{4}x^{125}+\frac{11}{5}\,a^{9}b^{3}x^{100}+\frac{22}{25}\,a^{10}b^{2}x^{75}+\frac{6}{25}\,a^{11}bx^{50}+\frac{1}{25}\,a^{12}x^{25}\end{aligned}$$

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="fricas")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{24}\left(a+bx^{25}\right)^{12}\,dx &= \frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}\\&\quad+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{225}}{5}\\&\quad+\frac{22a^{3}b^{9}x^{250}}{25}+\frac{6a^{2}b^{10}x^{275}}{25}+\frac{ab^{11}x^{300}}{25}+\frac{b^{12}x^{325}}{325}\end{aligned}$$

```
[In] integrate(x**24*(b*x**25+a)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="maxima")
[Out] 1/325*(b*x^25 + a)^13/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="giac")
[Out] 1/325*(b*x^25 + a)^13/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{24}\left(a+bx^{25}\right)^{12}\,dx=\frac{\left(b\,x^{25}+a\right)^{13}}{325\,b}$$

```
[In] int(x^24*(a + b*x^25)^12,x)
[Out] (a + b*x^25)^13/(325*b)
```

3.353 $\int x^{24}(ax+bx^{38})^{12}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1598, 267}

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[In] Int[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\text{integral} = \int x^{36}\left(a+bx^{37}\right)^{12}\,dx$$

$$= \frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = {} & \frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}\\ & +\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}\\ & +\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}\end{aligned}$$

```
[In] Integrate[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(14) = 28.

Time = 2.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 8.44

method	result
default	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}+$
parallelrisch	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}+$
gosper	$\frac{x^{37}\left(b^{12}x^{444}+13a\,b^{11}x^{407}+78a^2b^{10}x^{370}+286a^3b^9x^{333}+715a^4b^8x^{296}+1287a^5b^7x^{259}+1716a^6b^6x^{222}+1716a^7b^5x^{185}+1287a^8b^4\right.}{481}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{132a^7b^5x^{222}}{37}+$

```
[In] int(x^24*(b*x^38+a*x)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 6/37*b*a^11*x^74+1/481*b^12*x^481+55/37*a^9*b^3*x^148+99/37*a^5*b^7*x^296+1
32/37*a^7*b^5*x^222+132/37*a^6*b^6*x^259+22/37*a^3*b^9*x^370+1/37*a^12*x^37
+99/37*a^8*b^4*x^185+22/37*a^10*b^2*x^111+55/37*a^4*b^8*x^333+1/37*a*b^11*x
^444+6/37*a^2*b^10*x^407
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time $= 0.28$ (sec) , antiderivative size $= 134$, normalized size of antiderivative $= 8.38$

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = \frac{1}{481}\,b^{12}x^{481} + \frac{1}{37}\,ab^{11}x^{444} + \frac{6}{37}\,a^{2}b^{10}x^{407} + \frac{22}{37}\,a^{3}b^{9}x^{370} + \frac{55}{37}\,a^{4}b^{8}x^{333} + \frac{99}{37}\,a^{5}b^{7}x^{296} + \frac{132}{37}\,a^{6}b^{6}x^{259} + \frac{132}{37}\,a^{7}b^{5}x^{222} + \frac{99}{37}\,a^{8}b^{4}x^{185} + \frac{55}{37}\,a^{9}b^{3}x^{148} + \frac{22}{37}\,a^{10}b^{2}x^{111} + \frac{6}{37}\,a^{11}bx^{74} + \frac{1}{37}\,a^{12}x^{37}$$

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time $= 0.05$ (sec) , antiderivative size $= 160$, normalized size of antiderivative $= 10.00$

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx = \frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^{2}x^{111}}{37} + \frac{55a^{9}b^{3}x^{148}}{37} + \frac{99a^{8}b^{4}x^{185}}{37} + \frac{132a^{7}b^{5}x^{222}}{37} + \frac{132a^{6}b^{6}x^{259}}{37} + \frac{99a^{5}b^{7}x^{296}}{37} + \frac{55a^{4}b^{8}x^{333}}{37} + \frac{22a^{3}b^{9}x^{370}}{37} + \frac{6a^{2}b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

```
[In] integrate(x**24*(b*x**38+a*x)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=&\ \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}\\&+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}\\&+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}\end{aligned}$$

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=&\ \frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}\\&+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}\\&+\frac{99}{37}\,a^{8}b^{4}x^{185}+\frac{55}{37}\,a^{9}b^{3}x^{148}+\frac{22}{37}\,a^{10}b^{2}x^{111}+\frac{6}{37}\,a^{11}bx^{74}+\frac{1}{37}\,a^{12}x^{37}\end{aligned}$$

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx=\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55\,a^4\,b^8\,x^{333}}{37}+\frac{22\,a^3\,b^9\,x^{370}}{37}+\frac{6\,a^2\,b^{10}\,x^{407}}{37}+\frac{a\,b^{11}\,x^{444}}{37}+\frac{b^{12}\,x^{481}}{481}$$

```
[In] int(x^24*(a*x + b*x^38)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

3.354 $\int x^{36}(a+bx^{37})^{12}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {267}

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[In] Int[x^36*(a + b*x^37)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. $2(16) = 32$.

Time = 0.00 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\begin{aligned}\int x^{36}\left(a+bx^{37}\right)^{12}\,dx = {}& \frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}\\ &+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{333}\\ &+\frac{22}{37}a^3b^9x^{370}+\frac{6}{37}a^2b^{10}x^{407}+\frac{1}{37}ab^{11}x^{444}+\frac{b^{12}x^{481}}{481}\end{aligned}$$

```
[In] Integrate[x^36*(a + b*x^37)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{\left(b\,x^{37}+a\right)^{13}}{481b}$
gosper	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
parallelrisch	$\frac{6}{37}b\,a^{11}x^{74}+\frac{1}{481}b^{12}x^{481}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{22}{37}a^3b^9x^{370}$
risch	$\frac{b^{12}x^{481}}{481}+\frac{a\,b^{11}x^{444}}{37}+\frac{6a^2b^{10}x^{407}}{37}+\frac{22a^3b^9x^{370}}{37}+\frac{55a^4b^8x^{333}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{132a^7b^5x^{222}}{37}+$

```
[In] int(x^36*(b*x^37+a)^12,x,method=_RETURNVERBOSE)
```

```
[Out] 1/481*(b*x^37+a)^13/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 8.38

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx = \frac{1}{481}\,b^{12}x^{481} + \frac{1}{37}\,ab^{11}x^{444} + \frac{6}{37}\,a^{2}b^{10}x^{407} + \frac{22}{37}\,a^{3}b^{9}x^{370} + \frac{55}{37}\,a^{4}b^{8}x^{333} + \frac{99}{37}\,a^{5}b^{7}x^{296} + \frac{132}{37}\,a^{6}b^{6}x^{259} + \frac{132}{37}\,a^{7}b^{5}x^{222} + \frac{99}{37}\,a^{8}b^{4}x^{185} + \frac{55}{37}\,a^{9}b^{3}x^{148} + \frac{22}{37}\,a^{10}b^{2}x^{111} + \frac{6}{37}\,a^{11}bx^{74} + \frac{1}{37}\,a^{12}x^{37}$$

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="fricas")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(10) = 20$.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 10.00

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx = \frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^{2}x^{111}}{37} + \frac{55a^{9}b^{3}x^{148}}{37} + \frac{99a^{8}b^{4}x^{185}}{37} + \frac{132a^{7}b^{5}x^{222}}{37} + \frac{132a^{6}b^{6}x^{259}}{37} + \frac{99a^{5}b^{7}x^{296}}{37} + \frac{55a^{4}b^{8}x^{333}}{37} + \frac{22a^{3}b^{9}x^{370}}{37} + \frac{6a^{2}b^{10}x^{407}}{37} + \frac{ab^{11}x^{444}}{37} + \frac{b^{12}x^{481}}{481}$$

```
[In] integrate(x**36*(b*x**37+a)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="maxima")
```

```
[Out] 1/481*(b*x^37 + a)^13/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="giac")
```

```
[Out] 1/481*(b*x^37 + a)^13/b
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^{36}\left(a+bx^{37}\right)^{12}\,dx=\frac{\left(b\,x^{37}+a\right)^{13}}{481\,b}$$

```
[In] int(x^36*(a + b*x^37)^12,x)
```

```
[Out] (a + b*x^37)^13/(481*b)
```

3.355 $\int \frac{1}{ax+bx^n}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{1}{ax+bx^n}\,dx=\frac{\log\left(b+ax^{1-n}\right)}{a(1-n)}$$

```
[Out] ln(b+a*x^(1-n))/a/(1-n)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 266}

$$\int \frac{1}{ax+bx^n}\,dx=\frac{\log\left(ax^{1-n}+b\right)}{a(1-n)}$$

```
[In] Int[(a*x + b*x^n)^(-1),x]
```

```
[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^{-n}}{b+ax^{1-n}}\,dx\\ &= \frac{\log\left(b+ax^{1-n}\right)}{a(1-n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx^n}\,dx=\frac{\log\left(b+ax^{1-n}\right)}{a(1-n)}$$

```
[In] Integrate[(a*x + b*x^n)^(-1),x]
[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

method	result	size
parallelrisch	$\frac{n\ln\left(x\right)-\ln\left(ax+b\,x^{n}\right)}{a\left(-1+n\right)}$	27
risch	$\frac{n\ln\left(x\right)}{a\left(-1+n\right)}-\frac{\ln\left(x^{n}+\frac{ax}{b}\right)}{a\left(-1+n\right)}$	35
norman	$\frac{n\ln\left(x\right)}{a\left(-1+n\right)}-\frac{\ln\left(ax+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{a\left(-1+n\right)}$	36

```
[In] int(1/(a*x+b*x^n),x,method=_RETURNVERBOSE)
[Out] (n*ln(x)-ln(a*x+b*x^n))/a/(-1+n)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{ax+bx^n}\,dx=\frac{n\log\left(x\right)-\log\left(ax+bx^{n}\right)}{an-a}$$

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="fricas")
[Out] (n*log(x) - log(a*x + b*x^n))/(a*n - a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(14) = 28$.

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{1}{ax + bx^n}\,dx = \begin{cases} \tilde{\infty}\log{(x)} & \text{for}\: a = 0 \wedge b = 0 \wedge n = 1 \\ \frac{\log{(x)}}{a} & \text{for}\: b = 0 \\ -\frac{x}{b(nx^n - x^n)} & \text{for}\: a = 0 \\ \frac{\log{(x)}}{a+b} & \text{for}\: n = 1 \\ \frac{n\log{(x)}}{an-a} - \frac{\log{\left(\frac{ax}{b} + x^n\right)}}{an-a} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x+b*x**n),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 1)), (log(x)/a, Eq(b, 0)
), (-x/(b*(n*x**n - x**n)), Eq(a, 0)), (log(x)/(a + b), Eq(n, 1)), (n*log(x
)/(a*n - a) - log(a*x/b + x**n)/(a*n - a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{1}{ax + bx^n}\,dx = \frac{n\log{(x)}}{a(n-1)} - \frac{\log{\left(\frac{ax+bx^n}{b}\right)}}{a(n-1)}$$

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="maxima")
```

```
[Out] n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1))
```

Giac [F]

$$\int \frac{1}{ax + bx^n}\,dx = \int \frac{1}{ax + bx^n}\,dx$$

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{1}{ax+bx^n}\,dx=-\frac{\ln\left(b\,x^n+a\,x\right)-n\,\ln\left(x\right)}{a\,\left(n-1\right)}$$

```
[In] int(1/(b*x^n + a*x),x)
```

```
[Out] -(log(b*x^n + a*x) - n*log(x))/(a*(n - 1))
```

3.356 $\int \frac{1}{ax+bx^{1+n}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}$$

```
[Out] ln(x)/a-ln(a+b*x^n)/a/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1607, 272, 36, 29, 31}

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}$$

```
[In] Int[(a*x + b*x^(1 + n))^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^n]/(a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{x\left(a+bx^n\right)}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{an}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^n\right)}{an}\\ &= \frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\frac{\log\left(x^n\right)-\log\left(an(a+bx^n)\right)}{an}$$

```
[In] Integrate[(a*x + b*x^(1 + n))^(-1),x]
[Out] (Log[x^n] - Log[a*n*(a + b*x^n)])/(a*n)
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

method	result	size
parallelrisch	$\frac{n\ln(x)+\ln(x)-\ln\left(ax+b\,x^{1+n}\right)}{an}$	29
norman	$\frac{(1+n)\ln(x)}{an}-\frac{\ln\left(ax+b\,\mathrm{e}^{(1+n)\ln(x)}\right)}{an}$	36
risch	$\frac{\ln(x)}{an}+\frac{\ln(x)}{a}-\frac{\ln\left(x^{1+n}+\frac{ax}{b}\right)}{an}$	38

```
[In] int(1/(a*x+b*x^(1+n)),x,method=_RETURNVERBOSE)
```

```
[Out] (n*ln(x)+ln(x)-ln(a*x+b*x^(1+n)))/a/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\frac{(n+1)\log\left(x\right)-\log\left(ax+bx^{n+1}\right)}{an}$$

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="fricas")
```

```
[Out] ((n + 1)*log(x) - log(a*x + b*x^(n + 1)))/(a*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(15) = 30$.

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int \frac{1}{ax+bx^{1+n}}\,dx=\begin{cases}\frac{\log\left(x\right)}{b} & \text{for}\ a=0\wedge n=0\\ -\frac{xx^{-n-1}}{bn} & \text{for}\ a=0\\ \frac{\log\left(x\right)}{a+b} & \text{for}\ n=0\\ \frac{\log\left(x\right)}{a}+\frac{\log\left(x\right)}{an}-\frac{\log\left(x+\frac{bx^{n+1}}{a}\right)}{an} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(a*x+b*x**(1+n)),x)
```

```
[Out] Piecewise((log(x)/b, Eq(a, 0) & Eq(n, 0)), (-x*x**(-n - 1)/(b*n), Eq(a, 0))
, (log(x)/(a + b), Eq(n, 0)), (log(x)/a + log(x)/(a*n) - log(x + b*x**(n +
1)/a)/(a*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{ax+bx^{1+n}}\,dx = \frac{\log{(x)}}{a} - \frac{\log{\left(\frac{bx^n+a}{b}\right)}}{an}$$

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="maxima")
```

```
[Out] log(x)/a - log((b*x^n + a)/b)/(a*n)
```

Giac [F]

$$\int \frac{1}{ax+bx^{1+n}}\,dx = \int \frac{1}{ax+bx^{n+1}}\,dx$$

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^(n + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{ax+bx^{1+n}}\,dx = \frac{\ln{(x)}\ (n+1)}{a\,n} - \frac{\ln{(x\,(a+b\,x^n))}}{a\,n}$$

```
[In] int(1/(a*x + b*x^(n + 1)),x)
```

```
[Out] (log(x)*(n + 1))/(a*n) - log(x*(a + b*x^n))/(a*n)
```

3.357 $\int \frac{1}{ax+bx^{1-n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\frac{\log\left(b+ax^n\right)}{an}$$

```
[Out] ln(b+a*x^n)/a/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1607, 266}

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\frac{\log\left(ax^n+b\right)}{an}$$

```
[In] Int[(a*x + b*x^(1 - n))^(-1),x]
```

```
[Out] Log[b + a*x^n]/(a*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x^{-1+n}}{b+ax^n}\,dx \\
&= \frac{\log(b+ax^n)}{an}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax+bx^{1-n}}\,dx = \frac{\log(b+ax^n)}{an}$$

```
[In] Integrate[(a*x + b*x^(1 - n))^(-1),x]
```

```
[Out] Log[b + a*x^n]/(a*n)
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

method	result	size
parallelrisch	$\frac{n\ln(x)-\ln(x)+\ln\left(ax+b\,x^{1-n}\right)}{an}$	31
norman	$\frac{(-1+n)\ln(x)}{an}+\frac{\ln\left(ax+b\,\mathrm{e}^{(1-n)\ln(x)}\right)}{an}$	37
risch	$-\frac{\ln(x)}{an}+\frac{\ln(x)}{a}+\frac{\ln\left(x^{1-n}+\frac{ax}{b}\right)}{an}$	40

```
[In] int(1/(a*x+b*x^(1-n)),x,method=_RETURNVERBOSE)
```

```
[Out] (n*ln(x)-ln(x)+ln(a*x+b*x^(1-n)))/a/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{1}{ax+bx^{1-n}}\,dx = \frac{(n-1)\log(x)+\log\left(ax+bx^{-n+1}\right)}{an}$$

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="fricas")
```

```
[Out] ((n - 1)*log(x) + log(a*x + b*x^(-n + 1)))/(a*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(10) = 20$.

Time = 0.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.40

$$\int \frac{1}{ax+bx^{1-n}}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ \frac{xx^{n-1}}{bn} & \text{for } a=0 \\ \frac{\log(x)}{a} & \text{for } b=0 \\ \frac{\log(x)}{a+b} & \text{for } n=0 \\ \frac{\log(x)}{a} - \frac{\log(x)}{an} + \frac{\log\left(\frac{ax}{b}+x^{1-n}\right)}{an} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a*x+b*x**(1-n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)/(b*n)
, Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/(a + b), Eq(n, 0)), (log(x)/a -
log(x)/(a*n) + log(a*x/b + x**(1 - n))/(a*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{1}{ax+bx^{1-n}}\,dx = \frac{\log\left(\frac{ax^n+b}{a}\right)}{an}$$

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="maxima")
```

```
[Out] log((a*x^n + b)/a)/(a*n)
```

Giac [F]

$$\int \frac{1}{ax+bx^{1-n}}\,dx = \int \frac{1}{ax+bx^{-n+1}}\,dx$$

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^(-n + 1)), x)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \frac{1}{ax+bx^{1-n}}\,dx=\frac{\ln\left(a\,x+b\,x^{1-n}\right)}{a\,n}+\frac{\ln\left(x\right)\,\left(n-1\right)}{a\,n}$$

```
[In] int(1/(a*x + b*x^(1 - n)),x)
```

```
[Out] log(a*x + b*x^(1 - n))/(a*n) + (log(x)*(n - 1))/(a*n)
```

3.358 $\int \frac{1}{2x+3x^{1+n}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{2x+3x^{1+n}}\,dx=\frac{\log(x)}{2}-\frac{\log\left(2+3x^n\right)}{2n}$$

```
[Out] 1/2*ln(x)-1/2*ln(2+3*x^n)/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1607, 272, 36, 29, 31}

$$\int \frac{1}{2x+3x^{1+n}}\,dx=\frac{\log(x)}{2}-\frac{\log\left(3x^n+2\right)}{2n}$$

```
[In] Int[(2*x + 3*x^(1 + n))^(-1),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x^n]/(2*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x\left(2+3x^n\right)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(2+3x)}\,dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{2n} - \frac{3\text{Subst}\left(\int \frac{1}{2+3x}\,dx, x, x^n\right)}{2n}\\
&= \frac{\log(x)}{2} - \frac{\log\left(2+3x^n\right)}{2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{2x+3x^{1+n}}\,dx = \frac{\log\left(x^n\right) - \log\left(n(2+3x^n)\right)}{2n}$$

```
[In] Integrate[(2*x + 3*x^(1 + n))^(-1),x]
[Out] (Log[x^n] - Log[n*(2 + 3*x^n)])/(2*n)
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

method	result	size
parallelrisch	$\frac{n\ln\left(x\right)+\ln\left(x\right)-\ln\left(x+\frac{3x^{1+n}}{2}\right)}{2n}$	25
meijerg	$\frac{n\ln\left(x\right)+\ln\left(3\right)-\ln\left(2\right)-\ln\left(1+\frac{3x^{n}}{2}\right)}{2n}$	27
risch	$\frac{\ln\left(x\right)}{2n}+\frac{\ln\left(x\right)}{2}-\frac{\ln\left(\frac{2x}{3}+x^{1+n}\right)}{2n}$	28
norman	$\frac{\left(1+n\right)\ln\left(x\right)}{2n}-\frac{\ln\left(2x+3\,{\mathrm e}^{\left(1+n\right)\ln\left(x\right)}\right)}{2n}$	31

```
[In] int(1/(2*x+3*x^(1+n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(n*ln(x)+ln(x)-ln(x+3/2*x^(1+n)))/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{2x+3x^{1+n}}\,dx=\frac{(n+1)\log\left(x\right)-\log\left(3\,x^{n+1}+2\,x\right)}{2\,n}$$

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="fricas")
```

```
[Out] 1/2*((n + 1)*log(x) - log(3*x^(n + 1) + 2*x))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{1}{2x+3x^{1+n}}\,dx=\begin{cases}\frac{\log\left(x\right)}{2}+\frac{\log\left(x\right)}{2n}-\frac{\log\left(2x+3x^{n+1}\right)}{2n} & \text{for}\ n\neq 0\\ \frac{\log\left(x\right)}{5} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/(2*x+3*x**(1+n)),x)
```

```
[Out] Piecewise((log(x)/2 + log(x)/(2*n) - log(2*x + 3*x**(n + 1))/(2*n), Ne(n, 0
)), (log(x)/5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{2x+3x^{1+n}}\,dx = -\frac{\log\left(x^n+\frac{2}{3}\right)}{2\,n} + \frac{1}{2}\,\log\left(x\right)$$

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="maxima")
[Out] -1/2*log(x^n + 2/3)/n + 1/2*log(x)
```

Giac [F]

$$\int \frac{1}{2x+3x^{1+n}}\,dx = \int \frac{1}{3\,x^{n+1}+2\,x}\,dx$$

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="giac")
[Out] integrate(1/(3*x^(n + 1) + 2*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{2x+3x^{1+n}}\,dx = \frac{\ln\left(x\right)\,\left(n+1\right)}{2\,n} - \frac{\ln\left(\frac{2\,x}{3}+x^{n+1}\right)}{2\,n}$$

```
[In] int(1/(2*x + 3*x^(n + 1)),x)
[Out] (log(x)*(n + 1))/(2*n) - log((2*x)/3 + x^(n + 1))/(2*n)
```

3.359 $\int \frac{1}{2x+3x^{1-n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{2x+3x^{1-n}}\,dx=\frac{\log(3+2x^n)}{2n}$$

```
[Out] 1/2*ln(3+2*x^n)/n
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1607, 266}

$$\int \frac{1}{2x+3x^{1-n}}\,dx=\frac{\log(2x^n+3)}{2n}$$

```
[In] Int[(2*x + 3*x^(1 - n))^(-1),x]
```

```
[Out] Log[3 + 2*x^n]/(2*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{x^{-1+n}}{3+2x^n}\,dx \\ &= \frac{\log\left(3+2x^n\right)}{2n} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{2x+3x^{1-n}}\,dx = \frac{\log\left(3+2x^n\right)}{2n}$$

```
[In] Integrate[(2*x + 3*x^(1 - n))^(-1),x]
```

```
[Out] Log[3 + 2*x^n]/(2*n)
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

method	result	size
parallelrisch	$\frac{n\ln(x)-\ln(x)+\ln\left(x+\frac{3x^{1-n}}{2}\right)}{2n}$	27
meijerg	$-\frac{-n\ln(x)+\ln(3)-\ln(2)-\ln\left(1+\frac{3x^{-n}}{2}\right)}{2n}$	30
risch	$-\frac{\ln(x)}{2n}+\frac{\ln(x)}{2}+\frac{\ln\left(\frac{2x}{3}+x^{1-n}\right)}{2n}$	30
norman	$\frac{(-1+n)\ln(x)}{2n}+\frac{\ln\left(2x+3\,\mathrm{e}^{(1-n)\ln(x)}\right)}{2n}$	33

```
[In] int(1/(2*x+3*x^(1-n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(n*ln(x)-ln(x)+ln(x+3/2*x^(1-n)))/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{1}{2x+3x^{1-n}}\,dx = \frac{(n-1)\log\left(x\right)+\log\left(3\,x^{-n+1}+2\,x\right)}{2\,n}$$

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="fricas")
```

```
[Out] 1/2*((n - 1)*log(x) + log(3*x^(-n + 1) + 2*x))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(10) = 20.

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{1}{2x+3x^{1-n}}\,dx = \begin{cases} \frac{\log\left(x\right)}{2} - \frac{\log\left(x\right)}{2n} + \frac{\log\left(2x+3x^{1-n}\right)}{2n} & \text{for}\: n \neq 0 \\ \frac{\log\left(x\right)}{5} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(2*x+3*x**(1-n)),x)
```

```
[Out] Piecewise((log(x)/2 - log(x)/(2*n) + log(2*x + 3*x**(1 - n))/(2*n), Ne(n, 0
)), (log(x)/5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{2x+3x^{1-n}}\,dx = \frac{\log\left(x^n+\frac{3}{2}\right)}{2\,n}$$

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^n + 3/2)/n
```

Giac [F]

$$\int \frac{1}{2x+3x^{1-n}}\,dx = \int \frac{1}{3\,x^{-n+1}+2\,x}\,dx$$

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="giac")
```

```
[Out] integrate(1/(3*x^(-n + 1) + 2*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{1}{2x+3x^{1-n}}\,dx = \frac{\ln\left(\frac{2\,x}{3}+x^{1-n}\right)}{2\,n} + \frac{\ln\left(x\right)\,\left(n-1\right)}{2\,n}$$

```
[In] int(1/(2*x + 3*x^(1 - n)),x)
```

```
[Out] log((2*x)/3 + x^(1 - n))/(2*n) + (log(x)*(n - 1))/(2*n)
```

3.360 $\int \frac{1}{-\sqrt{x}+x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{1}{-\sqrt{x}+x} \, dx = 2 \log \left(1-\sqrt{x}\right)$$

```
[Out] 2*ln(1-x^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 266}

$$\int \frac{1}{-\sqrt{x}+x} \, dx = 2 \log \left(1-\sqrt{x}\right)$$

```
[In] Int[(-Sqrt[x] + x)^(-1),x]
```

```
[Out] 2*Log[1 - Sqrt[x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\left(-1+\sqrt{x}\right)\sqrt{x}}\,dx\\ &= 2\log\left(1-\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\log\left(-1+\sqrt{x}\right)$$

```
[In] Integrate[(-Sqrt[x] + x)^(-1),x]
[Out] 2*Log[-1 + Sqrt[x]]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$2\ln\left(\sqrt{x}-1\right)$	9
meijerg	$2\ln\left(1-\sqrt{x}\right)$	11
default	$\ln\left(-1+x\right)-2\ \operatorname{arctanh}\left(\sqrt{x}\right)$	12
trager	$\ln\left(2\sqrt{x}-1-x\right)$	12

```
[In] int(1/(x-x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*ln(x^(1/2)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\ \log\left(\sqrt{x}-1\right)$$

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="fricas")
[Out] 2*log(sqrt(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\log\left(\sqrt{x}-1\right)$$

```
[In] integrate(1/(x-x**(1/2)),x)
[Out] 2*log(sqrt(x) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\,\log\left(\sqrt{x}-1\right)$$

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="maxima")
[Out] 2*log(sqrt(x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\,\log\left(\left|\sqrt{x}-1\right|\right)$$

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="giac")
[Out] 2*log(abs(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{-\sqrt{x}+x}\,dx = 2\,\ln\left(\sqrt{x}-1\right)$$

```
[In] int(1/(x - x^(1/2)),x)
[Out] 2*log(x^(1/2) - 1)
```

3.361 $\int \frac{1}{-x^{3/5}+x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{1}{-x^{3/5}+x} \, dx = \frac{5}{2} \log \left(1-x^{2/5}\right)$$

```
[Out] 5/2*ln(1-x^(2/5))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 266}

$$\int \frac{1}{-x^{3/5}+x} \, dx = \frac{5}{2} \log \left(1-x^{2/5}\right)$$

```
[In] Int[(-x^(3/5) + x)^(-1),x]
```

```
[Out] (5*Log[1 - x^(2/5)])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\left(-1+x^{2/5}\right)x^{3/5}}\,dx\\ &= \frac{5}{2}\log\left(1-x^{2/5}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5}{2}\log\left(-1+\sqrt[5]{x}\right)+\frac{5}{2}\log\left(1+\sqrt[5]{x}\right)$$

```
[In] Integrate[(-x^(3/5) + x)^(-1),x]
```

```
[Out] (5*Log[-1 + x^(1/5)])/2 + (5*Log[1 + x^(1/5)])/2
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result
meijerg	$\frac{5\ln\left(1-x^{\frac{2}{5}}\right)}{2}$
derivativedivides	$\frac{5\ln\left(x^{\frac{1}{5}}-1\right)}{2}+\frac{5\ln\left(1+x^{\frac{1}{5}}\right)}{2}$
trager	$\frac{\ln\left(-10x^{\frac{4}{5}}-5x^{\frac{8}{5}}+5x^{\frac{2}{5}}+10x^{\frac{6}{5}}+x^{2}-1\right)}{2}$
default	$2\ln\left(1+x^{\frac{1}{5}}\right)+\frac{\left(-\sqrt{5}-1\right)\ln\left(\sqrt{5}\,x^{\frac{1}{5}}+2x^{\frac{2}{5}}+x^{\frac{1}{5}}+2\right)}{4}-\frac{\left(-\sqrt{5}+1\right)\ln\left(-\sqrt{5}\,x^{\frac{1}{5}}+2x^{\frac{2}{5}}+x^{\frac{1}{5}}+2\right)}{4}-\left(-\sqrt{5}+1\right)\ln$

```
[In] int(1/(-x^(3/5)+x),x,method=_RETURNVERBOSE)
```

```
[Out] 5/2*ln(1-x^(2/5))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5}{2}\log\left(x^{\frac{2}{5}}-1\right)$$

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="fricas")
```

```
[Out] 5/2*log(x^(2/5) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(10) = 20.

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5\log\left(\sqrt[5]{x}-1\right)}{2}+\frac{5\log\left(\sqrt[5]{x}+1\right)}{2}$$

```
[In] integrate(1/(-x**(3/5)+x),x)
```

```
[Out] 5*log(x**(1/5) - 1)/2 + 5*log(x**(1/5) + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5}{2}\log\left(x^{\frac{1}{5}}+1\right)+\frac{5}{2}\log\left(x^{\frac{1}{5}}-1\right)$$

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="maxima")
```

```
[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(x^(1/5) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5}{2}\,\log\left(x^{\frac{1}{5}}+1\right)+\frac{5}{2}\,\log\left(\left|x^{\frac{1}{5}}-1\right|\right)$$

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="giac")
```

```
[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(abs(x^(1/5) - 1))
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{-x^{3/5}+x}\,dx = \frac{5\,\ln\left(x^{2/5}-1\right)}{2}$$

```
[In] int(1/(x - x^(3/5)),x)
```

```
[Out] (5*log(x^(2/5) - 1))/2
```

3.362 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx = \frac{3}{4} \log \left(1+x^{4/3}\right)$$

```
[Out] 3/4*ln(1+x^(4/3))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1607, 266}

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx = \frac{3}{4} \log \left(x^{4/3}+1\right)$$

```
[In] Int[(x^(-1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(4/3)])/4
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{\sqrt[3]{x}}{1+x^{4/3}}\,dx \\ &= \frac{3}{4}\log\left(1+x^{4/3}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx = \frac{3}{4}\log\left(1+x^{4/3}\right)$$

```
[In] Integrate[(x^(-1/3) + x)^(-1),x]
[Out] (3*Log[1 + x^(4/3)])/4
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
default	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
meijerg	$\frac{3\ln\left(1+x^{\frac{4}{3}}\right)}{4}$	9
trager	$-\frac{\ln\left(\frac{3x^{\frac{20}{3}}-x^{8}-6x^{\frac{16}{3}}-6x^{\frac{8}{3}}+7x^{4}+3x^{\frac{4}{3}}-1}{\left(x^{4}+1\right)^{3}}\right)}{4}$	44

```
[In] int(1/(1/x^(1/3)+x),x,method=_RETURNVERBOSE)
[Out] 3/4*ln(1+x^(4/3))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx=\frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="fricas")
[Out] 3/4*log(x^(4/3) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx=\frac{3\log\left(x^{\frac{4}{3}}+1\right)}{4}$$

```
[In] integrate(1/(1/x**(1/3)+x),x)
[Out] 3*log(x**(4/3) + 1)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx=\frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="maxima")
[Out] 3/4*log(x^(4/3) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(8) = 16$.

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}} + x}\, dx = \frac{3}{4} \log\left(\sqrt{2}x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right) + \frac{3}{4} \log\left(-\sqrt{2}x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right)$$

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="giac")
```

```
[Out] 3/4*log(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*log(-sqrt(2)*x^(1/3) + x^(2/3)
 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}} + x}\, dx = \frac{3 \ln\left(x^{4/3} + 1\right)}{4}$$

```
[In] int(1/(x + 1/x^(1/3)),x)
```

```
[Out] (3*log(x^(4/3) + 1))/4
```

3.363 $\int \frac{1}{x+x^{\sqrt{2}}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=\log(x)-\left(1+\sqrt{2}\right)\log\left(1+x^{-1+\sqrt{2}}\right)$$

```
[Out] ln(x)-ln(1+x^(2^(1/2)-1))*(1+2^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {1607, 272, 36, 29, 31}

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=\log(x)-\left(1+\sqrt{2}\right)\log\left(x^{\sqrt{2}-1}+1\right)$$

```
[In] Int[(x + x^Sqrt[2])^(-1),x]
```

```
[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{x\left(1+x^{-1+\sqrt{2}}\right)}\,dx\\
&= \left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^{-1+\sqrt{2}}\right)\\
&= \left(-1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^{-1+\sqrt{2}}\right)+\left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^{-1+\sqrt{2}}\right)\\
&= \log(x)-\left(1+\sqrt{2}\right)\log\left(1+x^{-1+\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx = \log(x)-\left(1+\sqrt{2}\right)\log\left(1+x^{-1+\sqrt{2}}\right)$$

```
[In] Integrate[(x + x^Sqrt[2])^(-1),x]
[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

method	result	size
norman	$(2+\sqrt{2})\ln(x)+(-\sqrt{2}-1)\ln\left(x+\mathrm{e}^{\sqrt{2}\ln(x)}\right)$	28
meijerg	$\frac{\left(\sqrt{2}-1\right)\ln(x)-\ln\left(1+x^{\sqrt{2}-1}\right)}{\sqrt{2}-1}$	30
risch	$2\ln(x)+\sqrt{2}\ln(x)-\ln\left(x+x^{\sqrt{2}}\right)\sqrt{2}-\ln\left(x+x^{\sqrt{2}}\right)$	35
parallelrisch	$2\ln(x)+\sqrt{2}\ln(x)-\ln\left(x+x^{\sqrt{2}}\right)\sqrt{2}-\ln\left(x+x^{\sqrt{2}}\right)$	35

```
[In] int(1/(x+x^(2^(1/2))),x,method=_RETURNVERBOSE)
[Out] (2+2^(1/2))*ln(x)+(-2^(1/2)-1)*ln(x+exp(2^(1/2)*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=-\left(\sqrt{2}+1\right)\log\left(x+x^{\left(\sqrt{2}\right)}\right)+\left(\sqrt{2}+2\right)\log(x)$$

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="fricas")
[Out] -(sqrt(2) + 1)*log(x + x^sqrt(2)) + (sqrt(2) + 2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=\frac{\sqrt{2}\log(x)}{-1+\sqrt{2}}-\frac{\log\left(x+x^{\sqrt{2}}\right)}{-1+\sqrt{2}}$$

```
[In] integrate(1/(x+x**(2**(1/2))),x)
[Out] sqrt(2)*log(x)/(-1 + sqrt(2)) - log(x + x**(sqrt(2)))/(-1 + sqrt(2))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=\frac{\sqrt{2}\log\left(x\right)}{\sqrt{2}-1}-\frac{\log\left(x+x^{\left(\sqrt{2}\right)}\right)}{\sqrt{2}-1}$$

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="maxima")
[Out] sqrt(2)*log(x)/(sqrt(2) - 1) - log(x + x^sqrt(2))/(sqrt(2) - 1)
```

Giac [F]

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=\int \frac{1}{x+x^{\left(\sqrt{2}\right)}}\,dx$$

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="giac")
[Out] integrate(1/(x + x^sqrt(2)), x)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x+x^{\sqrt{2}}}\,dx=\ln\left(x\right)\,\left(\sqrt{2}+2\right)-\frac{\ln\left(x+x^{\sqrt{2}}\right)}{\sqrt{2}-1}$$

```
[In] int(1/(x + x^(2^(1/2))),x)
[Out] log(x)*(2^(1/2) + 2) - log(x + x^(2^(1/2)))/(2^(1/2) - 1)
```

3.364 $\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{2\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}$$

```
[Out] 2*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))*a^(1/2)/(j-n)-2*(a*x^j+b*x
^n)^(1/2)/(j-n)/(x^(1/2*j))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2053, 2054, 212}

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\frac{2\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

```
[In] Int[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x^
(j/2))/Sqrt[a*x^j + b*x^n]])/(j - n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+a\int\frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx\\
&= -\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{(2a)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}\\
&= -\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=-\frac{2x^{-j/2}\left(ax^j+bx^n-\sqrt{a}\sqrt{b}x^{\frac{j+n}{2}}\sqrt{1+\frac{ax^{j-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{(j-n)\sqrt{ax^j+bx^n}}$$

```
[In] Integrate[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b
]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/((j - n)*x^(j/2)*Sqrt[a*x^j +
b*x^n])
```

Maple [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{a\,x^{j}+b\,x^{n}}dx$$

```
[In] int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
[Out] int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^{j}+bx^{n}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^{j}+bx^{n}}\,dx=\int x^{-\frac{j}{2}-1}\sqrt{ax^{j}+bx^{n}}\,dx$$

```
[In] integrate(x**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
[Out] Integral(x**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^{j}+bx^{n}}\,dx=\int\sqrt{ax^{j}+bx^{n}}x^{-\frac{1}{2}\,j-1}\,dx$$

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

Giac [F]

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int\sqrt{ax^j+bx^n}x^{-\frac{1}{2}\,j-1}\,dx$$

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int\frac{\sqrt{a\,x^j+b\,x^n}}{x^{\frac{j}{2}+1}}\,dx$$

```
[In] int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1),x)
[Out] int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1), x)
```

3.365 $\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=-\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{2\sqrt{a}x^{j/2}(cx)^{-j/2}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

```
[Out] 2*x^(1/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))*a^(1/2)/c/(j-n)/
((c*x)^(1/2*j))-2*(a*x^j+b*x^n)^(1/2)/c/(j-n)/((c*x)^(1/2*j))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2056, 2053, 2054, 212}

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\frac{2\sqrt{a}x^{j/2}(cx)^{-j/2}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}-\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

```
[In] Int[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^(j/2)) + (2*Sqrt[a]*x^(j/2)*ArcTa
nh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^(j/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x^{j/2}(cx)^{-j/2}\right)\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx}{c}\\ &= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{\left(ax^{j/2}(cx)^{-j/2}\right)\int\frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{c}\\ &= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{\left(2ax^{j/2}(cx)^{-j/2}\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}\\ &= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{2\sqrt{a}x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\begin{aligned}&\int(cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx\\ &=-\frac{2(cx)^{-j/2}\left(ax^j+bx^n-\sqrt{a}\sqrt{b}x^{\frac{j+n}{2}}\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{c(j-n)\sqrt{ax^j+bx^n}}\end{aligned}$$

```
[In] Integrate[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b
]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(c*(j - n)*(c*x)^(j/2)*Sqrt[a*
x^j + b*x^n])
```

Maple [F]

$$\int (cx)^{-1-\frac{j}{2}} \sqrt{a\,x^{j}+b\,x^{n}}dx$$

```
[In] int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \int (cx)^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

```
[In] integrate((c*x)**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral((c*x)**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \int \sqrt{ax^j+bx^n}(cx)^{-\frac{1}{2}\,j-1}\,dx$$

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

Giac [F]

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}(cx)^{-\frac{1}{2}\,j-1}\,dx$$

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx=\int \frac{\sqrt{a\,x^j+b\,x^n}}{(c\,x)^{\frac{j}{2}+1}}\,dx$$

```
[In] int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1),x)
[Out] int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1), x)
```

3.366 $\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx=-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{2\sqrt{a}\sqrt{cx}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}$$

```
[Out] 2*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*a^(1/2)*(c*x)^(1/2)/c^3/(3-n
)/x^(1/2)-2*(a*x^3+b*x^n)^(1/2)/c/(3-n)/(c*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2053, 2056, 2054, 212}

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx=\frac{2\sqrt{a}\sqrt{cx}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

```
[In] Int[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^n])/(c*(3 - n)*(c*x)^(3/2)) + (2*Sqrt[a]*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(c^3*(3 - n)*Sqrt[x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{a\int\frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{c^3}\\
&= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{\left(a\sqrt{cx}\right)\int\frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{c^3\sqrt{x}}\\
&= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{\left(2a\sqrt{cx}\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}\\
&= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{2\sqrt{a}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$
\int\frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx=\frac{2x\left(ax^3+bx^n-\sqrt{a}\sqrt{b}x^{\frac{3+n}{2}}\sqrt{1+\frac{ax^{3-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{(-3+n)(cx)^{5/2}\sqrt{ax^3+bx^n}}
$$

```
[In] Integrate[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

```
[Out] (2*x*(a*x^3 + b*x^n - Sqrt[a]*Sqrt[b]*x^((3 + n)/2)*Sqrt[1 + (a*x^(3 - n))/
b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/((-3 + n)*(c*x)^(5/2)*Sqrt[a*
x^3 + b*x^n])
```

Maple [F]

$$\int \frac{\sqrt{a\,x^{3}+b\,x^{n}}}{\left(c x\right)^{\frac{5}{2}}}dx$$

```
[In] int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

```
[Out] int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{ax^3+bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*x**3+b*x**n)**(1/2)/(c*x)**(5/2),x)
```

```
[Out] Integral(sqrt(a*x**3 + b*x**n)/(c*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{ax^3+bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx=\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx=\int \frac{\sqrt{b\,x^n+a\,x^3}}{{(c\,x)}^{5/2}}\,dx$$

```
[In] int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2),x)
```

```
[Out] int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2), x)
```

3.367 $\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx=-\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}+\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}$$

```
[Out] 2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))*a^(1/2)/c^2/(2-n)-2*(a*x^2+b*x^n)^
(1/2)/c^2/(2-n)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2053, 2033, 212}

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx=\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}-\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}$$

```
[In] Int[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^n])/(c^2*(2 - n)*x) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/S
qrt[a*x^2 + b*x^n]])/(c^2*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^2}\\
&= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}+\frac{a\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{c^2}\\
&= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}+\frac{(2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}\\
&= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx = \frac{2\left(ax^2+bx^n-\sqrt{a}\sqrt{b}x^{1+\frac{n}{2}}\sqrt{1+\frac{ax^{2-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{c^2(-2+n)x\sqrt{ax^2+bx^n}}$$

```
[In] Integrate[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

```
[Out] (2*(a*x^2 + b*x^n - Sqrt[a]*Sqrt[b]*x^(1 + n/2)*Sqrt[1 + (a*x^(2 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(c^2*(-2 + n)*x*Sqrt[a*x^2 + b*x^n]
)
```

Maple [F]

$$\int \frac{\sqrt{a\,x^{2}+b\,x^{n}}}{c^{2}x^{2}}dx$$

```
[In] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

```
[Out] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^{2}+bx^{n}}}{c^{2}x^{2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{ax^{2}+bx^{n}}}{c^{2}x^{2}}\,dx = \frac{\int \frac{\sqrt{ax^{2}+bx^{n}}}{x^{2}}\,dx}{c^{2}}$$

```
[In] integrate((a*x**2+b*x**n)**(1/2)/c**2/x**2,x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x**n)/x**2, x)/c**2
```

Maxima [F]

$$\int \frac{\sqrt{ax^{2}+bx^{n}}}{c^{2}x^{2}}\,dx = \int \frac{\sqrt{ax^{2}+bx^{n}}}{c^{2}x^{2}}\,dx$$

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + b*x^n)/x^2, x)/c^2
```

Giac [F]

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx=\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$$

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="giac")
[Out] integrate(sqrt(a*x^2 + b*x^n)/(c^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx=\int \frac{\sqrt{b\,x^n+a\,x^2}}{c^2\,x^2}\,dx$$

```
[In] int((b*x^n + a*x^2)^(1/2)/(c^2*x^2),x)
[Out] int((b*x^n + a*x^2)^(1/2)/(c^2*x^2), x)
```

3.368 $\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}} + \frac{2\sqrt{a}\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}$$

```
[Out] 2*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/c/(1-n)/(c*x)^
(1/2)-2*(a*x+b*x^n)^(1/2)/c/(1-n)/(c*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2053, 2056, 2054, 212}

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx = \frac{2\sqrt{a}\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}} - \frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

```
[In] Int[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^n])/(c*(1 - n)*Sqrt[c*x]) + (2*Sqrt[a]*Sqrt[x]*ArcTanh[(
Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(c*(1 - n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{a\int\frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx}{c}\\
&= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{\left(a\sqrt{x}\right)\int\frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{c\sqrt{cx}}\\
&= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{\left(2a\sqrt{x}\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}\\
&= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int\frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx=\frac{x\left(2ax+2bx^n-2\sqrt{a}\sqrt{b}x^{\frac{1+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{(-1+n)(cx)^{3/2}\sqrt{ax+bx^n}}$$

```
[In] Integrate[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

```
[Out] (x*(2*a*x + 2*b*x^n - 2*Sqrt[a]*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n)
)/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/((-1 + n)*(c*x)^(3/2)*Sqrt[
a*x + b*x^n])
```

Maple [F]

$$\int \frac{\sqrt{a x+b\,x^{n}}}{\left(c x\right)^{\frac{3}{2}}}d x$$

```
[In] int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

```
[Out] int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx=\int \frac{\sqrt{ax+bx^{n}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*x+b*x**n)**(1/2)/(c*x)**(3/2),x)
```

```
[Out] Integral(sqrt(a*x + b*x**n)/(c*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx=\int \frac{\sqrt{ax+bx^{n}}}{\left(cx\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx=\int \frac{\sqrt{ax+bx^n}}{(cx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^n+a\,x}}{{\left(c\,x\right)}^{3/2}} \,dx$$

```
[In] int((b*x^n + a*x)^(1/2)/(c*x)^(3/2),x)
[Out] int((b*x^n + a*x)^(1/2)/(c*x)^(3/2), x)
```

3.369 $\int \frac{\sqrt{a+bx^n}}{cx} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 51

$$\int \frac{\sqrt{a+bx^n}}{cx} \, dx = \frac{2\sqrt{a+bx^n}}{cn} - \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))*a^(1/2)/c/n+2*(a+b*x^n)^(1/2)/c/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {12, 272, 52, 65, 214}

$$\int \frac{\sqrt{a+bx^n}}{cx} \, dx = \frac{2\sqrt{a+bx^n}}{cn} - \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

```
[In] Int[Sqrt[a + b*x^n]/(c*x),x]
```

```
[Out] (2*Sqrt[a + b*x^n])/(c*n) - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{a+bx^n}}{x}\,dx}{c} \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^n\right)}{cn} \\
&= \frac{2\sqrt{a+bx^n}}{cn} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{cn} \\
&= \frac{2\sqrt{a+bx^n}}{cn} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bcn} \\
&= \frac{2\sqrt{a+bx^n}}{cn} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx=\frac{2\left(\sqrt{a+bx^n}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{cn}$$

```
[In] Integrate[Sqrt[a + b*x^n]/(c*x),x]
[Out] (2*(Sqrt[a + b*x^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]]))/(c*n)
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	39
default	$\frac{2\sqrt{a+b\,x^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn}$	39
risch	$\frac{2\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{nc}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{\sqrt{a}}\right)}{nc}$	48

```
[In] int((a+b*x^n)^(1/2)/c/x,x,method=_RETURNVERBOSE)
[Out] 1/c/n*(2*(a+b*x^n)^(1/2)-2*a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.90

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \left[\frac{\sqrt{a}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\sqrt{bx^n+a}}{cn}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)+\sqrt{bx^n+a}\right)}{cn}\right]$$

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="fricas")
[Out] [(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^n
 + a))/(c*n), 2*(sqrt(-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n +
 a))/(c*n)]
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \frac{-\frac{2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{n} + \frac{2ax^{-\frac{n}{2}}}{\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}} + \frac{2\sqrt{b}x^{\frac{n}{2}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}}{c}$$

```
[In] integrate((a+b*x**n)**(1/2)/c/x,x)
```

```
[Out] (-2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/n + 2*a/(sqrt(b)*n*x**(n/2)*s
qrt(a/(b*x**n) + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a/(b*x**n) + 1)))/c
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \frac{\frac{\sqrt{a}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\sqrt{bx^n+a}}{n}}{c}$$

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="maxima")
```

```
[Out] (sqrt(a)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2
*sqrt(b*x^n + a)/n)/c
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \int \frac{\sqrt{bx^n+a}}{cx}\,dx$$

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)/(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}}{cx}\,dx = \int \frac{\sqrt{a+b\,x^n}}{c\,x}\,dx$$

```
[In] int((a + b*x^n)^(1/2)/(c*x),x)
[Out] int((a + b*x^n)^(1/2)/(c*x), x)
```

3.370 $\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}-\frac{2\sqrt{a}\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/(1+n)/(c*x)^(
1/2)+2*(c*x)^(1/2)*(a/x+b*x^n)^(1/2)/c/(1+n)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2053, 2056, 2054, 212}

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\sqrt{x}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}$$

```
[In] Int[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[Sq
rt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} + (ac)\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx \\
&= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} + \frac{\left(a\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{\sqrt{cx}} \\
&= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} - \frac{\left(2a\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}} \\
&= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} - \frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\frac{2x\sqrt{\frac{a}{x}+bx^n}\left(\sqrt{a+bx^{1+n}}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{(1+n)\sqrt{cx}\sqrt{a+bx^{1+n}}}$$

```
[In] Integrate[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

```
[Out] (2*x*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)] - Sqrt[a]*ArcTanh[Sqrt[a + b*
x^(1 + n)]/Sqrt[a]]))/((1 + n)*Sqrt[c*x]*Sqrt[a + b*x^(1 + n)])
```

Maple [F]

$$\int \frac{\sqrt{\frac{a}{x}+b\,x^{n}}}{\sqrt{cx}}dx$$

```
[In] int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$$

```
[In] integrate((a/x+b*x**n)**(1/2)/(c*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x + b*x**n)/sqrt(c*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{bx^n+\frac{a}{x}}}{\sqrt{cx}}\,dx$$

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{bx^n+\frac{a}{x}}}{\sqrt{cx}}\,dx$$

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{b\,x^n+\frac{a}{x}}}{\sqrt{c\,x}}\,dx$$

```
[In] int((b*x^n + a/x)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((b*x^n + a/x)^(1/2)/(c*x)^(1/2), x)
```

3.371 $\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))*a^(1/2)/(2+n)+2*x*(a/x^2+b*x^n)^(
1/2)/(2+n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2032, 2054, 212}

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}$$

```
[In] Int[Sqrt[a/x^2 + b*x^n],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^
2 + b*x^n])])/(2 + n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n} + a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n} - \frac{(2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx = \frac{2x\sqrt{\frac{a}{x^2}+bx^n}\left(\sqrt{a+bx^{2+n}}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{(2+n)\sqrt{a+bx^{2+n}}}$$

```
[In] Integrate[Sqrt[a/x^2 + b*x^n],x]
[Out] (2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)] - Sqrt[a]*ArcTanh[Sqrt[a +
b*x^(2 + n)]/Sqrt[a]]))/((2 + n)*Sqrt[a + b*x^(2 + n)])
```

Maple [F]

$$\int \sqrt{\frac{a}{x^2}+b\,x^n}dx$$

```
[In] int((a/x^2+b*x^n)^(1/2),x)
```

```
[Out] int((a/x^2+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx = \int \sqrt{\frac{a}{x^2}+bx^n}\,dx$$

```
[In] integrate((a/x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2 + b*x**n), x)
```

Maxima [F]

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx = \int \sqrt{bx^n+\frac{a}{x^2}}\,dx$$

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x^2), x)
```

Giac [F]

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\int \sqrt{bx^n+\frac{a}{x^2}}\,dx$$

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx=\frac{x\,\sqrt{b\,x^n+\frac{a}{x^2}}}{\frac{n}{2}+1}+\frac{\sqrt{a}\,x\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x^{\frac{n}{2}+1}}\right)\,\sqrt{b\,x^n+\frac{a}{x^2}}\,1\mathrm{i}}{\sqrt{b}\,x^{\frac{n}{2}+1}\,\left(\frac{n}{2}+1\right)\,\sqrt{\frac{a}{b\,x^{n+2}}+1}}$$

```
[In] int((b*x^n + a/x^2)^(1/2),x)
```

```
[Out] (x*(b*x^n + a/x^2)^(1/2))/(n/2 + 1) + (a^(1/2)*x*asin((a^(1/2)*1i)/(b^(1/2)
*x^(n/2 + 1)))*(b*x^n + a/x^2)^(1/2)*1i)/(b^(1/2)*x^(n/2 + 1)*(n/2 + 1)*(a/
(b*x^(n + 2)) + 1)^(1/2))
```

3.372 $\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)}-\frac{2\sqrt{a}c\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}$$

```
[Out] -2*c*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/(3+n)/(c*
x)^(1/2)+2*(c*x)^(3/2)*(a/x^3+b*x^n)^(1/2)/c/(3+n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2053, 2056, 2054, 212}

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}c\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}$$

```
[In] Int[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

```
[Out] (2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(c*(3 + n)) - (2*Sqrt[a]*c*Sqrt[x]*ArcT
anh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} + \left(ac^3\right)\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx \\
&= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} + \frac{\left(ac\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{\sqrt{cx}} \\
&= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} - \frac{\left(2ac\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}} \\
&= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} - \frac{2\sqrt{a}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\frac{2x\sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\left(\sqrt{a+bx^{3+n}}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{(3+n)\sqrt{a+bx^{3+n}}}$$

```
[In] Integrate[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

```
[Out] (2*x*Sqrt[c*x]*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)] - Sqrt[a]*ArcTanh
[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/((3 + n)*Sqrt[a + b*x^(3 + n)])
```

Maple [F]

$$\int \sqrt{cx}\,\sqrt{\frac{a}{x^{3}}+b\,x^{n}}dx$$

```
[In] int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$$

```
[In] integrate((c*x)**(1/2)*(a/x**3+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(c*x)*sqrt(a/x**3 + b*x**n), x)
```

Maxima [F]

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\int \sqrt{bx^n+\frac{a}{x^3}}\sqrt{cx}\,dx$$

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

Giac [F]

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\int \sqrt{bx^n+\frac{a}{x^3}}\sqrt{cx}\,dx$$

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx=\int \sqrt{c\,x}\,\sqrt{b\,x^n+\frac{a}{x^3}}\,dx$$

```
[In] int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2),x)
[Out] int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2), x)
```

3.373 $\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 141

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx = -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

```
[Out] -2/3*(a*x^j+b*x^n)^(3/2)/c/(j-n)/((c*x)^(3/2*j))+2*a^(3/2)*x^(3/2*j)*arctan
h(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/c/(j-n)/((c*x)^(3/2*j))-2*a*x^j*(a
*x^j+b*x^n)^(1/2)/c/(j-n)/((c*x)^(3/2*j))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2056, 2053, 2054, 212}

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx = \frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\text{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} - \frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

```
[In] Int[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*a*x^j*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^((3*j)/2)) - (2*(a*x^j + b*
x^n)^(3/2))/(3*c*(j - n)*(c*x)^((3*j)/2)) + (2*a^(3/2)*x^((3*j)/2)*ArcTanh[
(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^((3*j)/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^{3j/2}(cx)^{-3j/2}\right)\int x^{-1-\frac{3j}{2}}\left(ax^j+bx^n\right)^{3/2}\,dx}{c}\\
&= -\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)}+\frac{\left(ax^{3j/2}(cx)^{-3j/2}\right)\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx}{c}\\
&= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}-\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)}+\frac{\left(a^2x^{3j/2}(cx)^{-3j/2}\right)\int\frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{c}\\
&= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}-\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)}\\
&\quad+\frac{\left(2a^2x^{3j/2}(cx)^{-3j/2}\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}
\end{aligned}$$

$$= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93

$$\int (cx)^{-1-\frac{3j}{2}}\left(ax^j+bx^n\right)^{3/2}\,dx = -\frac{2(cx)^{-3j/2}\left(4a^2x^{2j}+b^2x^{2n}+5abx^{j+n}-3a^{3/2}\sqrt{b}x^{\frac{1}{2}(3j+n)}\sqrt{1+\frac{ax^{j-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{3c(j-n)\sqrt{ax^j+bx^n}}$$

```
[In] Integrate[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
[Out] (-2*(4*a^2*x^(2*j) + b^2*x^(2*n) + 5*a*b*x^(j + n) - 3*a^(3/2)*Sqrt[b]*x^((
3*j + n)/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[
b]]))/(3*c*(j - n)*(c*x)^((3*j)/2)*Sqrt[a*x^j + b*x^n])
```

Maple [F]

$$\int (cx)^{-1-\frac{3j}{2}}\left(a\,x^j+b\,x^n\right)^{\frac{3}{2}}dx$$

```
[In] int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
[Out] int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^{-1-\frac{3j}{2}}\left(ax^j+bx^n\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx = \int (cx)^{-\frac{3j}{2}-1} \left(ax^j+bx^n\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate((c*x)**(-1-3/2*j)*(a*x**j+b*x**n)**(3/2),x)
[Out] Integral((c*x)**(-3*j/2 - 1)*(a*x**j + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx = \int \left(ax^j+bx^n\right)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1} \, dx$$

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

Giac [F]

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx = \int \left(ax^j+bx^n\right)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1} \, dx$$

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} \, dx = \int \frac{\left(a\,x^j+b\,x^n\right)^{3/2}}{(c\,x)^{\frac{3\,j}{2}+1}} \, dx$$

```
[In] int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1),x)
[Out] int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1), x)
```

3.374 $\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{2a^{3/2}\sqrt{cx}\,\mathrm{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}$$

```
[Out] -2/3*(a*x^3+b*x^n)^(3/2)/c/(3-n)/(c*x)^(9/2)+2*a^(3/2)*arctanh(x^(3/2)*a^(1
/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/c^6/(3-n)/x^(1/2)-2*a*(a*x^3+b*x^n)^(1
/2)/c^4/(3-n)/(c*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2053, 2056, 2054, 212}

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx = \frac{2a^{3/2}\sqrt{cx}\,\mathrm{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}} - \frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

```
[In] Int[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (-2*a*Sqrt[a*x^3 + b*x^n])/(c^4*(3 - n)*(c*x)^(3/2)) - (2*(a*x^3 + b*x^n)^(
3/2))/(3*c*(3 - n)*(c*x)^(9/2)) + (2*a^(3/2)*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(
3/2))/Sqrt[a*x^3 + b*x^n]])/(c^6*(3 - n)*Sqrt[x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}+\frac{a\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx}{c^3}\\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}+\frac{a^2\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{c^6}\\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}+\frac{\left(a^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{c^6\sqrt{x}}\\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}+\frac{\left(2a^2\sqrt{cx}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}\\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}+\frac{2a^{3/2}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx=\frac{2\sqrt{cx}\left(4a^2x^6+b^2x^{2n}+5abx^{3+n}-3a^{3/2}\sqrt{b}x^{\frac{9+n}{2}}\sqrt{1+\frac{ax^{3-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3c^6(-3+n)x^5\sqrt{ax^3+bx^n}}$$

```
[In] Integrate[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[c*x]*(4*a^2*x^6 + b^2*x^(2*n) + 5*a*b*x^(3 + n) - 3*a^(3/2)*Sqrt[b]
*x^((9 + n)/2)*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sq
rt[b]]))/(3*c^6*(-3 + n)*x^5*Sqrt[a*x^3 + b*x^n])
```

Maple [F]

$$\int \frac{(a\,x^3+b\,x^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}dx$$

```
[In] int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

```
[Out] int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a*x**3+b*x**n)**(3/2)/(c*x)**(11/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{(ax^3+bx^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
[Out] integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

Giac [F]

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{(ax^3+bx^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
[Out] integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{\left(b\,x^n+a\,x^3\right)^{3/2}}{(c\,x)^{11/2}}\,dx$$

```
[In] int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2),x)
[Out] int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2), x)
```

3.375 $\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x} - \frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3} + \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}$$

```
[Out] -2/3*(a*x^2+b*x^n)^(3/2)/c^4/(2-n)/x^3+2*a^(3/2)*arctanh(x*a^(1/2)/(a*x^2+b
*x^n)^(1/2))/c^4/(2-n)-2*a*(a*x^2+b*x^n)^(1/2)/c^4/(2-n)/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2053, 2033, 212}

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)} - \frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x} - \frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3}$$

```
[In] Int[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

```
[Out] (-2*a*Sqrt[a*x^2 + b*x^n])/(c^4*(2 - n)*x) - (2*(a*x^2 + b*x^n)^(3/2))/(3*c
^4*(2 - n)*x^3) + (2*a^(3/2)*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(c^4
*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{(ax^2+bx^n)^{3/2}}{x^4}\,dx}{c^4}\\
&= -\frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3}+\frac{a\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^4}\\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3}+\frac{a^2\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{c^4}\\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3}+\frac{(2a^2)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}\\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2(ax^2+bx^n)^{3/2}}{3c^4(2-n)x^3}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = \frac{2\left(4a^2x^4+b^2x^{2n}+5abx^{2+n}-3a^{3/2}\sqrt{b}x^{3+\frac{n}{2}}\sqrt{1+\frac{ax^{2-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3c^4(-2+n)x^3\sqrt{ax^2+bx^n}}$$

```
[In] Integrate[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
[Out] (2*(4*a^2*x^4 + b^2*x^(2*n) + 5*a*b*x^(2 + n) - 3*a^(3/2)*Sqrt[b]*x^(3 + n/
2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(3*c^
4*(-2 + n)*x^3*Sqrt[a*x^2 + b*x^n])
```

Maple [F]

$$\int \frac{(a\,x^2+b\,x^n)^{\frac{3}{2}}}{c^4x^4}dx$$

```
[In] int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
[Out] int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = \frac{\int \frac{a\sqrt{ax^2+bx^n}}{x^2}\,dx + \int \frac{bx^n\sqrt{ax^2+bx^n}}{x^4}\,dx}{c^4}$$

```
[In] integrate((a*x**2+b*x**n)**(3/2)/c**4/x**4,x)
[Out] (Integral(a*sqrt(a*x**2 + b*x**n)/x**2, x) + Integral(b*x**n*sqrt(a*x**2 +
b*x**n)/x**4, x))/c**4
```

Maxima [F]

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = \int \frac{(ax^2+bx^n)^{\frac{3}{2}}}{c^4x^4}\,dx$$

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="maxima")
[Out] integrate((a*x^2 + b*x^n)^(3/2)/x^4, x)/c^4
```

Giac [F]

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = \int \frac{(ax^2+bx^n)^{\frac{3}{2}}}{c^4x^4}\,dx$$

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="giac")
[Out] integrate((a*x^2 + b*x^n)^(3/2)/(c^4*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^n)^{3/2}}{c^4x^4}\,dx = \int \frac{(b\,x^n+a\,x^2)^{3/2}}{c^4\,x^4}\,dx$$

```
[In] int((b*x^n + a*x^2)^(3/2)/(c^4*x^4),x)
[Out] int((b*x^n + a*x^2)^(3/2)/(c^4*x^4), x)
```

3.376 $\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}} \, dx = -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{2a^{3/2}\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}$$

```
[Out] -2/3*(a*x+b*x^n)^(3/2)/c/(1-n)/(c*x)^(3/2)+2*a^(3/2)*arctanh(a^(1/2)*x^(1/2
)/(a*x+b*x^n)^(1/2))*x^(1/2)/c^2/(1-n)/(c*x)^(1/2)-2*a*(a*x+b*x^n)^(1/2)/c^
2/(1-n)/(c*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2053, 2056, 2054, 212}

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}} \, dx = \frac{2a^{3/2}\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}} - \frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

```
[In] Int[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a*x + b*x^n])/(c^2*(1 - n)*Sqrt[c*x]) - (2*(a*x + b*x^n)^(3/2))/
(3*c*(1 - n)*(c*x)^(3/2)) + (2*a^(3/2)*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sq
rt[a*x + b*x^n]])/(c^2*(1 - n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}+\frac{a\int\frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx}{c}\\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}+\frac{a^2\int\frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx}{c^2}\\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}+\frac{\left(a^2\sqrt{x}\right)\int\frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{c^2\sqrt{cx}}\\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}+\frac{\left(2a^2\sqrt{x}\right)\operatorname{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}\\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}+\frac{2a^{3/2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx=\frac{x\left(8a^2x^2+2b^2x^{2n}+10abx^{1+n}-6a^{3/2}\sqrt{b}x^{\frac{3+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{3(-1+n)(cx)^{5/2}\sqrt{ax+bx^n}}$$

```
[In] Integrate[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (x*(8*a^2*x^2 + 2*b^2*x^(2*n) + 10*a*b*x^(1 + n) - 6*a^(3/2)*Sqrt[b]*x^((3
+ n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]])
)/(3*(-1 + n)*(c*x)^(5/2)*Sqrt[a*x + b*x^n])
```

Maple [F]

$$\int \frac{(ax+b\,x^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}dx$$

```
[In] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

```
[Out] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx=\int \frac{(ax+bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*x+b*x**n)**(3/2)/(c*x)**(5/2),x)
```

```
[Out] Integral((a*x + b*x**n)**(3/2)/(c*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx=\int \frac{(ax+bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx=\int \frac{(ax+bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx=\int \frac{(b\,x^n+a\,x)^{3/2}}{(c\,x)^{5/2}}\,dx$$

```
[In] int((b*x^n + a*x)^(3/2)/(c*x)^(5/2),x)
[Out] int((b*x^n + a*x)^(3/2)/(c*x)^(5/2), x)
```

3.377 $\int \frac{(a+bx^n)^{3/2}}{cx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx=\frac{2a\sqrt{a+bx^n}}{cn}+\frac{2(a+bx^n)^{3/2}}{3cn}-\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

```
[Out] 2/3*(a+b*x^n)^(3/2)/c/n-2*a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n+2*a*
(a+b*x^n)^(1/2)/c/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 272, 52, 65, 214}

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx=-\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}+\frac{2a\sqrt{a+bx^n}}{cn}+\frac{2(a+bx^n)^{3/2}}{3cn}$$

```
[In] Int[(a + b*x^n)^(3/2)/(c*x),x]
```

```
[Out] (2*a*Sqrt[a + b*x^n])/(c*n) + (2*(a + b*x^n)^(3/2))/(3*c*n) - (2*a^(3/2)*Ar
cTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{(a+bx^n)^{3/2}}{x}\,dx}{c}\\
&= \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^n\right)}{cn}\\
&= \frac{2(a+bx^n)^{3/2}}{3cn}+\frac{a\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^n\right)}{cn}\\
&= \frac{2a\sqrt{a+bx^n}}{cn}+\frac{2(a+bx^n)^{3/2}}{3cn}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^n\right)}{cn}\\
&= \frac{2a\sqrt{a+bx^n}}{cn}+\frac{2(a+bx^n)^{3/2}}{3cn}+\frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^n}\right)}{bcn}\\
&= \frac{2a\sqrt{a+bx^n}}{cn}+\frac{2(a+bx^n)^{3/2}}{3cn}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx=\frac{2\sqrt{a+bx^n}(4a+bx^n)-6a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{3cn}$$

```
[In] Integrate[(a + b*x^n)^(3/2)/(c*x),x]
```

```
[Out] (2*Sqrt[a + b*x^n]*(4*a + b*x^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a
]])/(3*c*n)
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\frac{2(a+b\,x^n)^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,x^n}-2a^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{cn}$	51
default	$\frac{\frac{2(a+b\,x^n)^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,x^n}-2a^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{cn}$	51
risch	$\frac{2\left(b\,\text{e}^{n\ln(x)}+4a\right)\sqrt{a+b\,\text{e}^{n\ln(x)}}}{3nc}-\frac{2a^{\frac{3}{2}}\,\text{arctanh}\left(\frac{\sqrt{a+b\,\text{e}^{n\ln(x)}}}{\sqrt{a}}\right)}{nc}$	59

```
[In] int((a+b*x^n)^(3/2)/c/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/n*(2/3*(a+b*x^n)^(3/2)+2*a*(a+b*x^n)^(1/2)-2*a^(3/2)*arctanh((a+b*x^n)^
(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.71 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx=\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{bx^n-2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\,(bx^n+4\,a)\sqrt{bx^n+a}}{3\,cn},\frac{2\left(3\,\sqrt{-a}a\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right.\right.}{3\,cn}\right.$$

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(b*x
^n + 4*a)*sqrt(b*x^n + a))/(c*n), 2/3*(3*sqrt(-a)*a*arctan(sqrt(b*x^n + a)*
sqrt(-a)/a) + (b*x^n + 4*a)*sqrt(b*x^n + a))/(c*n)]
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \frac{\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^n}{a}}}{3n} + \frac{a^{\frac{3}{2}}\log\left(\frac{bx^n}{a}\right)}{n} - \frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{n} + \frac{2\sqrt{a}bx^n\sqrt{1+\frac{bx^n}{a}}}{3n}}{c}$$

```
[In] integrate((a+b*x**n)**(3/2)/c/x,x)
```

```
[Out] (8*a**(3/2)*sqrt(1 + b*x**n/a)/(3*n) + a**(3/2)*log(b*x**n/a)/n - 2*a**(3/2
)*log(sqrt(1 + b*x**n/a) + 1)/n + 2*sqrt(a)*b*x**n*sqrt(1 + b*x**n/a)/(3*n)
)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \frac{\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\left((bx^n+a)^{\frac{3}{2}}+3\,\sqrt{bx^n+a}a\right)}{n}}{3\,c}$$

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^(3/2)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))
/n + 2*((b*x^n + a)^(3/2) + 3*sqrt(b*x^n + a)*a)/n)/c
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}}{cx}\,dx$$

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)/(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \int \frac{(a+b\,x^n)^{3/2}}{c\,x}\,dx$$

```
[In] int((a + b*x^n)^(3/2)/(c*x),x)
[Out] int((a + b*x^n)^(3/2)/(c*x), x)
```

3.378 $\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}-\frac{2a^{3/2}c\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

```
[Out] 2/3*(c*x)^(3/2)*(a/x+b*x^n)^(3/2)/c/(1+n)-2*a^(3/2)*c*arctanh(a^(1/2)/x^(1/
2)/(a/x+b*x^n)^(1/2))*x^(1/2)/(1+n)/(c*x)^(1/2)+2*a*(c*x)^(1/2)*(a/x+b*x^n)
^(1/2)/(1+n)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2053, 2056, 2054, 212}

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=-\frac{2a^{3/2}c\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}+\frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{n+1}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

```
[In] Int[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

```
[Out] (2*a*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(1 + n) + (2*(c*x)^(3/2)*(a/x + b*x^n)^(3
/2))/(3*c*(1 + n)) - (2*a^(3/2)*c*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x
 + b*x^n])])/((1 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} + (ac)\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx \\
&= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} + \left(a^2c^2\right)\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx \\
&= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} + \frac{\left(a^2c\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{\sqrt{cx}} \\
&= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} - \frac{\left(2a^2c\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}} \\
&= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} - \frac{2a^{3/2}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}\left(\sqrt{a+bx^{1+n}}(4a+bx^{1+n})-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{3(1+n)\sqrt{a+bx^{1+n}}}$$

```
[In] Integrate[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)]*(4*a + b*x^(1 + n)) -
 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]]))/(3*(1 + n)*Sqrt[a + b*x
^(1 + n)])
```

Maple [F]

$$\int \sqrt{cx}\,\left(\frac{a}{x}+b\,x^{n}\right)^{\frac{3}{2}}d x$$

```
[In] int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((c*x)**(1/2)*(a/x+b*x**n)**(3/2),x)
```

```
[Out] Integral(sqrt(c*x)*(a/x + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

Giac [F]

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx=\int \sqrt{c\,x}\,\left(b\,x^n+\frac{a}{x}\right)^{3/2}\,dx$$

```
[In] int((c*x)^(1/2)*(b*x^n + a/x)^(3/2),x)
[Out] int((c*x)^(1/2)*(b*x^n + a/x)^(3/2), x)
```

3.379 $\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{2a^{3/2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

```
[Out] 2/3*c^2*x^3*(a/x^2+b*x^n)^(3/2)/(2+n)-2*a^(3/2)*c^2*arctanh(a^(1/2)/x/(a/x^
2+b*x^n)^(1/2))/(2+n)+2*a*c^2*x*(a/x^2+b*x^n)^(1/2)/(2+n)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {12, 2053, 2032, 2054, 212}

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=-\frac{2a^{3/2}c^2\operatorname{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}+\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}$$

```
[In] Int[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) + (2*c^2*x^3*(a/x^2 + b*x^n)^(3/2))
/(3*(2 + n)) - (2*a^(3/2)*c^2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2
+ n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\text{integral} = c^2 \int x^2 \left(\frac{a}{x^2}+bx^n\right)^{3/2} dx$$

$$= \frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)} + \left(ac^2\right) \int \sqrt{\frac{a}{x^2}+bx^n}\, dx$$

$$= \frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n} + \frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)} + \left(a^2c^2\right) \int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\, dx$$

$$= \frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n} + \frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)} - \frac{\left(2a^2c^2\right) \operatorname{Subst}\left(\int \frac{1}{1-ax^2}\, dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

$$=\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\frac{2c^2x\sqrt{\frac{a}{x^2}+bx^n}\left(\sqrt{a+bx^{2+n}}(4a+bx^{2+n})-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{3(2+n)\sqrt{a+bx^{2+n}}}$$

```
[In] Integrate[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)]*(4*a + b*x^(2 + n)) - 3
*a^(3/2)*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]]))/(3*(2 + n)*Sqrt[a + b*x^(
2 + n)])
```

Maple [F]

$$\int c^2x^2\left(\frac{a}{x^2}+b\,x^n\right)^{\frac{3}{2}}dx$$

```
[In] int(c^2*x^2*(a/x^2+b*x^n)^(3/2),x)
```

```
[Out] int(c^2*x^2*(a/x^2+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=c^2\left(\int a\sqrt{\frac{a}{x^2}+bx^n}\,dx+\int bx^2x^n\sqrt{\frac{a}{x^2}+bx^n}\,dx\right)$$

```
[In] integrate(c**2*x**2*(a/x**2+b*x**n)**(3/2),x)
[Out] c**2*(Integral(a*sqrt(a/x**2 + b*x**n), x) + Integral(b*x**2*x**n*sqrt(a/x*
*2 + b*x**n), x))
```

Maxima [F]

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\int\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^2x^2\,dx$$

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
[Out] c^2*integrate((b*x^n + a/x^2)^(3/2)*x^2, x)
```

Giac [F]

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\int\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^2x^2\,dx$$

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a/x^2)^(3/2)*c^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx=\int c^2\,x^2\,\left(b\,x^n+\frac{a}{x^2}\right)^{3/2}\,dx$$

```
[In] int(c^2*x^2*(b*x^n + a/x^2)^(3/2),x)
[Out] int(c^2*x^2*(b*x^n + a/x^2)^(3/2), x)
```

3.380 $\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 122

$$\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} \, dx = \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n} + \frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)} - \frac{2a^{3/2}c^4\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}$$

```
[Out] 2/3*(c*x)^(9/2)*(a/x^3+b*x^n)^(3/2)/c/(3+n)-2*a^(3/2)*c^4*arctanh(a^(1/2)/x
^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/(3+n)/(c*x)^(1/2)+2*a*c^2*(c*x)^(3/2)*(
a/x^3+b*x^n)^(1/2)/(3+n)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2053, 2056, 2054, 212}

$$\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} \, dx = -\frac{2a^{3/2}c^4\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}} + \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{n+3} + \frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

```
[In] Int[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(3 + n) + (2*(c*x)^(9/2)*(a/x^3 +
 b*x^n)^(3/2))/(3*c*(3 + n)) - (2*a^(3/2)*c^4*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3
/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\left(ac^3\right)\int\sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx\\
&= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\left(a^2c^6\right)\int\frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx\\
&= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\frac{\left(a^2c^4\sqrt{x}\right)\int\frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{\sqrt{cx}}\\
&= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}\\
&\quad-\frac{\left(2a^2c^4\sqrt{x}\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}\\
&= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}dx=\frac{2c^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}\left(\sqrt{a+bx^{3+n}}(4a+bx^{3+n})-3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{3(3+n)\sqrt{a+bx^{3+n}}}$$

```
[In] Integrate[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)]*(4*a + b*x^(3
 + n)) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/(3*(3 + n)*Sqrt
[a + b*x^(3 + n)])
```

Maple [F]

$$\int (cx)^{\frac{7}{2}}\left(\frac{a}{x^3}+b\,x^n\right)^{\frac{3}{2}}dx$$

```
[In] int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}dx=\text{Timed out}$$

```
[In] integrate((c*x)**(7/2)*(a/x**3+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}\,dx$$

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

Giac [F]

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\int \left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}\,dx$$

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx=\int (c\,x)^{7/2}\left(b\,x^n+\frac{a}{x^3}\right)^{3/2}\,dx$$

```
[In] int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2),x)
[Out] int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2), x)
```

3.381 $\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{2a^{3/2}c^5\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}$$

```
[Out] 2/3*c^5*x^6*(a/x^4+b*x^n)^(3/2)/(4+n)-2*a^(3/2)*c^5*arctanh(a^(1/2)/x^2/(a/
x^4+b*x^n)^(1/2))/(4+n)+2*a*c^5*x^2*(a/x^4+b*x^n)^(1/2)/(4+n)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2053, 2054, 212}

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=-\frac{2a^{3/2}c^5\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}+\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}$$

```
[In] Int[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^5*x^2*Sqrt[a/x^4 + b*x^n])/(4 + n) + (2*c^5*x^6*(a/x^4 + b*x^n)^(3/2
))/(3*(4 + n)) - (2*a^(3/2)*c^5*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])
/(4 + n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2053

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*p*(n - j))), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c^5 \int x^5 \left(\frac{a}{x^4}+bx^n\right)^{3/2} dx \\
&= \frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}+\left(ac^5\right)\int x\sqrt{\frac{a}{x^4}+bx^n}\,dx \\
&= \frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}+\left(a^2c^5\right)\int \frac{1}{x^3\sqrt{\frac{a}{x^4}+bx^n}}\,dx \\
&= \frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{\left(2a^2c^5\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n} \\
&= \frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}dx=\frac{2c^5x^2\sqrt{\frac{a}{x^4}+bx^n}\left(\sqrt{a+bx^{4+n}}(4a+bx^{4+n})-3a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{4+n}}}{\sqrt{a}}\right)\right)}{3(4+n)\sqrt{a+bx^{4+n}}}$$

```
[In] Integrate[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^5*x^2*Sqrt[a/x^4 + b*x^n]*(Sqrt[a + b*x^(4 + n)]*(4*a + b*x^(4 + n)) -
 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]]))/(3*(4 + n)*Sqrt[a + b*x
^(4 + n)])
```

Maple [F]

$$\int c^5x^5\left(\frac{a}{x^4}+b\,x^n\right)^{\frac{3}{2}}dx$$

```
[In] int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

```
[Out] int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}dx=\text{Exception raised: TypeError}$$

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}dx=c^5\left(\int ax\sqrt{\frac{a}{x^4}+bx^n}\,dx+\int bx^5x^n\sqrt{\frac{a}{x^4}+bx^n}\,dx\right)$$

```
[In] integrate(c**5*x**5*(a/x**4+b*x**n)**(3/2),x)
```

```
[Out] c**5*(Integral(a*x*sqrt(a/x**4 + b*x**n), x) + Integral(b*x**5*x**n*sqrt(a/
x**4 + b*x**n), x))
```

Maxima [F]

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\int\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^5x^5\,dx$$

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
[Out] c^5*integrate((b*x^n + a/x^4)^(3/2)*x^5, x)
```

Giac [F]

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\int\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^5x^5\,dx$$

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^n + a/x^4)^(3/2)*c^5*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=\int c^5\,x^5\,\left(b\,x^n+\frac{a}{x^4}\right)^{3/2}\,dx$$

```
[In] int(c^5*x^5*(b*x^n + a/x^4)^(3/2),x)
[Out] int(c^5*x^5*(b*x^n + a/x^4)^(3/2), x)
```

3.382 $\int \sqrt{\frac{a+bx}{x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}x - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x}\right)$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b/x)^(1/2))*a^(1/2)+2*x*(a/x^2+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2004, 2032, 2038, 634, 212}

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = 2x\sqrt{\frac{a}{x^2}+\frac{b}{x}} - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+\frac{b}{x}}}\right)$$

```
[In] Int[Sqrt[(a + b*x)/x^2],x]
```

```
[Out] 2*Sqrt[a/x^2 + b/x]*x - 2*Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[a/x^2 + b/x]*x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{\frac{a}{x^2}+\frac{b}{x}}\,dx \\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}x + a\int \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x^2}\,dx \\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}x - a\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \frac{1}{x}\right) \\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}x - (2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x}\right) \\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}x - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \frac{2x\sqrt{\frac{a+bx}{x^2}}\left(\sqrt{a+bx}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a+bx}}$$

```
[In] Integrate[Sqrt[(a + b*x)/x^2],x]
```

```
[Out] (2*x*Sqrt[(a + b*x)/x^2]*(Sqrt[a + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqr
t[a]]))/Sqrt[a + b*x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{2\sqrt{\frac{bx+a}{x^2}}\,x\left(\sqrt{bx+a}-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{\sqrt{bx+a}}$	47

```
[In] int(((b*x+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((b*x+a)/x^2)^(1/2)*x*((b*x+a)^(1/2)-a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2
)))/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.82

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \left[2\,x\sqrt{\frac{bx+a}{x^2}}+\sqrt{a}\log\left(\frac{bx-2\,\sqrt{a}x\sqrt{\frac{bx+a}{x^2}}+2\,a}{x}\right), 2\,x\sqrt{\frac{bx+a}{x^2}} + 2\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx+a}{x^2}}}{a}\right)\right]$$

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*x*sqrt((b*x + a)/x^2) + sqrt(a)*log((b*x - 2*sqrt(a)*x*sqrt((b*x + a)/x^
2) + 2*a)/x), 2*x*sqrt((b*x + a)/x^2) + 2*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((
b*x + a)/x^2)/a)]
```

Sympy [F]

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \int \sqrt{\frac{a+bx}{x^2}}\,dx$$

```
[In] integrate(((b*x+a)/x**2)**(1/2),x)
[Out] Integral(sqrt((a + b*x)/x**2), x)
```

Maxima [F]

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \int \sqrt{\frac{bx+a}{x^2}}\,dx$$

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((b*x + a)/x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + 2\,\sqrt{bx+a}\operatorname{sgn}(x) - \frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + \sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="giac")
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x)
 - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx = 2\,x\,\sqrt{\frac{a}{x^2}+\frac{b}{x}}+\frac{\sqrt{a}\,\sqrt{x}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,\sqrt{x}}\right)\,\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

```
[In] int(((a + b*x)/x^2)^(1/2),x)
```

```
[Out] 2*x*(a/x^2 + b/x)^(1/2) + (a^(1/2)*x^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(1/
2)))*(a/x^2 + b/x)^(1/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

3.383 $\int \sqrt{\frac{a+bx^2}{x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx=\sqrt{b+\frac{a}{x^2}}x-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}x}\right)$$

```
[Out] -arctanh(a^(1/2)/x/(b+a/x^2)^(1/2))*a^(1/2)+x*(b+a/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1997, 248, 283, 223, 212}

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx=x\sqrt{\frac{a}{x^2}+b}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+b}}\right)$$

```
[In] Int[Sqrt[(a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b + a/x^2]*x - Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[b + a/x^2]*x)]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{b+\frac{a}{x^2}}\,dx\\
&= -\text{Subst}\left(\int \frac{\sqrt{b+ax^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{b+\frac{a}{x^2}}x - a\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{b+\frac{a}{x^2}}x - a\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{b+\frac{a}{x^2}}x}\right)\\
&= \sqrt{b+\frac{a}{x^2}}x - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \sqrt{b+\frac{a}{x^2}}x - \frac{\sqrt{a}\sqrt{b+\frac{a}{x^2}}x\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a+bx^2}}$$

```
[In] Integrate[Sqrt[(a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b + a/x^2]*x - (Sqrt[a]*Sqrt[b + a/x^2]*x*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]])/Sqrt[a + b*x^2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{\sqrt{\frac{b\,x^{2}+a}{x^{2}}}\,x\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{\sqrt{b\,x^{2}+a}}$	61

```
[In] int(((b*x^2+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)/x^2)^(1/2)*x/(b*x^2+a)^(1/2)*((b*x^2+a)^(1/2)-a^(1/2)*ln(2*(a^(1
/2)*(b*x^2+a)^(1/2)+a)/x))
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.57

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \left[x\sqrt{\frac{bx^2+a}{x^2}} + \frac{1}{2}\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{a}x\sqrt{\frac{bx^2+a}{x^2}}+2a}{x^2}\right), x\sqrt{\frac{bx^2+a}{x^2}} + \sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^2+a}{x^2}}}{bx^2+a}\right)\right]$$

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt((b*x^2 + a)/x^2) + 1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(a)*x*sqrt((b*x^
2 + a)/x^2) + 2*a)/x^2), x*sqrt((b*x^2 + a)/x^2) + sqrt(-a)*arctan(sqrt(-a)
*x*sqrt((b*x^2 + a)/x^2)/(b*x^2 + a))]
```

Sympy [F]

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \int \sqrt{\frac{a+bx^2}{x^2}}\,dx$$

```
[In] integrate(((b*x**2+a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**2)/x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \sqrt{b+\frac{a}{x^2}}x + \frac{1}{2}\sqrt{a}\log\left(\frac{\sqrt{b+\frac{a}{x^2}}x-\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}x+\sqrt{a}}\right)$$

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b + a/x^2)*x + 1/2*sqrt(a)*log((sqrt(b + a/x^2)*x - sqrt(a))/(sqrt(b +
 a/x^2)*x + sqrt(a)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(34) = 68.

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = \frac{a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \sqrt{bx^2+a}\operatorname{sgn}(x) - \frac{\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*arctan(sqrt(b*x^2 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + sqrt(b*x^2 + a)*sgn(x)
 - (a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \sqrt{\frac{a+bx^2}{x^2}}\,dx = x\,\sqrt{b+\frac{a}{x^2}}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x}\right)\,\sqrt{b+\frac{a}{x^2}}\,1\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x^2}+1}}$$

```
[In] int(((a + b*x^2)/x^2)^(1/2),x)
```

```
[Out] x*(b + a/x^2)^(1/2) + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x))*(b + a/x^2)^(
1/2)*1i)/(b^(1/2)*(a/(b*x^2) + 1)^(1/2))
```

3.384 $\int \sqrt{\frac{a+bx^3}{x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx=\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

```
[Out] -2/3*arctanh(a^(1/2)/x/(a/x^2+b*x)^(1/2))*a^(1/2)+2/3*x*(a/x^2+b*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2004, 2032, 2054, 212}

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx=\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

```
[In] Int[Sqrt[(a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x]
)])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{\frac{a}{x^2}+bx}\,dx\\ &= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}+a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx}}\,dx\\ &= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{1}{3}(2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx}}\right)\\ &= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx=\frac{2x\sqrt{\frac{a}{x^2}+bx}\left(\sqrt{a+bx^3}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{a+bx^3}}$$

```
[In] Integrate[Sqrt[(a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]]))/(3*Sqrt[a + b*x^3])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\sqrt{\frac{b\,x^{3}+a}{x^{2}}}\,x\left(-\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\sqrt{a}+\sqrt{b\,x^{3}+a}\right)}{3\sqrt{b\,x^{3}+a}}$	55

```
[In] int(((b*x^3+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*((b*x^3+a)/x^2)^(1/2)*x*(-arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+(b*x
^3+a)^(1/2))/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.04

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \left[\frac{2}{3}\,x\sqrt{\frac{bx^3+a}{x^2}}+\frac{1}{3}\,\sqrt{a}\log\left(\frac{bx^3-2\,\sqrt{a}x\sqrt{\frac{bx^3+a}{x^2}}+2\,a}{x^3}\right), \frac{2}{3}\,x\sqrt{\frac{bx^3+a}{x^2}} + \frac{2}{3}\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^3+a}{x^2}}}{a}\right)\right]$$

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*x*sqrt((b*x^3 + a)/x^2) + 1/3*sqrt(a)*log((b*x^3 - 2*sqrt(a)*x*sqrt((b
*x^3 + a)/x^2) + 2*a)/x^3), 2/3*x*sqrt((b*x^3 + a)/x^2) + 2/3*sqrt(-a)*arct
an(sqrt(-a)*x*sqrt((b*x^3 + a)/x^2)/a)]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \text{Timed out}$$

```
[In] integrate(((b*x**3+a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \int \sqrt{\frac{bx^3+a}{x^2}}\,dx$$

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^3 + a)/x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{3\,\sqrt{-a}} + \frac{2}{3}\,\sqrt{bx^3+a}\operatorname{sgn}(x) - \frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{3\,\sqrt{-a}}$$

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*sqrt(b*x^3 + a
)*sgn(x) - 2/3*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(
-a)
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \sqrt{\frac{a+bx^3}{x^2}}\,dx = \frac{2\,x\,\sqrt{b\,x+\frac{a}{x^2}}}{3} + \frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x^{3/2}}\right)\,\sqrt{b\,x+\frac{a}{x^2}}\,2\mathrm{i}}{3\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a}{b\,x^3}+1}}$$

```
[In] int(((a + b*x^3)/x^2)^(1/2),x)
```

```
[Out] (2*x*(b*x + a/x^2)^(1/2))/3 + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(3/2)))
*(b*x + a/x^2)^(1/2)*2i)/(3*b^(1/2)*x^(1/2)*(a/(b*x^3) + 1)^(1/2))
```

3.385 $\int \sqrt{\frac{a+bx^n}{x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx=\frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^(-2+n))^(1/2))*a^(1/2)/n+2*x*(a/x^2+b*x^(-2
+n))^(1/2)/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2004, 2032, 2054, 212}

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx=\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{n-2}}}\right)}{n}$$

```
[In] Int[Sqrt[(a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x^(-2 + n)])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x
^2 + b*x^(-2 + n)])])/n
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{\frac{a}{x^2}+bx^{-2+n}}\,dx\\
&= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}+a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\,dx\\
&= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}-\frac{(2a)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}\\
&= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$
\int \sqrt{\frac{a+bx^n}{x^2}}\,dx=\frac{2x\sqrt{\frac{a+bx^n}{x^2}}\left(\sqrt{a+bx^n}-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{a+bx^n}}
$$

```
[In] Integrate[Sqrt[(a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[(a + b*x^n)/x^2]*(Sqrt[a + b*x^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^
n]/Sqrt[a]]))/(n*Sqrt[a + b*x^n])
```

Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

method	result	size
risch	$\frac{2\sqrt{\frac{a+b\,\mathrm{e}^{n\ln(x)}}{x^2}}\,x}{n}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{\sqrt{a}}\right)\sqrt{\frac{a+b\,\mathrm{e}^{n\ln(x)}}{x^2}}\,x}{n\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}$	74

```
[In] int(((a+b*x^n)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/n*((a+b*exp(n*ln(x)))/x^2)^(1/2)*x-2*a^(1/2)/n*arctanh((a+b*exp(n*ln(x)))
^(1/2)/a^(1/2))*((a+b*exp(n*ln(x)))/x^2)^(1/2)/(a+b*exp(n*ln(x)))^(1/2)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.84

$$\int\sqrt{\frac{a+bx^n}{x^2}}\,dx = \left[\frac{2\,x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{a}\log\left(\frac{bx^n-2\,\sqrt{a}x\sqrt{\frac{bx^n+a}{x^2}}+2\,a}{x^n}\right)}{n}, \frac{2\left(x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^n+a}{x^2}}}{a}\right)\right)}{n}\right]$$

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*x*sqrt((b*x^n + a)/x^2) + sqrt(a)*log((b*x^n - 2*sqrt(a)*x*sqrt((b*x^n
+ a)/x^2) + 2*a)/x^n))/n, 2*(x*sqrt((b*x^n + a)/x^2) + sqrt(-a)*arctan(sqrt
(-a)*x*sqrt((b*x^n + a)/x^2)/a))/n]
```

Sympy [F]

$$\int\sqrt{\frac{a+bx^n}{x^2}}\,dx=\int\sqrt{\frac{a+bx^n}{x^2}}\,dx$$

```
[In] integrate(((a+b*x**n)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**n)/x**2), x)
```

Maxima [F]

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{bx^n+a}{x^2}}\,dx$$

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((b*x^n + a)/x^2), x)
```

Giac [F]

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{bx^n+a}{x^2}}\,dx$$

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^n + a)/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{a+bx^n}{x^2}}\,dx = \int \sqrt{\frac{a+b\,x^n}{x^2}}\,dx$$

```
[In] int(((a + b*x^n)/x^2)^(1/2),x)
[Out] int(((a + b*x^n)/x^2)^(1/2), x)
```

3.386 $\int \sqrt{\frac{-a+bx}{x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx=2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x+2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x}\right)$$

```
[Out] 2*arctan(a^(1/2)/x/(-a/x^2+b/x)^(1/2))*a^(1/2)+2*x*(-a/x^2+b/x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2004, 2032, 2038, 634, 209}

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx=2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x}-\frac{a}{x^2}}}\right)+2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}$$

```
[In] Int[Sqrt[(-a + b*x)/x^2],x]
```

```
[Out] 2*Sqrt[-(a/x^2) + b/x]*x + 2*Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[-(a/x^2) + b/x]*x
)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,dx \\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x - a\int \frac{1}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x^2}\,dx \\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x + a\text{Subst}\left(\int \frac{1}{\sqrt{bx-ax^2}}\,dx, x, \frac{1}{x}\right) \\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x + (2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x}\right) \\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x + 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx=\frac{2x\sqrt{\frac{-a+bx}{x^2}}\left(\sqrt{-a+bx}-\sqrt{a}\arctan\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{-a+bx}}$$

```
[In] Integrate[Sqrt[(-a + b*x)/x^2],x]
[Out] (2*x*Sqrt[(-a + b*x)/x^2]*(Sqrt[-a + b*x] - Sqrt[a]*ArcTan[Sqrt[-a + b*x]/S
qrt[a]]))/Sqrt[-a + b*x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{2\sqrt{-\frac{-bx+a}{x^2}}\,x\left(-\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\right)}{\sqrt{bx-a}}$	55

```
[In] int(((b*x-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(-(-b*x+a)/x^2)^(1/2)*x*(-a^(1/2)*arctan((b*x-a)^(1/2)/a^(1/2))+(b*x-a)^(
1/2))/(b*x-a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.85

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx=\left[2\,x\sqrt{\frac{bx-a}{x^2}}+\sqrt{-a}\log\left(\frac{bx-2\sqrt{-a}x\sqrt{\frac{bx-a}{x^2}}-2\,a}{x}\right),2\,x\sqrt{\frac{bx-a}{x^2}}-2\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx-a}{x^2}}}{\sqrt{a}}\right)\right]$$

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="fricas")
[Out] [2*x*sqrt((b*x - a)/x^2) + sqrt(-a)*log((b*x - 2*sqrt(-a)*x*sqrt((b*x - a)/
x^2) - 2*a)/x), 2*x*sqrt((b*x - a)/x^2) - 2*sqrt(a)*arctan(x*sqrt((b*x - a)
/x^2)/sqrt(a))]
```

Sympy [F]

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = \int \sqrt{\frac{-a+bx}{x^2}}\,dx$$

```
[In] integrate(((b*x-a)/x**2)**(1/2),x)
[Out] Integral(sqrt((-a + b*x)/x**2), x)
```

Maxima [F]

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = \int \sqrt{\frac{bx-a}{x^2}}\,dx$$

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((b*x - a)/x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = -2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\operatorname{sgn}(x) + 2\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right) - \sqrt{-a}\right)\operatorname{sgn}(x) + 2\sqrt{bx-a}\operatorname{sgn}(x)$$

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a))*sgn(x) + 2*(sqrt(a)*arctan(sqrt(-a
)/sqrt(a)) - sqrt(-a))*sgn(x) + 2*sqrt(b*x - a)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx = 2x\sqrt{\frac{b}{x}-\frac{a}{x^2}} + \frac{2\sqrt{a}\sqrt{x}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)\sqrt{\frac{b}{x}-\frac{a}{x^2}}}{\sqrt{b}\sqrt{1-\frac{a}{bx}}}$$

```
[In] int((-(a - b*x)/x^2)^(1/2),x)
[Out] 2*x*(b/x - a/x^2)^(1/2) + (2*a^(1/2)*x^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(1/2))
)*(b/x - a/x^2)^(1/2))/(b^(1/2)*(1 - a/(b*x))^(1/2))
```

3.387 $\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx=\sqrt{b-\frac{a}{x^2}}x+\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{b-\frac{a}{x^2}}x}\right)$$

```
[Out] arctan(a^(1/2)/x/(b-a/x^2)^(1/2))*a^(1/2)+x*(b-a/x^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1997, 248, 283, 223, 209}

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx=\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)+x\sqrt{b-\frac{a}{x^2}}$$

```
[In] Int[Sqrt[(-a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b - a/x^2]*x + Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[b - a/x^2]*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{b-\frac{a}{x^2}}\,dx\\
&= -\text{Subst}\left(\int \frac{\sqrt{b-ax^2}}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= \sqrt{b-\frac{a}{x^2}}x + a\text{Subst}\left(\int \frac{1}{\sqrt{b-ax^2}}\,dx,x,\frac{1}{x}\right)\\
&= \sqrt{b-\frac{a}{x^2}}x + a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,\frac{1}{\sqrt{b-\frac{a}{x^2}}x}\right)\\
&= \sqrt{b-\frac{a}{x^2}}x + \sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{b-\frac{a}{x^2}}x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx=\sqrt{b-\frac{a}{x^2}}x-\frac{\sqrt{a}\sqrt{b-\frac{a}{x^2}}x\arctan\left(\frac{\sqrt{-a+bx^2}}{\sqrt{a}}\right)}{\sqrt{-a+bx^2}}$$

```
[In] Integrate[Sqrt[(-a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b - a/x^2]*x - (Sqrt[a]*Sqrt[b - a/x^2]*x*ArcTan[Sqrt[-a + b*x^2]/Sqrt
[a]])/Sqrt[-a + b*x^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(35) = 70.

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

method	result	size
default	$\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}\,x\left(\sqrt{-a}\,\sqrt{b\,x^2-a}+a\ln\left(\frac{-2a+2\sqrt{-a}\,\sqrt{b\,x^2-a}}{x}\right)\right)}{\sqrt{-a}\,\sqrt{b\,x^2-a}}$	81

```
[In] int(((b*x^2-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*x^2+a)/x^2)^(1/2)*x*((-a)^(1/2)*(b*x^2-a)^(1/2)+a*ln(2*((-a)^(1/2)*(b
*x^2-a)^(1/2)-a)/x))/(-a)^(1/2)/(b*x^2-a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.74

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx=\left[x\sqrt{\frac{bx^2-a}{x^2}}+\frac{1}{2}\sqrt{-a}\log\left(-\frac{bx^2-2\sqrt{-a}x\sqrt{\frac{bx^2-a}{x^2}}-2a}{x^2}\right),x\sqrt{\frac{bx^2-a}{x^2}}+\sqrt{a}\arctan\left(\frac{\sqrt{a}x\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}\right)\right]$$

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt((b*x^2 - a)/x^2) + 1/2*sqrt(-a)*log(-(b*x^2 - 2*sqrt(-a)*x*sqrt((b*
x^2 - a)/x^2) - 2*a)/x^2), x*sqrt((b*x^2 - a)/x^2) + sqrt(a)*arctan(sqrt(a)
*x*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a))]
```

Sympy [F]

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = \int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$$

```
[In] integrate(((b*x**2-a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x**2)/x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = \sqrt{b-\frac{a}{x^2}}\,x - \sqrt{a}\arctan\left(\frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a}}\right)$$

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b - a/x^2)*x - sqrt(a)*arctan(sqrt(b - a/x^2)*x/sqrt(a))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47

$$\begin{aligned}\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = {} & -\sqrt{a}\arctan\left(\frac{\sqrt{bx^2-a}}{\sqrt{a}}\right)\mathrm{sgn}(x) \\ & + \left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right) - \sqrt{-a}\right)\mathrm{sgn}(x) + \sqrt{bx^2-a}\,\mathrm{sgn}(x)\end{aligned}$$

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(a)*arctan(sqrt(b*x^2 - a)/sqrt(a))*sgn(x) + (sqrt(a)*arctan(sqrt(-a)/
sqrt(a)) - sqrt(-a))*sgn(x) + sqrt(b*x^2 - a)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx = x\,\sqrt{b-\frac{a}{x^2}}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)\,\sqrt{b-\frac{a}{x^2}}}{\sqrt{b}\,\sqrt{1-\frac{a}{b\,x^2}}}$$

```
[In] int((-(a - b*x^2)/x^2)^(1/2),x)
```

```
[Out] x*(b - a/x^2)^(1/2) + (a^(1/2)*asin(a^(1/2)/(b^(1/2)*x))*(b - a/x^2)^(1/2))
/(b^(1/2)*(1 - a/(b*x^2))^(1/2))
```

3.388 $\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx=\frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx}+\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx}}\right)$$

```
[Out] 2/3*arctan(a^(1/2)/x/(-a/x^2+b*x)^(1/2))*a^(1/2)+2/3*x*(-a/x^2+b*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2004, 2032, 2054, 209}

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx=\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{bx-\frac{a}{x^2}}}\right)+\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}$$

```
[In] Int[Sqrt[(-a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x])/3 + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2) +
 b*x])])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \sqrt{-\frac{a}{x^2}+bx}\,dx\\
&= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx}-a\int \frac{1}{x^2\sqrt{-\frac{a}{x^2}+bx}}\,dx\\
&= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx}+\frac{1}{3}(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,\frac{1}{x\sqrt{-\frac{a}{x^2}+bx}}\right)\\
&= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx}+\frac{2}{3}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx=\frac{2x\sqrt{-\frac{a}{x^2}+bx}\left(\sqrt{-a+bx^3}-\sqrt{a}\arctan\left(\frac{\sqrt{-a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{-a+bx^3}}$$

```
[In] Integrate[Sqrt[(-a + b*x^3)/x^2],x]
[Out] (2*x*Sqrt[-(a/x^2) + b*x]*(Sqrt[-a + b*x^3] - Sqrt[a]*ArcTan[Sqrt[-a + b*x^
3]/Sqrt[a]]))/(3*Sqrt[-a + b*x^3])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{2\sqrt{-\frac{-b\,x^{3}+a}{x^{2}}}\,x\left(\sqrt{b\,x^{3}-a}\,\sqrt{-a}+a\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}-a}}{\sqrt{-a}}\right)\right)}{3\sqrt{b\,x^{3}-a}\,\sqrt{-a}}$	73

```
[In] int(((b*x^3-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-(-b*x^3+a)/x^2)^(1/2)*x*((b*x^3-a)^(1/2)*(-a)^(1/2)+a*arctanh((b*x^3-
a)^(1/2)/(-a)^(1/2)))/(b*x^3-a)^(1/2)/(-a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.06

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \left[\frac{2}{3}\,x\sqrt{\frac{bx^3-a}{x^2}} + \frac{1}{3}\,\sqrt{-a}\log\left(\frac{bx^3-2\,\sqrt{-a}x\sqrt{\frac{bx^3-a}{x^2}}-2\,a}{x^3}\right), \frac{2}{3}\,x\sqrt{\frac{bx^3-a}{x^2}} - \frac{2}{3}\,\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx^3-a}{x^2}}}{\sqrt{a}}\right)\right]$$

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*x*sqrt((b*x^3 - a)/x^2) + 1/3*sqrt(-a)*log((b*x^3 - 2*sqrt(-a)*x*sqrt(
(b*x^3 - a)/x^2) - 2*a)/x^3), 2/3*x*sqrt((b*x^3 - a)/x^2) - 2/3*sqrt(a)*arc
tan(x*sqrt((b*x^3 - a)/x^2)/sqrt(a))]
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \text{Timed out}$$

```
[In] integrate(((b*x**3-a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx=\int \sqrt{\frac{bx^3-a}{x^2}}\,dx$$

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^3 - a)/x^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx=-\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{bx^3-a}}{\sqrt{a}}\right)\mathrm{sgn}(x)+\frac{2}{3}\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\mathrm{sgn}(x)+\frac{2}{3}\sqrt{bx^3-a}\mathrm{sgn}(x)$$

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(a)*arctan(sqrt(b*x^3 - a)/sqrt(a))*sgn(x) + 2/3*(sqrt(a)*arctan(s
qrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x) + 2/3*sqrt(b*x^3 - a)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx=\frac{2\,x\,\sqrt{b\,x-\frac{a}{x^2}}}{3}+\frac{2\,\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{3/2}}\right)\,\sqrt{b\,x-\frac{a}{x^2}}}{3\,\sqrt{b}\,\sqrt{x}\,\sqrt{1-\frac{a}{b\,x^3}}}$$

```
[In] int((-(a - b*x^3)/x^2)^(1/2),x)
```

```
[Out] (2*x*(b*x - a/x^2)^(1/2))/3 + (2*a^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(3/2)))*(b
*x - a/x^2)^(1/2))/(3*b^(1/2)*x^(1/2)*(1 - a/(b*x^3))^(1/2))
```

3.389 $\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx=\frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n}+\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}$$

```
[Out] 2*arctan(a^(1/2)/x/(-a/x^2+b*x^(-2+n))^(1/2))*a^(1/2)/n+2*x*(-a/x^2+b*x^(-2
+n))^(1/2)/n
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2004, 2032, 2054, 209}

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx=\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}\right)}{n}+\frac{2x\sqrt{bx^{n-2}-\frac{a}{x^2}}}{n}$$

```
[In] Int[Sqrt[(-a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x^(-2 + n)])/n + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-
(a/x^2) + b*x^(-2 + n)])])/n
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2032

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(p*(n - j))), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{-\frac{a}{x^2}+bx^{-2+n}}\,dx \\
&= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} - a\int \frac{1}{x^2\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\,dx \\
&= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} + \frac{(2a)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n} \\
&= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} + \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$
\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx = \frac{2x\sqrt{\frac{-a+bx^n}{x^2}}\left(\sqrt{-a+bx^n}-\sqrt{a}\arctan\left(\frac{\sqrt{-a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{-a+bx^n}}
$$

```
[In] Integrate[Sqrt[(-a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[(-a + b*x^n)/x^2]*(Sqrt[-a + b*x^n] - Sqrt[a]*ArcTan[Sqrt[-a + b*
x^n]/Sqrt[a]]))/(n*Sqrt[-a + b*x^n])
```

Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

method	result	size
risch	$-\frac{2\left(a-b\,\mathrm{e}^{n\ln\left(x\right)}\right)\sqrt{\frac{b\,\mathrm{e}^{n\ln\left(x\right)}-a}{x^{2}}}\,x}{n\left(b\,\mathrm{e}^{n\ln\left(x\right)}-a\right)}-\frac{2\sqrt{a}\,\arctan\left(\frac{\sqrt{b\,\mathrm{e}^{n\ln\left(x\right)}-a}}{\sqrt{a}}\right)\sqrt{\frac{b\,\mathrm{e}^{n\ln\left(x\right)}-a}{x^{2}}}\,x}{n\sqrt{b\,\mathrm{e}^{n\ln\left(x\right)}-a}}$	105

```
[In] int(((b*x^n-a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(a-b*exp(n*ln(x)))/n/(b*exp(n*ln(x))-a)*((b*exp(n*ln(x))-a)/x^2)^(1/2)*x
-2*a^(1/2)/n*arctan((b*exp(n*ln(x))-a)^(1/2)/a^(1/2))*((b*exp(n*ln(x))-a)/x
^2)^(1/2)/(b*exp(n*ln(x))-a)^(1/2)*x
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.87

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$$
$$=\left[\frac{2\,x\sqrt{\frac{bx^n-a}{x^2}}+\sqrt{-a}\log\left(\frac{bx^n-2\,\sqrt{-a}x\sqrt{\frac{bx^n-a}{x^2}}-2\,a}{x^n}\right)}{n},\ \frac{2\left(x\sqrt{\frac{bx^n-a}{x^2}}-\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx^n-a}{x^2}}}{\sqrt{a}}\right)\right)}{n}\right]$$

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*x*sqrt((b*x^n - a)/x^2) + sqrt(-a)*log((b*x^n - 2*sqrt(-a)*x*sqrt((b*x^
n - a)/x^2) - 2*a)/x^n))/n, 2*(x*sqrt((b*x^n - a)/x^2) - sqrt(a)*arctan(x*s
qrt((b*x^n - a)/x^2)/sqrt(a)))/n]
```

Sympy [F]

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx=\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$$

```
[In] integrate(((-a+b*x**n)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x**n)/x**2), x)
```

Maxima [F]

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx=\int \sqrt{\frac{bx^n-a}{x^2}}\,dx$$

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((b*x^n - a)/x^2), x)
```

Giac [F]

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx=\int \sqrt{\frac{bx^n-a}{x^2}}\,dx$$

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^n - a)/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx=\int \sqrt{-\frac{a-b\,x^n}{x^2}}\,dx$$

```
[In] int((-(a - b*x^n)/x^2)^(1/2),x)
[Out] int((-(a - b*x^n)/x^2)^(1/2), x)
```

3.390 $\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 62

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx=\frac{2x^{-j/2}(cx)^{j/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}c(j-n)}$$

```
[Out] 2*(c*x)^(1/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/c/(j-n)/(x^(
1/2*j))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2056, 2054, 212}

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx=\frac{2x^{-j/2}(cx)^{j/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}c(j-n)}$$

```
[In] Int[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*(c*x)^(j/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(Sqrt[a]*c*(
j - n)*x^(j/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^{-j/2}(cx)^{j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{c}\\
&= \frac{\left(2x^{-j/2}(cx)^{j/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}\\
&= \frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}c(j-n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \frac{2\sqrt{b}x^{\frac{1}{2}(-j+n)}(cx)^{j/2}\sqrt{1+\frac{ax^{j-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}c(j-n)\sqrt{ax^j+bx^n}}$$

```
[In] Integrate[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
[Out] (2*Sqrt[b]*x^((-j + n)/2)*(c*x)^(j/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sq
rt[a]*x^((j - n)/2))/Sqrt[b]])/(Sqrt[a]*c*(j - n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{a\,x^j+b\,x^n}}dx$$

```
[In] int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
[Out] int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{ax^j+bx^n}}\,dx$$

```
[In] integrate((c*x)**(-1+1/2*j)/(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral((c*x)**(j/2 - 1)/sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j+bx^n}}\,dx$$

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

Giac [F]

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j+bx^n}}\,dx$$

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx=\int \frac{{\left(c\,x\right)}^{\frac{j}{2}-1}}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

```
[In] int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2),x)
[Out] int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2), x)
```

3.391 $\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx=\frac{2\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}$$

```
[Out] 2*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/(3-n)/a^(1/2)/x^
(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2056, 2054, 212}

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx=\frac{2\sqrt{cx}\,\text{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}$$

```
[In] Int[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

```
[Out] (2*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(Sqrt[a]*(3 -
n)*Sqrt[x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{\sqrt{x}}\\ &= \frac{\left(2\sqrt{cx}\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{(3-n)\sqrt{x}}\\ &= \frac{2\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int\frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = -\frac{2\sqrt{b}x^{\frac{1}{2}(-1+n)}\sqrt{cx}\sqrt{1+\frac{ax^{3-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(-3+n)\sqrt{ax^3+bx^n}}$$

```
[In] Integrate[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

```
[Out] (-2*Sqrt[b]*x^((-1 + n)/2)*Sqrt[c*x]*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqr
t[a]*x^(3/2 - n/2))/Sqrt[b]])/(Sqrt[a]*(-3 + n)*Sqrt[a*x^3 + b*x^n])
```

Maple [F]

$$\int\frac{\sqrt{cx}}{\sqrt{a\,x^3+b\,x^n}}dx$$

```
[In] int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

```
[In] integrate((c*x)**(1/2)/(a*x**3+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(c*x)/sqrt(a*x**3 + b*x**n), x)
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx=\int \frac{\sqrt{c\,x}}{\sqrt{b\,x^n+a\,x^3}}\,dx$$

```
[In] int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2),x)
[Out] int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2), x)
```

3.392 $\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)}$$

```
[Out] 2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))/(2-n)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2033, 212}

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)}$$

```
[In] Int[1/Sqrt[a*x^2 + b*x^n],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(Sqrt[a]*(2 - n))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
```

```
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{2-n} \\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx = -\frac{2\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{2-n}}{b}}\text{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(-2+n)\sqrt{ax^2+bx^n}}$$

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^n],x]
[Out] (-2*Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))
/Sqrt[b]])/(Sqrt[a]*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

Maple [F]

$$\int \frac{1}{\sqrt{a\,x^2+b\,x^n}}dx$$

```
[In] int(1/(a*x^2+b*x^n)^(1/2),x)
[Out] int(1/(a*x^2+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx=\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$$

```
[In] integrate(1/(a*x**2+b*x**n)**(1/2),x)
[Out] Integral(1/sqrt(a*x**2 + b*x**n), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx=\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$$

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx=\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$$

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx=\frac{\sqrt{b}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{b}}\right)\,\sqrt{\frac{a\,x^{2-n}}{b}+1}\,1\mathrm{i}}{\sqrt{a}\,\left(\frac{n}{2}-1\right)\,\sqrt{b\,x^n+a\,x^2}}$$

```
[In] int(1/(b*x^n + a*x^2)^(1/2),x)
[Out] (b^(1/2)*x^(n/2)*asin((a^(1/2)*x^(1 - n/2)*1i)/b^(1/2))*((a*x^(2 - n))/b +
1)^(1/2)*1i)/(a^(1/2)*(n/2 - 1)*(b*x^n + a*x^2)^(1/2))
```

3.393 $\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}} \, dx = \frac{2\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}$$

```
[Out] 2*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*x^(1/2)/(1-n)/a^(1/2)/(c*x)^(1
/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2056, 2054, 212}

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}} \, dx = \frac{2\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}$$

```
[In] Int[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

```
[Out] (2*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(Sqrt[a]*(1 - n)*S
qrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{\sqrt{cx}}\\
&= \frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{(1-n)\sqrt{cx}}\\
&= \frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = -\frac{2\sqrt{b}x^{\frac{1+n}{2}}\sqrt{1+\frac{ax^{1-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(-1+n)\sqrt{cx}\sqrt{ax+bx^n}}$$

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
[Out] (-2*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2
 - n/2))/Sqrt[b]])/(Sqrt[a]*(-1 + n)*Sqrt[c*x]*Sqrt[a*x + b*x^n])
```

Maple [F]

$$\int \frac{1}{\sqrt{cx}\,\sqrt{ax+b\,x^n}}dx$$

```
[In] int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
[Out] int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx$$

```
[In] integrate(1/(c*x)**(1/2)/(a*x+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*x)*sqrt(a*x + b*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \int \frac{1}{\sqrt{ax+bx^n}\sqrt{cx}}\,dx$$

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \int \frac{1}{\sqrt{ax+bx^n}\sqrt{cx}}\,dx$$

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx = \int \frac{1}{\sqrt{c\,x}\,\sqrt{b\,x^n+a\,x}}\,dx$$

```
[In] int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)),x)
[Out] int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)), x)
```

3.394 $\int \frac{1}{cx\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {12, 272, 65, 214}

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

```
[In] Int[1/(c*x*Sqrt[a + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{x\sqrt{a+bx^n}}\,dx}{c}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{cn}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bcn}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

```
[In] Integrate[1/(c*x*Sqrt[a + b*x^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn\sqrt{a}}$	26
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{cn\sqrt{a}}$	26

```
[In] int(1/c/x/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n/a^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \left[\frac{\log\left(\frac{bx^n-2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)}{\sqrt{a}cn}, \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)}{acn}\right]$$

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*c*n), 2*sqrt(-
a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a)/(a*c*n)]
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = -\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}cn}$$

```
[In] integrate(1/c/x/(a+b*x**n)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*x**(n/2)))/(sqrt(a)*c*n)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{\sqrt{a}cn}$$

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="maxima")
[Out] log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(sqrt(a)*c*n)
```

Giac [F]

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+a}cx}\,dx$$

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^n + a)*c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{cx\sqrt{a+bx^n}}\,dx = \int \frac{1}{c\,x\,\sqrt{a+b\,x^n}}\,dx$$

```
[In] int(1/(c*x*(a + b*x^n)^(1/2)),x)
[Out] int(1/(c*x*(a + b*x^n)^(1/2)), x)
```

3.395 $\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx = -\frac{2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}c(1+n)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*x^(1/2)/c/(1+n)/a^(1/2)/(c*x)
^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2056, 2054, 212}

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx = -\frac{2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}c(n+1)\sqrt{cx}}$$

```
[In] Int[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(Sqrt[a]*c*(1 + n
)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{c\sqrt{cx}}\\
&= -\frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{c(1+n)\sqrt{cx}}\\
&= -\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}c(1+n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx = -\frac{2x\sqrt{a+bx^{1+n}}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)}{\sqrt{a}(1+n)(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}$$

```
[In] Integrate[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])/(Sqrt[a
]*(1 + n)*(c*x)^(3/2)*Sqrt[a/x + b*x^n])
```

Maple [F]

$$\int \frac{1}{\left(c x\right)^{\frac{3}{2}} \sqrt{\frac{a}{x}+b\,x^{n}}}d x$$

```
[In] int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)
```

```
[Out] int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx = \int \frac{1}{\left(cx\right)^{\frac{3}{2}}\sqrt{\frac{a}{x}+bx^{n}}}\,dx$$

```
[In] integrate(1/(c*x)**(3/2)/(a/x+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/((c*x)**(3/2)*sqrt(a/x + b*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^{n}+\frac{a}{x}}\left(cx\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=\int \frac{1}{\sqrt{bx^n+\frac{a}{x}}\,(cx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=\int \frac{1}{{\left(c\,x\right)}^{3/2}\,\sqrt{b\,x^n+\frac{a}{x}}}\,dx$$

```
[In] int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)),x)
[Out] int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)), x)
```

3.396 $\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 40

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}c^2(2+n)}$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))/c^2/(2+n)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {12, 2054, 212}

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}c^2(n+2)}$$

```
[In] Int[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(Sqrt[a]*c^2*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}\\ &= -\frac{2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{c^2(2+n)}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}c^2(2+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = -\frac{2\sqrt{a+bx^{2+n}}\text{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)}{\sqrt{a}c^2(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}$$

```
[In] Integrate[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
[Out] (-2*Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])/(Sqrt[a]*
c^2*(2 + n)*x*Sqrt[a/x^2 + b*x^n])
```

Maple [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+b\,x^n}}dx$$

```
[In] int(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x)
[Out] int(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}$$

```
[In] integrate(1/c**2/x**2/(a/x**2+b*x**n)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a/x**2 + b*x**n)), x)/c**2
```

Maxima [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+\frac{a}{x^2}}c^2x^2}\,dx$$

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^n + a/x^2)*x^2), x)/c^2
```

Giac [F]

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n+\frac{a}{x^2}}c^2x^2}\,dx$$

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^n + a/x^2)*c^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx=\int \frac{1}{c^2\,x^2\,\sqrt{b\,x^n+\frac{a}{x^2}}}\,dx$$

```
[In] int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)),x)
[Out] int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)), x)
```

3.397 $\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx=-\frac{2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}c^2(3+n)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/c^2/(3+n)/a^(1/2)/(
c*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2056, 2054, 212}

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx=-\frac{2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}c^2(n+3)\sqrt{cx}}$$

```
[In] Int[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(Sqrt[a]*c^2*(3
 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{c^2\sqrt{cx}}\\ &= -\frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{c^2(3+n)\sqrt{cx}}\\ &= -\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}c^2(3+n)\sqrt{cx}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx = -\frac{2x\sqrt{a+bx^{3+n}}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)}{\sqrt{a}(3+n)(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

```
[In] Integrate[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
[Out] (-2*x*Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])/(Sqrt[a
]*(3 + n)*(c*x)^(5/2)*Sqrt[a/x^3 + b*x^n])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{2}} \sqrt{\frac{a}{x^3} + b\,x^n}}dx$$

```
[In] int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)
[Out] int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3} + bx^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3} + bx^n}}\,dx = \int \frac{1}{(cx)^{\frac{5}{2}} \sqrt{\frac{a}{x^3} + bx^n}}\,dx$$

```
[In] integrate(1/(c*x)**(5/2)/(a/x**3+b*x**n)**(1/2),x)
[Out] Integral(1/((c*x)**(5/2)*sqrt(a/x**3 + b*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3} + bx^n}}\,dx = \int \frac{1}{\sqrt{bx^n + \frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx=\int \frac{1}{\sqrt{bx^n+\frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx=\int \frac{1}{{(c\,x)}^{5/2}\,\sqrt{b\,x^n+\frac{a}{x^3}}}\,dx$$

```
[In] int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)), x)
```

3.398 $\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=-\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}+\frac{2x^{-3j/2}(cx)^{3j/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}$$

```
[Out] 2*(c*x)^(3/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/a^(3/2)/c/(j
-n)/(x^(3/2*j))-2*(c*x)^(3/2*j)/a/c/(j-n)/(x^j)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2056, 2055, 2054, 212}

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=\frac{2x^{-3j/2}(cx)^{3j/2}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}-\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}$$

```
[In] Int[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(c*x)^((3*j)/2))/(a*c*(j - n)*x^j*Sqrt[a*x^j + b*x^n]) + (2*(c*x)^((3*j
)/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(a^(3/2)*c*(j - n)*x^(
(3*j)/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^{-3j/2}(cx)^{3j/2}\right)\int \frac{x^{-1+\frac{3j}{2}}}{\left(ax^j+bx^n\right)^{3/2}}\,dx}{c}\\
&= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}+\frac{\left(x^{-3j/2}(cx)^{3j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{ac}\\
&= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}+\frac{\left(2x^{-3j/2}(cx)^{3j/2}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{ac(j-n)}\\
&= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}+\frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx = -\frac{2x^{-3j/2}(cx)^{3j/2}\left(\sqrt{a}x^{j/2}-\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}c(j-n)\sqrt{ax^j+bx^n}}$$

```
[In] Integrate[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(c*x)^((3*j)/2)*(Sqrt[a]*x^(j/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(j - n
))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(a^(3/2)*c*(j - n)*x^((3*j
)/2)*Sqrt[a*x^j + b*x^n])
```

Maple [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(a\,x^j+b\,x^n)^{\frac{3}{2}}}dx$$

```
[In] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3j}{2}-1}}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x)**(-1+3/2*j)/(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((c*x)**(3*j/2 - 1)/(a*x**j + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{(cx)^{\frac{3}{2}\,j-1}}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{(cx)^{\frac{3}{2}\,j-1}}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{(ax^j+bx^n)^{3/2}}\,dx=\int \frac{(c\,x)^{\frac{3\,j}{2}-1}}{(a\,x^j+b\,x^n)^{3/2}}\,dx$$

```
[In] int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2),x)
[Out] int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2), x)
```

3.399 $\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx = -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}} + \frac{2c^3\sqrt{cx}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}$$

```
[Out] 2*c^3*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/a^(3/2)/(3-n
)/x^(1/2)-2*c^2*(c*x)^(3/2)/a/(3-n)/(a*x^3+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2055, 2056, 2054, 212}

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx = \frac{2c^3\sqrt{cx}\operatorname{arctanh}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}} - \frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

```
[In] Int[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

```
[Out] (-2*c^2*(c*x)^(3/2))/(a*(3 - n)*Sqrt[a*x^3 + b*x^n]) + (2*c^3*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(a^(3/2)*(3 - n)*Sqrt[x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{c^3\int\frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{a}\\
&= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{\left(c^3\sqrt{cx}\right)\int\frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{a\sqrt{x}}\\
&= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{\left(2c^3\sqrt{cx}\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a(3-n)\sqrt{x}}\\
&= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{2c^3\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.16

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx = \frac{2c^3\sqrt{cx}\left(\sqrt{a}x^{3/2}-\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{3-n}}{b}}\,\text{arcsinh}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-3+n)\sqrt{x}\sqrt{ax^3+bx^n}}$$

```
[In] Integrate[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^3*Sqrt[c*x]*(Sqrt[a]*x^(3/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(3 - n))/
b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-3 + n)*Sqrt[x]*Sqr
t[a*x^3 + b*x^n])
```

Maple [F]

$$\int \frac{(cx)^{\frac{7}{2}}}{(a\,x^3+b\,x^n)^{\frac{3}{2}}}dx$$

```
[In] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((c*x)**(7/2)/(a*x**3+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx=\int \frac{(cx)^{\frac{7}{2}}}{(ax^3+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx=\int \frac{(cx)^{\frac{7}{2}}}{(ax^3+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(ax^3+bx^n)^{3/2}}\,dx=\int \frac{{(c\,x)}^{7/2}}{{\left(b\,x^n+a\,x^3\right)}^{3/2}}\,dx$$

```
[In] int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2),x)
[Out] int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2), x)
```

3.400 $\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx=-\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{2c^2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}$$

```
[Out] 2*c^2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))/a^(3/2)/(2-n)-2*c^2*x/a/(2-n)/
(a*x^2+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2055, 2033, 212}

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx=\frac{2c^2\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}-\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}$$

```
[In] Int[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

```
[Out] (-2*c^2*x)/(a*(2 - n)*Sqrt[a*x^2 + b*x^n]) + (2*c^2*ArcTanh[(Sqrt[a]*x)/Sqr
t[a*x^2 + b*x^n]])/(a^(3/2)*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^2\int \frac{x^2}{(ax^2+bx^n)^{3/2}}\,dx\\
&= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{c^2\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{a}\\
&= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{(2c^2)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{a(2-n)}\\
&= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \frac{2c^2\left(\sqrt{a}x-\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{2-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-2+n)\sqrt{ax^2+bx^n}}$$

```
[In] Integrate[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*(Sqrt[a]*x - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt
[a]*x^(1 - n/2))/Sqrt[b]]))/(a^(3/2)*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

Maple [F]

$$\int \frac{c^{2} x^{2}}{\left(a\,x^{2}+b\,x^{n}\right)^{\frac{3}{2}}}dx$$

```
[In] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

```
[Out] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = c^2\int \frac{x^2}{ax^2\sqrt{ax^2+bx^n}+bx^n\sqrt{ax^2+bx^n}}\,dx$$

```
[In] integrate(c**2*x**2/(a*x**2+b*x**n)**(3/2),x)
```

```
[Out] c**2*Integral(x**2/(a*x**2*sqrt(a*x**2 + b*x**n) + b*x**n*sqrt(a*x**2 + b*x
**n)), x)
```

Maxima [F]

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \int \frac{c^2x^2}{(ax^2+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^2/(a*x^2 + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \int \frac{c^2x^2}{(ax^2+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(c^2*x^2/(a*x^2 + b*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c^2x^2}{(ax^2+bx^n)^{3/2}}\,dx = \int \frac{c^2\,x^2}{(b\,x^n+a\,x^2)^{3/2}}\,dx$$

```
[In] int((c^2*x^2)/(b*x^n + a*x^2)^(3/2),x)
[Out] int((c^2*x^2)/(b*x^n + a*x^2)^(3/2), x)
```

3.401 $\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx=-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{2c\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}$$

```
[Out] 2*c*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/(1-n)/(c*x)^
(1/2)-2*(c*x)^(1/2)/a/(1-n)/(a*x+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2055, 2056, 2054, 212}

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx=\frac{2c\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

```
[In] Int[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

```
[Out] (-2*Sqrt[c*x])/(a*(1 - n)*Sqrt[a*x + b*x^n]) + (2*c*Sqrt[x]*ArcTanh[(Sqrt[a
]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(a^(3/2)*(1 - n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{c\int\frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx}{a}\\
&= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{\left(c\sqrt{x}\right)\int\frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{a\sqrt{cx}}\\
&= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{\left(2c\sqrt{x}\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a(1-n)\sqrt{cx}}\\
&= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{2c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \frac{2\sqrt{cx}\left(\sqrt{a}\sqrt{x}-\sqrt{b}x^{n/2}\sqrt{1+\frac{ax^{1-n}}{b}}\operatorname{arcsinh}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(-1+n)\sqrt{x}\sqrt{ax+bx^n}}$$

```
[In] Integrate[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

```
[Out] (2*Sqrt[c*x]*(Sqrt[a]*Sqrt[x] - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(1 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-1 + n)*Sqrt[x]*Sqrt[a*
x + b*x^n])
```

Maple [F]

$$\int \frac{\sqrt{cx}}{(ax+b\,x^n)^{\frac{3}{2}}}dx$$

```
[In] int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x)**(1/2)/(a*x+b*x**n)**(3/2),x)
```

```
[Out] Integral(sqrt(c*x)/(a*x + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{c\,x}}{(b\,x^n+a\,x)^{3/2}}\,dx$$

```
[In] int((c*x)^(1/2)/(b*x^n + a*x)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)/(b*x^n + a*x)^(3/2), x)
```

3.402 $\int \frac{1}{cx(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx=\frac{2}{acn\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(3/2)/c/n+2/a/c/n/(a+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 272, 53, 65, 214}

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx=\frac{2}{acn\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

```
[In] Int[1/(c*x*(a + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*c
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{x(a+bx^n)^{3/2}}\,dx}{c}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^n\right)}{cn}\\
&= \frac{2}{acn\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{acn}\\
&= \frac{2}{acn\sqrt{a+bx^n}} + \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{abcn}\\
&= \frac{2}{acn\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx = \frac{\frac{2}{an\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}}{c}$$

```
[In] Integrate[1/(c*x*(a + b*x^n)^(3/2)),x]
```

```
[Out] (2/(a*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*n)
)/c
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{2}{a\sqrt{a+b\,x^{n}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{cn}$	42
default	$\frac{\frac{2}{a\sqrt{a+b\,x^{n}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{cn}$	42

```
[In] int(1/c/x/(a+b*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/n*(2/a/(a+b*x^n)^(1/2)-2/a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.74

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx = \left[\frac{\left(\sqrt{a}bx^n+a^{\frac{3}{2}}\right)\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\sqrt{bx^n+a}a}{a^2bcnx^n+a^3cn}, \frac{2\left(\left(\sqrt{-a}bx^n+\sqrt{-a}a\right)\mathrm{ar}}{a^2bc}\right.$$

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [((sqrt(a)*b*x^n + a^(3/2))*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x
^n) + 2*sqrt(b*x^n + a)*a)/(a^2*b*c*n*x^n + a^3*c*n), 2*((sqrt(-a)*b*x^n +
sqrt(-a)*a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n + a)*a)/(a^2*b*
c*n*x^n + a^3*c*n)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(42) = 84$.

Time = 1.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.43

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx = \frac{\frac{2a^3\sqrt{1+\frac{bx^n}{a}}}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n} + \frac{a^3\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n} + \frac{a^2bx^n\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n} - \frac{2a^2bx^n\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}}{c}$$

```
[In] integrate(1/c/x/(a+b*x**n)**(3/2),x)
```

```
[Out] (2*a**3*sqrt(1 + b*x**n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**3*log(b*x*
*n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) - 2*a**3*log(sqrt(1 + b*x**n/a) + 1)
/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**2*b*x**n*log(b*x**n/a)/(a**(9/2)*n +
 a**(7/2)*b*n*x**n) - 2*a**2*b*x**n*log(sqrt(1 + b*x**n/a) + 1)/(a**(9/2)*n
 + a**(7/2)*b*n*x**n))/c
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx = \frac{\frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}n} + \frac{2}{\sqrt{bx^n+a}an}}{c}$$

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] (log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(3/2)*n) +
 2/(sqrt(b*x^n + a)*a*n))/c
```

Giac [F]

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^n+a\right)^{\frac{3}{2}}cx}\,dx$$

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(3/2)*c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{cx\left(a+bx^n\right)^{3/2}}\,dx=\int \frac{1}{c\,x\,\left(a+b\,x^n\right)^{3/2}} \,dx$$

```
[In] int(1/(c*x*(a + b*x^n)^(3/2)),x)
```

```
[Out] int(1/(c*x*(a + b*x^n)^(3/2)), x)
```

3.403 $\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(1+n)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/c^2/(1+n)/(c*
x)^(1/2)+2/a/c^2/(1+n)/(c*x)^(1/2)/(a/x+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2055, 2056, 2054, 212}

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

```
[In] Int[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt[a]/
(Sqrt[x]*Sqrt[a/x + b*x^n])])/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}+\frac{\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{ac}\\
&= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}+\frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{ac^2\sqrt{cx}}\\
&= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{ac^2(1+n)\sqrt{cx}}\\
&= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(1+n)\sqrt{cx}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{1+n}}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{1+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}$$

```
[In] Integrate[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

```
[Out] (2*(Sqrt[a] - Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])
)/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{2}}\left(\frac{a}{x}+b\,x^{n}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)
```

```
[Out] int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{(cx)^{\frac{5}{2}}\left(\frac{a}{x}+bx^{n}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c*x)**(5/2)/(a/x+b*x**n)**(3/2),x)
```

```
[Out] Integral(1/((c*x)**(5/2)*(a/x + b*x**n)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{(c\,x)^{5/2}\left(b\,x^n+\frac{a}{x}\right)^{3/2}}\,dx$$

```
[In] int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)),x)
[Out] int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)), x)
```

3.404 $\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(2+n)}$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))/a^(3/2)/c^4/(2+n)+2/a/c^4/(2+n)/x
/(a/x^2+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2055, 2054, 212}

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^4(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(n+2)}$$

```
[In] Int[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2
+ b*x^n])])/(a^(3/2)*c^4*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx}{c^4} \\
&= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}+\frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{ac^4} \\
&= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{ac^4(2+n)} \\
&= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(2+n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{2+n}}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{2+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}}$$

```
[In] Integrate[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

```
[Out] (2*(Sqrt[a] - Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])
)/(a^(3/2)*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n])
```

Maple [F]

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+b\,x^n\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x)
```

```
[Out] int(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\frac{\int \frac{1}{ax^2\sqrt{\frac{a}{x^2}+bx^n}+bx^4x^n\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^4}$$

```
[In] integrate(1/c**4/x**4/(a/x**2+b*x**n)**(3/2),x)
```

```
[Out] Integral(1/(a*x**2*sqrt(a/x**2 + b*x**n) + b*x**4*x**n*sqrt(a/x**2 + b*x**n
)), x)/c**4
```

Maxima [F]

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^4x^4}\,dx$$

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*x^4), x)/c^4
```

Giac [F]

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^4x^4}\,dx$$

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*c^4*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{c^4\,x^4\,\left(b\,x^n+\frac{a}{x^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)),x)
[Out] int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)), x)
```

3.405 $\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 90

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(3+n)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/c^5/(3+n)/(
c*x)^(1/2)+2/a/c^4/(3+n)/(c*x)^(3/2)/(a/x^3+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2055, 2056, 2054, 212}

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(n+3)\sqrt{cx}}$$

```
[In] Int[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt
[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(a^(3/2)*c^5*(3 + n)*Sqrt[c*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rule 2056

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}+\frac{\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{ac^3}\\
&= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}+\frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{ac^5\sqrt{cx}}\\
&= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{ac^5(3+n)\sqrt{cx}}\\
&= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(3+n)\sqrt{cx}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{3+n}}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{3+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

```
[In] Integrate[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

```
[Out] (2*(Sqrt[a] - Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])
)/(a^(3/2)*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{11}{2}}\left(\frac{a}{x^3}+b\,x^n\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)
```

```
[Out] int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(c*x)**(11/2)/(a/x**3+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{11}{2}}}\,dx$$

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{11}{2}}}\,dx$$

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{(c\,x)^{11/2}\left(b\,x^n+\frac{a}{x^3}\right)^{3/2}}\,dx$$

```
[In] int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)),x)
[Out] int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)), x)
```

3.406 $\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(4+n)}$$

```
[Out] -2*arctanh(a^(1/2)/x^2/(a/x^4+b*x^n)^(1/2))/a^(3/2)/c^7/(4+n)+2/a/c^7/(4+n)
/x^2/(a/x^4+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2055, 2054, 212}

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\frac{2}{ac^7(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(n+4)}$$

```
[In] Int[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/
x^4 + b*x^n])])/(a^(3/2)*c^7*(4 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2055

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Dist[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1))), Int
[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n},
 x] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (I
ntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx}{c^7} \\
&= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}+\frac{\int \frac{1}{x^3\sqrt{\frac{a}{x^4}+bx^n}}\,dx}{ac^7} \\
&= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{ac^7(4+n)} \\
&= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(4+n)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\frac{2\left(\sqrt{a}-\sqrt{a+bx^{4+n}}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^{4+n}}}{\sqrt{a}}\right)\right)}{a^{3/2}c^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}$$

```
[In] Integrate[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

```
[Out] (2*(Sqrt[a] - Sqrt[a + b*x^(4 + n)]*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]])
)/(a^(3/2)*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n])
```

Maple [F]

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+b\,x^n\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)
```

```
[Out] int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\frac{\int \frac{1}{ax^3\sqrt{\frac{a}{x^4}+bx^n}+bx^7x^n\sqrt{\frac{a}{x^4}+bx^n}}\,dx}{c^7}$$

```
[In] integrate(1/c**7/x**7/(a/x**4+b*x**n)**(3/2),x)
```

```
[Out] Integral(1/(a*x**3*sqrt(a/x**4 + b*x**n) + b*x**7*x**n*sqrt(a/x**4 + b*x**n
)), x)/c**7
```

Maxima [F]

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^7x^7}\,dx$$

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^n + a/x^4)^(3/2)*x^7), x)/c^7
```

Giac [F]

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^7x^7}\,dx$$

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a/x^4)^(3/2)*c^7*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx=\int \frac{1}{c^7\,x^7\,\left(b\,x^n+\frac{a}{x^4}\right)^{3/2}}\,dx$$

```
[In] int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)),x)
[Out] int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)), x)
```

3.407 $\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

`[Out] 2/3*arctanh(x*b^(1/2)/(a/x+b*x^2)^(1/2))/b^(1/2)`

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2004, 2033, 212}

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

`[In] Int[1/Sqrt[(a + b*x^3)/x],x]`

`[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x + b*x^2]])/(3*Sqrt[b])`

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{\frac{a}{x}+bx^2}}\,dx \\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x}+bx^2}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \frac{2\sqrt{a+bx^3}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a+bx^3}{x}}}$$

```
[In] Integrate[1/Sqrt[(a + b*x^3)/x],x]
[Out] (2*Sqrt[a + b*x^3]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(3*Sqrt[b]*Sqrt[
x]*Sqrt[(a + b*x^3)/x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(24) = 48.

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.75

method	result	size
default	$\frac{2\left(b\,x^{3}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{x\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{\frac{b\,x^{3}+a}{x}}\,\sqrt{x\left(b\,x^{3}+a\right)}\,\sqrt{b}}$	56

```
[In] int(1/((b*x^3+a)/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/((b*x^3+a)/x)^(1/2)*(b*x^3+a)/(x*(b*x^3+a))^(1/2)/b^(1/2)*arctanh(1/x^2
*(x*(b*x^3+a))^(1/2)/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-a^2-4\,(2\,bx^5+ax^2)\sqrt{b}\sqrt{\frac{bx^3+a}{x}}\right)}{6\,\sqrt{b}},\ -\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{-b}x^2\sqrt{\frac{bx^3+a}{x}}}{2\,bx^3+a}\right)}{3\,b}\right]$$

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="fricas")
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^5 + a*x^2)*sqrt(b)*sqrt((b
*x^3 + a)/x))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(-b)*x^2*sqrt((b*x^3 + a)
/x)/(2*b*x^3 + a))/b]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$$

```
[In] integrate(1/((b*x**3+a)/x)**(1/2),x)
[Out] Integral(1/sqrt((a + b*x**3)/x), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{bx^3+a}{x}}}\,dx$$

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^3 + a)/x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{b\,x^3+a}{x}}}\,dx$$

```
[In] int(1/((a + b*x^3)/x)^(1/2),x)
```

```
[Out] int(1/((a + b*x^3)/x)^(1/2), x)
```

3.408 $\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] 1/2*arctanh(x*b^(1/2)/(a/x^2+b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2004, 2033, 212}

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

```
[In] Int[1/Sqrt[(a + b*x^4)/x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^2 + b*x^2]]/(2*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{\frac{a}{x^2}+bx^2}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \frac{\sqrt{a+bx^4}\log\left(\sqrt{b}x^2+\sqrt{a+bx^4}\right)}{2\sqrt{b}x\sqrt{\frac{a+bx^4}{x^2}}}$$

```
[In] Integrate[1/Sqrt[(a + b*x^4)/x^2],x]
[Out] (Sqrt[a + b*x^4]*Log[Sqrt[b]*x^2 + Sqrt[a + b*x^4]])/(2*Sqrt[b]*x*Sqrt[(a +
 b*x^4)/x^2])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

method	result	size
default	$\frac{\sqrt{b\,x^4+a}\,\ln\left(x^2\sqrt{b}+\sqrt{b\,x^4+a}\right)}{2\sqrt{\frac{b\,x^4+a}{x^2}}\,x\sqrt{b}}$	49

```
[In] int(1/((b*x^4+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/((b*x^4+a)/x^2)^(1/2)/x*(b*x^4+a)^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))
/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \left[\frac{\log\left(-2\,bx^4 - 2\,\sqrt{b}x^3\sqrt{\frac{bx^4+a}{x^2}} - a\right)}{4\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x^3\sqrt{\frac{bx^4+a}{x^2}}}{bx^4+a}\right)}{2\,b}\right]$$

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
[Out] [1/4*log(-2*b*x^4 - 2*sqrt(b)*x^3*sqrt((b*x^4 + a)/x^2) - a)/sqrt(b), -1/2*
sqrt(-b)*arctan(sqrt(-b)*x^3*sqrt((b*x^4 + a)/x^2)/(b*x^4 + a))/b]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$$

```
[In] integrate(1/((b*x**4+a)/x**2)**(1/2),x)
[Out] Integral(1/sqrt((a + b*x**4)/x**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{bx^4+a}{x^2}}}\,dx$$

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
[Out] b*integrate(x^5/(b*x^4 + a)^(3/2), x) + 1/2*x^2/sqrt(b*x^4 + a)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \frac{\log\left(|a|\right)\operatorname{sgn}(x)}{4\sqrt{b}} - \frac{\log\left(\left|-\sqrt{b}x^2+\sqrt{bx^4+a}\right|\right)}{2\sqrt{b}\operatorname{sgn}\left(x\right)}$$

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
[Out] 1/4*log(abs(a))*sgn(x)/sqrt(b) - 1/2*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a)
))/(sqrt(b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{b\,x^4+a}{x^2}}}\,dx$$

```
[In] int(1/((a + b*x^4)/x^2)^(1/2),x)
[Out] int(1/((a + b*x^4)/x^2)^(1/2), x)
```

3.409 $\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

```
[Out] 2/5*arctanh(x*b^(1/2)/(a/x^3+b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2004, 2033, 212}

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

```
[In] Int[1/Sqrt[(a + b*x^5)/x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^3 + b*x^2]])/(5*Sqrt[b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{\frac{a}{x^3}+bx^2}}\,dx\\ &= \frac{2}{5}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \frac{2\sqrt{a+bx^5}\log\left(\sqrt{b}x^{5/2}+\sqrt{a+bx^5}\right)}{5\sqrt{b}x^{3/2}\sqrt{\frac{a+bx^5}{x^3}}}$$

```
[In] Integrate[1/Sqrt[(a + b*x^5)/x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^5]*Log[Sqrt[b]*x^(5/2) + Sqrt[a + b*x^5]])/(5*Sqrt[b]*x^(3/
2)*Sqrt[(a + b*x^5)/x^3])
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{b\,x^5+a}{x^3}}}dx$$

```
[In] int(1/((b*x^5+a)/x^3)^(1/2),x)
```

```
[Out] int(1/((b*x^5+a)/x^3)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \left[\frac{\log\left(-8\,b^2x^{10} - 8\,abx^5 - a^2 - 4\,(2\,bx^9 + ax^4)\sqrt{b}\sqrt{\frac{bx^5+a}{x^3}}\right)}{10\,\sqrt{b}},\; -\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{-b}x^4\sqrt{\frac{bx^5+a}{x^3}}}{2\,bx^5+a}\right)}{5\,b}\right]$$

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/10*log(-8*b^2*x^10 - 8*a*b*x^5 - a^2 - 4*(2*b*x^9 + a*x^4)*sqrt(b)*sqrt(
(b*x^5 + a)/x^3))/sqrt(b), -1/5*sqrt(-b)*arctan(2*sqrt(-b)*x^4*sqrt((b*x^5
+ a)/x^3)/(2*b*x^5 + a))/b]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((b*x**5+a)/x**3)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{\frac{bx^5+a}{x^3}}}\,dx$$

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^5 + a)/x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{\frac{b\,x^5+a}{x^3}}}\,dx$$

```
[In] int(1/((a + b*x^5)/x^3)^(1/2),x)
```

```
[Out] int(1/((a + b*x^5)/x^3)^(1/2), x)
```

3.410 $\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^{2-n}}}\right)}{\sqrt{b}n}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^(2-n))^(1/2))/n/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2004, 2033, 212}

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{b}n}$$

```
[In] Int[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{bx^2+ax^{2-n}}}\,dx\\ &= \frac{2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^{2-n}}}\right)}{n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^{2-n}}}\right)}{\sqrt{b}n}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 76 vs. 2(37) = 74.

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}}\,dx = \frac{2\sqrt{a}x^{1-\frac{n}{2}}\sqrt{1+\frac{bx^n}{a}}\,\text{arcsinh}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}n\sqrt{x^{2-n}(a+bx^n)}}$$

```
[In] Integrate[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

```
[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a
]])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a + b*x^n)])
```

Maple [F]

$$\int \frac{1}{\sqrt{x^{2-n}(a+b\,x^n)}}dx$$

```
[In] int(1/(x^(2-n)*(a+b*x^n))^(1/2),x)
```

```
[Out] int(1/(x^(2-n)*(a+b*x^n))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.76

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx = \left[\frac{\log\left(\frac{2\,bxx^n+ax+2\,\sqrt{b}x^n\sqrt{\frac{bx^2x^n+ax^2}{x^n}}}{x}\right)}{\sqrt{b}n},\ -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{bx^2x^n+ax^2}{x^n}}}{bx}\right)}{bn}\right]$$

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x*x^n + a*x + 2*sqrt(b)*x^n*sqrt((b*x^2*x^n + a*x^2)/x^n))/x)/(sq
rt(b)*n), -2*sqrt(-b)*arctan(sqrt(-b)*sqrt((b*x^2*x^n + a*x^2)/x^n)/(b*x))/
(b*n)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx = \int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx$$

```
[In] integrate(1/(x**(2-n)*(a+b*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**(2 - n)*(a + b*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx = \int \frac{1}{\sqrt{(bx^n+a)x^{-n+2}}}\,dx$$

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx=\int \frac{1}{\sqrt{(bx^n+a)x^{-n+2}}}\,dx$$

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx=\int \frac{1}{\sqrt{x^{2-n}\,\left(a+b\,x^n\right)}}\,dx$$

```
[In] int(1/(x^(2 - n)*(a + b*x^n))^(1/2),x)
[Out] int(1/(x^(2 - n)*(a + b*x^n))^(1/2), x)
```

3.411 $\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

```
[Out] 2/3*arctan(x*b^(1/2)/(a/x-b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2004, 2033, 209}

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

```
[In] Int[1/Sqrt[(a - b*x^3)/x],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x - b*x^2]])/(3*Sqrt[b])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{\frac{a}{x}-bx^2}}\,dx \\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x}-bx^2}}\right) \\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = -\frac{2i\sqrt{a-bx^3}\log\left(i\sqrt{b}x^{3/2}+\sqrt{a-bx^3}\right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a-bx^3}{x}}}$$

```
[In] Integrate[1/Sqrt[(a - b*x^3)/x],x]
[Out] (((-2*I)/3)*Sqrt[a - b*x^3]*Log[I*Sqrt[b]*x^(3/2) + Sqrt[a - b*x^3]])/(Sqrt
[b]*Sqrt[x]*Sqrt[(a - b*x^3)/x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(25) = 50.

Time = 2.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

method	result	size
default	$-\frac{2\left(-b\,x^{3}+a\right)\arctan\left(\frac{\sqrt{x\left(-b\,x^{3}+a\right)}}{x^{2}\sqrt{b}}\right)}{3\sqrt{\frac{-b\,x^{3}+a}{x}}\,\sqrt{x\left(-b\,x^{3}+a\right)}\,\sqrt{b}}$	60

```
[In] int(1/((-b*x^3+a)/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/((-b*x^3+a)/x)^(1/2)*(-b*x^3+a)/(x*(-b*x^3+a))^(1/2)/b^(1/2)*arctan((x
*(-b*x^3+a))^(1/2)/x^2/b^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-8\,b^{2}x^{6}+8\,abx^{3}-a^{2}+4\,(2\,bx^{5}-ax^{2})\sqrt{-b}\sqrt{-\frac{bx^{3}-a}{x}}\right)}{6\,b},\ -\frac{\arctan\left(\frac{2\,\sqrt{b}x^{2}\sqrt{-\frac{bx^{3}-a}{x}}}{2\,bx^{3}-a}\right)}{3\,\sqrt{b}}\right]$$

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*sqrt(-b)*log(-8*b^2*x^6 + 8*a*b*x^3 - a^2 + 4*(2*b*x^5 - a*x^2)*sqrt(
-b)*sqrt(-(b*x^3 - a)/x))/b, -1/3*arctan(2*sqrt(b)*x^2*sqrt(-(b*x^3 - a)/x)
/(2*b*x^3 - a))/sqrt(b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$$

```
[In] integrate(1/((-b*x**3+a)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a - b*x**3)/x), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{-\frac{bx^3-a}{x}}}\,dx$$

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^3 - a)/x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{a-b\,x^3}{x}}}\,dx$$

```
[In] int(1/((a - b*x^3)/x)^(1/2),x)
```

```
[Out] int(1/((a - b*x^3)/x)^(1/2), x)
```

3.412 $\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] 1/2*arctan(x*b^(1/2)/(a/x^2-b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2004, 2033, 209}

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

```
[In] Int[1/Sqrt[(a - b*x^4)/x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a/x^2 - b*x^2]]/(2*Sqrt[b])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{\frac{a}{x^2}-bx^2}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = -\frac{i\sqrt{a-bx^4}\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)}{2\sqrt{b}x\sqrt{\frac{a-bx^4}{x^2}}}$$

```
[In] Integrate[1/Sqrt[(a - b*x^4)/x^2],x]
```

```
[Out] ((-1/2*I)*Sqrt[a - b*x^4]*Log[I*Sqrt[b]*x^2 + Sqrt[a - b*x^4]])/(Sqrt[b]*x*
Sqrt[(a - b*x^4)/x^2])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\sqrt{-b\,x^{4}+a}\,\arctan\left(\frac{x^{2}\sqrt{b}}{\sqrt{-b\,x^{4}+a}}\right)}{2\sqrt{\frac{-b\,x^{4}+a}{x^{2}}}\,x\sqrt{b}}$	51

```
[In] int(1/((-b*x^4+a)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((-b*x^4+a)/x^2)^(1/2)/x*(-b*x^4+a)^(1/2)*arctan(x^2*b^(1/2)/(-b*x^4+a)
^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.67

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \left[-\frac{\sqrt{-b}\log\left(2\,bx^4 - 2\,\sqrt{-b}x^3\sqrt{-\frac{bx^4-a}{x^2}} - a\right)}{4\,b}, -\frac{\arctan\left(\frac{\sqrt{b}x^3\sqrt{-\frac{bx^4-a}{x^2}}}{bx^4-a}\right)}{2\,\sqrt{b}}\right]$$

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b)*x^3*sqrt(-(b*x^4 - a)/x^2) - a)/b,
-1/2*arctan(sqrt(b)*x^3*sqrt(-(b*x^4 - a)/x^2)/(b*x^4 - a))/sqrt(b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$$

```
[In] integrate(1/((-b*x**4+a)/x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a - b*x**4)/x**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{-\frac{bx^4-a}{x^2}}}\,dx$$

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x^5/((b*x^4 - a)*sqrt(-b*x^4 + a)), x) + 1/2*x^2/sqrt(-b*x^4 +
a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \frac{\log\left(|a|\right)\operatorname{sgn}(x)}{4\sqrt{-b}} - \frac{\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{2\sqrt{-b}\operatorname{sgn}(x)}$$

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a))*sgn(x)/sqrt(-b) - 1/2*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4 +
 a)))/(sqrt(-b)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{a-b\,x^4}{x^2}}}\,dx$$

```
[In] int(1/((a - b*x^4)/x^2)^(1/2),x)
```

```
[Out] int(1/((a - b*x^4)/x^2)^(1/2), x)
```

3.413 $\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

```
[Out] 2/5*arctan(x*b^(1/2)/(a/x^3-b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2004, 2033, 209}

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

```
[In] Int[1/Sqrt[(a - b*x^5)/x^3],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x^3 - b*x^2]])/(5*Sqrt[b])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{\frac{a}{x^3}-bx^2}}\,dx \\
&= \frac{2}{5}\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^3}-bx^2}}\right) \\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.18

$$
\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = -\frac{2i\sqrt{a-bx^5}\log\left(i\sqrt{b}x^{5/2}+\sqrt{a-bx^5}\right)}{5\sqrt{b}x^{3/2}\sqrt{\frac{a-bx^5}{x^3}}}
$$

```
[In] Integrate[1/Sqrt[(a - b*x^5)/x^3],x]
```

```
[Out] (((-2*I)/5)*Sqrt[a - b*x^5]*Log[I*Sqrt[b]*x^(5/2) + Sqrt[a - b*x^5]])/(Sqrt
[b]*x^(3/2)*Sqrt[(a - b*x^5)/x^3])
```

Maple [F]

$$\int \frac{1}{\sqrt{\frac{-b\,x^{5}+a}{x^{3}}}}dx$$

```
[In] int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

```
[Out] int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.92 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.36

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^{10}+8\,abx^5-a^2+4\,(2\,bx^9-ax^4)\sqrt{-b}\sqrt{-\frac{bx^5-a}{x^3}}\right)}{10\,b},\; -\frac{\arctan\left(\frac{2\,\sqrt{b}x^4\sqrt{-\frac{bx^5-a}{x^3}}}{2\,bx^5-a}\right)}{5\,\sqrt{b}}\right]$$

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/10*sqrt(-b)*log(-8*b^2*x^10 + 8*a*b*x^5 - a^2 + 4*(2*b*x^9 - a*x^4)*sqr
t(-b)*sqrt(-(b*x^5 - a)/x^3))/b, -1/5*arctan(2*sqrt(b)*x^4*sqrt(-(b*x^5 - a
)/x^3)/(2*b*x^5 - a))/sqrt(b)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \text{Timed out}$$

```
[In] integrate(1/((-b*x**5+a)/x**3)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{-\frac{bx^5-a}{x^3}}}\,dx$$

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^5 - a)/x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx = \int \frac{1}{\sqrt{\frac{a-b\,x^5}{x^3}}}\,dx$$

```
[In] int(1/((a - b*x^5)/x^3)^(1/2),x)
```

```
[Out] int(1/((a - b*x^5)/x^3)^(1/2), x)
```

3.414 $\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^n\right)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{\sqrt{b}n}$$

```
[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^(2-n))^(1/2))/n/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2004, 2033, 209}

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^n\right)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{b}n}$$

```
[In] Int[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{-bx^2+ax^{2-n}}}\,dx\\ &= \frac{2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{\sqrt{b}n}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(38) = 76.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \frac{2\sqrt{a}x^{1-\frac{n}{2}}\sqrt{1-\frac{bx^n}{a}}\arcsin\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}n\sqrt{x^{2-n}(a-bx^n)}}$$

```
[In] Integrate[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 - (b*x^n)/a]*ArcSin[(Sqrt[b]*x^(n/2))/Sqrt[a]
])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a - b*x^n)])
```

Maple [F]

$$\int \frac{1}{\sqrt{x^{2-n}(a-b\,x^n)}}dx$$

```
[In] int(1/(x^(2-n)*(a-b*x^n))^(1/2),x)
[Out] int(1/(x^(2-n)*(a-b*x^n))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^{n}\right)}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-\frac{2\,bxx^{n}-ax-2\,\sqrt{-b}x^{n}\sqrt{-\frac{bx^{2}x^{n}-ax^{2}}{x^{n}}}}{x}\right)}{bn},\; -\frac{2\,\arctan\left(\frac{\sqrt{-\frac{bx^{2}x^{n}-ax^{2}}{x^{n}}}}{\sqrt{b}x}\right)}{\sqrt{b}n}\right]$$

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-(2*b*x*x^n - a*x - 2*sqrt(-b)*x^n*sqrt(-(b*x^2*x^n - a*x^2)
/x^n))/x)/(b*n), -2*arctan(sqrt(-(b*x^2*x^n - a*x^2)/x^n)/(sqrt(b)*x))/(sqr
t(b)*n)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^{n}\right)}}\,dx = \int \frac{1}{\sqrt{x^{2-n}\left(a-bx^{n}\right)}}\,dx$$

```
[In] integrate(1/(x**(2-n)*(a-b*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**(2 - n)*(a - b*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^{n}\right)}}\,dx = \int \frac{1}{\sqrt{-\left(bx^{n}-a\right)x^{-n+2}}}\,dx$$

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \int \frac{1}{\sqrt{-(bx^n-a)x^{-n+2}}}\,dx$$

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx = \int \frac{1}{\sqrt{x^{2-n}\,(a-b\,x^n)}}\,dx$$

```
[In] int(1/(x^(2 - n)*(a - b*x^n))^(1/2),x)
[Out] int(1/(x^(2 - n)*(a - b*x^n))^(1/2), x)
```

3.415 $\int \frac{1}{\sqrt{x^n(a+bx^{2-n})}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2004, 2033, 212}

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[In] Int[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{bx^2+ax^n}}\,dx\\ &= \frac{2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx = -\frac{2\sqrt{a}x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\,\text{arcsinh}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{bx^2+ax^n}}$$

```
[In] Integrate[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

Maple [F]

$$\int \frac{1}{\sqrt{x^n\left(a+b\,x^{2-n}\right)}}dx$$

```
[In] int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)
[Out] int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx=\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx$$

```
[In] integrate(1/(x**n*(a+b*x**(2-n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**n*(a + b*x**(2 - n))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx=\int \frac{1}{\sqrt{(bx^{-n+2}+a)x^n}}\,dx$$

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx=\int \frac{1}{\sqrt{(bx^{-n+2}+a)x^n}}\,dx$$

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{x^n\,(a+bx^{2-n})}}\,dx=\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

```
[In] int(1/(x^n*(a + b*x^(2 - n)))^(1/2),x)
```

```
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

3.416 $\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2004, 2033, 212}

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[In] Int[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{bx^2+ax^n}}\,dx\\ &= \frac{2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx = -\frac{2\sqrt{a}x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{bx^2+ax^n}}$$

```
[In] Integrate[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

Maple [F]

$$\int \frac{1}{\sqrt{x^2\left(b+a\,x^{-2+n}\right)}}dx$$

```
[In] int(1/(x^2*(b+a*x^(-2+n)))^(1/2),x)
[Out] int(1/(x^2*(b+a*x^(-2+n)))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.95

$$\int \frac{1}{\sqrt{x^2 \left(b + a x^{-2+n}\right)}}\, dx = \left[\frac{\sqrt{b}\log\left(\frac{a x x^{n-2} + 2\, b x - 2\, \sqrt{a x^2 x^{n-2} + b x^2}\sqrt{b}}{x x^{n-2}}\right)}{b n - 2\, b}, \frac{2\, \sqrt{-b}\arctan\left(\frac{\sqrt{a x^2 x^{n-2} + b x^2}\sqrt{-b}}{b x}\right)}{b n - 2\, b}\right]$$

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b)*log((a*x*x^(n - 2) + 2*b*x - 2*sqrt(a*x^2*x^(n - 2) + b*x^2)*sqrt(
b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(-b)*arctan(sqrt(a*x^2*x^(n - 2) + b*
x^2)*sqrt(-b)/(b*x))/(b*n - 2*b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^2 \left(b + a x^{-2+n}\right)}}\, dx = \int \frac{1}{\sqrt{x^2 \left(a x^{n-2} + b\right)}}\, dx$$

```
[In] integrate(1/(x**2*(b+a*x**(-2+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a*x**(n - 2) + b)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^2 \left(b + a x^{-2+n}\right)}}\, dx = \int \frac{1}{\sqrt{(a x^{n-2} + b) x^2}}\, dx$$

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-2}+b)x^2}}\,dx$$

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx=\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

```
[In] int(1/(x^2*(b + a*x^(n - 2)))^(1/2),x)
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

3.417 $\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2004, 2033, 212}

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[In] Int[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{bx^2+ax^n}}\,dx\\ &= \frac{2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x\,(bx+ax^{-1+n})}}\,dx = -\frac{2\sqrt{a}x^{n/2}\sqrt{1+\frac{bx^{2-n}}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{bx^2+ax^n}}$$

```
[In] Integrate[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

Maple [F]

$$\int \frac{1}{\sqrt{x\,(bx+a\,x^{-1+n})}}dx$$

```
[In] int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)
```

```
[Out] int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\int \frac{1}{\sqrt{x\left(ax^{n-1}+bx\right)}}\,dx$$

```
[In] integrate(1/(x*(b*x+a*x**(-1+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(a*x**(n - 1) + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-1}+bx)x}}\,dx$$

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-1}+bx)x}}\,dx$$

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx=\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

```
[In] int(1/(x*(b*x + a*x^(n - 1)))^(1/2),x)
```

```
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

3.418 $\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2004, 2033, 209}

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[In] Int[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\ &= \frac{2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx = -\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\arcsin\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{-bx^2+ax^n}}$$

```
[In] Integrate[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

Maple [F]

$$\int \frac{1}{\sqrt{x^n\left(a-b\,x^{2-n}\right)}}dx$$

```
[In] int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)
[Out] int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx = \int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx$$

```
[In] integrate(1/(x**n*(a-b*x**(2-n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**n*(a - b*x**(2 - n))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx = \int \frac{1}{\sqrt{-(bx^{-n+2}-a)x^n}}\,dx$$

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx = \int \frac{1}{\sqrt{-(bx^{-n+2}-a)x^n}}\,dx$$

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{x^n\,(a-bx^{2-n})}}\,dx=-\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

```
[In] int(1/(x^n*(a - b*x^(2 - n)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

3.419 $\int \frac{1}{\sqrt{x^2(-b+ax^{-2+n})}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 38

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2004, 2033, 209}

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx=\frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[In] Int[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\ &= \frac{2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx = -\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\arcsin\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{-bx^2+ax^n}}$$

```
[In] Integrate[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

Maple [F]

$$\int \frac{1}{\sqrt{x^2\left(-b+a\,x^{-2+n}\right)}}dx$$

```
[In] int(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x)
```

```
[Out] int(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.87

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx = \left[-\frac{\sqrt{-b}\log\left(\frac{axx^{n-2}-2\,bx-2\,\sqrt{ax^2x^{n-2}-bx^2}\sqrt{-b}}{xx^{n-2}}\right)}{bn-2\,b}, \frac{2\,\sqrt{b}\arctan\left(\frac{\sqrt{ax^2x^{n-2}-bx^2}}{\sqrt{b}x}\right)}{bn-2\,b}\right]$$

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log((a*x*x^(n - 2) - 2*b*x - 2*sqrt(a*x^2*x^(n - 2) - b*x^2)*sqr
t(-b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(b)*arctan(sqrt(a*x^2*x^(n - 2) -
b*x^2)/(sqrt(b)*x))/(b*n - 2*b)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx = \int \frac{1}{\sqrt{x^2\left(ax^{n-2}-b\right)}}\,dx$$

```
[In] integrate(1/(x**2*(-b+a*x**(-2+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a*x**(n - 2) - b)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx = \int \frac{1}{\sqrt{(ax^{n-2}-b)x^2}}\,dx$$

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx=\int \frac{1}{\sqrt{(ax^{n-2}-b)x^2}}\,dx$$

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx=-\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

```
[In] int(1/(-x^2*(b - a*x^(n - 2)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

3.420 $\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2004, 2033, 209}

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[In] Int[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2033

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\ &= \frac{2\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = -\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\arcsin\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(-2+n)\sqrt{-bx^2+ax^n}}$$

```
[In] Integrate[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

Maple [F]

$$\int \frac{1}{\sqrt{x\left(-bx+a\,x^{-1+n}\right)}}dx$$

```
[In] int(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x)
[Out] int(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = \int \frac{1}{\sqrt{x\left(ax^{n-1}-bx\right)}}\,dx$$

```
[In] integrate(1/(x*(-b*x+a*x**(-1+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(a*x**(n - 1) - b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = \int \frac{1}{\sqrt{(ax^{n-1}-bx)x}}\,dx$$

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx = \int \frac{1}{\sqrt{(ax^{n-1}-bx)x}}\,dx$$

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx=-\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

```
[In] int(1/(-x*(b*x - a*x^(n - 1)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

3.421 $\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx$

Optimal result

Integrand size = 21, antiderivative size = 107

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx = \frac{2bx^{1+n}(cx)^m\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m+\frac{3n}{2}}{j-n},1+\frac{1+m+\frac{3n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{(2+2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

```
[Out] 2*b*x^(1+n)*(c*x)^m*hypergeom([-3/2, (1+m+3/2*n)/(j-n)],[1+(1+m+3/2*n)/(j-n
)],-a*x^(j-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+2*m+3*n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2057, 372, 371}

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx = \frac{2bx^{n+1}(cx)^m\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+\frac{3n}{2}+1}{j-n},\frac{m+\frac{3n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{(2m+3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

```
[In] Int[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(1 + n)*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + m +
 (3*n)/2)/(j - n), 1 + (1 + m + (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/((2
+ 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{3n}{2}}\left(b+ax^{j-n}\right)^{3/2}\,dx}{\sqrt{b+ax^{j-n}}}\\
&= \frac{\left(bx^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{3n}{2}}\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}\\
&= \frac{2bx^{1+n}(cx)^m\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m+\frac{3n}{2}}{j-n};1+\frac{1+m+\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2+2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 218 vs. 2(107) = 214.

Time = 0.57 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.04

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2}\,dx = \frac{2(cx)^m\left((2+4j+2m-n)x^{-m}(ax^j+bx^n)\left(a(2-j+2m+4n)x^{1+j+m}+b(2+2j+2m+\right.\right.}{(2+4j+2m-n)(2+2j+}$$

```
[In] Integrate[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*(c*x)^m*(((2 + 4*j + 2*m - n)*(a*x^j + b*x^n)*(a*(2 - j + 2*m + 4*n)*x^(
1 + j + m) + b*(2 + 2*j + 2*m + n)*x^(1 + m + n)))/x^m + 3*a^2*(j - n)^2*x^
(1 + 2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 4*j + 2*m -
 n)/(2*j - 2*n), (2 + 6*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(
(2 + 4*j + 2*m - n)*(2 + 2*j + 2*m + n)*(2 + 2*m + 3*n)*Sqrt[a*x^j + b*x^n]
)
```

Maple [F]

$$\int \left(c x \right)^{m} \left(a \,x^{j}+b \,x^{n}\right)^{\frac{3}{2}}d x$$

```
[In] int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} \, dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} \, dx=\int \left(cx\right)^{m} \left(ax^{j} + bx^{n}\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate((c*x)**m*(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((c*x)**m*(a*x**j + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx = \int \left(ax^j+bx^n\right)^{\frac{3}{2}}(cx)^m dx$$

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx = \int \left(ax^j+bx^n\right)^{\frac{3}{2}}(cx)^m dx$$

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx = \int (c\,x)^m \left(a\,x^j+b\,x^n\right)^{3/2} dx$$

```
[In] int((c*x)^m*(a*x^j + b*x^n)^(3/2),x)
[Out] int((c*x)^m*(a*x^j + b*x^n)^(3/2), x)
```

3.422 $\int (cx)^m\sqrt{ax^j+bx^n}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx = \frac{2x(cx)^m\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m+\frac{n}{2}}{j-n},1+\frac{2+2m+n}{2j-2n},-\frac{ax^{j-n}}{b}\right)}{(2+2m+n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

```
[Out] 2*x*(c*x)^m*hypergeom([-1/2, (1+m+1/2*n)/(j-n)],[1+(2+2*m+n)/(2*j-2*n)],-a*
x^(j-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+2*m+n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2057, 372, 371}

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx = \frac{2x(cx)^m\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+\frac{n}{2}+1}{j-n},\frac{2m+n+2}{2j-2n}+1,-\frac{ax^{j-n}}{b}\right)}{(2m+n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

```
[In] Int[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (1 + m + n/2)/(j -
 n), 1 + (2 + 2*m + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m + n)*Sqr
t[1 + (a*x^(j - n))/b])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{n}{2}}\sqrt{b+ax^{j-n}}\,dx}{\sqrt{b+ax^{j-n}}} \\
&= \frac{\left(x^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{n}{2}}\sqrt{1+\frac{ax^{j-n}}{b}}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}} \\
&= \frac{2x(cx)^m\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1+m+\frac{n}{2}}{j-n};1+\frac{2+2m+n}{2j-2n};-\frac{ax^{j-n}}{b}\right)}{(2+2m+n)\sqrt{1+\frac{ax^{j-n}}{b}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.56

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx$$

$$= \frac{2x(cx)^m\left((2+2j+2m-n)\left(ax^j+bx^n\right)-a(j-n)x^j\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+2j+2m-n}{2j-2n},\right.\right.}{(2+2j+2m-n)(2+2m+n)\sqrt{ax^j+bx^n}}$$

```
[In] Integrate[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*((2 + 2*j + 2*m - n)*(a*x^j + b*x^n) - a*(j - n)*x^j*Sqrt[1 +
(a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*j + 2*m - n)/(2*j - 2*n), (2
 + 4*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/((2 + 2*j + 2*m - n)
*(2 + 2*m + n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]

$$\int (cx)^{m} \sqrt{a\,x^{j}+b\,x^{n}}dx$$

```
[In] int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx=\int (cx)^{m}\sqrt{ax^{j}+bx^{n}}\,dx$$

```
[In] integrate((c*x)**m*(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral((c*x)**m*sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}(cx)^m\,dx$$

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}(cx)^m\,dx$$

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx=\int (c\,x)^m\,\sqrt{a\,x^j+b\,x^n}\,dx$$

```
[In] int((c*x)^m*(a*x^j + b*x^n)^(1/2),x)
[Out] int((c*x)^m*(a*x^j + b*x^n)^(1/2), x)
```

3.423 $\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m-\frac{n}{2}}{j-n},1+\frac{1+m-\frac{n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x*(c*x)^m*hypergeom([1/2, (1+m-1/2*n)/(j-n)],[1+(1+m-1/2*n)/(j-n)],-a*x^(
j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/(2+2*m-n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2057, 372, 371}

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \frac{2x(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m-\frac{n}{2}+1}{j-n},\frac{m-\frac{n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

```
[In] Int[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (1 + m - n/2)
/(j - n), 1 + (1 + m - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 + 2*m - n)*Sq
rt[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int \frac{x^{m-\frac{n}{2}}}{\sqrt{b+ax^{j-n}}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{m-\frac{n}{2}}}{\sqrt{1+\frac{ax^{j-n}}{b}}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$= \frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m-\frac{n}{2}}{j-n};1+\frac{1+m-\frac{n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+2m-n}{2j-2n},1+\frac{2+2m-n}{2j-2n},-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

```
[In] Integrate[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*m - n)
/(2*j - 2*n), 1 + (2 + 2*m - n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m
 - n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]

$$\int \frac{(cx)^m}{\sqrt{a\,x^j+b\,x^n}}dx$$

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$$

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral((c*x)**m/sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$$

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$$

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \int \frac{(c\,x)^m}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

```
[In] int((c*x)^m/(a*x^j + b*x^n)^(1/2),x)
[Out] int((c*x)^m/(a*x^j + b*x^n)^(1/2), x)
```

3.424 $\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx = \frac{2x^{1-n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m-\frac{3n}{2}}{j-n},1+\frac{1+m-\frac{3n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b(2+2m-3n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-n)*(c*x)^m*hypergeom([3/2, (1+m-3/2*n)/(j-n)],[1+(1+m-3/2*n)/(j-n)],
-a*x^(j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/b/(2+2*m-3*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2057, 372, 371}

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx = \frac{2x^{1-n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m-\frac{3n}{2}+1}{j-n},\frac{m-\frac{3n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b(2m-3n+2)\sqrt{ax^j+bx^n}}$$

```
[In] Int[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*x^(1 - n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 +
m - (3*n)/2)/(j - n), 1 + (1 + m - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(
b*(2 + 2*m - 3*n)*Sqrt[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int \frac{x^{m-\frac{3n}{2}}}{(b+ax^{j-n})^{3/2}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{m-\frac{3n}{2}}}{\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}}\,dx}{b\sqrt{ax^j+bx^n}}$$

$$= \frac{2x^{1-n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m-\frac{3n}{2}}{j-n};1+\frac{1+m-\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b(2+2m-3n)\sqrt{ax^j+bx^n}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx = \frac{2x^{1-j}(cx)^m\left(-1+\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-2j+2m-n}{2j-2n},\frac{2+2m-3n}{2j-2n},-\frac{ax^{j-n}}{b}\right.\right.}{a(j-n)\sqrt{ax^j+bx^n}}$$

```
[In] Integrate[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*x^(1 - j)*(c*x)^m*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2,
 (2 - 2*j + 2*m - n)/(2*j - 2*n), (2 + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j -
n))/b)]))/(a*(j - n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]

$$\int \frac{(cx)^m}{\left(a\,x^j+b\,x^n\right)^{\frac{3}{2}}}dx$$

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx = \int \frac{(cx)^m}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((c*x)**m/(a*x**j + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx = \int \frac{(cx)^m}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx = \int \frac{(cx)^m}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{3/2}}\,dx = \int \frac{(c\,x)^m}{(a\,x^j+b\,x^n)^{3/2}}\,dx$$

```
[In] int((c*x)^m/(a*x^j + b*x^n)^(3/2),x)
[Out] int((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

3.425 $\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx=\frac{2x^{1-2n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m-\frac{5n}{2}}{j-n},1+\frac{1+m-\frac{5n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b^2(2+2m-5n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-2*n)*(c*x)^m*hypergeom([5/2, (1+m-5/2*n)/(j-n)],[1+(1+m-5/2*n)/(j-n)
],-a*x^(j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/b^2/(2+2*m-5*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2057, 372, 371}

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx=\frac{2x^{1-2n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m-\frac{5n}{2}+1}{j-n},\frac{m-\frac{5n}{2}+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b^2(2m-5n+2)\sqrt{ax^j+bx^n}}$$

```
[In] Int[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

```
[Out] (2*x^(1 - 2*n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1
+ m - (5*n)/2)/(j - n), 1 + (1 + m - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])
/(b^2*(2 + 2*m - 5*n)*Sqrt[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int\frac{x^{m-\frac{5n}{2}}}{(b+ax^{j-n})^{5/2}}\,dx}{\sqrt{ax^j+bx^n}}\\ &= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int\frac{x^{m-\frac{5n}{2}}}{\left(1+\frac{ax^{j-n}}{b}\right)^{5/2}}\,dx}{b^2\sqrt{ax^j+bx^n}}\\ &= \frac{2x^{1-2n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{5}{2},\frac{1+m-\frac{5n}{2}}{j-n};1+\frac{1+m-\frac{5n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b^2(2+2m-5n)\sqrt{ax^j+bx^n}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.50

$$\int\frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx = \frac{2x^{1-2j}(cx)^m\left(-2+2j-2m+3n-\frac{a(j-n)x^j}{ax^j+bx^n}-(-2+2j-2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}\,\mathrm{Hy}\right.}{3a^2(j-n)^2\sqrt{ax^j+bx^n}}$$

```
[In] Integrate[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

```
[Out] (2*x^(1 - 2*j)*(c*x)^m*(-2 + 2*j - 2*m + 3*n - (a*(j - n)*x^j)/(a*x^j + b*x
^n) - (-2 + 2*j - 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/
2, (2 - 4*j + 2*m - n)/(2*j - 2*n), (2 - 2*j + 2*m - 3*n)/(2*j - 2*n), -((a
*x^(j - n))/b)]))/(3*a^2*(j - n)^2*Sqrt[a*x^j + b*x^n])
```

Maple [F]

$$\int \frac{(cx)^{m}}{\left(a\,x^{j}+b\,x^{n}\right)^{\frac{5}{2}}}d x$$

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

```
[Out] int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx=\int \frac{(cx)^{m}}{\left(ax^{j}+bx^{n}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(5/2),x)
```

```
[Out] Integral((c*x)**m/(a*x**j + b*x**n)**(5/2), x)
```

Maxima [F]

$$\int \frac{(cx)^m}{(ax^j+bx^n)^{5/2}}\,dx=\int \frac{(cx)^{m}}{\left(ax^{j}+bx^{n}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx=\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx=\int \frac{{\left(c\,x\right)}^m}{{\left(a\,x^j+b\,x^n\right)}^{5/2}}\,dx$$

```
[In] int((c*x)^m/(a*x^j + b*x^n)^(5/2),x)
[Out] int((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

3.426 $\int \left(ax^j+bx^n\right)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\frac{2bx^{1+n}\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+\frac{3n}{2}}{j-n},\frac{2+2j+n}{2(j-n)},-\frac{ax^{j-n}}{b}\right)}{(2+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

```
[Out] 2*b*x^(1+n)*hypergeom([-3/2, (1+3/2*n)/(j-n)],[1/2*(2+2*j+n)/(j-n)],-a*x^(j
-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+3*n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2036, 372, 371}

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\frac{2bx^{n+1}\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{\frac{3n}{2}+1}{j-n},\frac{2j+n+2}{2(j-n)},-\frac{ax^{j-n}}{b}\right)}{(3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

```
[In] Int[(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(1 + n)*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + (3*n)/2)/(j
 - n), (2 + 2*j + n)/(2*(j - n)), -((a*x^(j - n))/b)])/((2 + 3*n)*Sqrt[1 +
(a*x^(j - n))/b])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{3n/2}\left(b+ax^{j-n}\right)^{3/2}\,dx}{\sqrt{b+ax^{j-n}}}\\
&= \frac{\left(bx^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{3n/2}\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}\\
&= \frac{2bx^{1+n}\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{1+\frac{3n}{2}}{j-n};\frac{2+2j+n}{2(j-n)};-\frac{ax^{j-n}}{b}\right)}{(2+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.82

$$\int \left(ax^j +bx^n\right)^{3/2}\,dx = \frac{2x\left((2+4j-n)\left(ax^j+bx^n\right)\left(a(2-j+4n)x^j+b(2+2j+n)x^n\right)+3a^2(j-n)^2x^{2j}\sqrt{1+\frac{ax}{}}\right.}{(2+4j-n)(2+2j+n)(2+3n)\sqrt{ax^j+}}$$

```
[In] Integrate[(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*x*((2 + 4*j - n)*(a*x^j + b*x^n)*(a*(2 - j + 4*n)*x^j + b*(2 + 2*j + n)*
x^n) + 3*a^2*(j - n)^2*x^(2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[
1/2, (2 + 4*j - n)/(2*j - 2*n), (2 + 6*j - 3*n)/(2*j - 2*n), -((a*x^(j - n)
)/b)]))/((2 + 4*j - n)*(2 + 2*j + n)*(2 + 3*n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]

$$\int \left(a\,x^{j}+b\,x^{n}\right)^{\frac{3}{2}}d x$$

```
[In] int((a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(ax^j + bx^n\right)^{3/2} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(ax^j + bx^n\right)^{3/2} dx = \int \left(ax^{j} + bx^{n}\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(3/2), x)
```

Maxima [F]

$$\int \left(ax^j + bx^n\right)^{3/2} dx = \int \left(ax^{j} + bx^{n}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2), x)
```

Giac [F]

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\int \left(ax^j+bx^n\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\frac{x\,\left(a\,x^j+b\,x^n\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{\frac{3\,n}{2}+1}{j-n};\ \frac{\frac{3\,n}{2}+1}{j-n}+1;\ -\frac{a\,x^{j-n}}{b}\right)}{\left(\frac{3\,n}{2}+1\right)\,\left(\frac{a\,x^{j-n}}{b}+1\right)^{3/2}}$$

```
[In] int((a*x^j + b*x^n)^(3/2),x)
```

```
[Out] (x*(a*x^j + b*x^n)^(3/2)*hypergeom([-3/2, ((3*n)/2 + 1)/(j - n)], ((3*n)/2
+ 1)/(j - n) + 1, -(a*x^(j - n))/b))/(((3*n)/2 + 1)*((a*x^(j - n))/b + 1)^(
3/2))
```

3.427 $\int \sqrt{ax^j+bx^n}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \sqrt{ax^j+bx^n}\,dx=\frac{2x\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2+n}{2(j-n)},1+\frac{2+n}{2j-2n},-\frac{ax^{j-n}}{b}\right)}{(2+n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

```
[Out] 2*x*hypergeom([-1/2, 1/2*(2+n)/(j-n)],[1+(2+n)/(2*j-2*n)],-a*x^(j-n)/b)*(a*
x^j+b*x^n)^(1/2)/(2+n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2036, 372, 371}

$$\int \sqrt{ax^j+bx^n}\,dx=\frac{2x\sqrt{ax^j+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{n+2}{2(j-n)},\frac{n+2}{2j-2n}+1,-\frac{ax^{j-n}}{b}\right)}{(n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

```
[In] Int[Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (2 + n)/(2*(j - n)), 1 + (
2 + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + n)*Sqrt[1 + (a*x^(j - n))/b]
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{n/2}\sqrt{b+ax^{j-n}}\,dx}{\sqrt{b+ax^{j-n}}}\\
&= \frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}\\
&= \frac{2x\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{2+n}{2(j-n)};1+\frac{2+n}{2j-2n};-\frac{ax^{j-n}}{b}\right)}{(2+n)\sqrt{1+\frac{ax^{j-n}}{b}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.54

$$
\int\sqrt{ax^j+bx^n}\,dx = \frac{2x\Bigg(-((2+2j-n)\,(ax^j+bx^n))+a(j-n)x^j\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+2j-n}{2j-2n},\frac{2+4j-3n}{2j-2n},-\frac{ax^{j-}}{b}\right.}{(2+n)(-2-2j+n)\sqrt{ax^j+bx^n}}
$$

```
[In] Integrate[Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(-((2 + 2*j - n)*(a*x^j + b*x^n)) + a*(j - n)*x^j*Sqrt[1 + (a*x^(j - n
))/b]*Hypergeometric2F1[1/2, (2 + 2*j - n)/(2*j - 2*n), (2 + 4*j - 3*n)/(2*
j - 2*n), -((a*x^(j - n))/b)]))/((2 + n)*(-2 - 2*j + n)*Sqrt[a*x^j + b*x^n]
)
```

Maple [F]

$$\int \sqrt{a\,x^{j}+b\,x^{n}}dx$$

```
[In] int((a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{ax^j+bx^n}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}\,dx$$

```
[In] integrate((a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int \sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}\,dx$$

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n), x)
```

Giac [F]

$$\int \sqrt{ax^j+bx^n}\,dx=\int \sqrt{ax^j+bx^n}\,dx$$

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \sqrt{ax^j+bx^n}\,dx=\frac{x\sqrt{a\,x^j+b\,x^n}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\,\frac{\frac{n}{2}+1}{j-n};\;\frac{\frac{n}{2}+1}{j-n}+1;\;-\frac{a\,x^{j-n}}{b}\right)}{\left(\frac{n}{2}+1\right)\,\sqrt{\frac{a\,x^{j-n}}{b}+1}}$$

```
[In] int((a*x^j + b*x^n)^(1/2),x)
```

```
[Out] (x*(a*x^j + b*x^n)^(1/2)*hypergeom([-1/2, (n/2 + 1)/(j - n)], (n/2 + 1)/(j
- n) + 1, -(a*x^(j - n))/b))/((n/2 + 1)*((a*x^(j - n))/b + 1)^(1/2))
```

3.428 $\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx=\frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2(j-n)},1+\frac{1-\frac{n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x*hypergeom([1/2, 1/2*(2-n)/(j-n)],[1+1/2*(2-n)/(j-n)],-a*x^(j-n)/b)*(1+a
*x^(j-n)/b)^(1/2)/(2-n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2036, 372, 371}

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx=\frac{2x\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2(j-n)},\frac{1-\frac{n}{2}}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

```
[In] Int[1/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 - n)/(2*(j - n)),
1 + (1 - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 - n)*Sqrt[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-n/2}}{\sqrt{b+ax^{j-n}}}\,dx}{\sqrt{ax^j+bx^n}}\\ &= \frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-n/2}}{\sqrt{1+\frac{ax^{j-n}}{b}}}\,dx}{\sqrt{ax^j+bx^n}}\\ &= \frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2(j-n)};1+\frac{1-\frac{n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx = -\frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{-2+n}{2(-j+n)},1+\frac{-2+n}{2(-j+n)},-\frac{ax^{j-n}}{b}\right)}{(-2+n)\sqrt{ax^j+bx^n}}$$

```
[In] Integrate[1/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (-2 + n)/(2*(-j + n)
), 1 + (-2 + n)/(2*(-j + n)), -((a*x^(j - n))/b)])/((-2 + n)*Sqrt[a*x^j + b
*x^n])
```

Maple [F]

$$\int \frac{1}{\sqrt{a\,x^{j}+b\,x^{n}}}dx$$

```
[In] int(1/(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int(1/(a*x^j+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{ax^{j}+bx^{n}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^{j}+bx^{n}}}\,dx = \int \frac{1}{\sqrt{ax^{j}+bx^{n}}}\,dx$$

```
[In] integrate(1/(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**j + b*x**n), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^{j}+bx^{n}}}\,dx = \int \frac{1}{\sqrt{ax^{j}+bx^{n}}}\,dx$$

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x^j + b*x^n), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx=\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$$

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x^j + b*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx=-\frac{x\,\sqrt{\frac{b\,x^{n-j}}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{\frac{j}{2}-1}{j-n};\ \frac{\frac{j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{j}{2}-1\right)\,\sqrt{a\,x^j+b\,x^n}}$$

```
[In] int(1/(a*x^j + b*x^n)^(1/2),x)
```

```
[Out] -(x*((b*x^(n - j))/a + 1)^(1/2)*hypergeom([1/2, (j/2 - 1)/(j - n)], (j/2 -
1)/(j - n) + 1, -(b*x^(n - j))/a))/((j/2 - 1)*(a*x^j + b*x^n)^(1/2))
```

3.429 $\int \frac{1}{\left(ax^j+bx^n\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{1}{\left(ax^j+bx^n\right)^{3/2}}\,dx=\frac{2x^{1-n}\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n},1+\frac{1-\frac{3n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-n)*hypergeom([3/2, (1-3/2*n)/(j-n)],[1+(2-3*n)/(2*j-2*n)],-a*x^(j-n)
/b)*(1+a*x^(j-n)/b)^(1/2)/b/(2-3*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2036, 372, 371}

$$\int \frac{1}{\left(ax^j+bx^n\right)^{3/2}}\,dx=\frac{2x^{1-n}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n},\frac{1-\frac{3n}{2}}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

```
[In] Int[(a*x^j + b*x^n)^(-3/2),x]
```

```
[Out] (2*x^(1 - n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 - (3*n)/2)
/(j - n), 1 + (1 - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b*(2 - 3*n)*Sqrt
[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-3n/2}}{\left(b+ax^{j-n}\right)^{3/2}}\,dx}{\sqrt{ax^j+bx^n}}\\
&= \frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-3n/2}}{\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}}\,dx}{b\sqrt{ax^j+bx^n}}\\
&= \frac{2x^{1-n}\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n};1+\frac{1-\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(ax^j+bx^n\right)^{3/2}}\,dx = \frac{2x^{1-j}\left(-1+\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{-2+2j+n}{2(j-n)},\frac{2-3n}{2j-2n},-\frac{ax^{j-n}}{b}\right)\right)}{a(j-n)\sqrt{ax^j+bx^n}}$$

```
[In] Integrate[(a*x^j + b*x^n)^(-3/2),x]
```

```
[Out] (2*x^(1 - j)*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, -1/2*(-
2 + 2*j + n)/(j - n), (2 - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(a*(j -
n)*Sqrt[a*x^j + b*x^n])
```

Maple [F]

$$\int \frac{1}{\left(a\,x^{j}+b\,x^{n}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int(1/(a*x^j+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx = \int \frac{1}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^j+bx^n)^{3/2}}\,dx = \int \frac{1}{(ax^j+bx^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(ax^j+bx^n\right)^{3/2}}\,dx=\int \frac{1}{\left(ax^j+bx^n\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(ax^j+bx^n\right)^{3/2}}\,dx=-\frac{x\,{\left(\frac{b\,x^{n-j}}{a}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{\frac{3\,j}{2}-1}{j-n};\ \frac{\frac{3\,j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{3\,j}{2}-1\right)\,{\left(a\,x^j+b\,x^n\right)}^{3/2}}$$

```
[In] int(1/(a*x^j + b*x^n)^(3/2),x)
```

```
[Out] -(x*((b*x^(n - j))/a + 1)^(3/2)*hypergeom([3/2, ((3*j)/2 - 1)/(j - n)], ((3
*j)/2 - 1)/(j - n) + 1, -(b*x^(n - j))/a))/(((3*j)/2 - 1)*(a*x^j + b*x^n)^(
3/2))
```

3.430 $\int \frac{1}{\left(ax^j+bx^n\right)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{1}{\left(ax^j+bx^n\right)^{5/2}}\,dx=\frac{2x^{1-2n}\sqrt{1+\frac{ax^{j-n}}{b}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n},1+\frac{1-\frac{5n}{2}}{j-n},-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-2*n)*hypergeom([5/2, (1-5/2*n)/(j-n)],[1+(2-5*n)/(2*j-2*n)],-a*x^(j-
n)/b)*(1+a*x^(j-n)/b)^(1/2)/b^2/(2-5*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2036, 372, 371}

$$\int \frac{1}{\left(ax^j+bx^n\right)^{5/2}}\,dx=\frac{2x^{1-2n}\sqrt{\frac{ax^{j-n}}{b}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n},\frac{1-\frac{5n}{2}}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

```
[In] Int[(a*x^j + b*x^n)^(-5/2),x]
```

```
[Out] (2*x^(1 - 2*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1 - (5*n)/
2)/(j - n), 1 + (1 - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b^2*(2 - 5*n)*
Sqrt[a*x^j + b*x^n])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-5n/2}}{\left(b+ax^{j-n}\right)^{5/2}}\,dx}{\sqrt{ax^j+bx^n}}\\
&= \frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-5n/2}}{\left(1+\frac{ax^{j-n}}{b}\right)^{5/2}}\,dx}{b^2\sqrt{ax^j+bx^n}}\\
&= \frac{2x^{1-2n}\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n};1+\frac{1-\frac{5n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.83

$$\int \frac{1}{\left(ax^j+bx^n\right)^{5/2}}\,dx = \frac{2x^{1-2j}\Bigg(-\left(\left(-2+4j+n\right)\left(a(-2+j+4n)x^j+b(-2+2j+3n)x^n\right)\right)+\left(4+8j^2-8\right.}{3a^2(2-4j}$$

```
[In] Integrate[(a*x^j + b*x^n)^(-5/2),x]
```

```
[Out] (2*x^(1 - 2*j)*(-((-2 + 4*j + n)*(a*(-2 + j + 4*n)*x^j + b*(-2 + 2*j + 3*n)
*x^n)) + (4 + 8*j^2 - 8*n + 3*n^2 + 2*j*(-6 + 7*n))*Sqrt[1 + (a*x^(j - n))/
b]*(a*x^j + b*x^n)*Hypergeometric2F1[1/2, -1/2*(-2 + 4*j + n)/(j - n), (2 -
 2*j - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(3*a^2*(2 - 4*j - n)*(j - n)
^2*(a*x^j + b*x^n)^(3/2))
```

Maple [F]

$$\int \frac{1}{\left(a\,x^{j}+b\,x^{n}\right)^{\frac{5}{2}}}d x$$

```
[In] int(1/(a*x^j+b*x^n)^(5/2),x)
```

```
[Out] int(1/(a*x^j+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx=\int \frac{1}{\left(ax^{j}+bx^{n}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*x**j+b*x**n)**(5/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^j+bx^n)^{5/2}}\,dx=\int \frac{1}{\left(ax^{j}+bx^{n}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(ax^j+bx^n\right)^{5/2}}\,dx=\int \frac{1}{\left(ax^j+bx^n\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(ax^j+bx^n\right)^{5/2}}\,dx=-\frac{x\,{\left(\frac{b\,x^{n-j}}{a}+1\right)}^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{\frac{5\,j}{2}-1}{j-n};\ \frac{\frac{5\,j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{5\,j}{2}-1\right)\,{\left(a\,x^j+b\,x^n\right)}^{5/2}}$$

```
[In] int(1/(a*x^j + b*x^n)^(5/2),x)
```

```
[Out] -(x*((b*x^(n - j))/a + 1)^(5/2)*hypergeom([5/2, ((5*j)/2 - 1)/(j - n)], ((5
*j)/2 - 1)/(j - n) + 1, -(b*x^(n - j))/a))/(((5*j)/2 - 1)*(a*x^j + b*x^n)^(
5/2))
```

3.431 $\int \sqrt{\frac{1+x}{x^5}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = -\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2} x^6$$

```
[Out] -2/3*(1/x^5+1/x^4)^(3/2)*x^6
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2004, 2025}

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = -\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2} x^6$$

```
[In] Int[Sqrt[(1 + x)/x^5],x]
```

```
[Out] (-2*(x^(-5) + x^(-4))^(3/2)*x^6)/3
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \sqrt{\frac{1}{x^5}+\frac{1}{x^4}}\,dx \\
&= -\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2} x^6
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = -\frac{2}{3}x(1+x)\sqrt{\frac{1+x}{x^5}}$$

```
[In] Integrate[Sqrt[(1 + x)/x^5],x]
[Out] (-2*x*(1 + x)*Sqrt[(1 + x)/x^5])/3
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{2x(1+x)\sqrt{\frac{1+x}{x^5}}}{3}$	16
trager	$-\frac{2x(1+x)\sqrt{-\frac{-x-1}{x^5}}}{3}$	19
default	$-\frac{2\sqrt{\frac{1+x}{x^5}}\,\left(x^2+x\right)^{\frac{3}{2}}}{3\sqrt{x(1+x)}}$	26
risch	$-\frac{2\sqrt{\frac{1+x}{x^5}}\,x\left(x^2+2x+1\right)}{3(1+x)}$	26

```
[In] int(((1+x)/x^5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*x*(1+x)*((1+x)/x^5)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = -\frac{2}{3}\left(x^2+x\right)\sqrt{\frac{x+1}{x^5}}$$

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="fricas")
[Out] -2/3*(x^2 + x)*sqrt((x + 1)/x^5)
```

Sympy [F]

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = \int \sqrt{\frac{x+1}{x^5}}\,dx$$

```
[In] integrate(((1+x)/x**5)**(1/2),x)
[Out] Integral(sqrt((x + 1)/x**5), x)
```

Maxima [F]

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = \int \sqrt{\frac{x+1}{x^5}}\,dx$$

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((x + 1)/x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(14) = 28.

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = \frac{2\left(3\left(x-\sqrt{x^2+x}\right)^2\mathrm{sgn}(x)+3\left(x-\sqrt{x^2+x}\right)\mathrm{sgn}(x)+\mathrm{sgn}(x)\right)}{3\left(x-\sqrt{x^2+x}\right)^3}$$

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*(x - sqrt(x^2 + x))^2*sgn(x) + 3*(x - sqrt(x^2 + x))*sgn(x) + sgn(x)
)/(x - sqrt(x^2 + x))^3
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{1+x}{x^5}}\,dx = -\frac{2\,x\,\sqrt{\frac{x+1}{x^5}}\,(x+1)}{3}$$

```
[In] int(((x + 1)/x^5)^(1/2),x)
[Out] -(2*x*((x + 1)/x^5)^(1/2)*(x + 1))/3
```

3.432 $\int \sqrt{x+x^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \sqrt{x+x^{5/2}}\,dx=\frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

```
[Out] 4/9*(x+x^(5/2))^(3/2)/x^(3/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2025}

$$\int \sqrt{x+x^{5/2}}\,dx=\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

```
[In] Int[Sqrt[x + x^(5/2)],x]
```

```
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\text{integral}=\frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{x+x^{5/2}}\,dx=\frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

```
[In] Integrate[Sqrt[x + x^(5/2)],x]
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{4\sqrt{x+x^{\frac{5}{2}}}\,\left(1+x^{\frac{3}{2}}\right)}{9\sqrt{x}}$	18
default	$\frac{4\sqrt{x+x^{\frac{5}{2}}}\,\left(1+x^{\frac{3}{2}}\right)}{9\sqrt{x}}$	18
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+2x^{\frac{3}{2}}\right)\sqrt{1+x^{\frac{3}{2}}}}{3}}{3\sqrt{\pi}}$	31

```
[In] int((x+x^(5/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/9*(x+x^(5/2))^(1/2)/x^(1/2)*(1+x^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{x+x^{5/2}}\,dx=\frac{4\,\sqrt{x^{\frac{5}{2}}+x}\left(x^2+\sqrt{x}\right)}{9\,x}$$

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="fricas")
[Out] 4/9*sqrt(x^(5/2) + x)*(x^2 + sqrt(x))/x
```

Sympy [F]

$$\int \sqrt{x+x^{5/2}}\,dx = \int \sqrt{x^{\frac{5}{2}}+x}\,dx$$

```
[In] integrate((x+x**(5/2))**(1/2),x)
[Out] Integral(sqrt(x**(5/2) + x), x)
```

Maxima [F]

$$\int \sqrt{x+x^{5/2}}\,dx = \int \sqrt{x^{\frac{5}{2}}+x}\,dx$$

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^(5/2) + x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

$$\int \sqrt{x+x^{5/2}}\,dx = \frac{4}{9}\left(x^{\frac{3}{2}}+1\right)^{\frac{3}{2}} - \frac{4}{9}$$

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="giac")
[Out] 4/9*(x^(3/2) + 1)^(3/2) - 4/9
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \sqrt{x+x^{5/2}}\,dx = \frac{2\,x\,\sqrt{x+x^{5/2}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},1;\ 2;\ -x^{3/2}\right)}{3\,\sqrt{x^{3/2}+1}}$$

```
[In] int((x + x^(5/2))^(1/2),x)
[Out] (2*x*(x + x^(5/2))^(1/2)*hypergeom([-1/2, 1], 2, -x^(3/2)))/(3*(x^(3/2) + 1
)^(1/2))
```

3.433 $\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx=2\arctan\left(\sqrt{x}\right)$$

```
[Out] 2*arctan(x^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1607, 65, 209}

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx=2\arctan\left(\sqrt{x}\right)$$

```
[In] Int[(Sqrt[x] + x^(3/2))^(-1),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{x}(1+x)}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

```
[In] Integrate[(Sqrt[x] + x^(3/2))^(-1),x]
[Out] 2*ArcTan[Sqrt[x]]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2\arctan\left(\sqrt{x}\right)$	7
default	$2\arctan\left(\sqrt{x}\right)$	7
meijerg	$2\arctan\left(\sqrt{x}\right)$	7
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{x}+x-1}{1+x}\right)$	29

```
[In] int(1/(x^(3/2)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

```
[In] integrate(1/(x**(3/2)+x**(1/2)),x)
[Out] 2*atan(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="giac")
[Out] 2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx=2\,\mathrm{atan}\left(\sqrt{x}\right)$$

```
[In] int(1/(x^(1/2) + x^(3/2)),x)
[Out] 2*atan(x^(1/2))
```

3.434 $\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

```
[Out] 2/9*(x^2*(b*x^3+a))^(3/2)/b/x^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1602}

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

```
[In] Int[x*Sqrt[x^2*(a + b*x^3)],x]
```

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\left(x^2(a+bx^3)\right)^{3/2}}{9bx^3}$$

```
[In] Integrate[x*Sqrt[x^2*(a + b*x^3)],x]
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{2\left(b\,x^{3}+a\right)\sqrt{x^{2}\left(b\,x^{3}+a\right)}}{9bx}$	29
default	$\frac{2\left(b\,x^{3}+a\right)\sqrt{x^{2}\left(b\,x^{3}+a\right)}}{9bx}$	29
trager	$\frac{2\left(b\,x^{3}+a\right)\sqrt{b\,x^{5}+a\,x^{2}}}{9bx}$	29
risch	$\frac{2\left(b\,x^{3}+a\right)\sqrt{x^{2}\left(b\,x^{3}+a\right)}}{9bx}$	29

```
[In] int(x*(x^2*(b*x^3+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/9*(b*x^3+a)*(x^2*(b*x^3+a))^(1/2)/b/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\sqrt{bx^5+ax^2}(bx^3+a)}{9\,bx}$$

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

Sympy [F(-1)]

Timed out.

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx = \text{Timed out}$$

```
[In] integrate(x*(x**2*(b*x**3+a))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx = \frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx = \frac{2\left(bx^3+a\right)^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b} - \frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b}$$

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx = \frac{2\left(b\,x^3+a\right)^{3/2}\sqrt{x^2}}{9\,b\,x}$$

```
[In] int(x*(x^2*(a + b*x^3))^(1/2),x)
```

```
[Out] (2*(a + b*x^3)^(3/2)*(x^2)^(1/2))/(9*b*x)
```

3.435 $\int x\sqrt{ax^2+bx^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2(ax^2+bx^5)^{3/2}}{9bx^3}$$

```
[Out] 2/9*(b*x^5+a*x^2)^(3/2)/b/x^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1602}

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2(ax^2+bx^5)^{3/2}}{9bx^3}$$

```
[In] Int[x*Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{2(ax^2+bx^5)^{3/2}}{9bx^3}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2(x^2(a+bx^3))^{3/2}}{9bx^3}$$

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^5],x]
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{2(b\,x^3+a)\sqrt{b\,x^5+a\,x^2}}{9bx}$	29
default	$\frac{2(b\,x^3+a)\sqrt{b\,x^5+a\,x^2}}{9bx}$	29
trager	$\frac{2(b\,x^3+a)\sqrt{b\,x^5+a\,x^2}}{9bx}$	29
risch	$\frac{2(b\,x^3+a)\,\sqrt{x^2(b\,x^3+a)}}{9bx}$	29

```
[In] int(x*(b*x^5+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/9*(b*x^3+a)/b/x*(b*x^5+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2\sqrt{bx^5+ax^2}(bx^3+a)}{9\,bx}$$

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

Sympy [F]

$$\int x\sqrt{ax^2+bx^5}\,dx=\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$$

```
[In] integrate(x*(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x*sqrt(x**2*(a + b*x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2\left(bx^3+a\right)^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b}-\frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{9\,b}$$

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] 2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{\left(\frac{2\,a}{9\,b}+\frac{2\,x^3}{9}\right)\,\sqrt{b\,x^5+a\,x^2}}{x}$$

```
[In] int(x*(a*x^2 + b*x^5)^(1/2),x)
[Out] (((2*a)/(9*b) + (2*x^3)/9)*(a*x^2 + b*x^5)^(1/2))/x
```

3.436 $\int \sqrt{x^4 \left(a+b x^3\right)} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \sqrt{x^4 \left(a+b x^3\right)} \, dx=\frac{2 {\left(a x^4+b x^7\right)}^{3/2}}{9 b x^6}$$

```
[Out] 2/9*(b*x^7+a*x^4)^(3/2)/b/x^6
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2004, 2025}

$$\int \sqrt{x^4 \left(a+b x^3\right)} \, dx=\frac{2 {\left(a x^4+b x^7\right)}^{3/2}}{9 b x^6}$$

```
[In] Int[Sqrt[x^4*(a + b*x^3)],x]
```

```
[Out] (2*(a*x^4 + b*x^7)^(3/2))/(9*b*x^6)
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\text{integral} = \int \sqrt{ax^4 + bx^7}\, dx$$
$$= \frac{2(ax^4 + bx^7)^{3/2}}{9bx^6}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sqrt{x^4\left(a + bx^3\right)}\, dx = \frac{2(x^4(a + bx^3))^{3/2}}{9bx^6}$$

```
[In] Integrate[Sqrt[x^4*(a + b*x^3)],x]
[Out] (2*(x^4*(a + b*x^3))^(3/2))/(9*b*x^6)
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

method	result	size
gosper	$\frac{2(b\,x^3+a)\sqrt{x^4(b\,x^3+a)}}{9b\,x^2}$	29
default	$\frac{2(b\,x^3+a)\sqrt{x^4(b\,x^3+a)}}{9b\,x^2}$	29
trager	$\frac{2(b\,x^3+a)\sqrt{b\,x^7+a\,x^4}}{9b\,x^2}$	29
risch	$\frac{2(b\,x^3+a)\sqrt{x^4(b\,x^3+a)}}{9b\,x^2}$	29

```
[In] int((x^4*(b*x^3+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/9*(b*x^3+a)*(x^4*(b*x^3+a))^(1/2)/b/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \sqrt{x^4\left(a+bx^3\right)}\,dx = \frac{2\sqrt{bx^7+ax^4}(bx^3+a)}{9\,bx^2}$$

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^7 + a*x^4)*(b*x^3 + a)/(b*x^2)
```

Sympy [F]

$$\int \sqrt{x^4\left(a+bx^3\right)}\,dx = \int \sqrt{x^4\left(a+bx^3\right)}\,dx$$

```
[In] integrate((x**4*(b*x**3+a))**(1/2),x)
[Out] Integral(sqrt(x**4*(a + b*x**3)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{x^4\left(a+bx^3\right)}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b}$$

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="maxima")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{x^4\left(a+bx^3\right)}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b}$$

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="giac")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \sqrt{x^4 (a + bx^3)}\, dx = \frac{2 \left(b\, x^3 + a\right)^{3/2} \sqrt{x^4}}{9\, b\, x^2}$$

```
[In] int((x^4*(a + b*x^3))^(1/2),x)
```

```
[Out] (2*(a + b*x^3)^(3/2)*(x^4)^(1/2))/(9*b*x^2)
```

3.437 $\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 988

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx = -\frac{45a^2\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}}{14\sqrt[3]{2}b^3\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$-\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$-\frac{45\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)^{2}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}}}{28\sqrt[3]{2}b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}\left(a\right.}}$$

$$+\frac{15\ 3^{3/4}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}\sqrt[3]{-\frac{b\left(\right.}{}}}}{7\ 2^{5/6}b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}\left(a\right.}}$$

```
[Out] -45/28*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a*x^(1/3)+b*x^(2/3))^(1/3)+9/7*(a+b*x^(
1/3))*x^(2/3)/b/(a*x^(1/3)+b*x^(2/3))^(1/3)-45/28*a^2*(a+2*b*x^(1/3))*(-b*(
a*x^(1/3)+b*x^(2/3))/a^2)^(1/3)*2^(2/3)/b^3/(a*x^(1/3)+b*x^(2/3))^(1/3)/(1-
2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))+15/14*3^(3/4)*a^4*(1-
2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3))/a^2
)^(1/3)*EllipticF((1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(1/2))/
```

```
(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1
+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+2*2^(1/3)*(-b*(a+b*x^(1/3))*x
^(1/3)/a^2)^(2/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))
^2)^(1/2)*2^(1/6)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x^(2/3))^(1/3)/((-1+2^(2
/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1
/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)-45/56*3^(1/4)*a^4*(1-2^(2/3)*(-b*(a+b*x^(1
/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3))/a^2)^(1/3)*EllipticE((1-
2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-b*(a+b*x
^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-b*(a+b*x^(
1/3))*x^(1/3)/a^2)^(1/3)+2*2^(1/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(2/3))/(1
-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2
)+1/2*2^(1/2))*2^(2/3)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x^(2/3))^(1/3)/((-1
+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))
*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules

used = {2036, 348, 52, 63, 636, 633, 241, 310, 225, 1893}

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx =$$

$$-\frac{45\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}}{28\sqrt[3]{2}b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}}$$

$$+\frac{15\ 3^{3/4}\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\sqrt[3]{-\frac{b\left(\sqrt[3]{}\right.}{}}}{7\ 2^{5/6}b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}}$$

$$-\frac{45\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{-\frac{b\left(\sqrt[3]{x}a+bx^{2/3}\right)}{a^2}}\,a^2}{14\sqrt[3]{2}b^3\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)\sqrt[3]{\sqrt[3]{x}a+bx^{2/3}}}$$

$$-\frac{45\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}a}{28b^2\sqrt[3]{\sqrt[3]{x}a+bx^{2/3}}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{\sqrt[3]{x}a+bx^{2/3}}}$$

```
[In] Int[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
```

```
[Out] (-45*a^2*(a + 2*b*x^(1/3))*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3))/(14*
2^(1/3)*b^3*(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/
3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) - (45*a*(a + b*x^(1/3))*x^(1/3))/(28*b^2
```

```
*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (9*(a + b*x^(1/3))*x^(2/3))/(7*b*(a*x^(1/
3) + b*x^(2/3))^(1/3)) - (45*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^4*(1 - 2^(2/3)*(-(
(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^
2]*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticE[ArcSin[(1 + Sqrt[3]
 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(28*2^(1/
3)*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 -
 Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b
*x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (15*3^(3/4)*a^4*(1 - 2^(2/3)*(-(
(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^
2]*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticF[ArcSin[(1 + Sqrt[3]
 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(7*2^(5/6
)*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 -
Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b*
x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/(a*c + (b*c + a*d)*x + b*d*x^2)^m), Int[(a*c + (b*c
 + a*d)*x + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d,
0] && LtQ[-1, m, 0] && LeQ[3, Denominator[m], 4] && AtomQ[b*c + a*d]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt[3]{a+b\sqrt[3]{x}}\,\sqrt[9]{x}\right)\int\frac{1}{\sqrt[3]{a+b\sqrt[3]{x}}\,\sqrt[9]{x}}\,dx}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= \frac{\left(3\sqrt[3]{a+b\sqrt[3]{x}}\,\sqrt[9]{x}\right)\text{Subst}\left(\int\frac{x^{5/3}}{\sqrt[3]{a+bx}}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(15a\sqrt[3]{a+b\sqrt[3]{x}}\,\sqrt[9]{x}\right)\text{Subst}\left(\int\frac{x^{2/3}}{\sqrt[3]{a+bx}}\,dx,x,\sqrt[3]{x}\right)}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(15a^2\sqrt[3]{a+b\sqrt[3]{x}}\,\sqrt[9]{x}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{x}\,\sqrt[3]{a+bx}}\,dx,x,\sqrt[3]{x}\right)}{14b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(15a^2\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{ax+bx^2}}\,dx,x,\sqrt[3]{x}\right)}{14b^2}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(15a^2\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}}\,dx,x,\sqrt[3]{x}\right)}{14b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(15a^4\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-\frac{a^2x^2}{b^2}}}\,dx, x, -\frac{b\left(a+2b\sqrt[3]{x}\right)}{a^2}\right)}{14\sqrt[3]{2}b^4\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(45a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)}{28\sqrt[3]{2}b^3\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(45a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}\left(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)}{28\sqrt[3]{2}b^3\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(45\left(1+\sqrt{3}\right)a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^3}}\,dx, x, \sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}}{a^2}}\right.}{28\sqrt[3]{2}b^3\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a^2\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}}{14\sqrt[3]{2}b^3\left(1-\sqrt{3}-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$-\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$-\frac{45\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^4\left(1-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}+\left(1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)^2}\sqrt[3]{-}}}{28\sqrt[3]{2}b^3\sqrt{-\frac{1-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)^2}\left(a+\right.}}$$

$$+\frac{15\ 3^{3/4}a^4\left(1-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}+\left(1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)^2}\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}\right.}{a}}}}{7\ 2^{5/6}b^3\sqrt{-\frac{1-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)^2}\left(a+2b\right.}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.06

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\frac{9\sqrt[3]{1+\frac{b\sqrt[3]{x}}{a}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{8\sqrt[3]{\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}}$$

```
[In] Integrate[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
```

```
[Out] (9*(1 + (b*x^(1/3))/a)^(1/3)*x*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^(1/
3))/a)])/(8*((a + b*x^(1/3))*x^(1/3))^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(a\,x^{\frac{1}{3}}+b\,x^{\frac{2}{3}}\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

```
[Out] int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{\frac{2}{3}}}}\,dx$$

```
[In] integrate(1/(a*x**(1/3)+b*x**(2/3))**(1/3),x)
```

```
[Out] Integral((a*x**(1/3) + b*x**(2/3))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="giac")
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.04

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx=\frac{9\,x\,{\left(\frac{b\,x^{1/3}}{a}+1\right)}^{1/3}\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{3},\frac{8}{3};\ \frac{11}{3};\ -\frac{b\,x^{1/3}}{a}\right)}{8\,{\left(a\,x^{1/3}+b\,x^{2/3}\right)}^{1/3}}$$

```
[In] int(1/(a*x^(1/3) + b*x^(2/3))^(1/3),x)
[Out] (9*x*((b*x^(1/3))/a + 1)^(1/3)*hypergeom([1/3, 8/3], 11/3, -(b*x^(1/3))/a))
/(8*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

3.438 $\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 487

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx = -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$+\frac{6\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{1+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)^{2/}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}}}{5b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(1-\sqrt{3}-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)^2}}\,(a}$$

```
[Out] -18/5*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a*x^(1/3)+b*x^(2/3))^(2/3)+9/5*(a+b*x^(1
/3))*x^(2/3)/b/(a*x^(1/3)+b*x^(2/3))^(2/3)+6/5*2^(1/3)*3^(3/4)*a^4*(1-2^(2/
3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3))/a^2)^(2/
3)*EllipticF((1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(1/2))/(1-2^
(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(
1/2)-1/2*2^(1/2))*((1+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+2*2^(1/3
)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(2/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3
)/a^2)^(1/3)-3^(1/2))^2)^(1/2)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x^(2/3))^(2
/3)/((-1+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3)*(-b*(a+b*
x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2036, 348, 52, 63, 636, 633, 242, 225}

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx = \frac{6\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}\right)^{2/3}+}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\,(}{}}\right.}}}{5b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{}}{\left(-2^{2/3}\sqrt[3]{-}\right.}}}$$

$$-\frac{18a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9x^{2/3}\left(a+b\sqrt[3]{x}\right)}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

```
[In] Int[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

```
[Out] (-18*a*(a + b*x^(1/3))*x^(1/3))/(5*b^2*(a*x^(1/3) + b*x^(2/3))^(2/3)) + (9*
(a + b*x^(1/3))*x^(2/3))/(5*b*(a*x^(1/3) + b*x^(2/3))^(2/3)) + (6*2^(1/3)*3
^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^
2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3) + 2
*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2]*(-((b*(a*x^(1/3) + b*x^(2/3
)))/a^2))^(2/3)*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3
))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/
3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b
*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3)
)*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b*x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(2/3)
)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
a + b*x)^m*((c + d*x)^m/(a*c + (b*c + a*d)*x + b*d*x^2)^m), Int[(a*c + (b*c
 + a*d)*x + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d,
0] && LtQ[-1, m, 0] && LeQ[3, Denominator[m], 4] && AtomQ[b*c + a*d]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 636

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p, Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 2036

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\text{integral} = \frac{\left(\left(a+b\sqrt[3]{x}\right)^{2/3} x^{2/9}\right) \int \frac{1}{\left(a+b\sqrt[3]{x}\right)^{2/3} x^{2/9}} \, dx}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$= \frac{\left(3\left(a+b\sqrt[3]{x}\right)^{2/3} x^{2/9}\right) \text{Subst}\left(\int \frac{x^{4/3}}{(a+bx)^{2/3}} \, dx, x, \sqrt[3]{x}\right)}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$= \frac{9\left(a+b\sqrt[3]{x}\right) x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} - \frac{\left(12a\left(a+b\sqrt[3]{x}\right)^{2/3} x^{2/9}\right) \text{Subst}\left(\int \frac{\sqrt[3]{x}}{(a+bx)^{2/3}} \, dx, x, \sqrt[3]{x}\right)}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{9\left(a+b\sqrt[3]{x}\right) x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{\left(6a^2\left(a+b\sqrt[3]{x}\right)^{2/3} x^{2/9}\right) \text{Subst}\left(\int \frac{1}{x^{2/3}(a+bx)^{2/3}} \, dx, x, \sqrt[3]{x}\right)}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{9\left(a+b\sqrt[3]{x}\right) x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{(6a^2) \text{Subst}\left(\int \frac{1}{(ax+bx^2)^{2/3}} \, dx, x, \sqrt[3]{x}\right)}{5b^2}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{9\left(a+b\sqrt[3]{x}\right) x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{\left(6a^2\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}\right) \text{Subst}\left(\int \frac{1}{\left(-\frac{bx}{a}-\frac{b^2x^2}{a^2}\right)^{2/3}} \, dx, x, \sqrt[3]{x}\right)}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{9\left(a+b\sqrt[3]{x}\right) x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} - \frac{\left(6\sqrt[3]{2}a^4\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}\right) \text{Subst}\left(\int \frac{1}{\left(1-\frac{a^2x^2}{b^2}\right)^{2/3}} \, dx, x, -\frac{b\left(a+2b\sqrt[3]{x}\right)}{a^2}\right)}{5b^4\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} - \frac{\left(9\sqrt[3]{2}a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-1+x^3}}\,dx,x,\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)}{5b^3\left(a+2b\sqrt[3]{x}\right)\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}} + \frac{6\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)\sqrt{\frac{1+\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}+\left(1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)^2}}\left(-\right.}{5b^3\sqrt{-\frac{1-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}}{\left(1-\sqrt{3}-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)^2}}\left(a+2b\right.}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.13

$$\int\frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\frac{9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2/3}x\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{b\sqrt[3]{x}}{a}\right)}{7\left(\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}\right)^{2/3}}$$

```
[In] Integrate[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

```
[Out] (9*(1 + (b*x^(1/3))/a)^(2/3)*x*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^(1/
3))/a)])/(7*((a + b*x^(1/3))*x^(1/3))^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(a\,x^{\frac{1}{3}}+b\,x^{\frac{2}{3}}\right)^{\frac{2}{3}}}dx$$

```
[In] int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)
[Out] int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx = \int \frac{1}{\left(a\sqrt[3]{x}+bx^{\frac{2}{3}}\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(a*x**(1/3)+b*x**(2/3))**(2/3),x)
[Out] Integral((a*x**(1/3) + b*x**(2/3))**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx = \int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="maxima")
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.09

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\frac{9\,x\,\left(\frac{b\,x^{1/3}}{a}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{7}{3};\ \frac{10}{3};\ -\frac{b\,x^{1/3}}{a}\right)}{7\,\left(a\,x^{1/3}+b\,x^{2/3}\right)^{2/3}}$$

```
[In] int(1/(a*x^(1/3) + b*x^(2/3))^(2/3),x)
```

```
[Out] (9*x*((b*x^(1/3))/a + 1)^(2/3)*hypergeom([2/3, 7/3], 10/3, -(b*x^(1/3))/a))
/(7*(a*x^(1/3) + b*x^(2/3))^(2/3))
```

3.439 $\int x^m \left(ax^j+bx^n\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int x^m \left(ax^j+bx^n\right)^p \, dx = \frac{x^{1+m}\left(ax^j+bx^n\right)^p \left(a+bx^{-j+n}\right) \text{Hypergeometric2F1}\left(1,1+p+\frac{1+m+jp}{-j+n},1+\frac{1+m+jp}{-j+n},-\frac{bx^{-j+n}}{a}\right)}{a(1+m+jp)}$$

```
[Out] x^(1+m)*(a*x^j+b*x^n)^p*(a+b*x^(-j+n))*hypergeom([1, 1+p+(j*p+m+1)/(-j+n)],
[1+(j*p+m+1)/(-j+n)],-b*x^(-j+n)/a)/a/(j*p+m+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2057, 372, 371}

$$\int x^m \left(ax^j+bx^n\right)^p \, dx = \frac{x^{m+1}\left(\frac{ax^{j-n}}{b}+1\right)^{-p}\left(ax^j+bx^n\right)^p \text{Hypergeometric2F1}\left(-p,\frac{m+np+1}{j-n},\frac{m+np+1}{j-n}+1,-\frac{ax^{j-n}}{b}\right)}{m+np+1}$$

```
[In] Int[x^m*(a*x^j + b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n), 1
 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(j
- n))/b)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{-np}\left(b+ax^{j-n}\right)^{-p}\left(ax^j+bx^n\right)^p\right)\int x^{m+np}\left(b+ax^{j-n}\right)^p\,dx\\ &= \left(x^{-np}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p\right)\int x^{m+np}\left(1+\frac{ax^{j-n}}{b}\right)^p\,dx\\ &= \frac{x^{1+m}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p\,{}_2F_1\left(-p,\frac{1+m+np}{j-n};1+\frac{1+m+np}{j-n};-\frac{ax^{j-n}}{b}\right)}{1+m+np}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int x^m\left(ax^j+bx^n\right)^p\,dx$$
$$=\frac{x^{1+m}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p\,\text{Hypergeometric2F1}\left(-p,\frac{1+m+np}{j-n},1+\frac{1+m+np}{j-n},-\frac{ax^{j-n}}{b}\right)}{1+m+np}$$

```
[In] Integrate[x^m*(a*x^j + b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n), 1
 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(j
- n))/b)^p)
```

Maple [F]

$$\int x^{m} \left(a\,x^{j}+b\,x^{n}\right)^{p}dx$$

```
[In] int(x^m*(a*x^j+b*x^n)^p,x)
[Out] int(x^m*(a*x^j+b*x^n)^p,x)
```

Fricas [F]

$$\int x^{m}{\left(ax^{j}+bx^{n}\right)}^{p}\,dx=\int{\left(ax^{j}+bx^{n}\right)}^{p}x^{m}\,dx$$

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="fricas")
[Out] integral((a*x^j + b*x^n)^p*x^m, x)
```

Sympy [F]

$$\int x^{m}{\left(ax^{j}+bx^{n}\right)}^{p}\,dx=\int x^{m}{\left(ax^{j}+bx^{n}\right)}^{p}\,dx$$

```
[In] integrate(x**m*(a*x**j+b*x**n)**p,x)
[Out] Integral(x**m*(a*x**j + b*x**n)**p, x)
```

Maxima [F]

$$\int x^{m}{\left(ax^{j}+bx^{n}\right)}^{p}\,dx=\int{\left(ax^{j}+bx^{n}\right)}^{p}x^{m}\,dx$$

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="maxima")
[Out] integrate((a*x^j + b*x^n)^p*x^m, x)
```

Giac [F]

$$\int x^m \left(ax^j+bx^n\right)^p\,dx=\int \left(ax^j+bx^n\right)^p x^m\,dx$$

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="giac")
[Out] integrate((a*x^j + b*x^n)^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(ax^j+bx^n\right)^p\,dx=\int x^m\,{\left(a\,x^j+b\,x^n\right)}^p\,dx$$

```
[In] int(x^m*(a*x^j + b*x^n)^p,x)
[Out] int(x^m*(a*x^j + b*x^n)^p, x)
```

3.440 $\int x^{-1-pq}(bx^n+ax^q)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int x^{-1-pq}(bx^n+ax^q)^p\,dx = -\frac{x^{-pq}(a+bx^{n-q})\,(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^{n-q}}{a}\right)}{a(1+p)(n-q)}$$

```
[Out] -(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([1, p+1],[2+p],1+b*x^(n-q)/a)/a/(p
+1)/(n-q)/(x^(p*q))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2057, 272, 67}

$$\int x^{-1-pq}(bx^n+ax^q)^p\,dx = -\frac{x^{-pq}(a+bx^{n-q})\,(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^{n-q}}{a}+1\right)}{a(p+1)(n-q)}$$

```
[In] Int[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -(((a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1
 + (b*x^(n - q))/a])/(a*(1 + p)*(n - q)*x^(p*q)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int\frac{\left(a+bx^{n-q}\right)^p}{x}\,dx\\ &= \frac{\left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\operatorname{Subst}\left(\int\frac{(a+bx)^p}{x}\,dx,x,x^{n-q}\right)}{n-q}\\ &= -\frac{x^{-pq}\left(a+bx^{n-q}\right)\left(bx^n+ax^q\right)^p\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^{n-q}}{a}\right)}{a(1+p)(n-q)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int x^{-1-pq}\left(bx^n+ax^q\right)^p\,dx = \frac{x^{-pq}\left(bx^n+ax^q\right)^p\left(1+\frac{ax^{-n+q}}{b}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{ax^{-n+q}}{b}\right)}{p(n-q)}$$

```
[In] Integrate[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] ((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((a*x^(-n + q))/b)])/(
p*(n - q)*x^(p*q)*(1 + (a*x^(-n + q))/b)^p)
```

Maple [F]

$$\int x^{-pq-1}(b\,x^{n}+a\,x^{q})^{p}\,dx$$

```
[In] int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
```

```
[Out] int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
```

Fricas [F]

$$\int x^{-1-pq}(bx^{n}+ax^{q})^{p}\ dx=\int(bx^{n}+ax^{q})^{p}x^{-pq-1}\,dx$$

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

Sympy [F]

$$\int x^{-1-pq}(bx^{n}+ax^{q})^{p}\ dx=\int x^{-pq-1}(ax^{q}+bx^{n})^{p}\ dx$$

```
[In] integrate(x**(-p*q-1)*(b*x**n+a*x**q)**p,x)
```

```
[Out] Integral(x**(-p*q - 1)*(a*x**q + b*x**n)**p, x)
```

Maxima [F]

$$\int x^{-1-pq}(bx^{n}+ax^{q})^{p}\ dx=\int(bx^{n}+ax^{q})^{p}x^{-pq-1}\,dx$$

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

Giac [F]

$$\int x^{-1-pq}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-pq-1}\,dx$$

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-pq}(bx^n+ax^q)^p\,dx=\int \frac{\left(b\,x^n+a\,x^q\right)^p}{x^{p\,q+1}}\,dx$$

```
[In] int((b*x^n + a*x^q)^p/x^(p*q + 1),x)
[Out] int((b*x^n + a*x^q)^p/x^(p*q + 1), x)
```

3.441 $\int x^{-1-np}(bx^n+ax^q)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx = -\frac{x^{-np}(a+bx^{n-q})\,(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(1,1,1-p,-\frac{bx^{n-q}}{a}\right)}{ap(n-q)}$$

```
[Out] -(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([1, 1],[1-p],-b*x^(n-q)/a)/a/p/(n-
q)/(x^(n*p))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2057, 372, 371}

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx = -\frac{x^{-np}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^{n-q}}{a}\right)}{p(n-q)}$$

```
[In] Int[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])/
(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}(bx^n+ax^q)^p\right)\int x^{-1-np+pq}\left(a+bx^{n-q}\right)^p\,dx\\
&= \left(x^{-pq}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\right)\int x^{-1-np+pq}\left(1+\frac{bx^{n-q}}{a}\right)^p\,dx\\
&= -\frac{x^{-np}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,{}_2F_1\left(-p,-p;1-p;-\frac{bx^{n-q}}{a}\right)}{p(n-q)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx = -\frac{x^{-np}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^{n-q}}{a}\right)}{p(n-q)}$$

```
[In] Integrate[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
[Out] -(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])/
(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))
```

Maple [F]

$$\int x^{-np-1}(b\,x^{n}+a\,x^{q})^{p}\,dx$$

```
[In] int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
[Out] int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
```

Fricas [F]

$$\int x^{-1-np}(bx^{n}+ax^{q})^{p}\,dx=\int (bx^{n}+ax^{q})^{p}x^{-np-1}\,dx$$

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

Sympy [F]

$$\int x^{-1-np}(bx^{n}+ax^{q})^{p}\,dx=\int x^{-np-1}(ax^{q}+bx^{n})^{p}\,dx$$

```
[In] integrate(x**(-n*p-1)*(b*x**n+a*x**q)**p,x)
[Out] Integral(x**(-n*p - 1)*(a*x**q + b*x**n)**p, x)
```

Maxima [F]

$$\int x^{-1-np}(bx^{n}+ax^{q})^{p}\,dx=\int (bx^{n}+ax^{q})^{p}x^{-np-1}\,dx$$

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

Giac [F]

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-np-1}\,dx$$

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-np}(bx^n+ax^q)^p\,dx=\int \frac{(b\,x^n+a\,x^q)^p}{x^{n\,p+1}}\,dx$$

```
[In] int((b*x^n + a*x^q)^p/x^(n*p + 1),x)
```

```
[Out] int((b*x^n + a*x^q)^p/x^(n*p + 1), x)
```

3.442 $\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx$

Optimal result

Integrand size = 27, antiderivative size = 69

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx = \frac{bx^{-pq}(a+bx^{n-q})\,(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(2,1+p,2+p,1+\frac{bx^{n-q}}{a}\right)}{a^2(1+p)(n-q)}$$

```
[Out] b*(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([2, p+1],[2+p],1+b*x^(n-q)/a)/a^2
/(p+1)/(n-q)/(x^(p*q))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2057, 272, 67}

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx = \frac{bx^{-pq}(a+bx^{n-q})\,(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx^{n-q}}{a}+1\right)}{a^2(p+1)(n-q)}$$

```
[In] Int[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (b*(a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[2, 1 + p, 2 + p, 1
 + (b*x^(n - q))/a])/(a^2*(1 + p)*(n - q)*x^(p*q))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
```

```
d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int x^{-1-n-(-1+p)q+pq}\left(a+bx^{n-q}\right)^p\,dx\\ &= \frac{\left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\operatorname{Subst}\left(\int\frac{(a+bx)^p}{x^2}\,dx,x,x^{n-q}\right)}{n-q}\\ &= \frac{bx^{-pq}\left(a+bx^{n-q}\right)\left(bx^n+ax^q\right)^p\,{}_2F_1\left(2,1+p;2+p;1+\frac{bx^{n-q}}{a}\right)}{a^2(1+p)(n-q)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int x^{-1-n-(-1+p)q}\left(bx^n+ax^q\right)^p\,dx$$
$$=\frac{x^{-n+q-pq}\left(bx^n+ax^q\right)^p\left(1+\frac{ax^{-n+q}}{b}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{ax^{-n+q}}{b}\right)}{(-1+p)(n-q)}$$

```
[In] Integrate[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (x^(-n + q - p*q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((
a*x^(-n + q))/b)])/((-1 + p)*(n - q)*(1 + (a*x^(-n + q))/b)^p)
```

Maple [F]

$$\int x^{-1-n-(p-1)q}(b\,x^{n}+a\,x^{q})^{p}\,dx$$

```
[In] int(x^(-1-n-(p-1)*q)*(b*x^n+a*x^q)^p,x)
[Out] int(x^(-1-n-(p-1)*q)*(b*x^n+a*x^q)^p,x)
```

Fricas [F]

$$\int x^{-1-n-(-1+p)q}(bx^{n}+ax^{q})^{p}\,dx=\int (bx^{n}+ax^{q})^{p}x^{-(p-1)q-n-1}\,dx$$

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

Sympy [F]

$$\int x^{-1-n-(-1+p)q}(bx^{n}+ax^{q})^{p}\,dx=\int x^{-n-q(p-1)-1}(ax^{q}+bx^{n})^{p}\,dx$$

```
[In] integrate(x**(-1-n-(-1+p)*q)*(b*x**n+a*x**q)**p,x)
[Out] Integral(x**(-n - q*(p - 1) - 1)*(a*x**q + b*x**n)**p, x)
```

Maxima [F]

$$\int x^{-1-n-(-1+p)q}(bx^{n}+ax^{q})^{p}\,dx=\int (bx^{n}+ax^{q})^{p}x^{-(p-1)q-n-1}\,dx$$

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

Giac [F]

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-(p-1)q-n-1}\,dx$$

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n-(-1+p)q}(bx^n+ax^q)^p\,dx=\int \frac{(b\,x^n+a\,x^q)^p}{x^{n+q\,(p-1)+1}}\,dx$$

```
[In] int((b*x^n + a*x^q)^p/x^(n + q*(p - 1) + 1),x)
[Out] int((b*x^n + a*x^q)^p/x^(n + q*(p - 1) + 1), x)
```

3.443 $\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx$

Optimal result

Integrand size = 27, antiderivative size = 84

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx = \frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}(bx^n+ax^q)^p\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

```
[Out] x^(-n*p+n-q)*(b*x^n+a*x^q)^p*hypergeom([-p, 1-p],[2-p],-b*x^(n-q)/a)/(1-p)/
(n-q)/((1+b*x^(n-q)/a)^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2057, 372, 371}

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx = \frac{x^{n(-p)+n-q}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}(ax^q+bx^n)^p\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

```
[In] Int[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b
*x^(n - q))/a)])/((1 - p)*(n - q)*(1 + (b*x^(n - q))/a)^p)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int x^{-1-n(-1+p)-q+pq}\left(a+bx^{n-q}\right)^p\,dx\\&=\left(x^{-pq}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int x^{-1-n(-1+p)-q+pq}\left(1+\frac{bx^{n-q}}{a}\right)^p\,dx\\&=\frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}\left(bx^n+ax^q\right)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\begin{aligned}&\int x^{-1-n(-1+p)-q}\left(bx^n+ax^q\right)^p\,dx\\&=-\frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}\left(bx^n+ax^q\right)^p\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^{n-q}}{a}\right)}{(-1+p)(n-q)}\end{aligned}$$

```
[In] Integrate[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -((x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -(
(b*x^(n - q))/a)])/((-1 + p)*(n - q)*(1 + (b*x^(n - q))/a)^p))
```

Maple [F]

$$\int x^{-1-n(p-1)-q}(b\,x^{n}+a\,x^{q})^{p}\,dx$$

```
[In] int(x^(-1-n*(p-1)-q)*(b*x^n+a*x^q)^p,x)
[Out] int(x^(-1-n*(p-1)-q)*(b*x^n+a*x^q)^p,x)
```

Fricas [F]

$$\int x^{-1-n(-1+p)-q}(bx^{n}+ax^{q})^{p}\,dx=\int (bx^{n}+ax^{q})^{p}x^{-n(p-1)-q-1}\,dx$$

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a*x^q)^p*x^(-n*p + n - q - 1), x)
```

Sympy [F]

$$\int x^{-1-n(-1+p)-q}(bx^{n}+ax^{q})^{p}\,dx=\int x^{-n(p-1)-q-1}(ax^{q}+bx^{n})^{p}\,dx$$

```
[In] integrate(x**(-1-n*(-1+p)-q)*(b*x**n+a*x**q)**p,x)
[Out] Integral(x**(-n*(p - 1) - q - 1)*(a*x**q + b*x**n)**p, x)
```

Maxima [F]

$$\int x^{-1-n(-1+p)-q}(bx^{n}+ax^{q})^{p}\,dx=\int (bx^{n}+ax^{q})^{p}x^{-n(p-1)-q-1}\,dx$$

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

Giac [F]

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx=\int (bx^n+ax^q)^p x^{-n(p-1)-q-1}\,dx$$

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-1+p)-q}(bx^n+ax^q)^p\,dx=\int \frac{(b\,x^n+a\,x^q)^p}{x^{q+n\,(p-1)+1}}\,dx$$

```
[In] int((b*x^n + a*x^q)^p/x^(q + n*(p - 1) + 1),x)
[Out] int((b*x^n + a*x^q)^p/x^(q + n*(p - 1) + 1), x)
```

3.444 $\int (ax^m+bx^{1+m+mp})^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \left(ax^m+bx^{1+m+mp}\right)^p \, dx = \frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}$$

```
[Out] (a*x^m+b*x^(m*p+m+1))^(p+1)/b/(p+1)/(m*p+1)/(x^(m*(p+1)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2025}

$$\int \left(ax^m+bx^{1+m+mp}\right)^p \, dx = \frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

```
[In] Int[(a*x^m + b*x^(1 + m + m*p))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}$$

Mathematica [A] (verified)

Time $=0.08$ (sec) , antiderivative size $=43$, normalized size of antiderivative $=0.98$

$$\int \left(ax^m+bx^{1+m+mp}\right)^p \, dx=\frac{x^{-m(1+p)}\left(x^m\left(a+bx^{1+mp}\right)\right)^{1+p}}{b(1+p)(1+mp)}$$

```
[In] Integrate[(a*x^m + b*x^(1 + m + m*p))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Maple [F]

$$\int \left(x^m a+b\,x^{mp+m+1}\right)^p dx$$

```
[In] int((x^m*a+b*x^(m*p+m+1))^p,x)
```

```
[Out] int((x^m*a+b*x^(m*p+m+1))^p,x)
```

Fricas [A] (verification not implemented)

none

Time $=0.47$ (sec) , antiderivative size $=64$, normalized size of antiderivative $=1.45$

$$\int \left(ax^m+bx^{1+m+mp}\right)^p \, dx=\frac{\left(bxx^{mp+m+1}+axx^m\right)\left(bx^{mp+m+1}+ax^m\right)^p}{\left(bmp^2+(bm+b)p+b\right)x^{mp+m+1}}$$

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + m + 1) + a*x*x^m)*(b*x^(m*p + m + 1) + a*x^m)^p/((b*m*p^2 + (
b*m + b)*p + b)*x^(m*p + m + 1))
```

Sympy [F]

$$\int \left(ax^m+bx^{1+m+mp}\right)^p \, dx=\int \left(ax^m+bx^{mp+m+1}\right)^p \, dx$$

```
[In] integrate((a*x**m+b*x**(m*p+m+1))**p,x)
```

```
[Out] Integral((a*x**m + b*x**(m*p + m + 1))**p, x)
```

Maxima [F]

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\int \left(bx^{mp+m+1}+ax^m\right)^p\,dx$$

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="maxima")
[Out] integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

Giac [F]

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\int \left(bx^{mp+m+1}+ax^m\right)^p\,dx$$

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="giac")
[Out] integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.73

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\frac{a\,\left(a\,x^m+b\,x^{m+m\,p+1}\right)^p\,\left(\frac{b\,x^{m\,p+1}}{a}-\frac{1}{{\left(\frac{b\,x^{m\,p+1}}{a}+1\right)}^p}+1\right)}{b\,x^{m\,p}\,\left(m\,p+1\right)\,\left(p+1\right)}$$

```
[In] int((a*x^m + b*x^(m + m*p + 1))^p,x)
[Out] (a*(a*x^m + b*x^(m + m*p + 1))^p*((b*x^(m*p + 1))/a - 1/((b*x^(m*p + 1))/a
+ 1)^p + 1))/(b*x^(m*p)*(m*p + 1)*(p + 1))
```

3.445 $\int (x^m(a+bx^{1+mp}))^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \left(x^m\left(a+bx^{1+mp}\right)\right)^p \, dx = \frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}$$

```
[Out] (a*x^m+b*x^(m*p+m+1))^(p+1)/b/(p+1)/(m*p+1)/(x^(m*(p+1)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2004, 2025}

$$\int \left(x^m\left(a+bx^{1+mp}\right)\right)^p \, dx = \frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

```
[In] Int[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ax^m + bx^{1+m+mp}\right)^p \, dx \\ &= \frac{x^{-m(1+p)}\left(ax^m + bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \left(x^m\left(a+bx^{1+mp}\right)\right)^p \, dx = \frac{x^{-m(1+p)}\left(x^m\left(a+bx^{1+mp}\right)\right)^{1+p}}{b(1+p)(1+mp)}$$

```
[In] Integrate[(x^m*(a + b*x^(1 + m*p)))^p,x]
[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Maple [F]

$$\int \left(x^m\left(a+b\,x^{mp+1}\right)\right)^p dx$$

```
[In] int((x^m*(a+b*x^(m*p+1)))^p,x)
[Out] int((x^m*(a+b*x^(m*p+1)))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \left(x^m\left(a+bx^{1+mp}\right)\right)^p \, dx = \frac{(bxx^{mp+1}+ax)(bx^{mp+1}x^m+ax^m)^p}{(bmp^2+(bm+b)p+b)x^{mp+1}}$$

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="fricas")
[Out] (b*x*x^(m*p + 1) + a*x)*(b*x^(m*p + 1)*x^m + a*x^m)^p/((b*m*p^2 + (b*m + b)
*p + b)*x^(m*p + 1))
```

Sympy [F]

$$\int \left(x^m\left(a+bx^{1+mp}\right)\right)^p\,dx=\int \left(x^m\left(a+bx^{mp+1}\right)\right)^p\,dx$$

```
[In] integrate((x**m*(a+b*x**(m*p+1)))**p,x)
[Out] Integral((x**m*(a + b*x**(m*p + 1)))**p, x)
```

Maxima [F]

$$\int \left(x^m\left(a+bx^{1+mp}\right)\right)^p\,dx=\int \left(\left(bx^{mp+1}+a\right)x^m\right)^p\,dx$$

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="maxima")
[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

Giac [F]

$$\int \left(x^m\left(a+bx^{1+mp}\right)\right)^p\,dx=\int \left(\left(bx^{mp+1}+a\right)x^m\right)^p\,dx$$

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="giac")
[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x^m\left(a+bx^{1+mp}\right)\right)^p\,dx=\int {\left(x^m\,\left(a+b\,x^{m\,p+1}\right)\right)}^p\,dx$$

```
[In] int((x^m*(a + b*x^(m*p + 1)))^p,x)
[Out] int((x^m*(a + b*x^(m*p + 1)))^p, x)
```

3.446 $\int x^n(x^m(a+bx^{1+n+mp}))^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}$$

```
[Out] (a*x^m+b*x^(m*p+m+n+1))^(p+1)/b/(p+1)/(m*p+n+1)/(x^(m*(p+1)))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2005, 2039}

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

```
[In] Int[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
p)))
```

Rule 2005

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
```

```
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^n\left(ax^m+bx^{1+m+n+mp}\right)^p\,dx\\ &= \frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\frac{x^{-m(1+p)}\left(x^m\left(a+bx^{1+n+mp}\right)\right)^{1+p}}{b(1+p)(1+n+mp)}$$

```
[In] Integrate[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)
))
```

Maple [F]

$$\int x^n\left(x^m\left(a+b\,x^{mp+n+1}\right)\right)^p dx$$

```
[In] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
[Out] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\frac{\left(bxx^{mp+n+1}x^n+axx^n\right)\left(bx^{mp+n+1}x^m+ax^m\right)^p}{(bmp^2+bn+(bm+bn+b)p+b)x^{mp+n+1}}$$

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="fricas")
[Out] (b*x*x^(m*p + n + 1)*x^n + a*x*x^n)*(b*x^(m*p + n + 1)*x^m + a*x^m)^p/((b*m
*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + n + 1))
```

Sympy [F(-1)]

Timed out.

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\text{Timed out}$$

```
[In] integrate(x**n*(x**m*(a+b*x**(m*p+n+1)))**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\int\left(\left(bx^{mp+n+1}+a\right)x^m\right)^p x^n\,dx$$

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

Giac [F]

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\int\left(\left(bx^{mp+n+1}+a\right)x^m\right)^p x^n\,dx$$

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^n\left(x^m\left(a+bx^{1+n+mp}\right)\right)^p\,dx=\int x^n\,\left(x^m\,\left(a+b\,x^{n+m\,p+1}\right)\right)^p\,dx$$

```
[In] int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p,x)
```

```
[Out] int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p, x)
```

3.447 $\int x^n(ax^m+bx^{1+m+n+mp})^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int x^n\left(ax^m+bx^{1+m+n+mp}\right)^p\,dx=\frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}$$

```
[Out] (a*x^m+b*x^(m*p+m+n+1))^(p+1)/b/(p+1)/(m*p+n+1)/(x^(m*(p+1)))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2039}

$$\int x^n\left(ax^m+bx^{1+m+n+mp}\right)^p\,dx=\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

```
[In] Int[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
p)))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral}=\frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int x^n \left(ax^m+bx^{1+m+n+mp}\right)^p\,dx=\frac{x^{-m(1+p)}\left(x^m\left(a+bx^{1+n+mp}\right)\right)^{1+p}}{b(1+p)(1+n+mp)}$$

```
[In] Integrate[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)
))
```

Maple [F]

$$\int x^n \left(x^m a+b\,x^{mp+m+n+1}\right)^p dx$$

```
[In] int(x^n*(x^m*a+b*x^(m*p+m+n+1))^p,x)
```

```
[Out] int(x^n*(x^m*a+b*x^(m*p+m+n+1))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int x^n \left(ax^m+bx^{1+m+n+mp}\right)^p\,dx=\frac{\left(bxx^{mp+m+n+1}x^n+axx^mx^n\right)\left(bx^{mp+m+n+1}+ax^m\right)^p}{(bmp^2+bn+(bm+bn+b)p+b)x^{mp+m+n+1}}$$

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + m + n + 1)*x^n + a*x*x^m*x^n)*(b*x^(m*p + m + n + 1) + a*x^m)
^p/((b*m*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + m + n + 1))
```

Sympy [F]

$$\int x^n \left(ax^m+bx^{1+m+n+mp}\right)^p\,dx=\int x^n\left(ax^m+bx^{mp+m+n+1}\right)^p\,dx$$

```
[In] integrate(x**n*(a*x**m+b*x**(m*p+m+n+1))**p,x)
```

```
[Out] Integral(x**n*(a*x**m + b*x**(m*p + m + n + 1))**p, x)
```

Maxima [F]

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \int \left(bx^{mp+m+n+1} + ax^m\right)^p x^n \, dx$$

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="maxima")
[Out] integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

Giac [F]

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \int \left(bx^{mp+m+n+1} + ax^m\right)^p x^n \, dx$$

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="giac")
[Out] integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \int x^n \left(a\,x^m + b\,x^{m+n+m\,p+1}\right)^p \, dx$$

```
[In] int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p,x)
[Out] int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p, x)
```

3.448 $\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx=\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{-2+3n}\right)^{3/2}}{3bn}$$

```
[Out] 2/3*x^(3-3*n)*(a/(x^(2-2*n))+b*x^(-2+3*n))^(3/2)/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2004, 2025}

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx=\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{3n-2}\right)^{3/2}}{3bn}$$

```
[In] Int[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

```
[Out] (2*x^(3*(1 - n))*(a/x^(2*(1 - n)) + b*x^(-2 + 3*n))^(3/2))/(3*b*n)
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{ax^{2(-1+n)}+bx^{2(-1+n)+n}}\,dx\\ &= \frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{-2+3n}\right)^{3/2}}{3bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx = \frac{2x^{3-3n}\left(x^{-2+2n}(a+bx^n)\right)^{3/2}}{3bn}$$

```
[In] Integrate[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
[Out] (2*x^(3 - 3*n)*(x^(-2 + 2*n)*(a + b*x^n))^(3/2))/(3*b*n)
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{2\sqrt{\frac{x^{2n}(a+b\,x^{n})}{x^{2}}}\,(a+b\,x^{n})x^{-n}x}{3bn}$	40

```
[In] int((x^(-2+2*n)*(a+b*x^n))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(1/x^2*(x^n)^2*(a+b*x^n))^(1/2)*(a+b*x^n)/(x^n)*x/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx = \frac{2\left(bxx^n+ax\right)\sqrt{\frac{bx^{3\,n}+ax^{2\,n}}{x^2}}}{3\,bnx^n}$$

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*x*x^n + a*x)*sqrt((b*x^(3*n) + a*x^(2*n))/x^2)/(b*n*x^n)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^{n}\right)}\,dx=\text{Timed out}$$

```
[In] integrate((x**(-2+2*n)*(a+b*x**n))**(1/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^{n}\right)}\,dx=\frac{2\,(bx^{n}+a)^{\frac{3}{2}}}{3\,bn}$$

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
[Out] 2/3*(b*x^n + a)^(3/2)/(b*n)
```

Giac [F]

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^{n}\right)}\,dx=\int \sqrt{(bx^{n}+a)x^{2\,n-2}}\,dx$$

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt((b*x^n + a)*x^(2*n - 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x^{2(-1+n)}\left(a+bx^{n}\right)}\,dx=\int \sqrt{x^{2\,n-2}\,\left(a+b\,x^{n}\right)}\,dx$$

```
[In] int((x^(2*n - 2)*(a + b*x^n))^(1/2),x)
[Out] int((x^(2*n - 2)*(a + b*x^n))^(1/2), x)
```

3.449 $\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{-3+4n}\right)^{4/3}}{4bn}$$

```
[Out] 3/4*x^(4-4*n)*(a/(x^(3-3*n))+b*x^(-3+4*n))^(4/3)/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2004, 2025}

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{4n-3}\right)^{4/3}}{4bn}$$

```
[In] Int[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
```

```
[Out] (3*x^(4*(1 - n))*(a/x^(3*(1 - n)) + b*x^(-3 + 4*n))^(4/3))/(4*b*n)
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt[3]{ax^{3(-1+n)}+bx^{3(-1+n)+n}}\,dx\\ &= \frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{-3+4n}\right)^{4/3}}{4bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\frac{3x^{4-4n}\left(x^{-3+3n}(a+bx^n)\right)^{4/3}}{4bn}$$

```
[In] Integrate[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
[Out] (3*x^(4 - 4*n)*(x^(-3 + 3*n)*(a + b*x^n))^(4/3))/(4*b*n)
```

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{3\left(\frac{x^{3n}(a+b\,x^{n})}{x^{3}}\right)^{\frac{1}{3}} x\,x^{-n}(a+b\,x^{n})}{4bn}$	40

```
[In] int((x^(-3+3*n)*(a+b*x^n))^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/4*(1/x^3*(x^n)^3*(a+b*x^n))^(1/3)*x/(x^n)*(a+b*x^n)/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\frac{3\left(bxx^n+ax\right)\left(\frac{bx^{4\,n}+ax^{3\,n}}{x^3}\right)^{\frac{1}{3}}}{4\,bnx^n}$$

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="fricas")
[Out] 3/4*(b*x*x^n + a*x)*((b*x^(4*n) + a*x^(3*n))/x^3)^(1/3)/(b*n*x^n)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\text{Timed out}$$

```
[In] integrate((x**(-3+3*n)*(a+b*x**n))**(1/3),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\frac{3\left(bx^n+a\right)^{\frac{4}{3}}}{4\,bn}$$

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="maxima")
[Out] 3/4*(b*x^n + a)^(4/3)/(b*n)
```

Giac [F]

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\int \left((bx^n+a)x^{3\,n-3}\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="giac")
[Out] integrate(((b*x^n + a)*x^(3*n - 3))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx=\int \left(x^{3\,n-3}\,\left(a+b\,x^n\right)\right)^{1/3}\,dx$$

```
[In] int((x^(3*n - 3)*(a + b*x^n))^(1/3),x)
[Out] int((x^(3*n - 3)*(a + b*x^n))^(1/3), x)
```

3.450 $\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{-4+5n}\right)^{5/4}}{5bn}$$

```
[Out] 4/5*x^(5-5*n)*(a/(x^(4-4*n))+b*x^(-4+5*n))^(5/4)/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2004, 2025}

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{5n-4}\right)^{5/4}}{5bn}$$

```
[In] Int[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

```
[Out] (4*x^(5*(1 - n))*(a/x^(4*(1 - n)) + b*x^(-4 + 5*n))^(5/4))/(5*b*n)
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt[4]{ax^{4(-1+n)}+bx^{4(-1+n)+n}}\,dx\\ &= \frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{-4+5n}\right)^{5/4}}{5bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx = \frac{4x^{5-5n}\left(x^{-4+4n}(a+bx^n)\right)^{5/4}}{5bn}$$

```
[In] Integrate[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
[Out] (4*x^(5 - 5*n)*(x^(-4 + 4*n)*(a + b*x^n))^(5/4))/(5*b*n)
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{4\left(\frac{x^{4n}(a+b\,x^n)}{x^4}\right)^{\frac{1}{4}} x\,x^{-n}(a+b\,x^n)}{5bn}$	40

```
[In] int((x^(-4+4*n)*(a+b*x^n))^(1/4),x,method=_RETURNVERBOSE)
[Out] 4/5*(1/x^4*(x^n)^4*(a+b*x^n))^(1/4)*x/(x^n)*(a+b*x^n)/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx = \frac{4\left(bxx^n+ax\right)\left(\frac{bx^{5\,n}+ax^{4\,n}}{x^4}\right)^{\frac{1}{4}}}{5\,bnx^n}$$

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="fricas")
[Out] 4/5*(b*x*x^n + a*x)*((b*x^(5*n) + a*x^(4*n))/x^4)^(1/4)/(b*n*x^n)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\text{Timed out}$$

```
[In] integrate((x**(-4+4*n)*(a+b*x**n))**(1/4),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\frac{4\left(bx^n+a\right)^{\frac{5}{4}}}{5\,bn}$$

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="maxima")
[Out] 4/5*(b*x^n + a)^(5/4)/(b*n)
```

Giac [F]

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\int \left((bx^n+a)x^{4\,n-4}\right)^{\frac{1}{4}}\,dx$$

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="giac")
[Out] integrate(((b*x^n + a)*x^(4*n - 4))^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{x^{4(-1+n)}\left(a+bx^n\right)}\,dx=\int {\left(x^{4\,n-4}\,\left(a+b\,x^n\right)\right)}^{1/4}\,dx$$

```
[In] int((x^(4*n - 4)*(a + b*x^n))^(1/4),x)
[Out] int((x^(4*n - 4)*(a + b*x^n))^(1/4), x)
```

3.451 $\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\frac{px^{(1-n)(1+p)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{1+\frac{1}{p}}}{bn(1+p)}$$

```
[Out] p*x^((1-n)*(p+1))*(a/(x^((1-n)*p))+b*x^(n-(1-n)*p))^(1+1/p)/b/n/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2004, 2025}

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\frac{px^{(1-n)(p+1)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{\frac{1}{p}+1}}{bn(p+1)}$$

```
[In] Int[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

```
[Out] (p*x^((1 - n)*(1 + p))*(a/x^((1 - n)*p) + b*x^(n - (1 - n)*p))^(1 + p^(-1))
)/(b*n*(1 + p))
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
```

```
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ax^{(-1+n)p}+bx^{n+(-1+n)p}\right)^{\frac{1}{p}}\,dx\\ &= \frac{px^{(1-n)(1+p)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{1+\frac{1}{p}}}{bn(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx = \frac{x^{1-n}(a+bx^n)\left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}}{bn\left(1+\frac{1}{p}\right)}$$

```
[In] Integrate[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)*p)*(a + b*x^n))^p^(-1))/(b*n*(1 + p^(-1
)))
```

Maple [F]

$$\int \left(x^{(-1+n)p}(a+b\,x^n)\right)^{\frac{1}{p}}\,dx$$

```
[In] int((x^((-1+n)*p)*(a+b*x^n))^(1/p),x)
[Out] int((x^((-1+n)*p)*(a+b*x^n))^(1/p),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx = \frac{(bpxx^n+apx)\left((bx^n+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}}{(bnp+bn)x^n}$$

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="fricas")
[Out] (b*p*x*x^n + a*p*x)*((b*x^n + a)*x^((n - 1)*p))^(1/p)/((b*n*p + b*n)*x^n)
```

Sympy [F]

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\int \left(x^{p(n-1)}(a+bx^n)\right)^{\frac{1}{p}}\,dx$$

```
[In] integrate((x**((-1+n)*p)*(a+b*x**n))**(1/p),x)
[Out] Integral((x**(p*(n - 1))*(a + b*x**n))**(1/p), x)
```

Maxima [F]

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\int \left((bx^n+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}\,dx$$

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="maxima")
[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

Giac [F]

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\int \left((bx^n+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}\,dx$$

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="giac")
[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}}\,dx=\int \left(x^{p\,(n-1)}\,\left(a+b\,x^n\right)\right)^{1/p}\,dx$$

```
[In] int((x^(p*(n - 1))*(a + b*x^n))^(1/p),x)
[Out] int((x^(p*(n - 1))*(a + b*x^n))^(1/p), x)
```

3.452 $\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p \, dx = \frac{x^{\frac{(1-n)(1+p)}{p}}\left(bx^{n-\frac{1-n}{p}}+ax^{-\frac{1-n}{p}}\right)^{1+p}}{bn(1+p)}$$

```
[Out] x^((1-n)*(p+1)/p)*(b*x^(n+(-1+n)/p)+a/(x^((1-n)/p)))^(p+1)/b/n/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2004, 2025}

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p \, dx = \frac{x^{\frac{(1-n)(p+1)}{p}}\left(ax^{-\frac{1-n}{p}}+bx^{n-\frac{1-n}{p}}\right)^{p+1}}{bn(p+1)}$$

```
[In] Int[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

```
[Out] (x^(((1 - n)*(1 + p))/p)*(b*x^(n - (1 - n)/p) + a/x^((1 - n)/p))^(1 + p))/(
b*n*(1 + p))
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
```

```
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(bx^{n+\frac{-1+n}{p}}+ax^{\frac{-1+n}{p}}\right)^p\,dx\\ &= \frac{x^{\frac{(1-n)(1+p)}{p}}\left(bx^{n-\frac{1-n}{p}}+ax^{-\frac{1-n}{p}}\right)^{1+p}}{bn(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx=\frac{x^{1-n}(a+bx^n)\left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p}{bn(1+p)}$$

```
[In] Integrate[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)/p)*(a + b*x^n))^p)/(b*n*(1 + p))
```

Maple [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+b\,x^n)\right)^p\,dx$$

```
[In] int((x^((-1+n)/p)*(a+b*x^n))^p,x)
[Out] int((x^((-1+n)/p)*(a+b*x^n))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx=\frac{(bxx^n+ax)\left(bx^nx^{\frac{n-1}{p}}+ax^{\frac{n-1}{p}}\right)^p}{(bnp+bn)x^n}$$

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="fricas")
[Out] (b*x*x^n + a*x)*(b*x^n*x^((n - 1)/p) + a*x^((n - 1)/p))^p/((b*n*p + b*n)*x^
n)
```

Sympy [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx=\int \left(x^{\frac{n-1}{p}}(a+bx^n)\right)^p\,dx$$

```
[In] integrate((x**((-1+n)/p)*(a+b*x**n))**p,x)
[Out] Integral((x**((n - 1)/p)*(a + b*x**n))**p, x)
```

Maxima [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx=\int \left((bx^n+a)x^{\frac{n-1}{p}}\right)^p\,dx$$

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="maxima")
[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

Giac [F]

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx=\int \left((bx^n+a)x^{\frac{n-1}{p}}\right)^p\,dx$$

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="giac")
[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx=\int \left(x^{\frac{n-1}{p}}\,\left(a+b\,x^n\right)\right)^p\,dx$$

```
[In] int((x^((n - 1)/p)*(a + b*x^n))^p,x)
[Out] int((x^((n - 1)/p)*(a + b*x^n))^p, x)
```

3.453 $\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx$

Optimal result

Integrand size = 25, antiderivative size = 39

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\frac{x^{-p(1+q)}(ax^n+bx^p)^{1+q}}{a(n-p)(1+q)}$$

```
[Out] (a*x^n+b*x^p)^(1+q)/a/(n-p)/(1+q)/(x^(p*(1+q)))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2039}

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\frac{x^{-p(q+1)}(ax^n+bx^p)^{q+1}}{a(q+1)(n-p)}$$

```
[In] Int[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

```
[Out] (a*x^n + b*x^p)^(1 + q)/(a*(n - p)*(1 + q)*x^(p*(1 + q)))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\text{integral}=\frac{x^{-p(1+q)}(ax^n+bx^p)^{1+q}}{a(n-p)(1+q)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=-\frac{x^{-p(1+q)}(ax^n+bx^p)^{1+q}}{a(-n+p)(1+q)}$$

```
[In] Integrate[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

```
[Out] -((a*x^n + b*x^p)^(1 + q)/(a*(-n + p)*(1 + q)*x^(p*(1 + q))))
```

Maple [F]

$$\int x^{-1+n-p(q+1)}(a\,x^n+b\,x^p)^q\,dx$$

```
[In] int(x^(-1+n-p*(q+1))*(a*x^n+b*x^p)^q,x)
```

```
[Out] int(x^(-1+n-p*(q+1))*(a*x^n+b*x^p)^q,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.95

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\frac{(axx^{-pq+n-p-1}x^n+bxx^{-pq+n-p-1}x^p)(ax^n+bx^p)^q}{(an-ap+(an-ap)q)x^n}$$

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="fricas")
```

```
[Out] (a*x*x^(-p*q + n - p - 1)*x^n + b*x*x^(-p*q + n - p - 1)*x^p)*(a*x^n + b*x^
p)^q/((a*n - a*p + (a*n - a*p)*q)*x^n)
```

Sympy [F]

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\int x^{n-p(q+1)-1}(ax^n+bx^p)^q\,dx$$

```
[In] integrate(x**(-1+n-p*(1+q))*(a*x**n+b*x**p)**q,x)
```

```
[Out] Integral(x**(n - p*(q + 1) - 1)*(a*x**n + b*x**p)**q, x)
```

Maxima [F]

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\int (ax^n+bx^p)^q x^{-p(q+1)+n-1}\,dx$$

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="maxima")
[Out] integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

Giac [F]

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\int (ax^n+bx^p)^q x^{-p(q+1)+n-1}\,dx$$

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="giac")
[Out] integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n-p(1+q)}(ax^n+bx^p)^q\,dx=\int x^{n-p\,(q+1)-1}\,(a\,x^n+b\,x^p)^q\,dx$$

```
[In] int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q,x)
[Out] int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q, x)
```

3.454 $\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx$

Optimal result

Integrand size = 28, antiderivative size = 40

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx=-\frac{x^{-((n+p)(1+q))}(ax^n+bx^{n+p})^{1+q}}{ap(1+q)}$$

```
[Out] -(a*x^n+b*x^(n+p))^(1+q)/a/p/(1+q)/(x^((n+p)*(1+q)))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2005, 2039}

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx=-\frac{x^{-((q+1)(n+p))}(ax^n+bx^{n+p})^{q+1}}{ap(q+1)}$$

```
[In] Int[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

```
[Out] -((a*x^n + b*x^(n + p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

Rule 2005

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !Generali
zedBinomialMatchQ[u, x]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
```

```
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int x^{-1-nq-p(1+q)}\left(ax^n+bx^{n+p}\right)^q\,dx\\ &= -\frac{x^{-((n+p)(1+q))}\left(ax^n+bx^{n+p}\right)^{1+q}}{ap(1+q)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = -\frac{x^{-((n+p)(1+q))}(x^n(a+bx^p))^{1+q}}{ap(1+q)}$$

```
[In] Integrate[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

```
[Out] -((x^n*(a + b*x^p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(41) = 82.

Time = 2.92 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.15

method	result	size
parallelrisch	$-\frac{x\,x^{p}x^{-qn-pq-p-1}(x^{n}(a+b\,x^{p}))^{q}b^{2}+x\,x^{-qn-pq-p-1}(x^{n}(a+b\,x^{p}))^{q}ab}{bp(q+1)a}$	86

```
[In] int(x^(-1-q*n-p*(q+1))*(x^n*(a+b*x^p))^q,x,method=_RETURNVERBOSE)
```

```
[Out] -(x*x^p*x^(-n*q-p*q-p-1)*(x^n*(a+b*x^p))^q*b^2+x*x^(-n*q-p*q-p-1)*(x^n*(a+b
*x^p))^q*a*b)/b/p/(q+1)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.60

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = -\frac{\left(bxx^{-(n+p)q-p-1}x^p+axx^{-(n+p)q-p-1}\right)(bx^nx^p+ax^n)^q}{apq+ap}$$

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="fricas")
```

```
[Out] -(b*x*x^(-(n + p)*q - p - 1)*x^p + a*x*x^(-(n + p)*q - p - 1))*(b*x^n*x^p +
 a*x^n)^q/(a*p*q + a*p)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = \text{Timed out}$$

```
[In] integrate(x**(-1-n*q-p*(1+q))*(x**n*(a+b*x**p))**q,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = \int ((bx^p+a)x^n)^q\,x^{-p(q+1)-nq-1}\,dx$$

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

Giac [F]

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx = \int ((bx^p+a)x^n)^q\,x^{-p(q+1)-nq-1}\,dx$$

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="giac")
```

```
[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-nq-p(1+q)}(x^n(a+bx^p))^q\,dx=\int\frac{(x^n\,(a+b\,x^p))^q}{x^{n\,q+p\,(q+1)+1}}\,dx$$

```
[In] int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1),x)
[Out] int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count i
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != −1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```